Chiral effects in meson phenomenology

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Quarks and Hadrons in strong QCD

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TECHNISCHE
UNIVERSITÄT
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Outline

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Introduction

DχSB important effects in low energy QCD

- driving force behind much phenomenology
- closely linked to the quark-gluon interaction

Wish to pin-down this interaction.

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- exhibits an infrared divergence

Q What impact does this have on meson observables?

Q What about unquenching effects?

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Q What impact does this have on meson observables?

Q What about unquenching effects?

In this talk focus on the impact of:

- **IR divergences** [C.S. Fischer, F. Llanes-Estrada, R. Alkofer, K. Schwenzer, *in prep.*] [RW, C.S. Fischer, R. Alkofer, *in prep.*]
- **Hadronic contributions** [C.S. Fischer, D. Nickel, J. Wambach, PRD **76** (2007) 094009]

Outline

IR Divergences (Landau Gauge): quark-gluon vertex

IR dynamics and Mesons

IR Divergences (Landau Gauge): quark-gluon vertex

$\uparrow$

three-gluon vertex

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ghost propagator

- Gribov-Zwanziger confinement scenario

  (Dominance of field configurations on Gribov Horizon)

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confinement $\leftrightarrow$ IR behaviour

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confinement $\leftrightarrow$ IR behaviour
$\updownarrow$
non-trivial topology

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confinement	$\leftrightarrow$	IR behaviour
$\updownarrow$		$\updownarrow$
non-trivial topology	$\leftrightarrow$	center vortices?

- removal reduces σ

[M. Engelhardt, Nucl. Phys. B **585** (2000) 614]
[R. Bertle M. Engelhardt and M. Faber, Phys. Rev. D **64** (2001) 074504]

- yields IR finite ghost

[J. Gattnar, K. Langfeld and H. Reinhardt, Phys. Rev. Lett. **93** (2004) 061601]

Non-trivial topology:

- Instantons $\rightarrow U_A(1)$ anomaly $\rightarrow \eta'$ mass

but $\chi \neq 0$ not restricted to Instantons.

- Other top. non-trivial gluonic conf. can contribute.
- resp. for confinement *and* $U_A(1)$ anomaly?
- encoded in IR behaviour of Green's fns?

asking: an η' mass through confinement?

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Kogut–Susskind mechanism

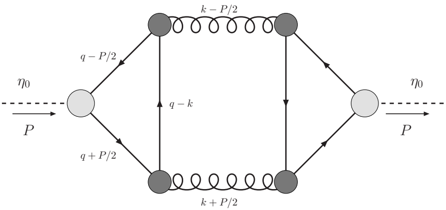
IR singular gluon ($D(p) \sim 1/p^4$)

- Screening of Goldstone pole

$\rightarrow$ Breaking axial symmetry – η' mass.

[J. B. Kogut and L. Susskind, Phys. Rev. D **10**(1974)]

Simplest suspect graph:



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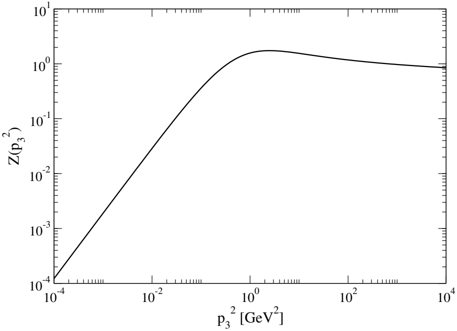
IR behaviour and topology

Gluon dressing function



$$D^{ab}_{\mu\nu} = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2} \ .$$

- $Z(k^2)/k^2$ give IR exponent $2\kappa - 1 \neq -2$
  NB: IR vanishing $(\kappa \simeq 0.595\ldots)$

IR behaviour and topology

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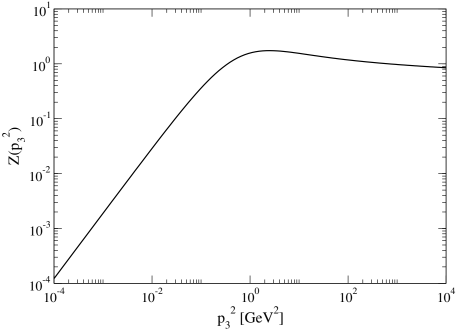

