

A photograph of the Schloss Rheinfels in St. Goar, Germany, a large stone castle built on a cliffside. The castle features a prominent tower with flags and a building with a gabled roof labeled 'HOTEL'. A small cartoon figure of a person in a red outfit is visible on the castle's wall. In the foreground, there is a dense forest of green trees and a small white building on a hillside. A colorful, multi-layered sphere with various particles is positioned in the upper right corner of the image.

Quarks & Hadrons in Strong **QCD**

Schloss Rheinfels
St Goar
March 2008

The background of the slide is a photograph of the Schloss Rheinfels in St. Goar, Germany. The castle is a large stone structure built on a hillside, featuring a prominent tower with flags and a building with a gabled roof labeled 'HOTEL'. The foreground is filled with lush green trees, and a small white building is visible on a lower slope. Overlaid on the image are three cartoonish figures with yellow heads and red bodies, and a colorful, multi-layered sphere with a rainbow gradient and small particles inside.

Quarks & Hadrons in Strong **QCD**

Schloss Rheinfels
St Goar
March 2008

QCD

1971

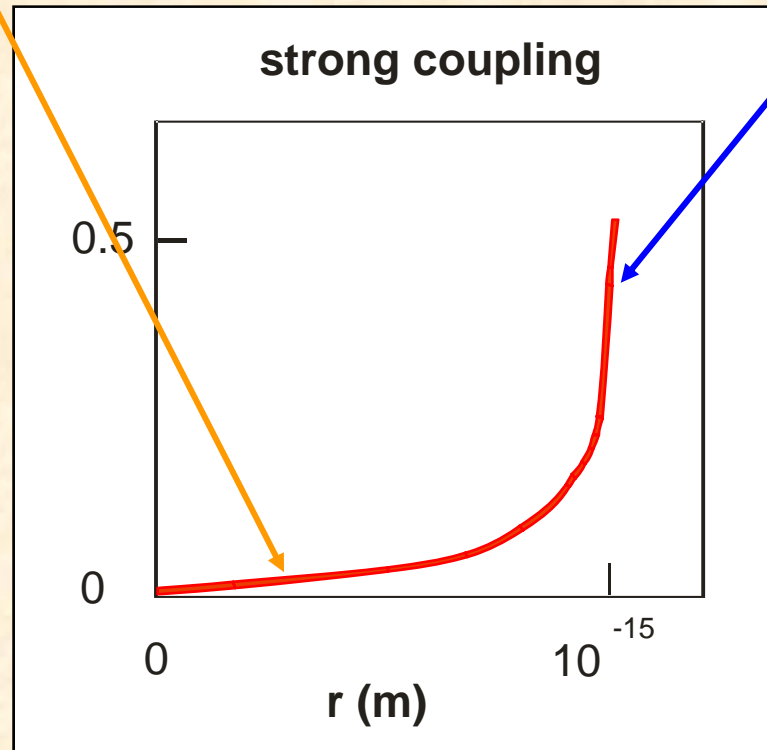
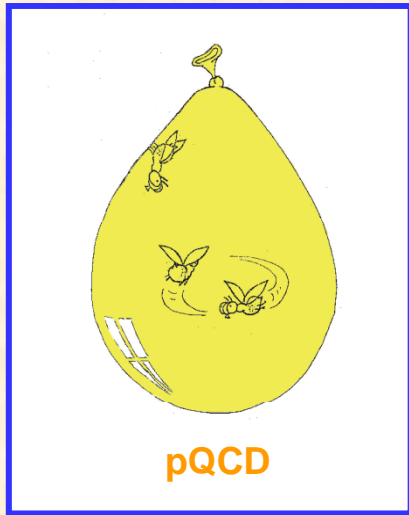


$$\mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b} \bar{q} (i\gamma_{\mu} D^{\mu} - m_q) q - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$

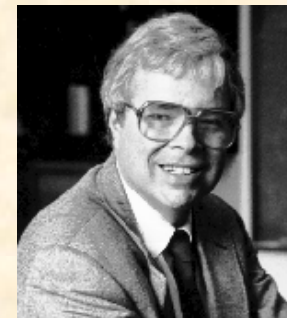


QCD

asymptotic freedom

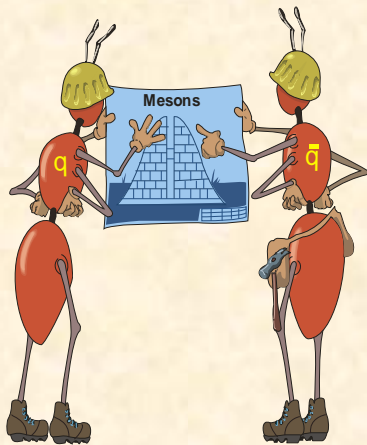


confinement

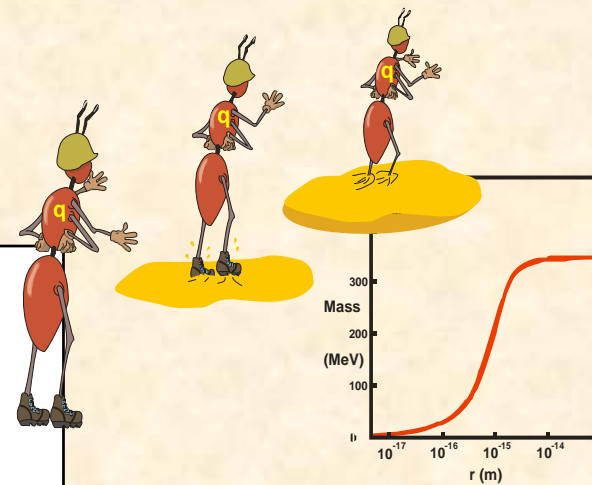


Strong physics problems

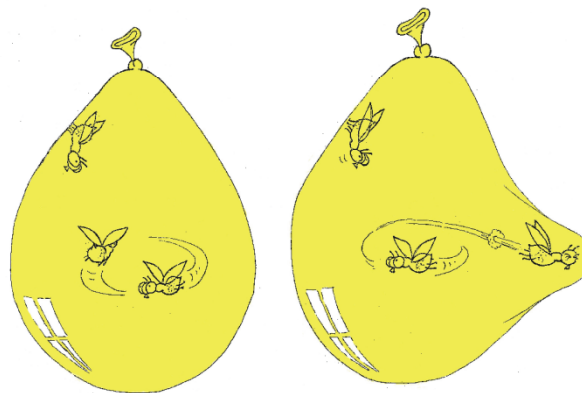
bound states



mass generation



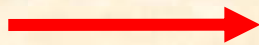
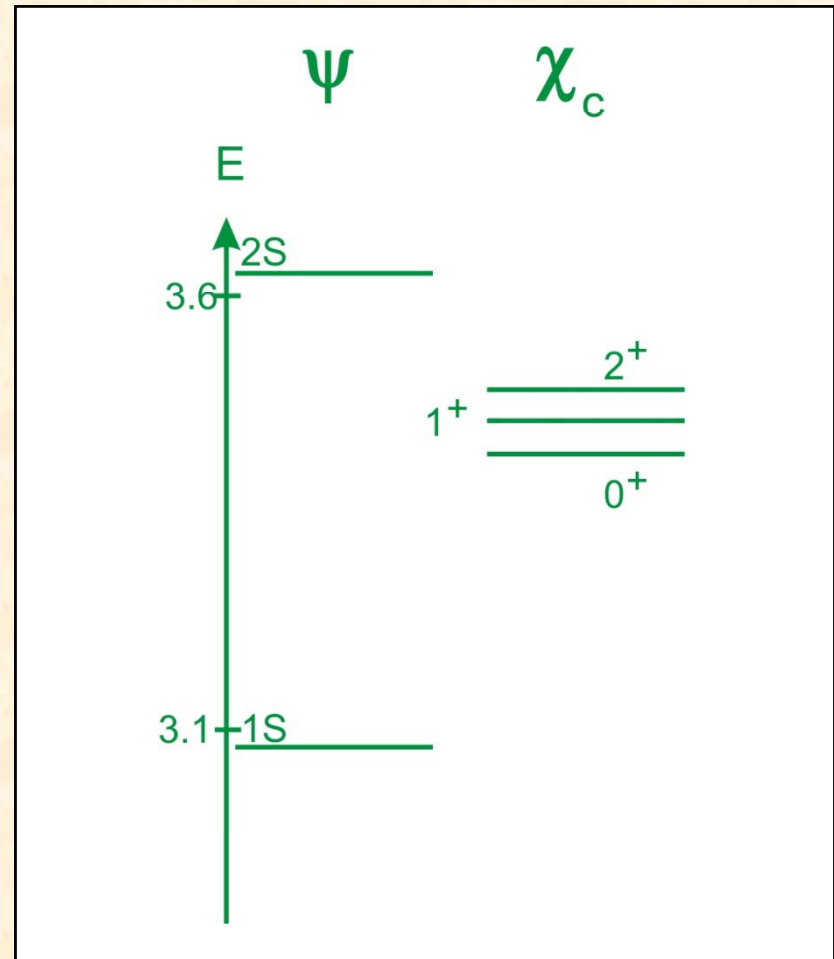
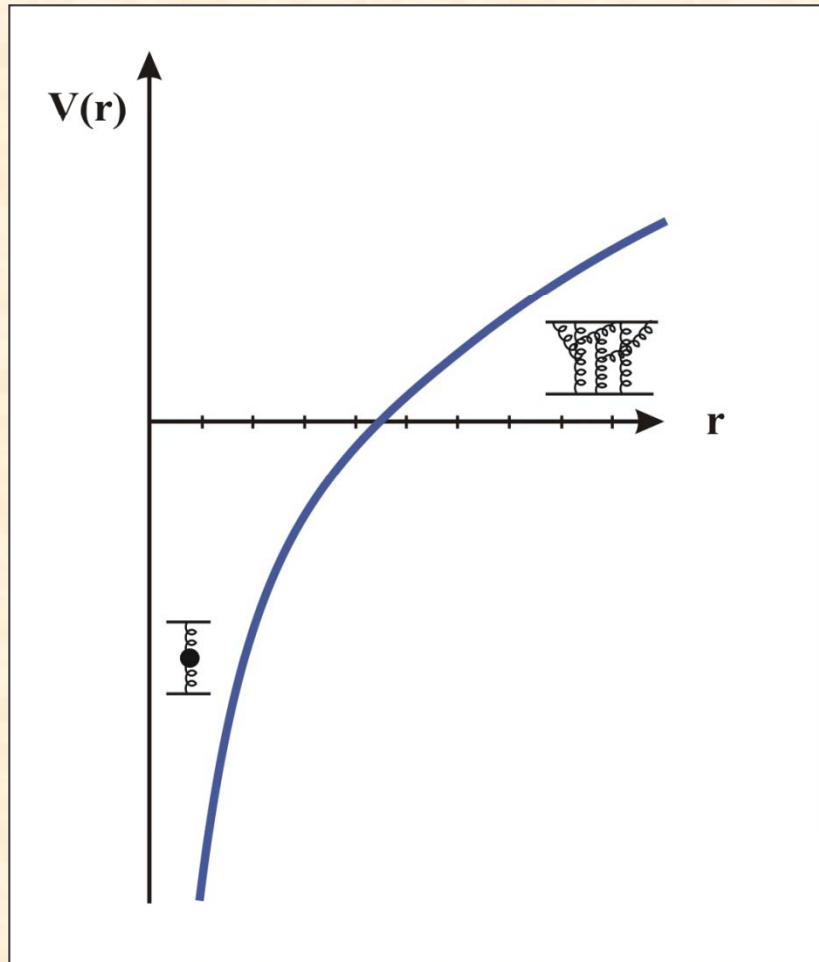
confinement



pQCD

strong QCD

interquark potential – spectrum



$$\underline{L} \cdot \underline{S}$$

$$\underline{S}_1 \cdot \underline{S}_2$$

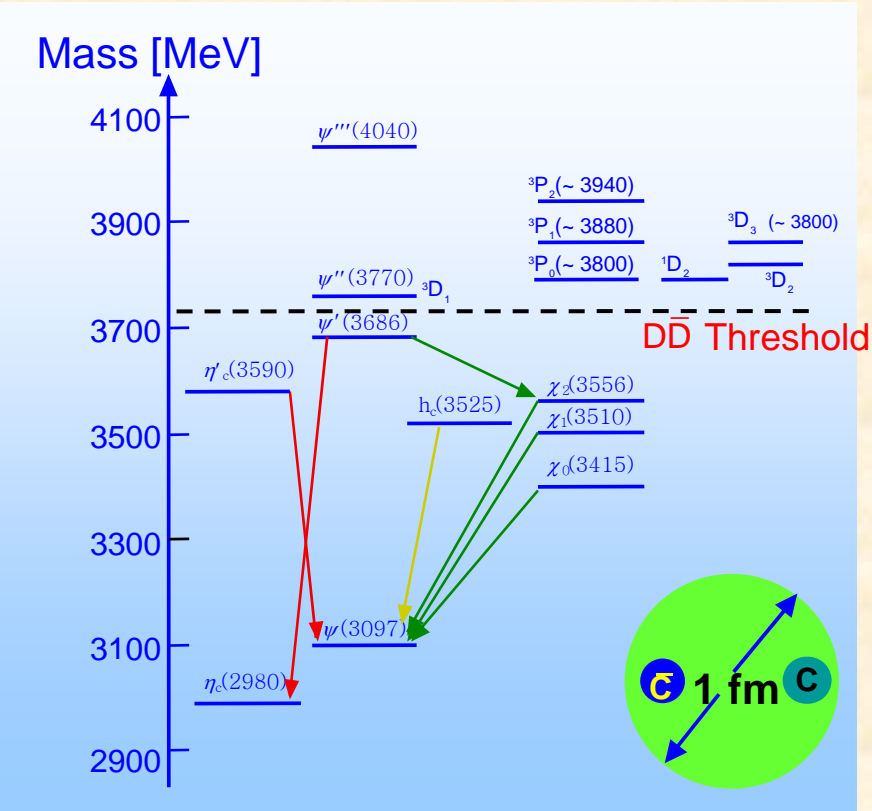
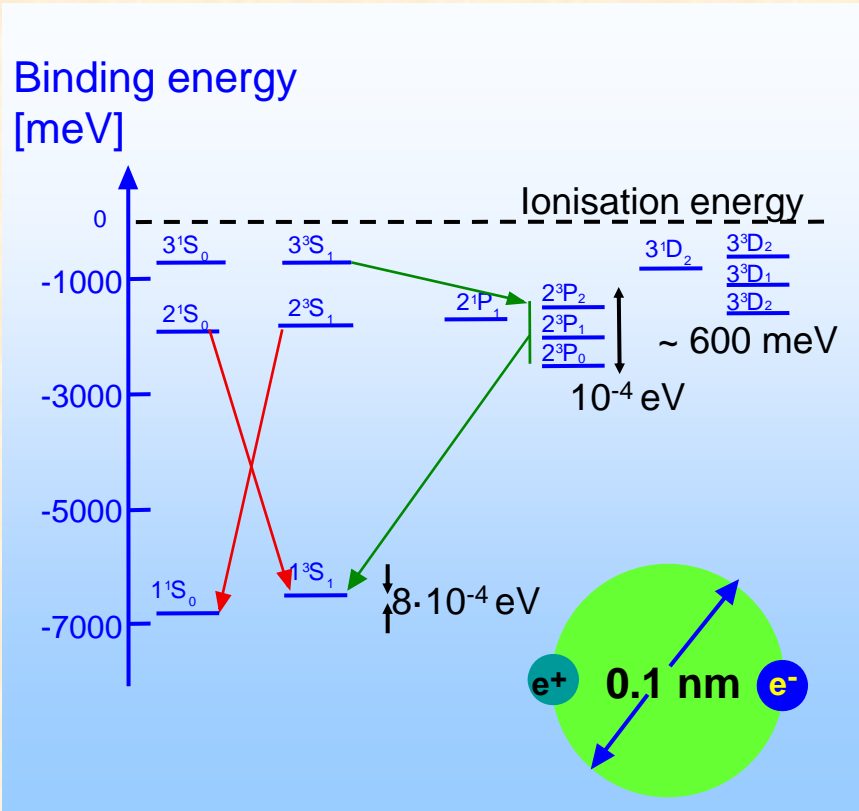
$$O(v/c)$$

Charmonium

positronium of QCD

- Positronium

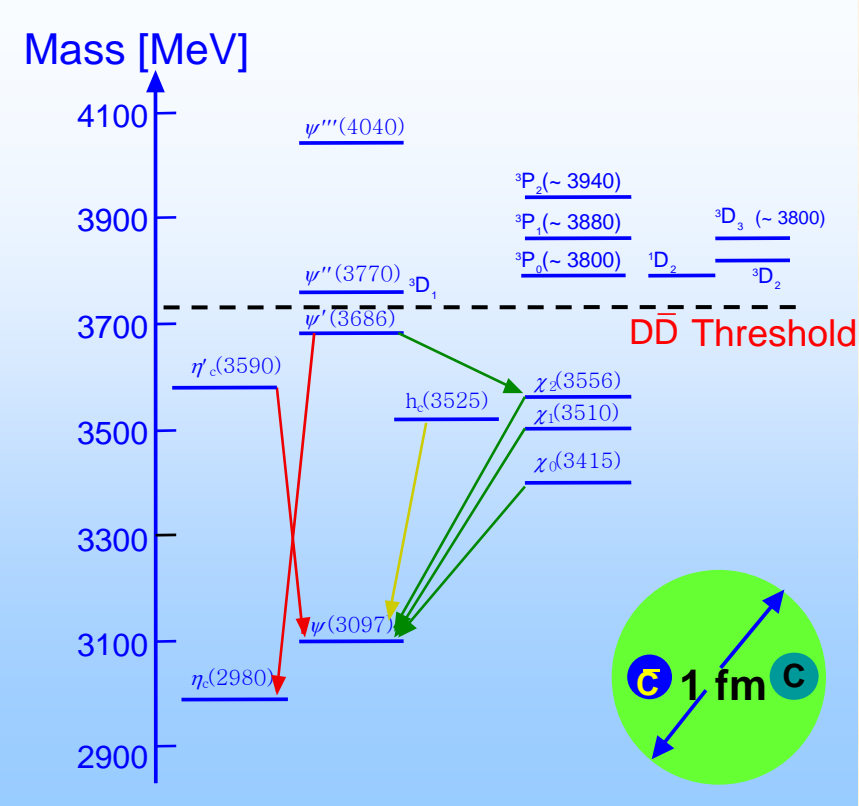
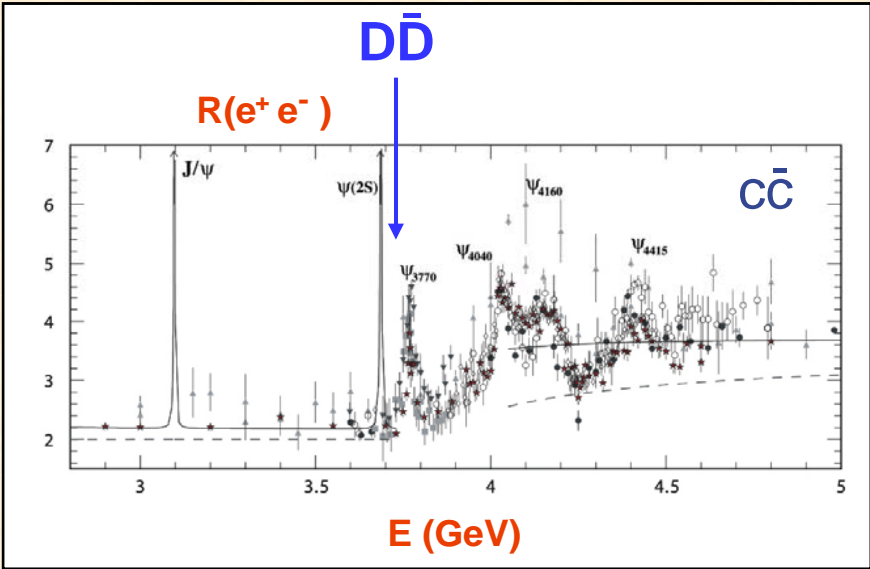
- Charmonium



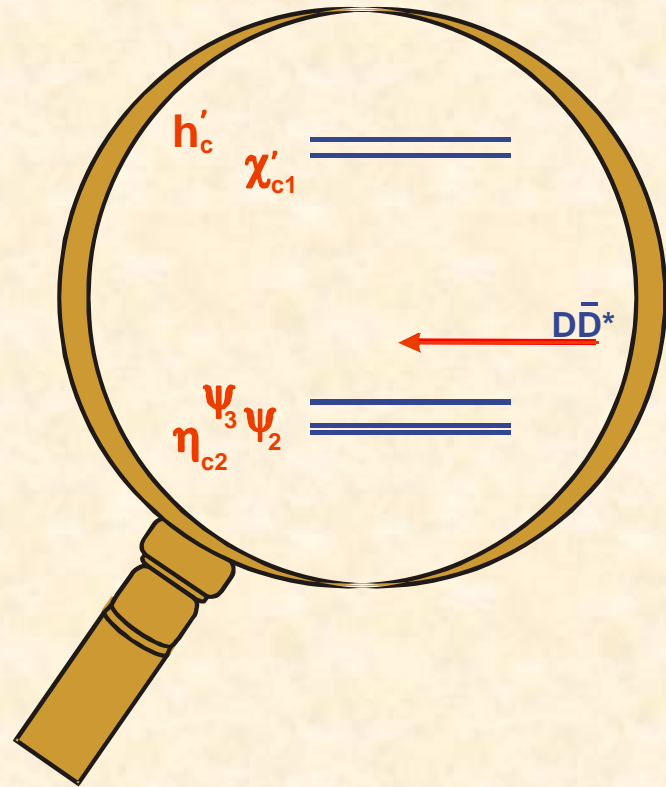
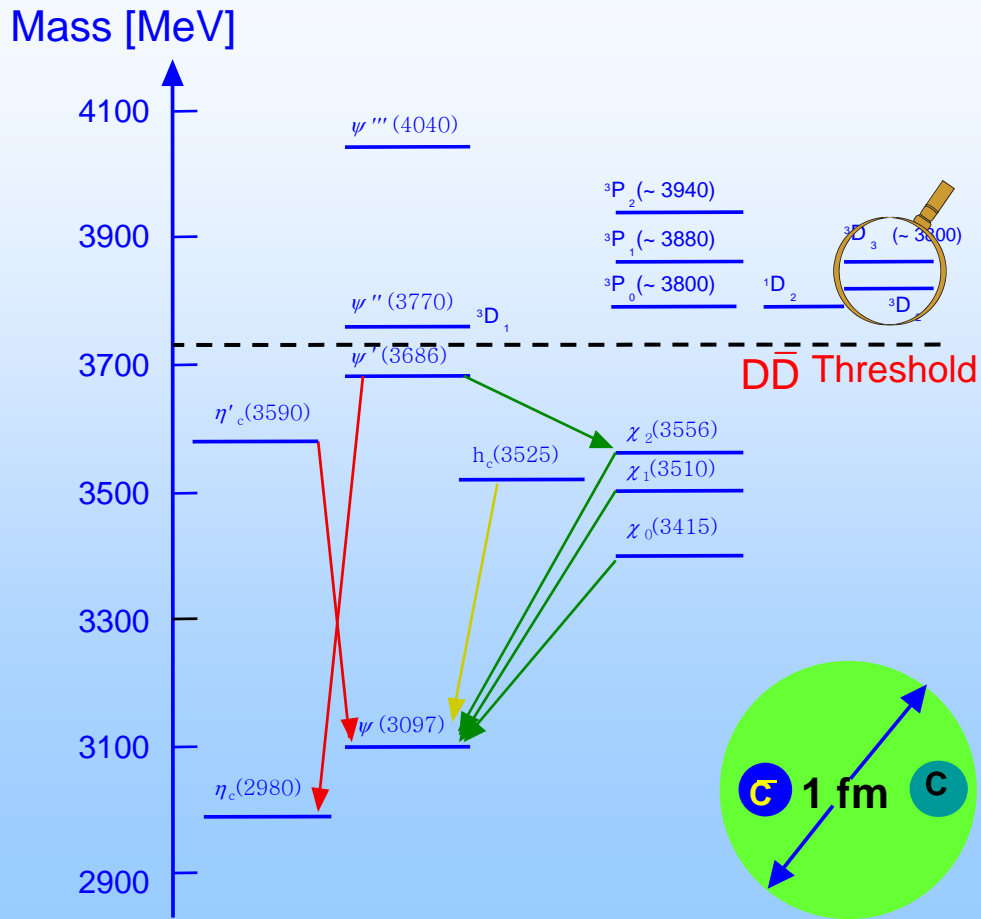
Charmonium

positronium of QCD

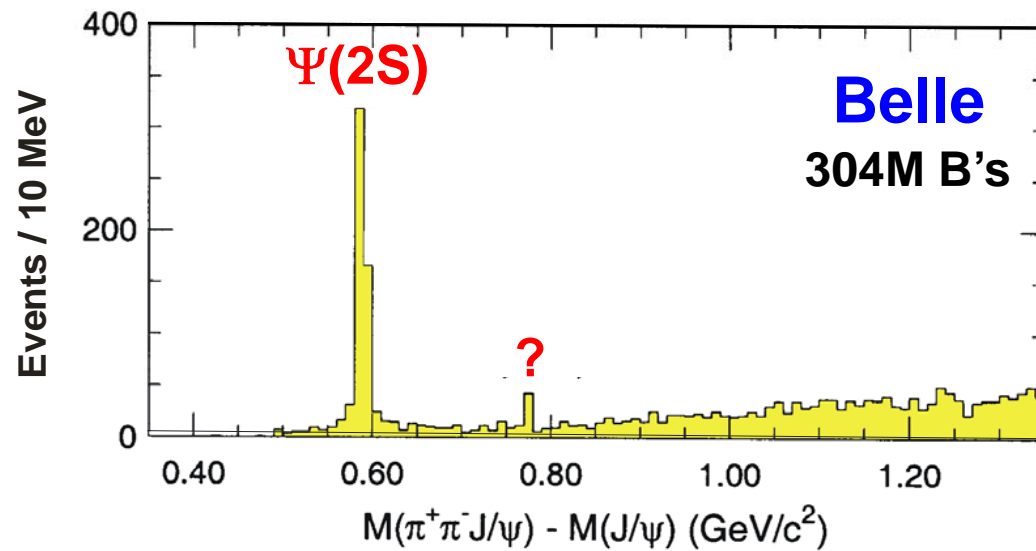
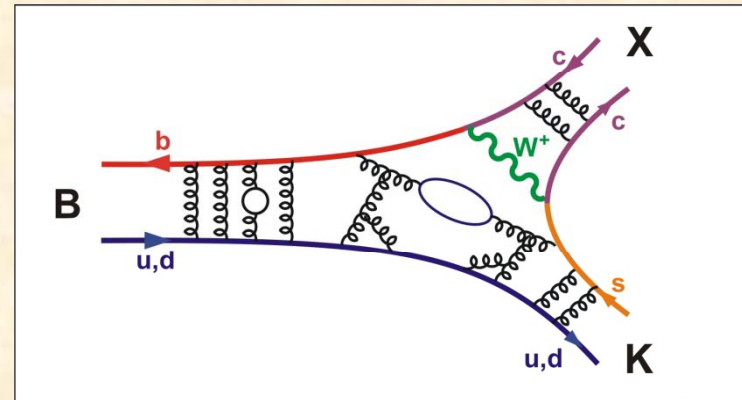
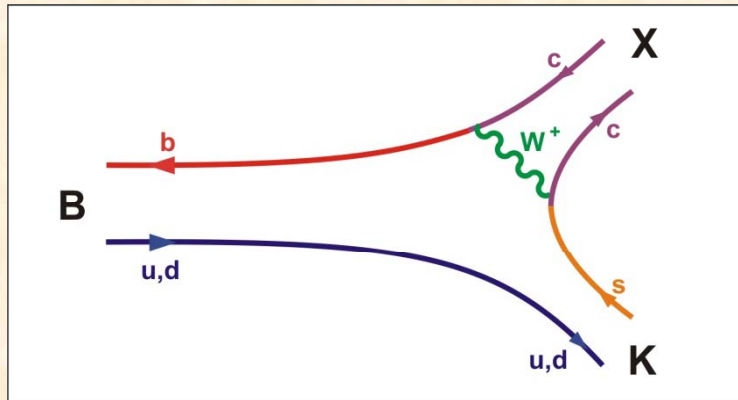
- Charmonium



Charmonium



charmonium from B-decay

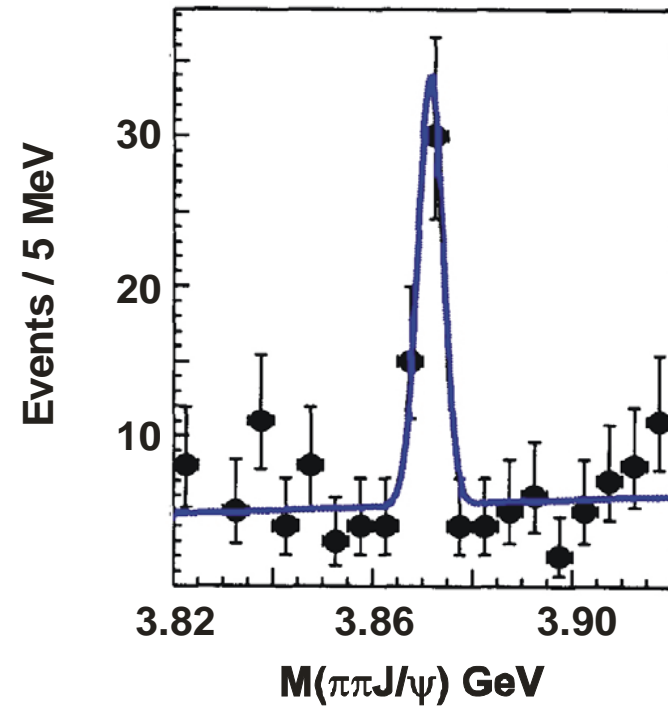
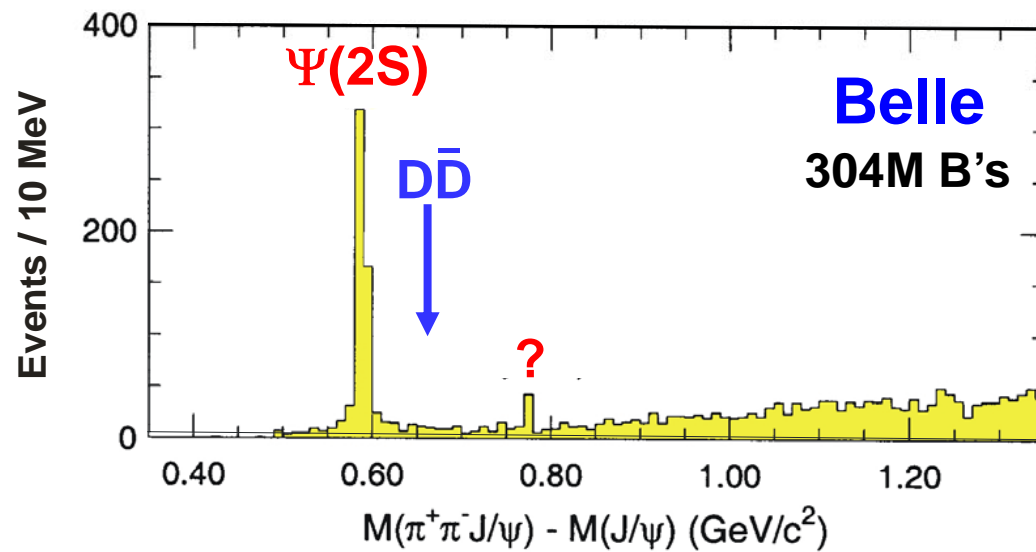
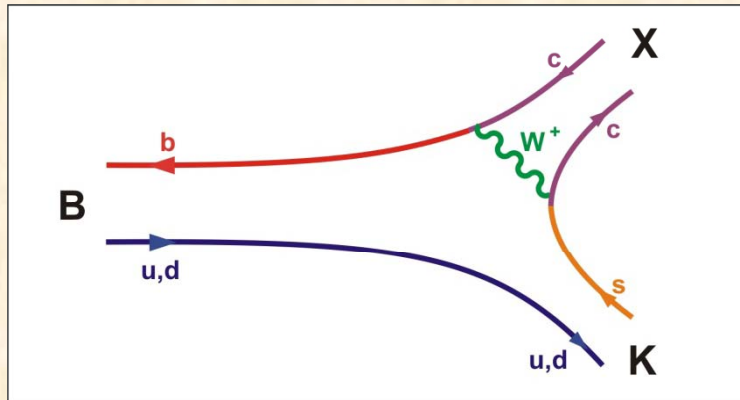


$B \rightarrow KX$

$X \rightarrow \pi^+ \pi^- J/\psi$

X(3872)