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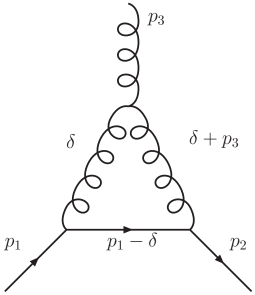
Combine with singular behaviour of QG-Vertex

Subtlety: *collinear divergences*

IR Collinear Singularities



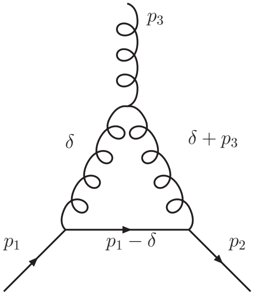
Choose kinematics so as to manifestly exhibit soft-divergence.

- Gluon momentum p_3 small.
- Loop dominated by small δ, $\delta + p_3$ internal gluon momenta.
- External quark mom $p_1 \simeq p_2$ can be large.

IR behaviour: p_3^{β}, with exponent $\beta = -\kappa - 1/2$

[Kai Schwenzer, *Priv. Comm.*]

IR Collinear Singularities



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Conspires with gluon propagator to yield the

$$D_{eff}(p) \sim (p^2)^{-2}$$

required by K-S to give $\chi^2 \neq 0$

[Kai Schwenzer, *Priv. Comm.*]

Bethe-Salpeter equation

$q\bar{q}$ bound state described by:

$$\Gamma_{tu}(p;P) = \int \frac{d^4k}{(2\pi)^4} K_{tu;rs}(p,k;P)\left[S(k_+)\Gamma(k;P)S(k_-)\right]_{sr}$$

- K quark-antiquark scattering kernel.
- Rest frame of meson: $P^2 = -M^2$ (Euclidean Space)

Pseudoscalar:

$$\Gamma(p,P) = \gamma_5 \left(E - i\not{P}\, F - i\not{p}\, p\cdot P G - [\gamma_\mu, \gamma_\nu]\, P_\mu p_\nu H\right)$$

Note E is leading component.

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av-WTI

$$\begin{aligned} P_\mu \Gamma^a_{5\mu}(k;P) &= S^{-1}(k_+)\frac{1}{2}\lambda^a_f i\gamma_5 + \frac{1}{2}\lambda^a_f i\gamma_5 S^{-1}(k_-) \\ &\quad - M_\zeta i\Gamma^a_5(k;P) - i\Gamma^a_5(k;P)M_\zeta \ . \end{aligned}$$

- Symmetry preserving truncation in DSE and BSE
  $\rightarrow$ preserve Goldstone character of the pion

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Only consistent known truncation
Rainbow-ladder

Must neglect all but leading structure of the DSE Quark-Gluon vertex study. Create a model:

- Qualitative IR features of quark-gluon vertex.
- UV from perturbation theory
- Fit scales using meson phenomenology.

Effective Gluon

Compose from $g^2 \times$ Gluon $\times$ Vertex Dressing.

$$\alpha_{eff}(z) = \alpha_\mu Z(z)\,\lambda_1(z)$$

$$\begin{aligned}
\lambda_1(z) &= \left(\frac{z}{z+d_2}\right)^{-1/2-\kappa} \\
&\times \left(\frac{d_1}{1+z/d_2} + z\frac{d_3}{d_2^2+(z-d_2)^2}\right. \\
&+ \left.\frac{z}{d_2+z}\left(\frac{4\pi}{\beta_0\alpha_\mu}\left(\frac{1}{\log(z/d_2)} - \frac{1}{z/d_2-1}\right)\right)^{-2\delta}\right)
\end{aligned}$$

Effective Gluon

z	:	gluon momentum
d_1	:	IR strength.
d_2	:	soft scale.
d_3	:	added integrated strength.