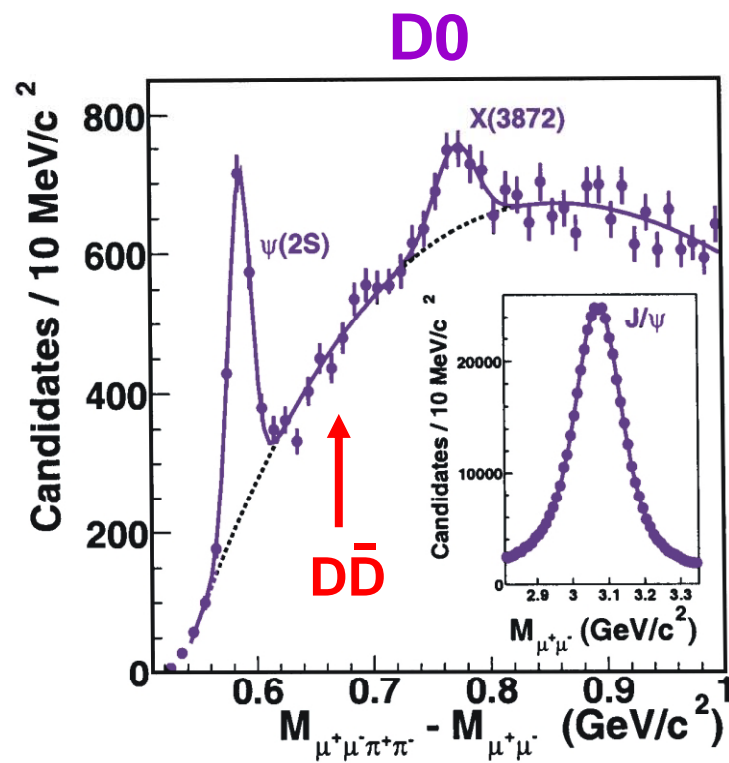
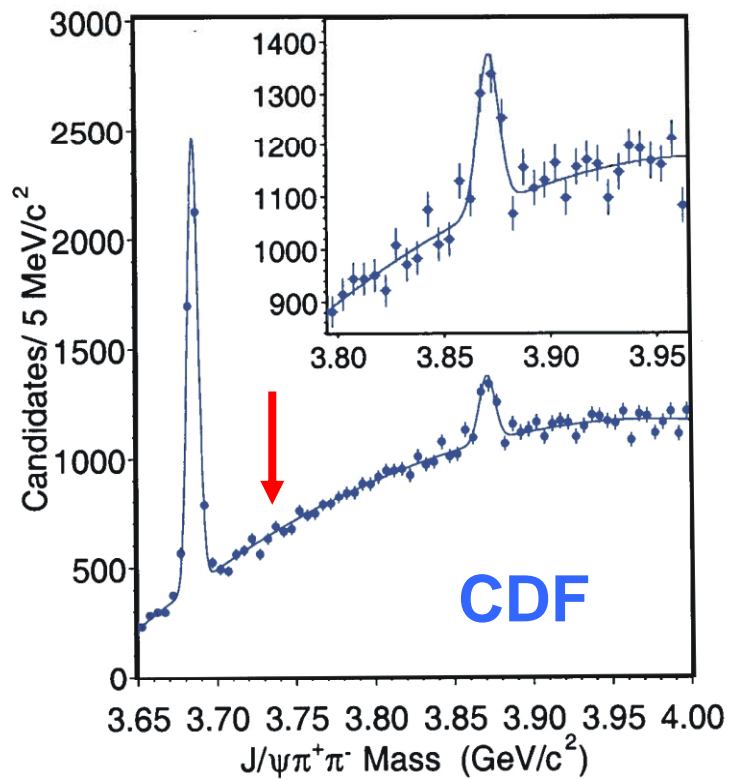
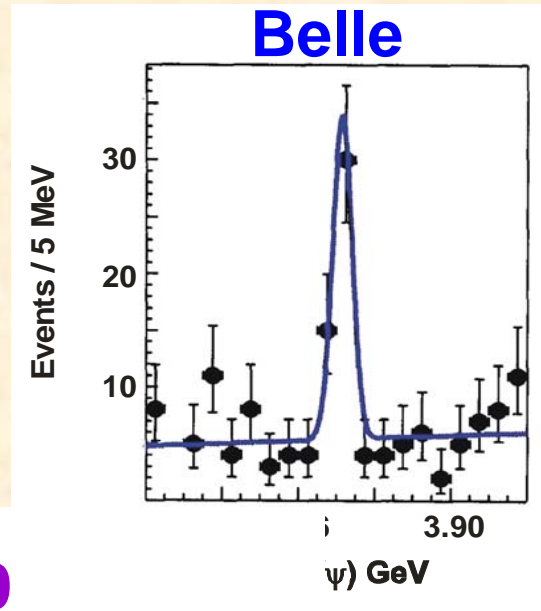
charmonium from B-decay



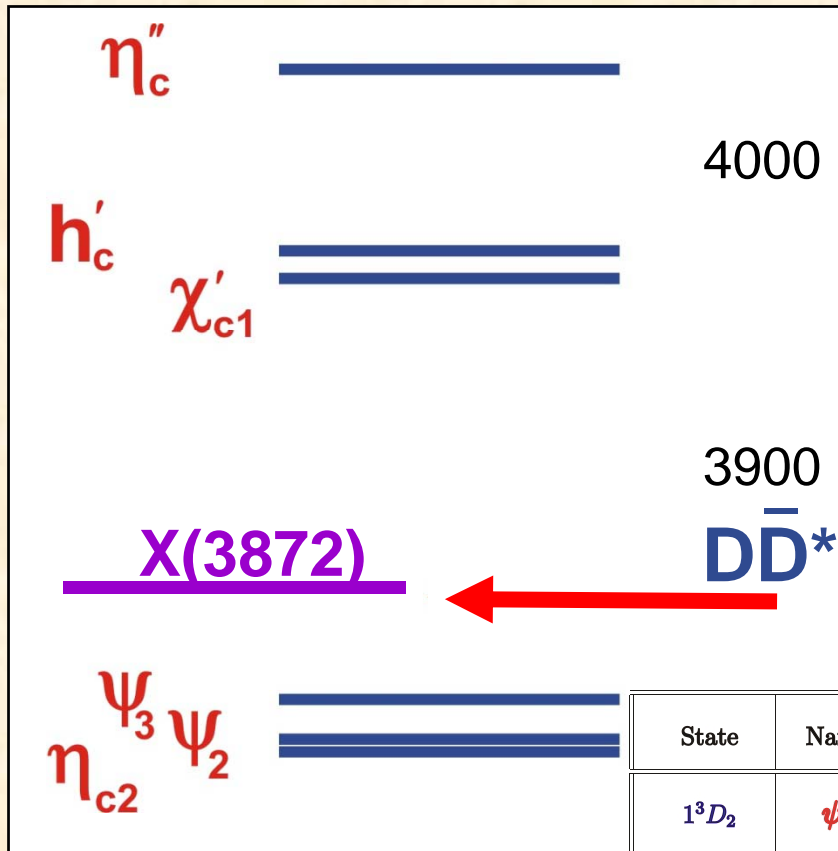
$\Gamma < 2.3$ MeV

X(3872)

X(3872) confirmed

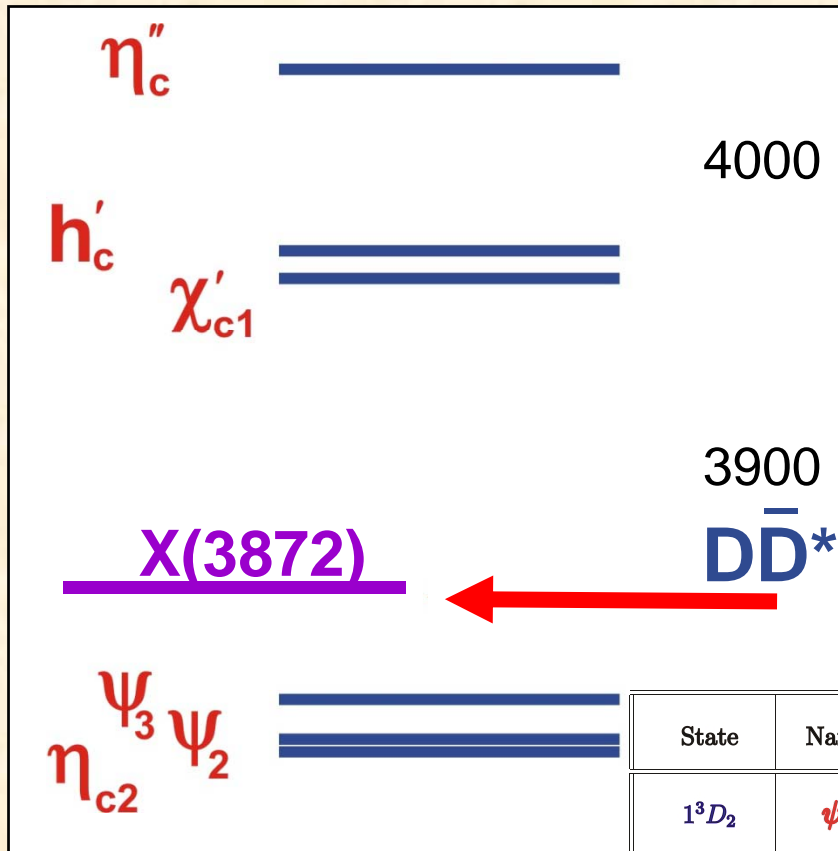


candidate assignments



State	Name	J^{PC}	$M_{predicted}$ MeV	$\Gamma_{predicted}$ MeV	comment
1^3D_2	ψ_2	2^{--}	3838	0.7	Mass wrong; $\Gamma(\rightarrow \gamma\chi_{c1})$ too small
2^1P_1	h_c'	1^{+-}	3963	1.6	Ruled out by $ \cos\theta_\psi $ distribution
1^3D_3	ψ_3	3^{--}	3849	4.8	$\Gamma(\rightarrow \gamma\chi_{c2})$ too small; spin too high
2^3P_1	χ_{c1}'	1^{++}	3956	1.7	$\Gamma(\rightarrow \gamma\psi)$ too small
1^1D_2	η_{c2}	2^{-+}	3837	0.9	$B(\pi^+\pi^-\psi)$ expected to be very small
3^1S_0	η_c'	0^{-+}	4069	~ 20	Mass and width wrong

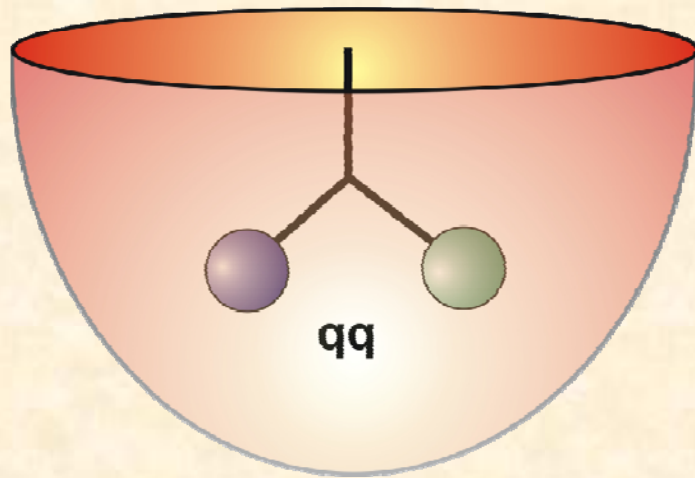
candidate assignments



Belle disfavours $P = -$
 (hep-ex/0505038) $\rightarrow J^{PC} = 1^{++}$

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2^3P_1	χ_{c1}	1^{++}	3956		
1^1D_2	η_{c2}	2^{-+}	3657	0.9	$B(\pi^+\pi^-\psi)$ expected to be very small
3^1S_0	η_c''	0^{-+}	4069	~ 20	Mass and width wrong

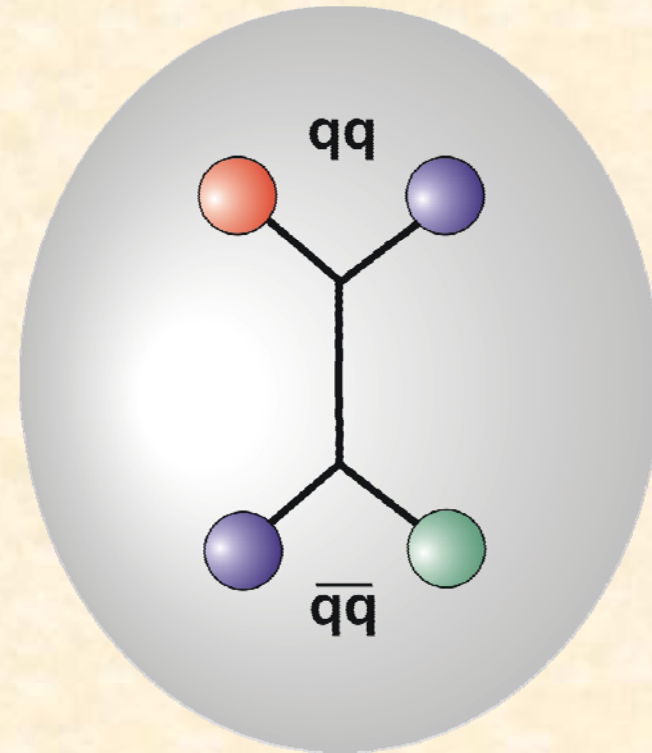
diquarks: colour



$$\bar{3}, 6$$

$$\bar{3} \otimes 3 = 1 \oplus \dots$$

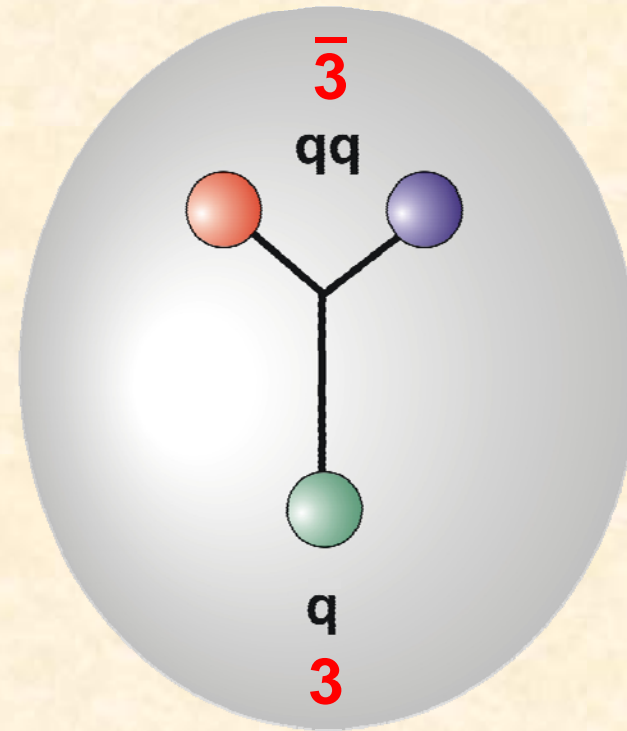
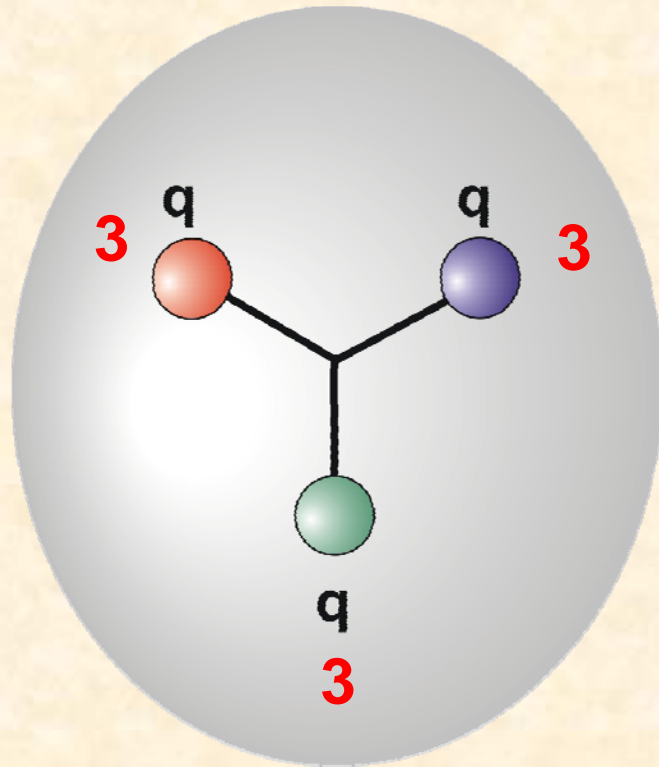
$$6 \otimes \bar{6} = 1 \oplus \dots$$



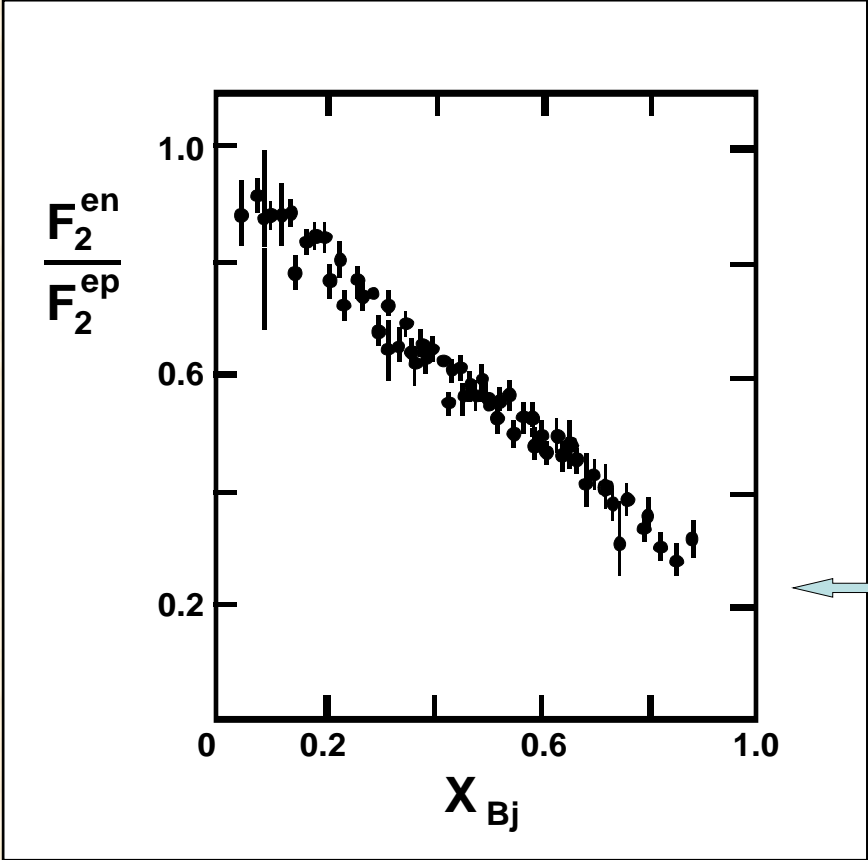
tetraquark

Rossi, Veneziano
Høgaasen, Sorba

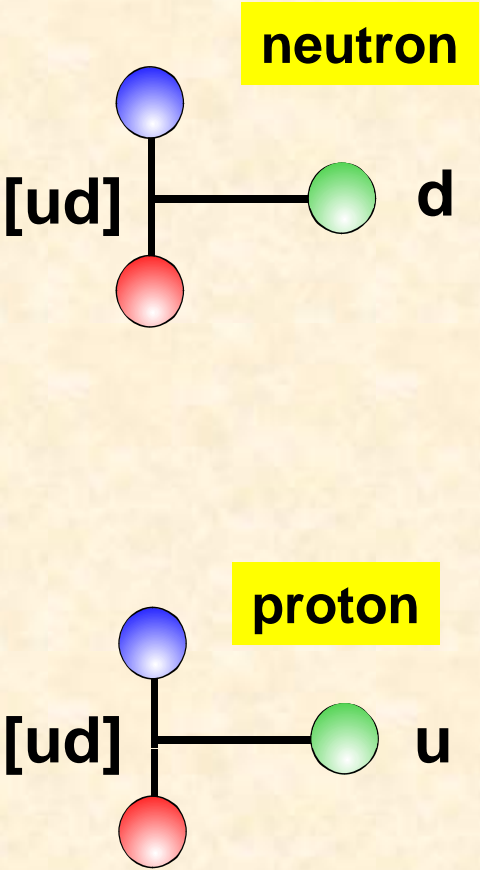
Baryons: colour



Deep inelastic scattering as $x \rightarrow 1$

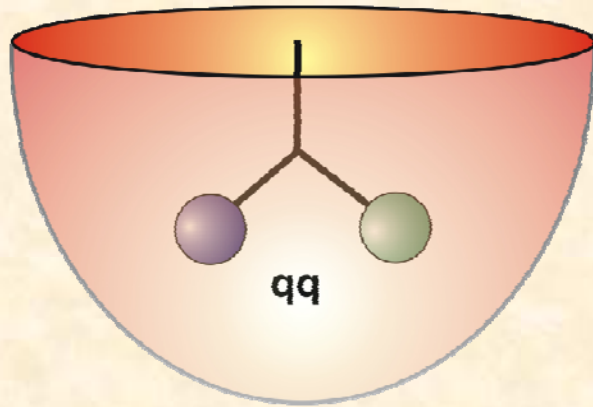


1/4

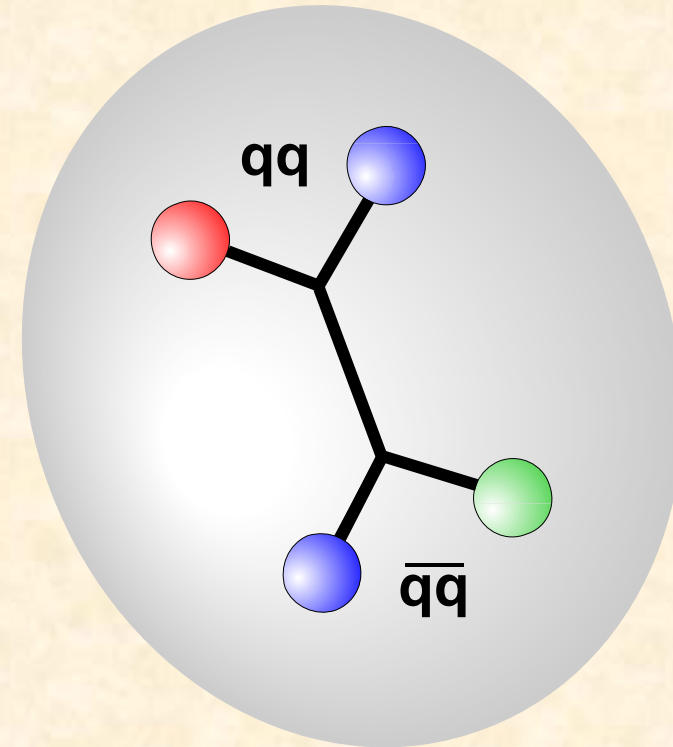


$$F_2 = x e_q^2 q(x)$$

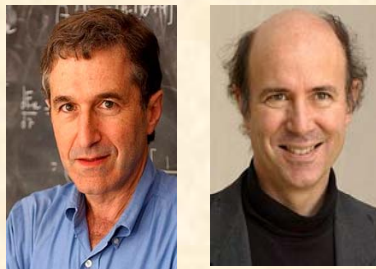
diquarks: colour



tetraquark



Jaffe & Wilczek

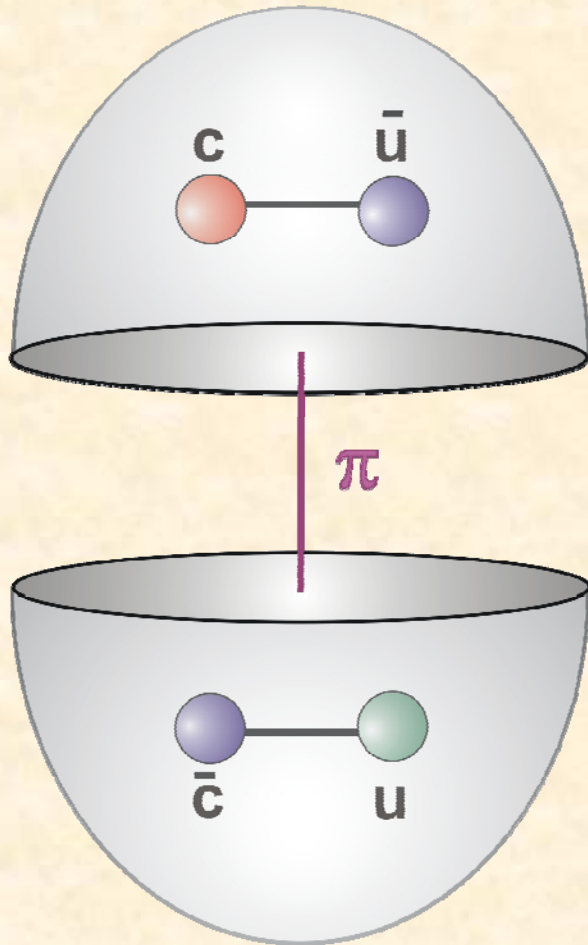


Scalar diquarks

[ud]	[us]	[ds]
[cd]	[cu]	[cs]

possible **X(3872)** structures

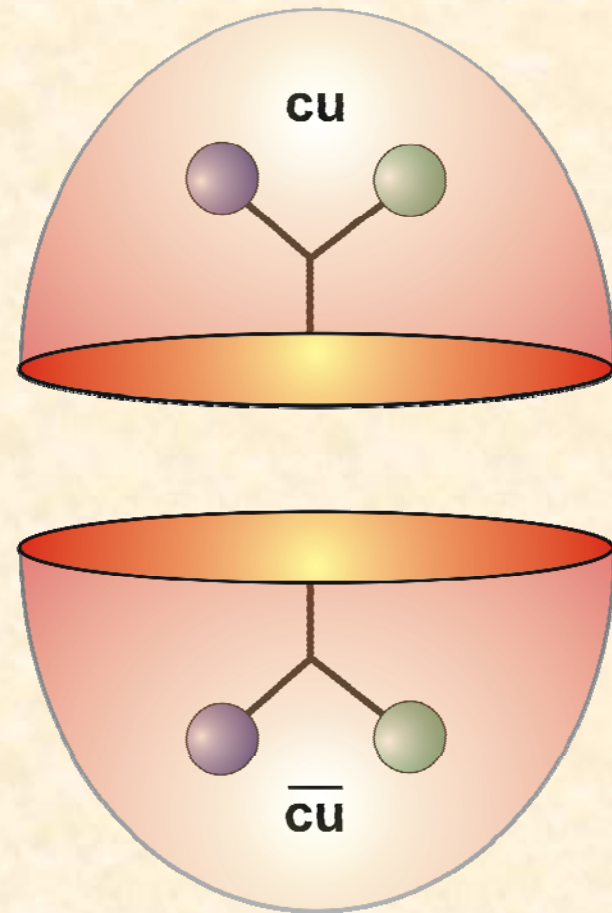
molecule



D^0

\bar{D}^{*0}

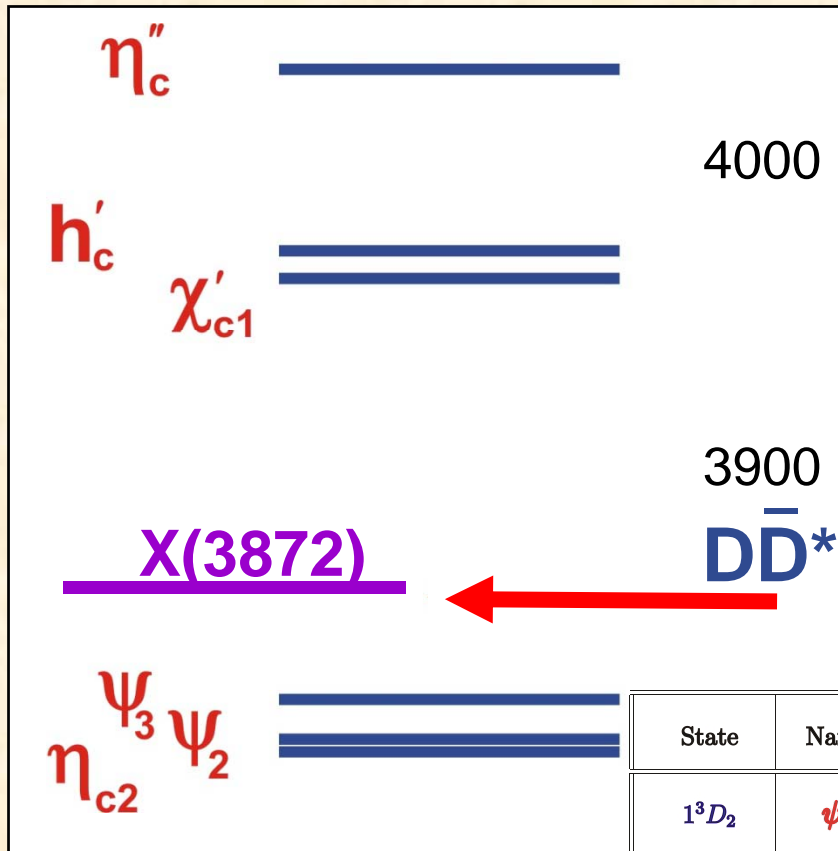
tetraquark



Tornqvist, ...

$D_{sJ}^*(2317)$, $D_{sJ}(2460)$, $D_{sJ}^*(2632)$ — $D^{(*)}K$

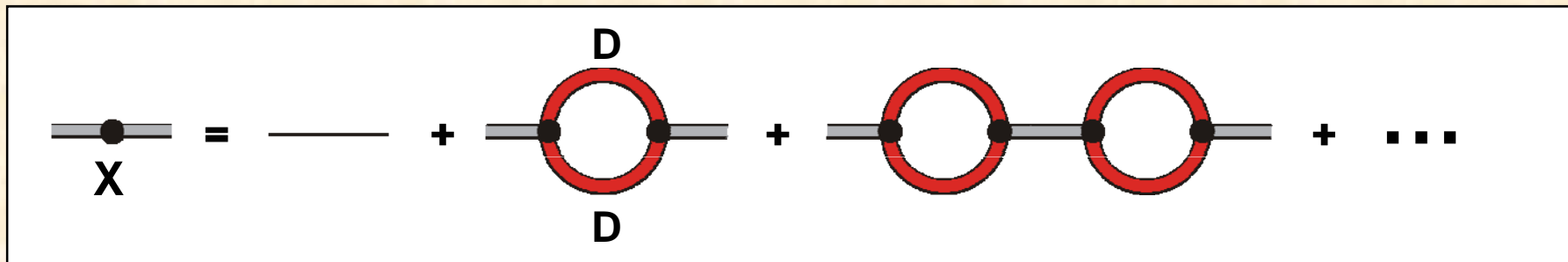
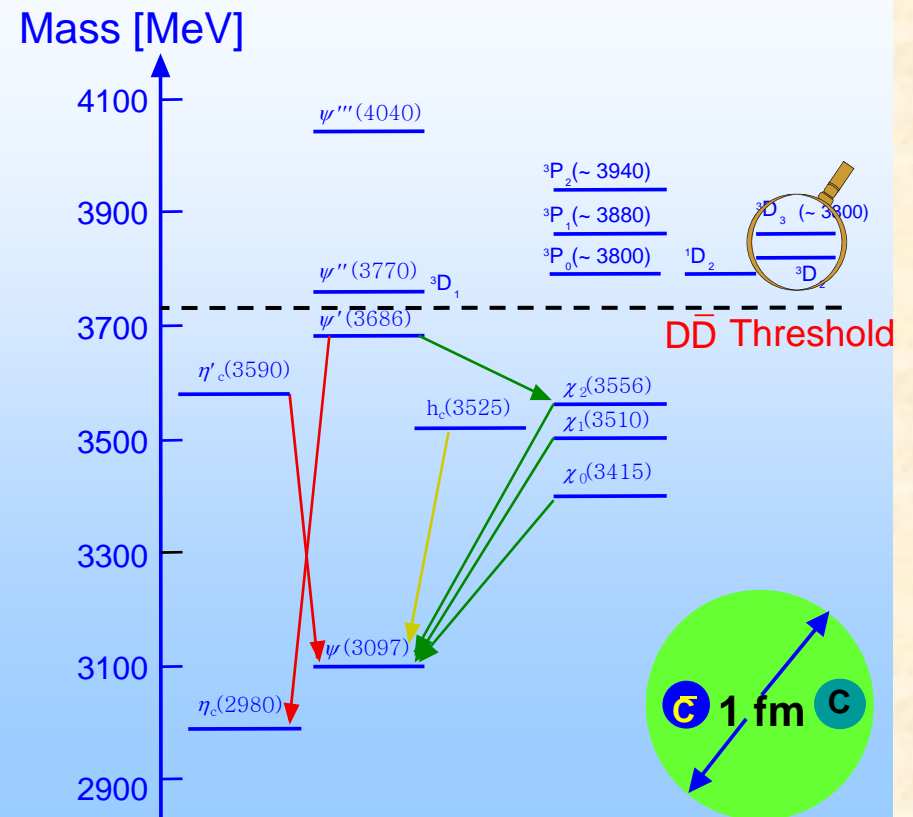
candidate assignments