$$
\begin{aligned}
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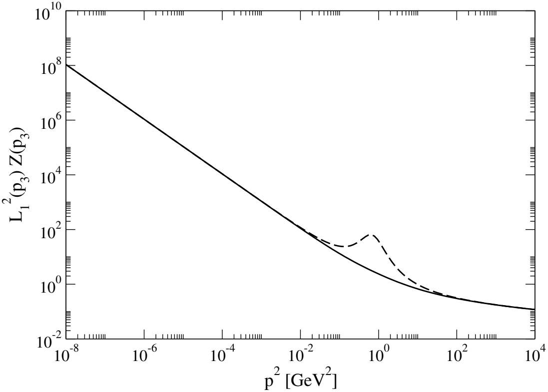
\end{aligned}
$$

Effective Gluon dressing in Diamond Diagram

c.f. a p^{-4} singular gluon

α_μVertex dressing^2 $\times$ Gluon



$d_1 = 1.67$, $d_2 = 0.5$, $d_3 = 2.6$

- Integrated strength added to obtain Meson observables: require $m_\pi \sim 138$, $f_\pi \sim 99$, $m_\rho \sim 747$.

Results

- Vary IR strength parameter.

  . . . meson phenomenology relatively unchanged.

  but anomalous mass very sensitive.

Obtain:

d_1 GeV^2	d_2 GeV^2	d_3 GeV^2	m_π MeV	m_ρ MeV	m_A^2 GeV^2
1.41	0.5	2.6	135	735	0.302
1.55	0.5	2.6	135	741	0.417
1.67	0.5	2.6	135	747	0.558

Results

$$M^2 = \begin{pmatrix} M_\pi^2 & 0 & 0 \\ 0 & M_{88}^2 & M_{80}^2 \\ 0 & M_{08}^2 & M_{00}^2 + m_A^2 \end{pmatrix}$$

Employ singlet-octet mass-squared mixing matrix. Diagonalise to obtain physical mass eigenstates.

Obtain:

d_1 GeV^2	m_A^2 GeV^2	θ	m_η MeV	$m_{\eta'}$ MeV
1.41	0.302	-35.3	412	790
1.55	0.417	-29.1	450	840
1.67	0.558	-23.2	479	906

Realistic η and η' masses eminently achievable with model.

Summary I

Summary

Quark-Gluon vertex IR divergences:

- tangible impact on meson phenomenology
  $\rightarrow \eta'$ through IR soft-collinear divergence

Topological susceptibility calculable:

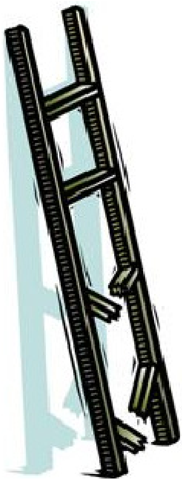
- through motivated model interaction
- in functional integral approach
- *without explicit breaking* through instantons

i.e. in Landau Gauge

- topological effects encoded in the IR behaviour of Green's functions. Complementary

Outline

Beyond Rainbow-Ladder



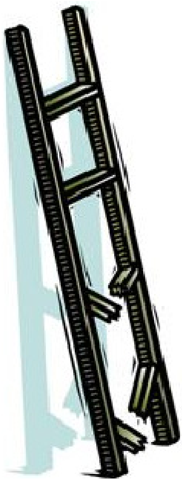
Rainbow-ladder very successful:

- Satisfies AV-WTI
- reproduces PS, V
- *whole range of quantities*

Still plenty of life left. Applied to:

- excited states
- baryons

Beyond Rainbow-Ladder



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cons:

- scalars, axial vectors
- no flavour mixing

Beyond Rainbow-Ladder

Moving beyond R-L difficult task:

- akin to looking for a pot of gold at the end of a *rainbow*

Every attempt ambitious:

- Many technical challenges
- Computationally involved
- Coupled integral equations

plain difficult

Beyond Rainbow-Ladder

Be more humble and ask for some pi at the end of our rainbow.