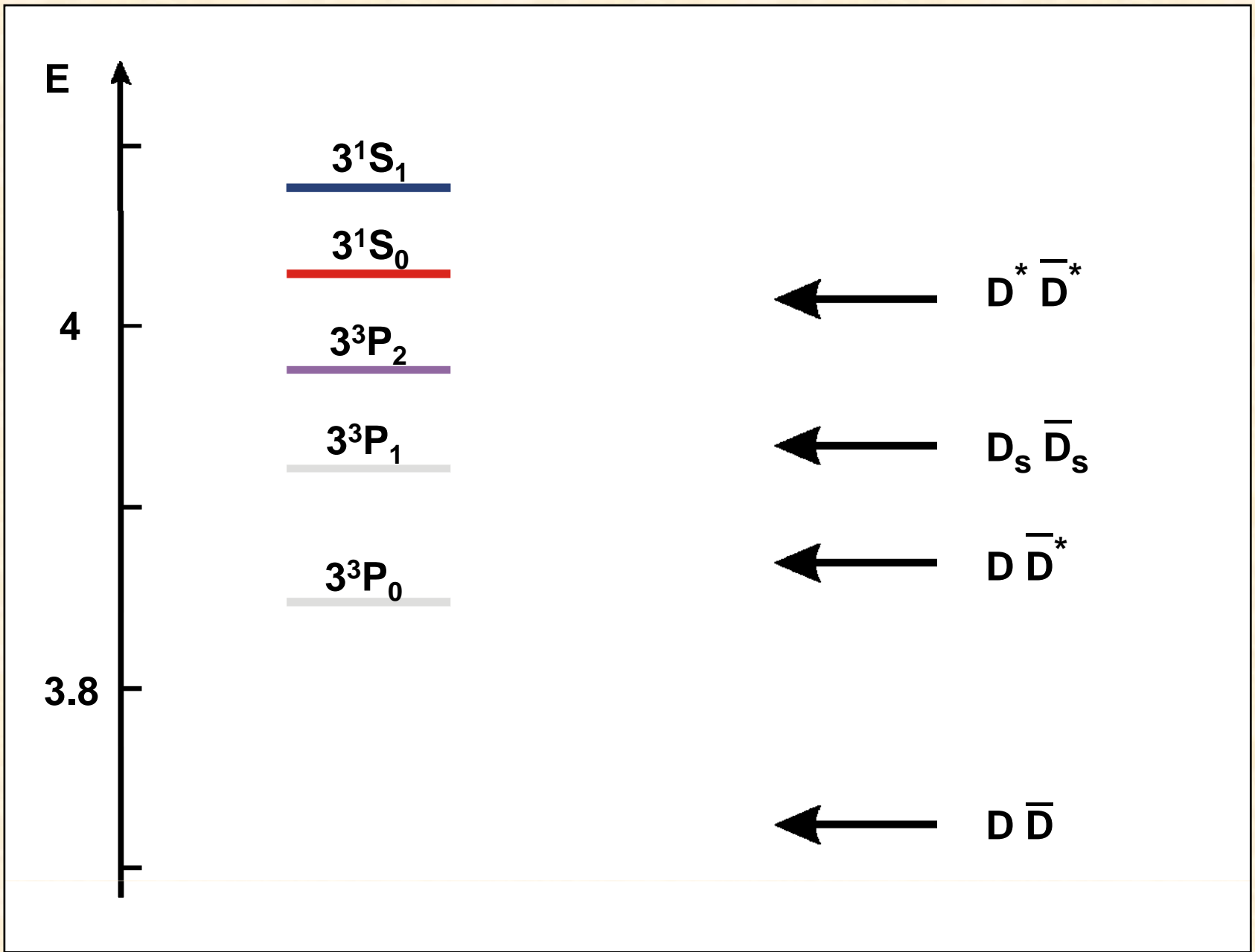


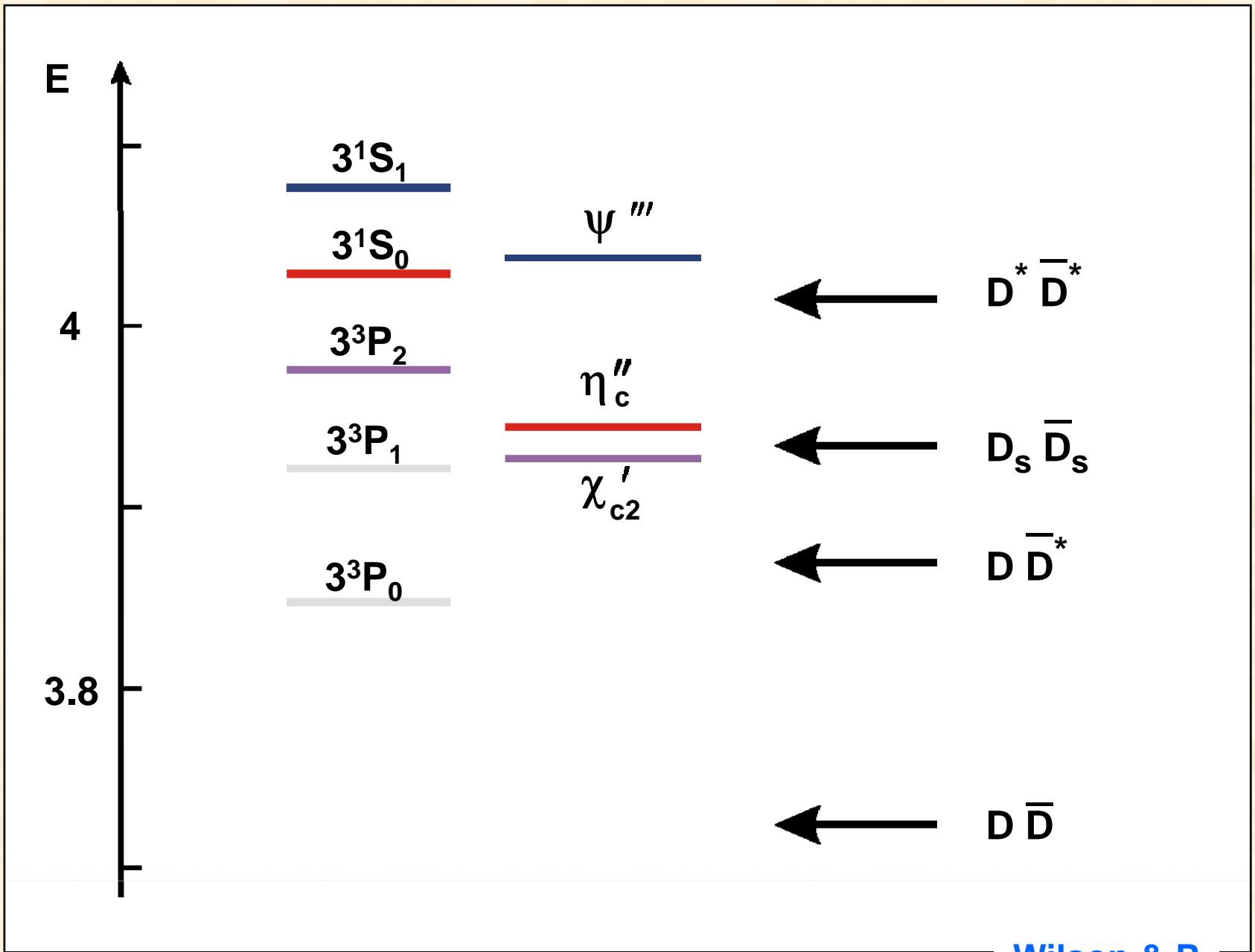
Belle disfavours $P = -$
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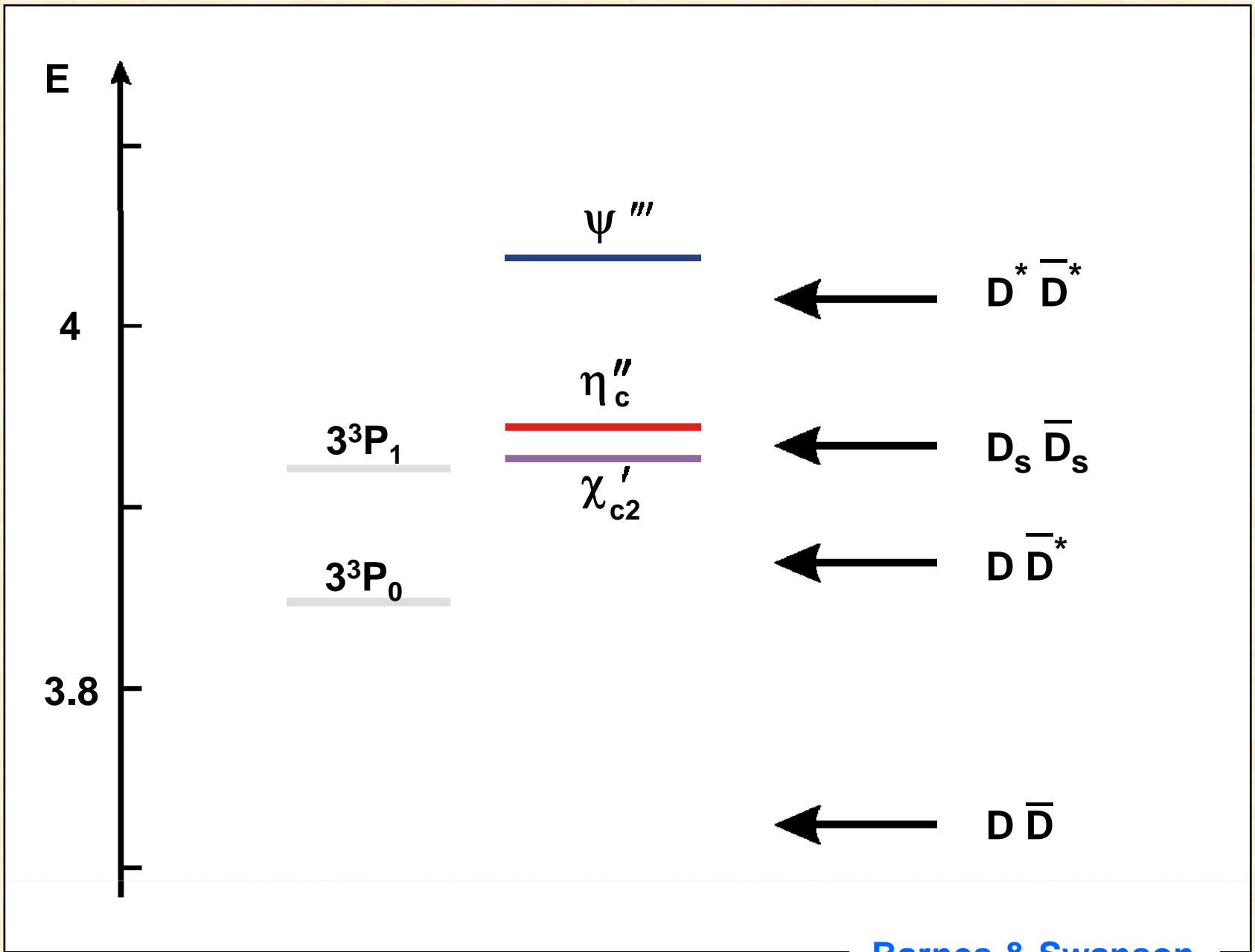
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Effect of Decay channels





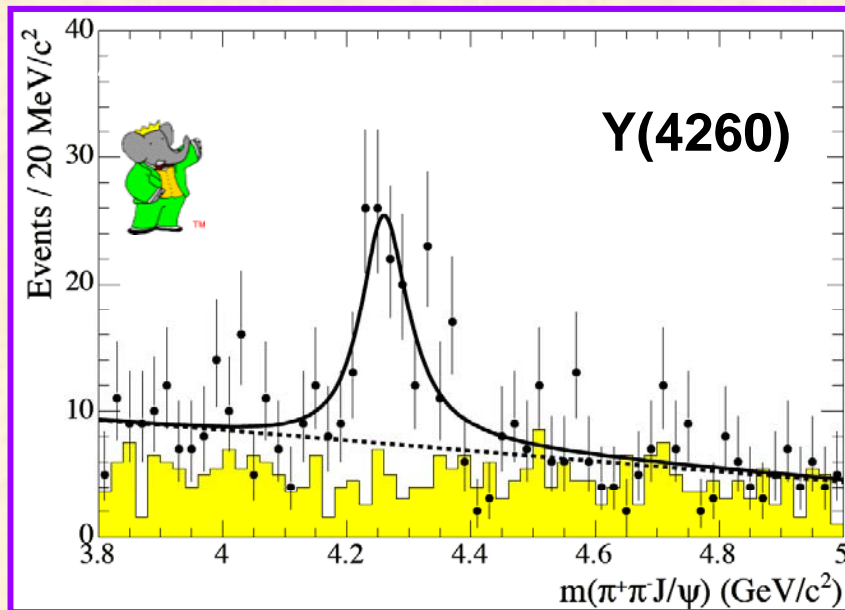




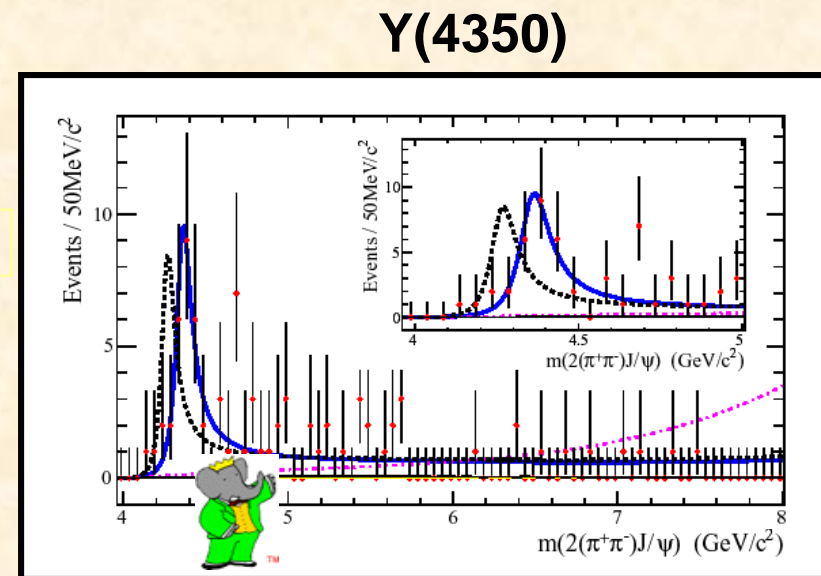
The 1^- family

Faccini

several resonances observed in $e^+e^- \rightarrow Y\gamma_{\text{ISR}}$



PRL 95, 142001 (2005)



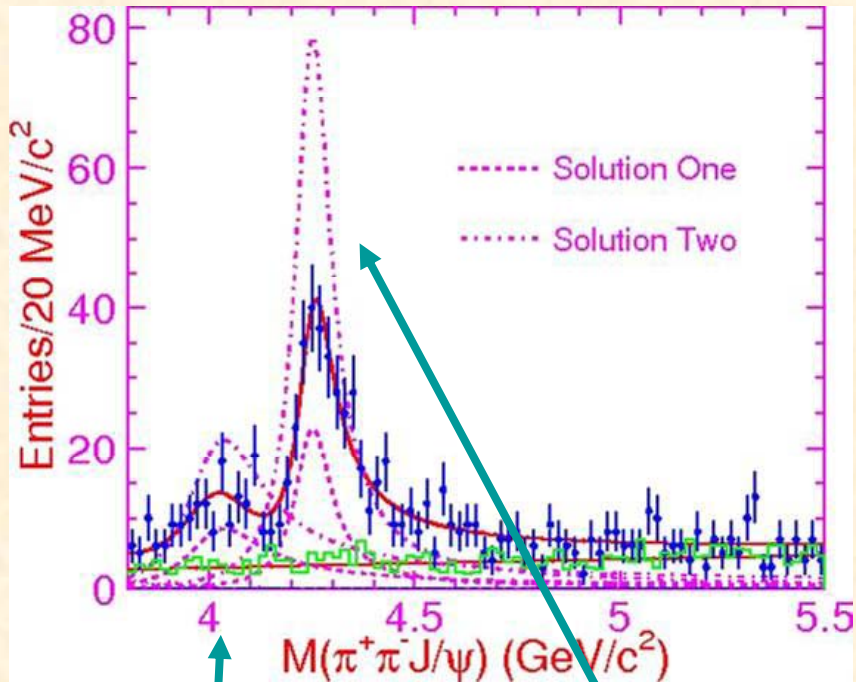
$Y(4350) \rightarrow \psi(2S)\pi\pi$

definitely $J^{PC}=1^{--}$

PRL 98, 212001 (2007)

The youngest of the 1^- family

arXiv:0707.2541

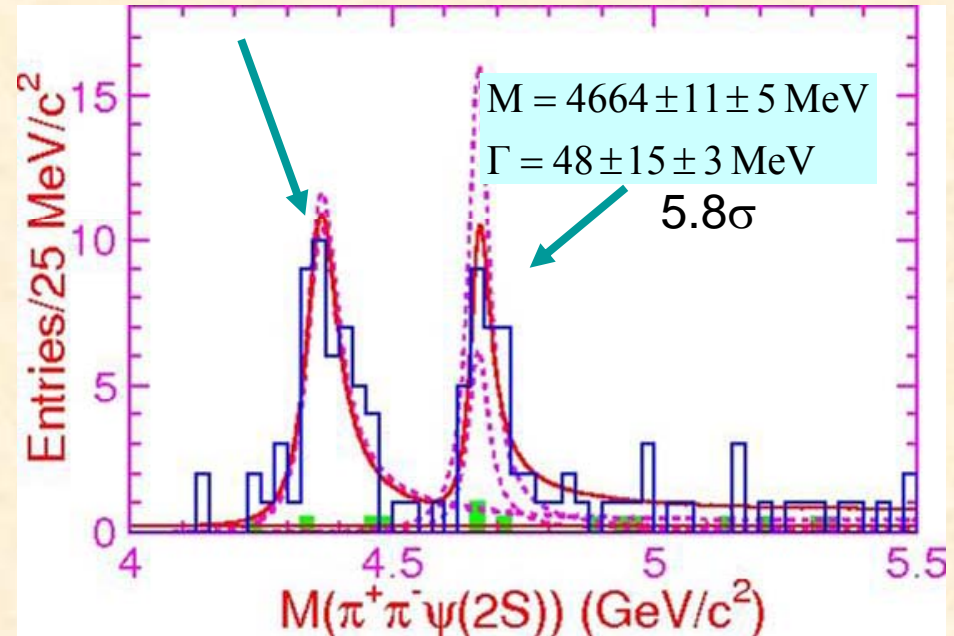


$M = 4361 \pm 9 \pm 9 \text{ MeV}$
 $\Gamma = 74 \pm 15 \pm 15 \text{ MeV}$



$M = 4008 \pm 40^{+72}_{-28} \text{ MeV}$
 $\Gamma = 226 \pm 44^{+87}_{-79} \text{ MeV}$