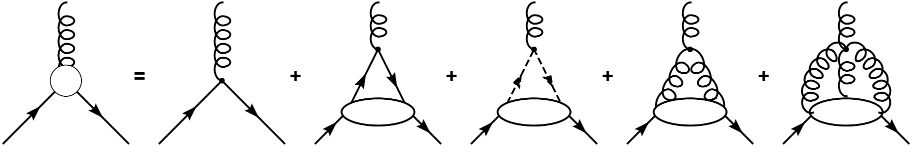
- (clearly) part of the pion cloud
- Hadronic contribution (decay widths)
- tensor structure

$\rightarrow$ beyond the rainbow

Still challenging.

Unquenching the Quark-Gluon Vertex

Full untruncated Dyson-Schwinger equation for the quark-gluon vertex:



- Hadronic contributions appear in the second diagram

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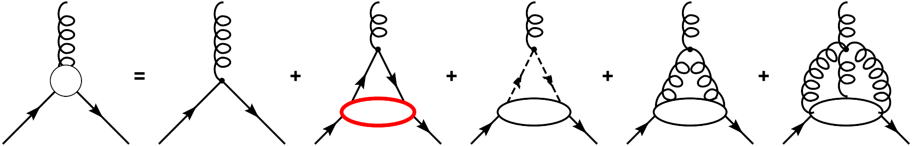
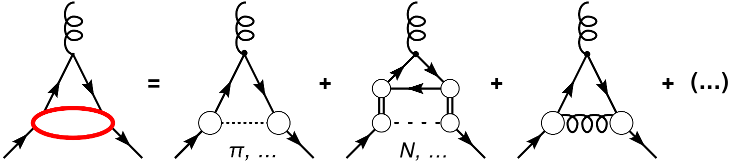
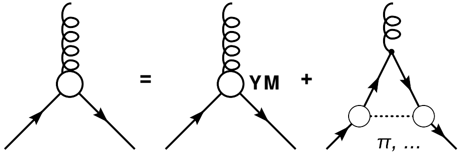


- Hadronic contributions appear in the second diagram
- Expand quark-antiquark scattering kernel in terms of resonance contributions to the kernel, and 1PI Green's functions



Unquenching the Quark-Gluon Vertex

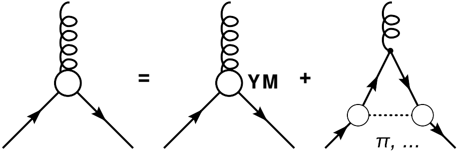
Approximate DSE:



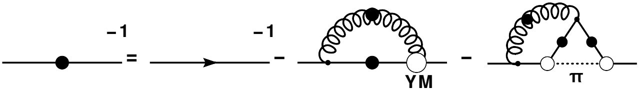
- Hadronic contributions appear in the second diagram
- Expand quark-antiquark scattering kernel in terms of resonance contributions to the kernel, and 1PI Green's functions
- Restrict ourselves to:
  - Yang-Mills interaction (dressed rainbow)
  - One pion exchange.

Quark Dyson-Schwinger equation

Take approximate DSE for vertex:



Yields the following quark DSE:



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Quark Dyson-Schwinger equation

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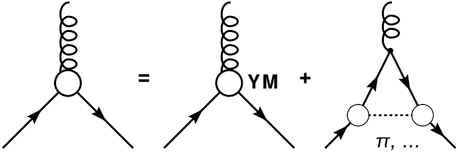

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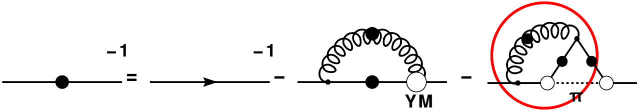

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- Pion-exchange diagram complicated.

Quark Dyson-Schwinger equation

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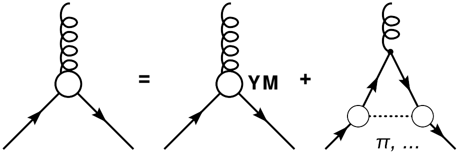

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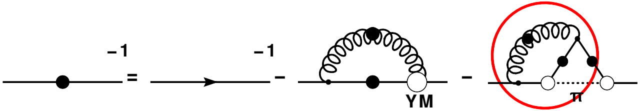

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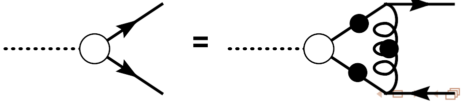