$M = 4247 \pm 12^{+17}_{-26} \text{ MeV}$
 $\Gamma = 108 \pm 19^{+8}_{-10} \text{ MeV}$



$M = 4664 \pm 11 \pm 5 \text{ MeV}$
 $\Gamma = 48 \pm 15 \pm 3 \text{ MeV}$
 5.8σ



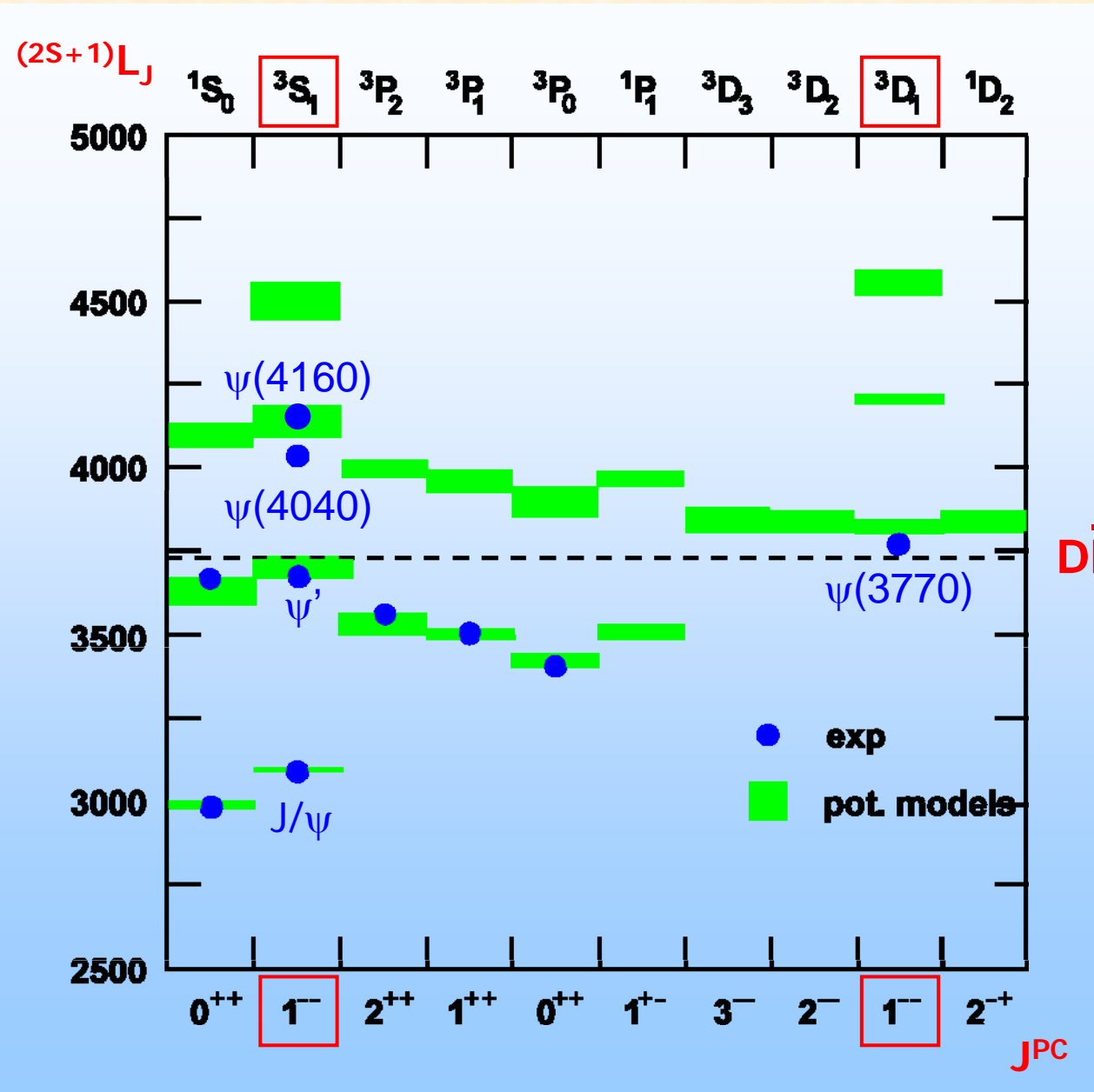
arXiv:0707.3699

1⁻ family

Only seen in $\psi(2S)\pi\pi$

- 4660
- 4350
- 4260
- 4008

4 Ys ?



The 3940 family

	Observed in	J^{PC} (?)	M (MeV)	Γ (MeV)
X	$e^+e^- \rightarrow J/\psi X$ ($X \rightarrow DD^*$)	$0^{--}, 1^{++}$	3943 ± 8	< 39
Y	$B \rightarrow YK$ ($Y \rightarrow J/\psi \omega$)	$1^{++}, \dots$	3943 ± 17	87 ± 34
Z	$\gamma\gamma \rightarrow Z$ ($Z \rightarrow DD$)	2^{++}	3929 ± 5	29 ± 10

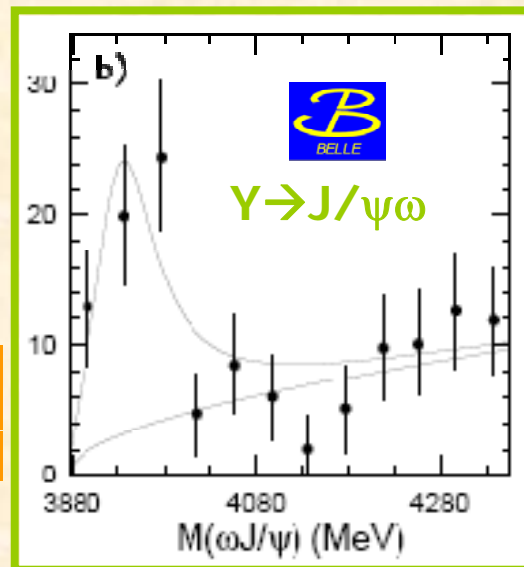
PRL 98, 082001 (2007)



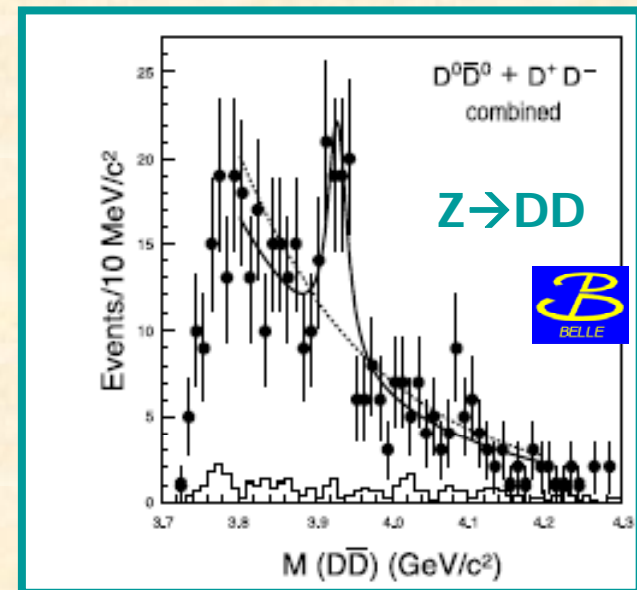
Y(3940) closer to X(3940)
Can they be the same state?

$X \rightarrow DD^*$

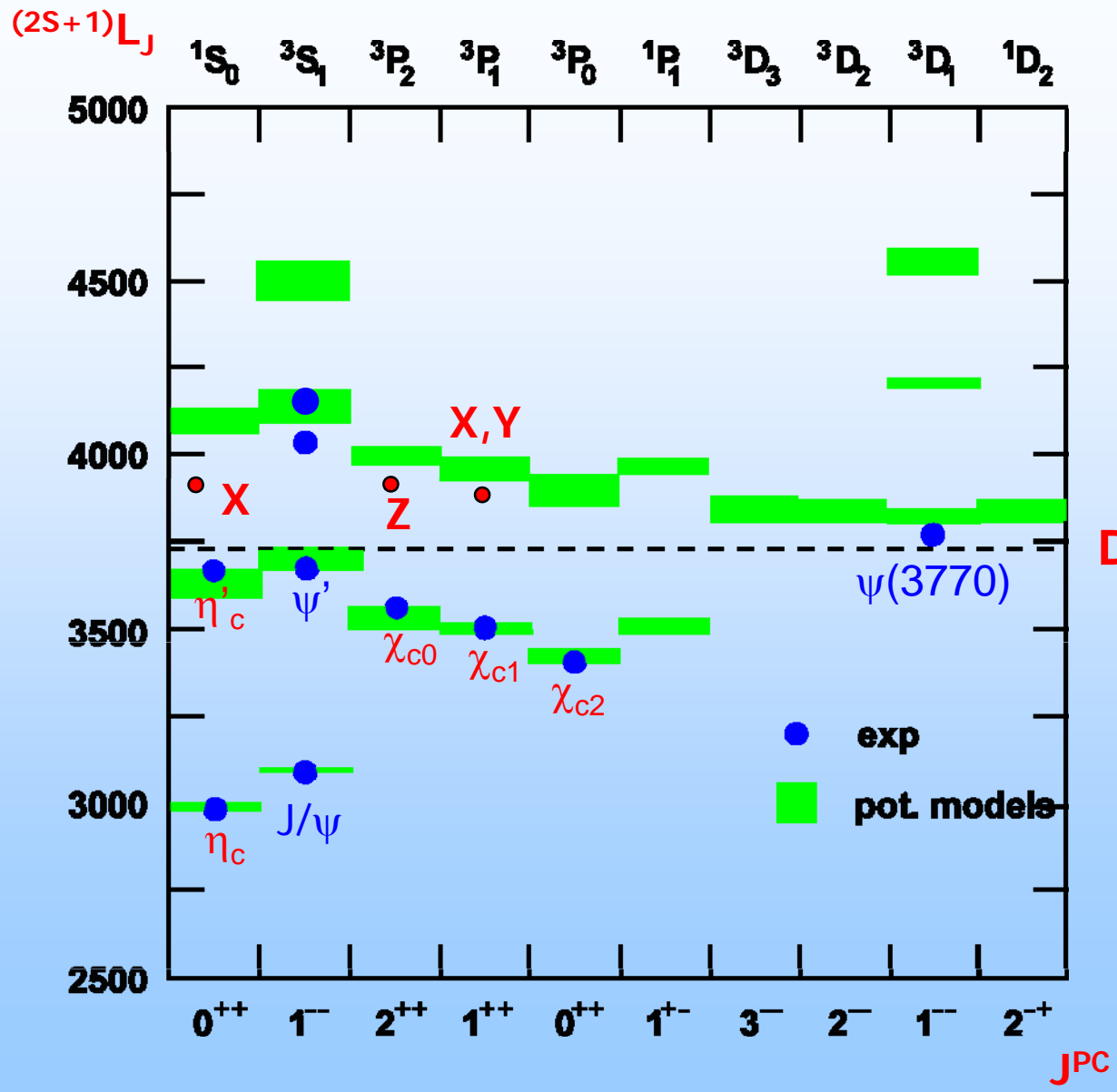
PRL 94, 182002 (2005)



PRL 96, 082003 (2006)



X, Y, Z states



D \bar{D} threshold

hybrid ?

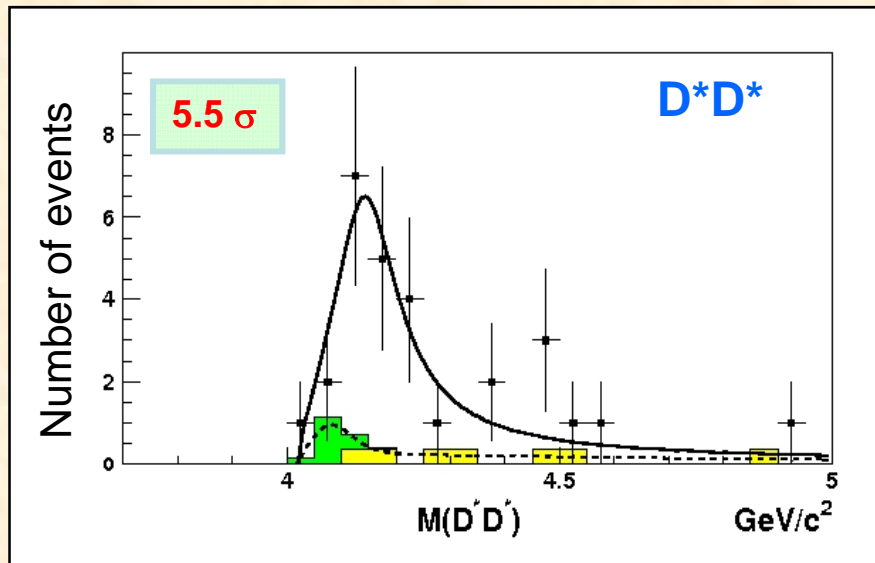
$X(4160) \rightarrow D^* D^*$

$$e^+ e^- \rightarrow J/\psi D^{(*)} D^{(*)}$$

$$M = 3942 \pm 6 \text{ MeV}$$

$$\Gamma_{\text{tot}} = 37 \pm 12 \text{ MeV}$$

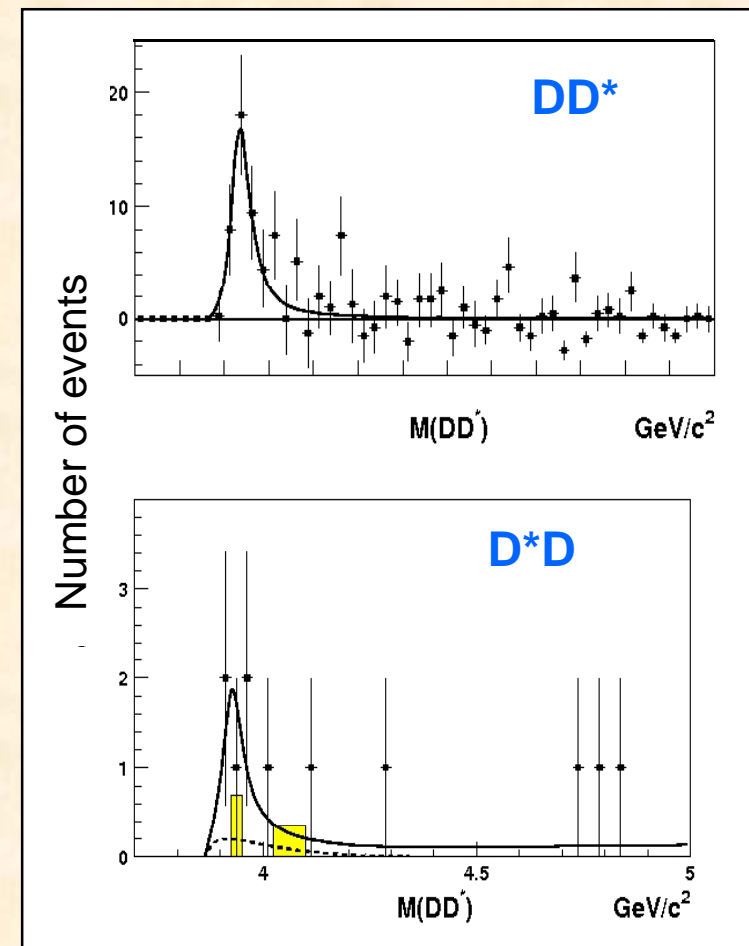
$$N_{\text{ev}} = 52 \pm 11$$



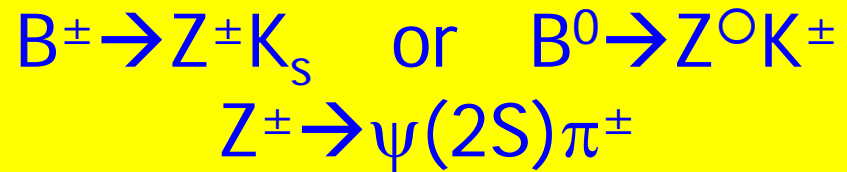
$$M = 4156^{+25}_{-20} \pm 15 \text{ MeV}$$