Quark Dyson-Schwinger equation

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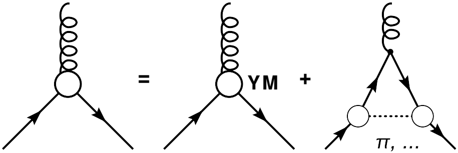

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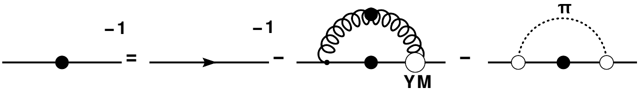
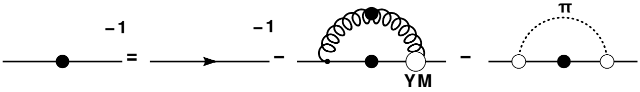


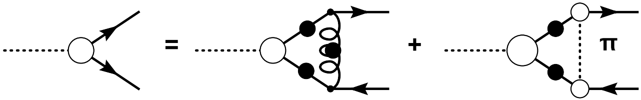
- Separates into YM part and Hadronic part.
- Pion-exchange diagram complicated.
- *Looks like* BSE for pion in R-L approx.
- Contributes rich tensor structure.

Bethe-Salpeter Equation

Find simultaneous solutions to Quark DSE:



and the Bethe-Salpeter equation:



Technicalities:

- Quark for complex Euclidean momenta.
- 'Off-shell pion' (solve inhom. BSE).

Second point tricky - approximations?

Bethe-Salpeter Equation

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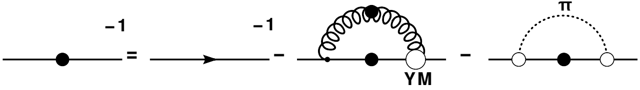

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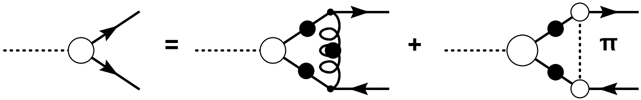

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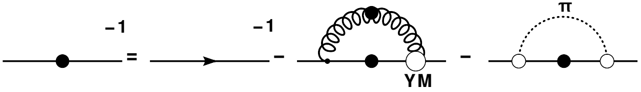

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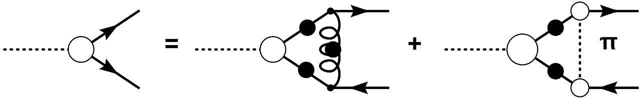

Technicalities:

- Quark for complex Euclidean momenta.
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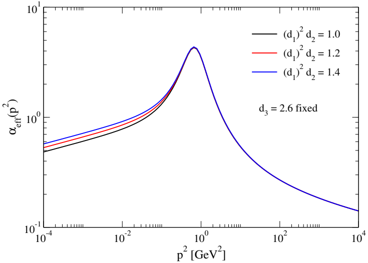
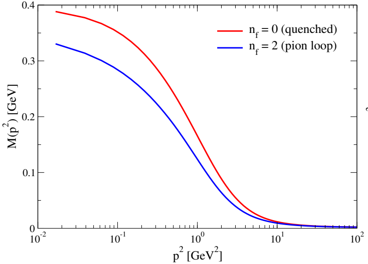
Second point tricky - approximations?

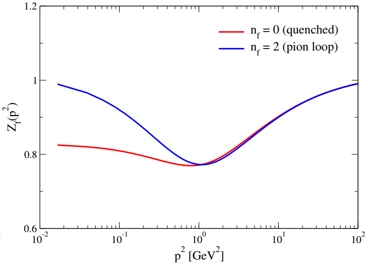
- Absolute value approximation?
- $\Gamma_\pi(k, P \to 0) \sim i\gamma_5 B_\chi(k^2)/f_\pi$, for k^2 complex (simpler kernel)

Effective running coupling

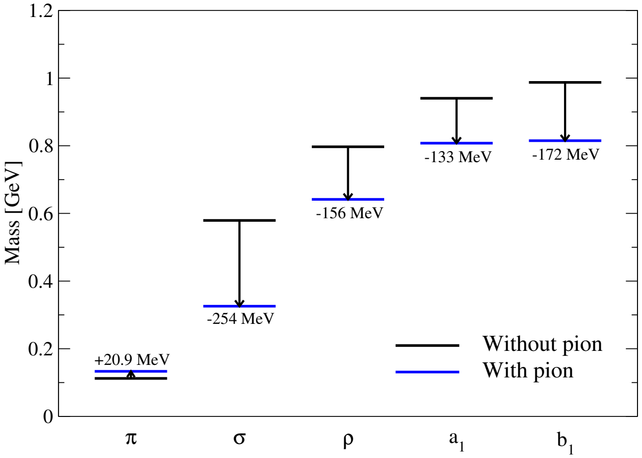
Take soft-divergent effective coupling for YM part







Results



Soft-divergence interaction

Outline

Summary

Summary

Including effects beyond Rainbow-Ladder *challenging*.
Investigation of Quark-Gluon Vertex DSE:

- Soft-divergences $\rightarrow$ $U_A(1)$ breaking – η'

  IR dynamics $\leftrightarrow$ topological non-trivial objects?

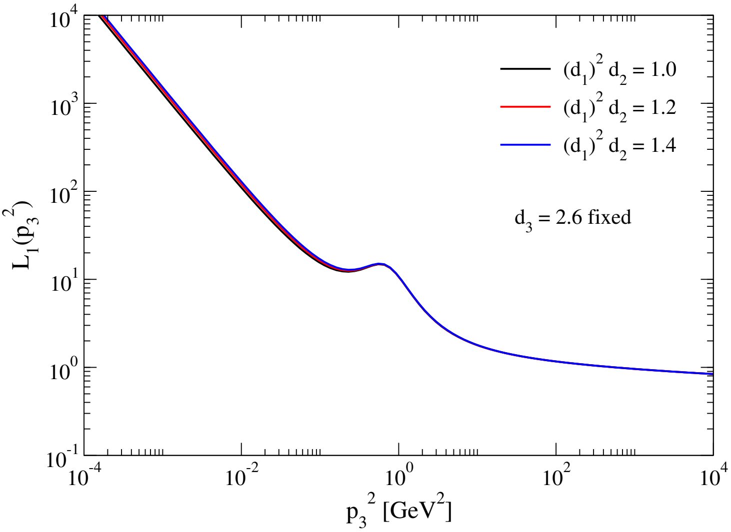
- Hadronic effects $\rightarrow$ eff. one-pion exchange

  $\rightarrow$ sizable effects on meson spectrum

Outlook

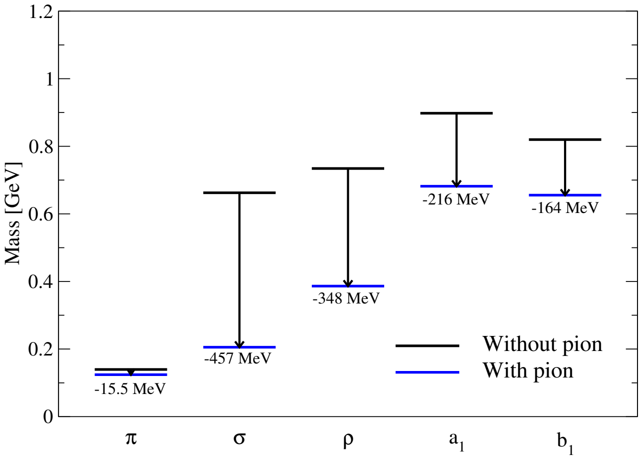
- Include off-shell 'pion' via inhom. BSE
- finite decay widths?
- Additional gluon-exchange contributions.

YM Vertex Dressing



Meson spectrum with Soft-Interaction

Meson	Mass (MeV)	Decay (MeV)	Mass (MeV)	Decay (MeV)
PS (0^-)	112.3	107.8	133.2	89.5
S (0^+)	579.3	-	325.8	-
V (1^-)	797.1	-	641.5	-
AV1 (1^{++})	940.3	-	807.6	-
AV2 (1^{+-})	987.4	-	815.1	-



Maris-Tandy interaction