$$\Gamma_{\text{tot}} = 37^{+111}_{-61} \pm 21 \text{ MeV}$$

$$N_{\text{ev}} = 24^{+12}_{-8}$$

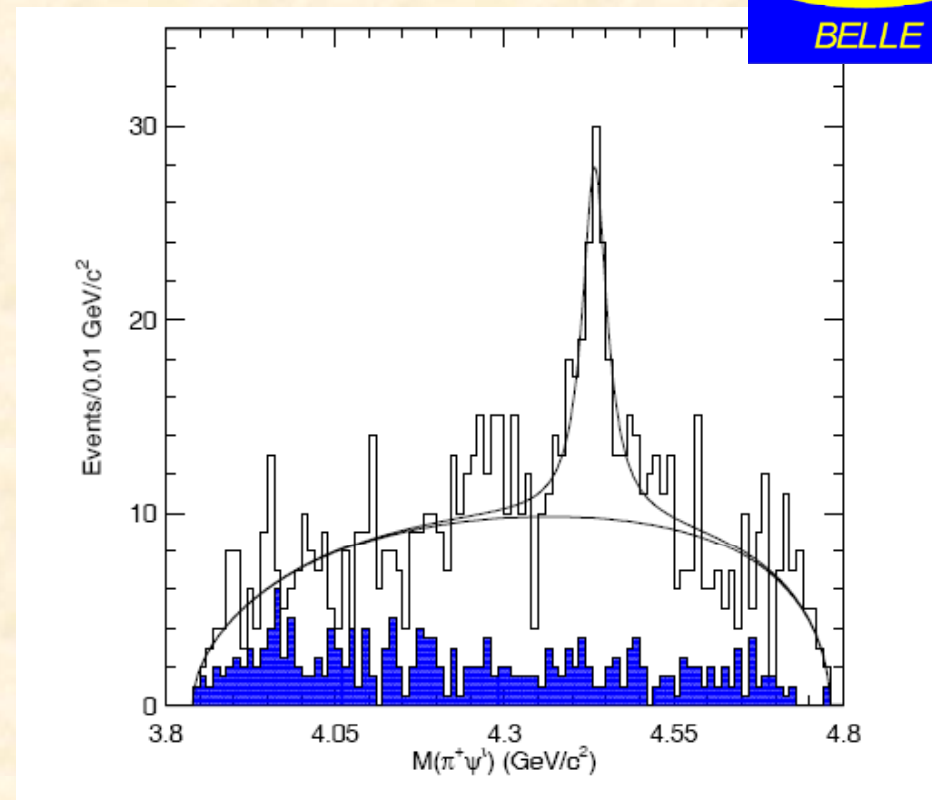


The first charged state: Z(4430)

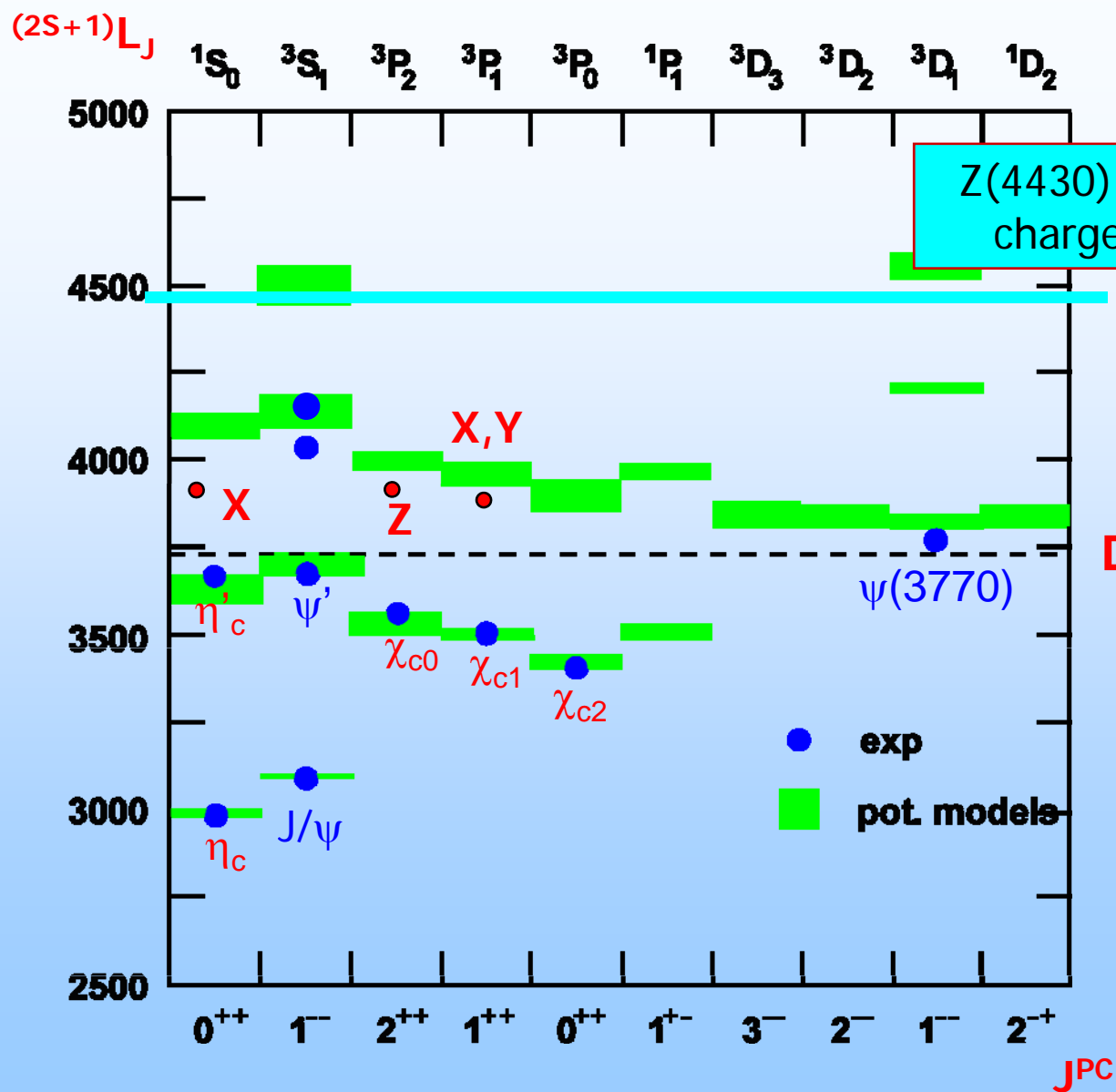


7.3σ

$$M = (4433 \pm 4) \text{ MeV}$$
$$\Gamma = (44^{+17}_{-13}) \text{ MeV}$$



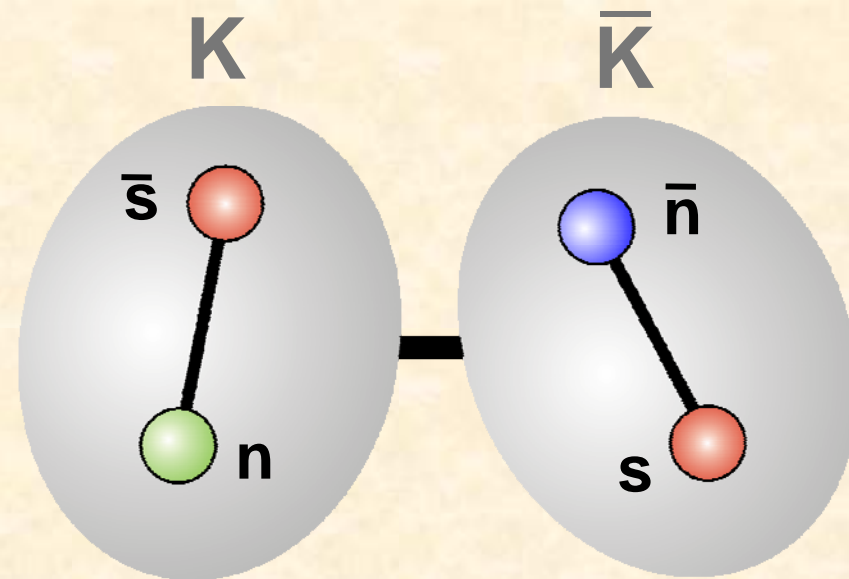
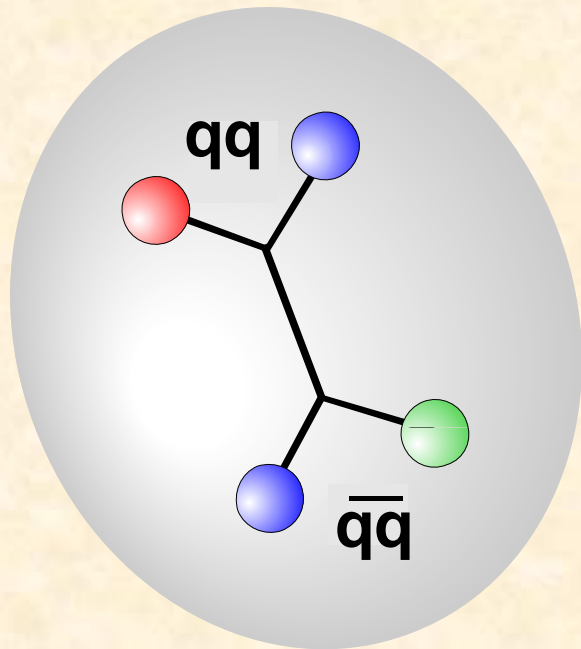
X, Y, Z states



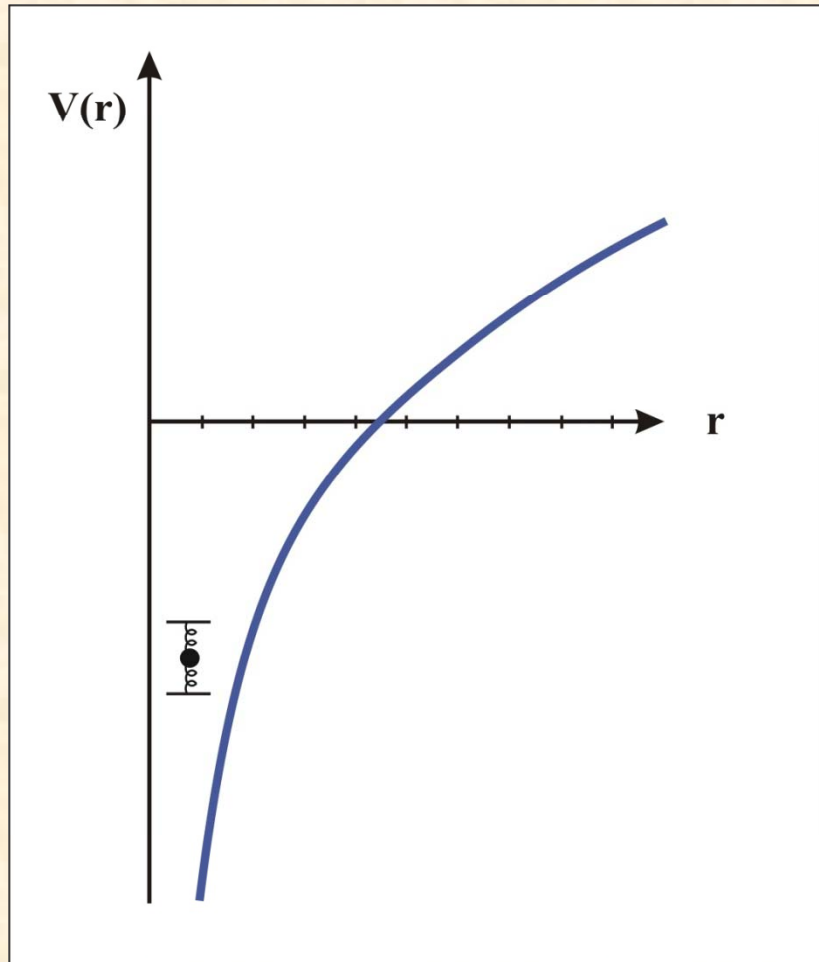
Z(4430): the first charged state

$D\bar{D}$ threshold

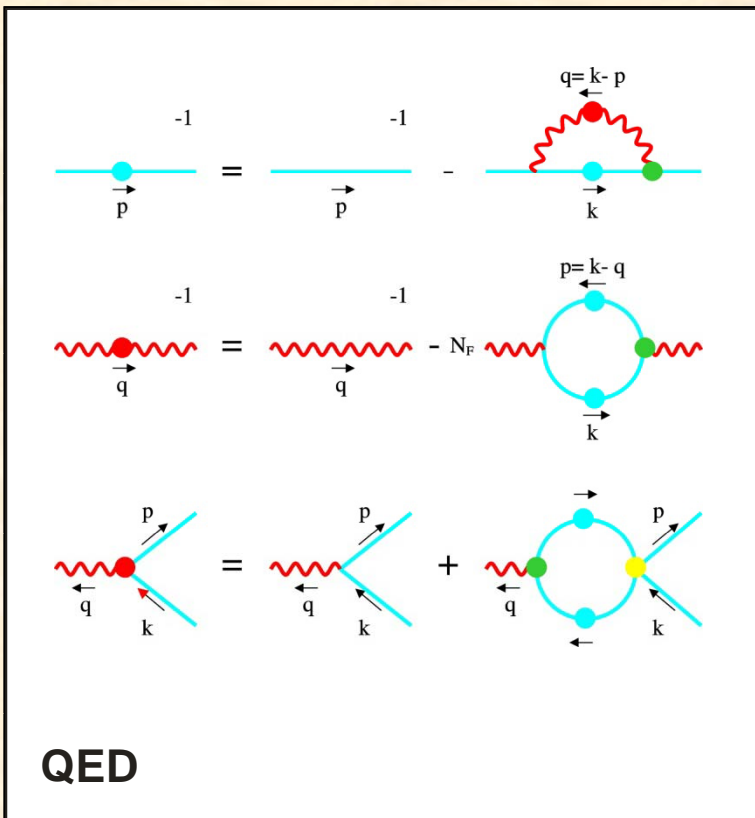
What is the $f_0(980)$?



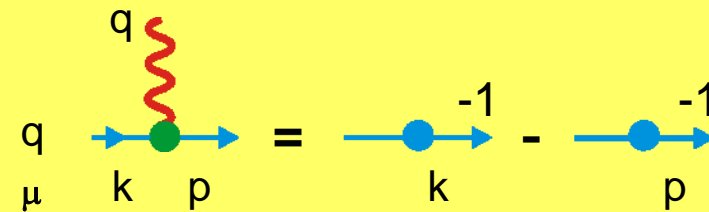
interquark potential – spectrum



Schwinger-Dyson Equations

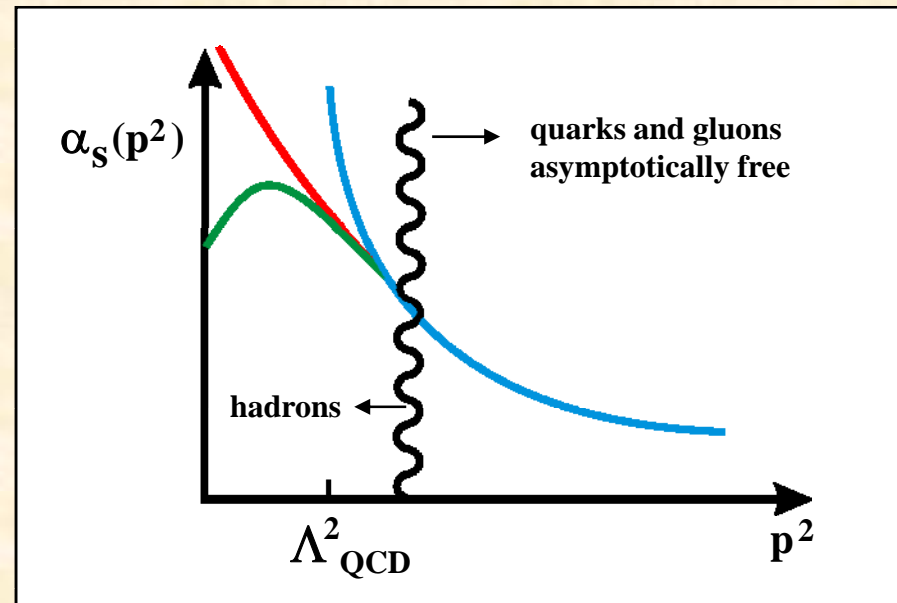
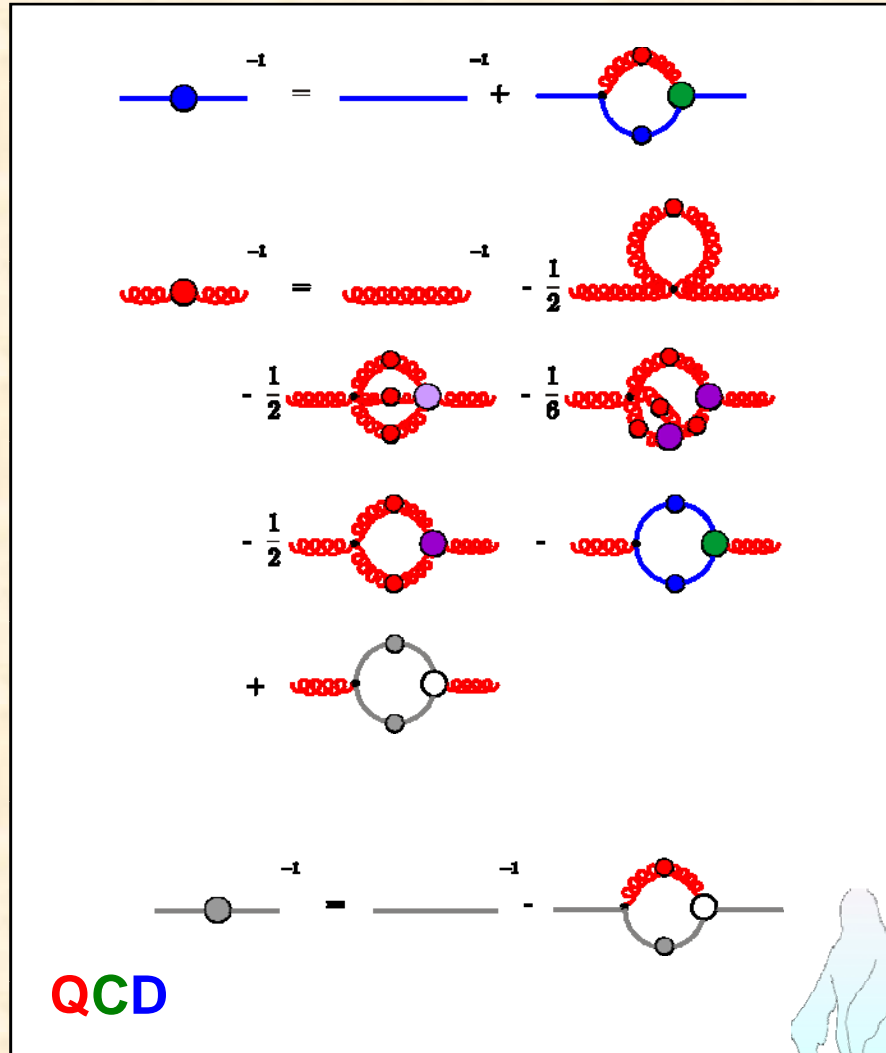


Ward-Green-Takahashi:

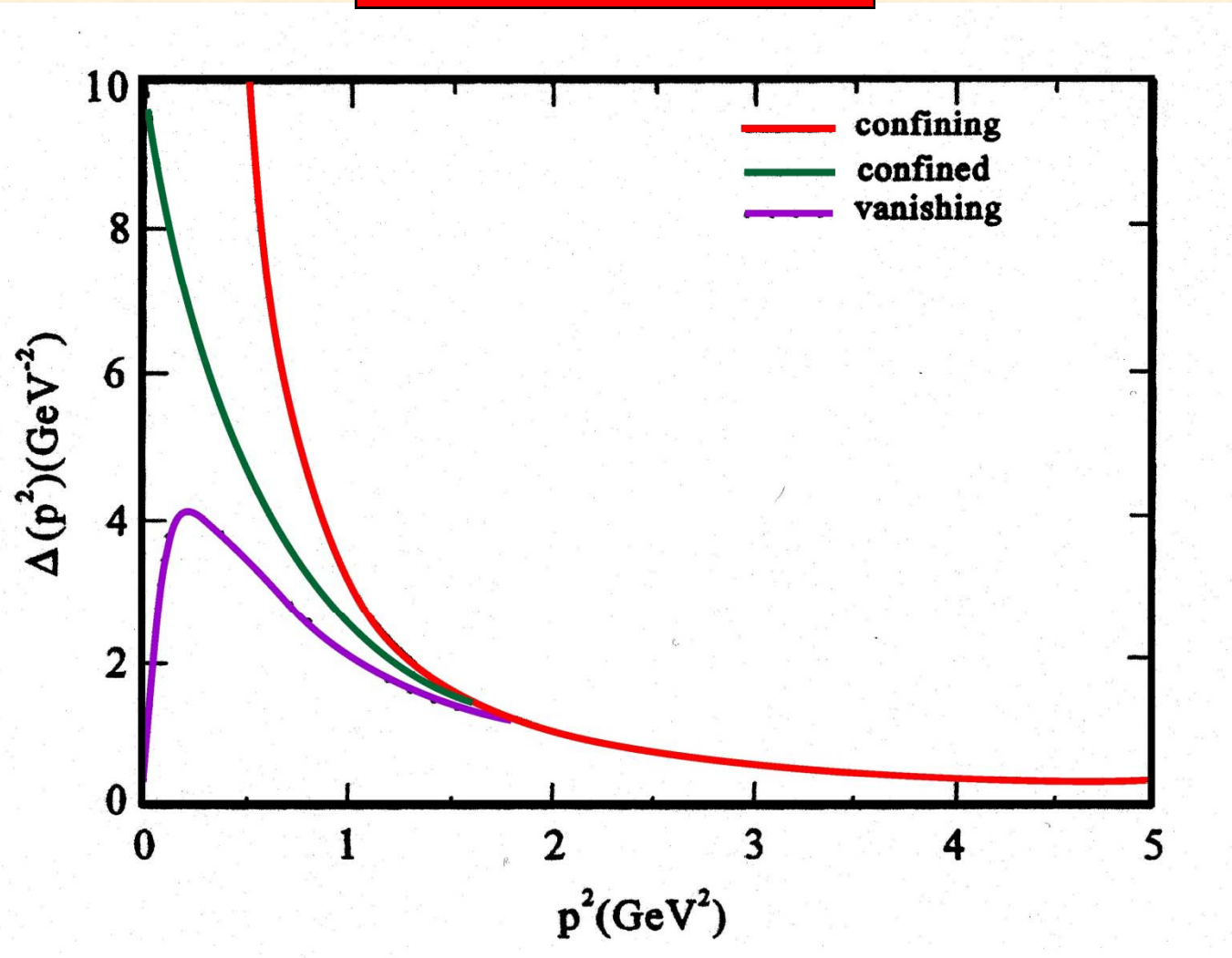


Gauge Invariance & Multiplicative Renormalizability

Schwinger-Dyson Equations

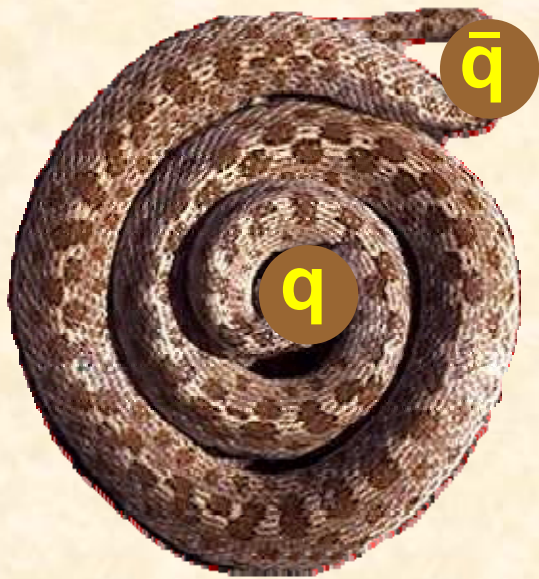


gluon propagator

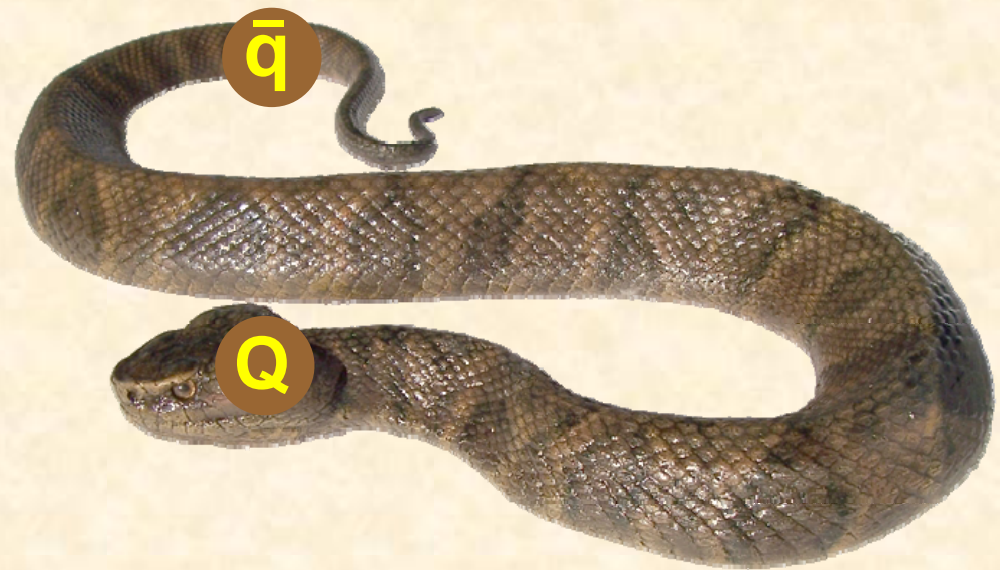
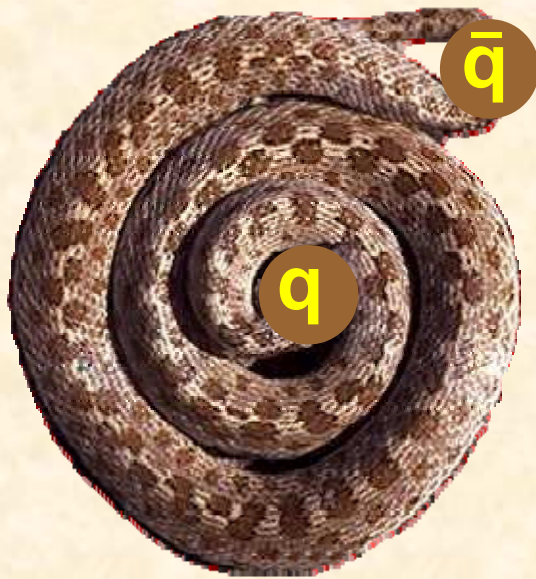


$$\Delta(\mathbf{p}) \equiv \Delta_{00}(\mathbf{p})$$

Hadrons & quark confinement



Hadrons & quark confinement



Wilson area law

interquark potential

$$V(r) \simeq \int d^3p \exp(ip \cdot r) \Delta_{00}(p)$$

$$V(r) \sim r^a \iff \Delta(p) \sim p^{-a-3}$$

$rp \sim 1$

Coulomb : OBE

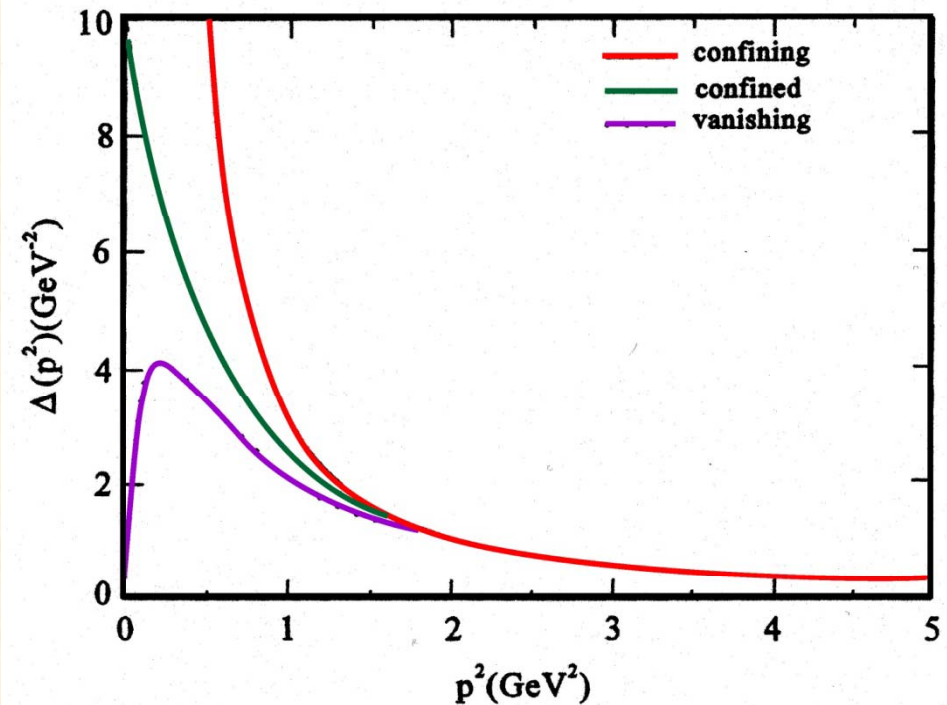
$$V(r) \sim \frac{1}{r} \iff \Delta(p) \sim \frac{1}{p^2}$$

$r \ll 1, p \gg 1$

$$V(r) \sim r \iff \Delta(p) \sim \frac{1}{p^4}$$

$r \gg 1, p \ll 1$

gluon propagator



interquark potential

$$V(r) \simeq \int d^3p \exp(ip \cdot r) \Delta_{00}(p)$$

$$V(r) \sim r^a \iff \Delta(p) \sim p^{-a-3}$$

$rp \sim 1$

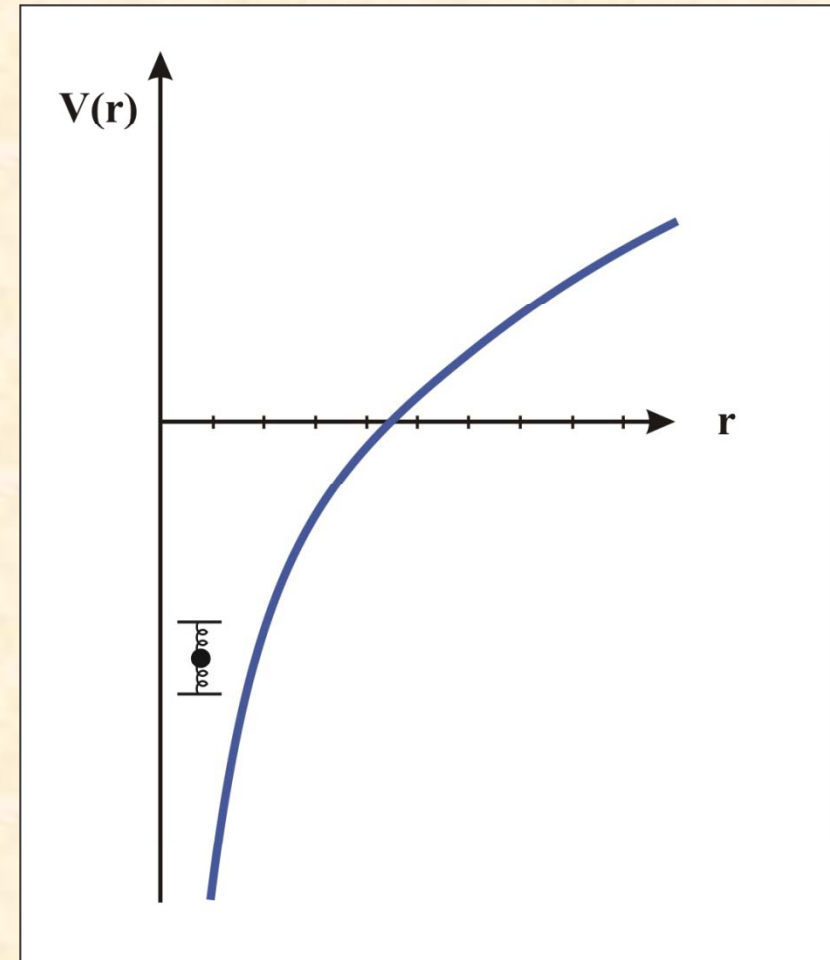
Coulomb : OBE

$$V(r) \sim \frac{1}{r} \iff \Delta(p) \sim \frac{1}{p^2}$$

$r \ll 1, p \gg 1$

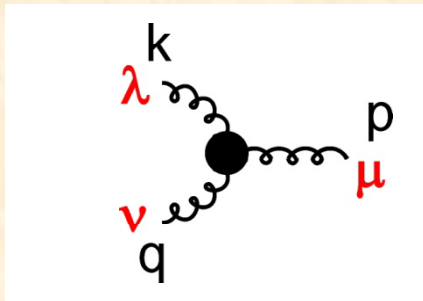
$$V(r) \sim r \iff \Delta(p) \sim \frac{1}{p^4}$$

$r \gg 1, p \ll 1$



Schwinger-Dyson Equations

Slavnov-Taylor Identity

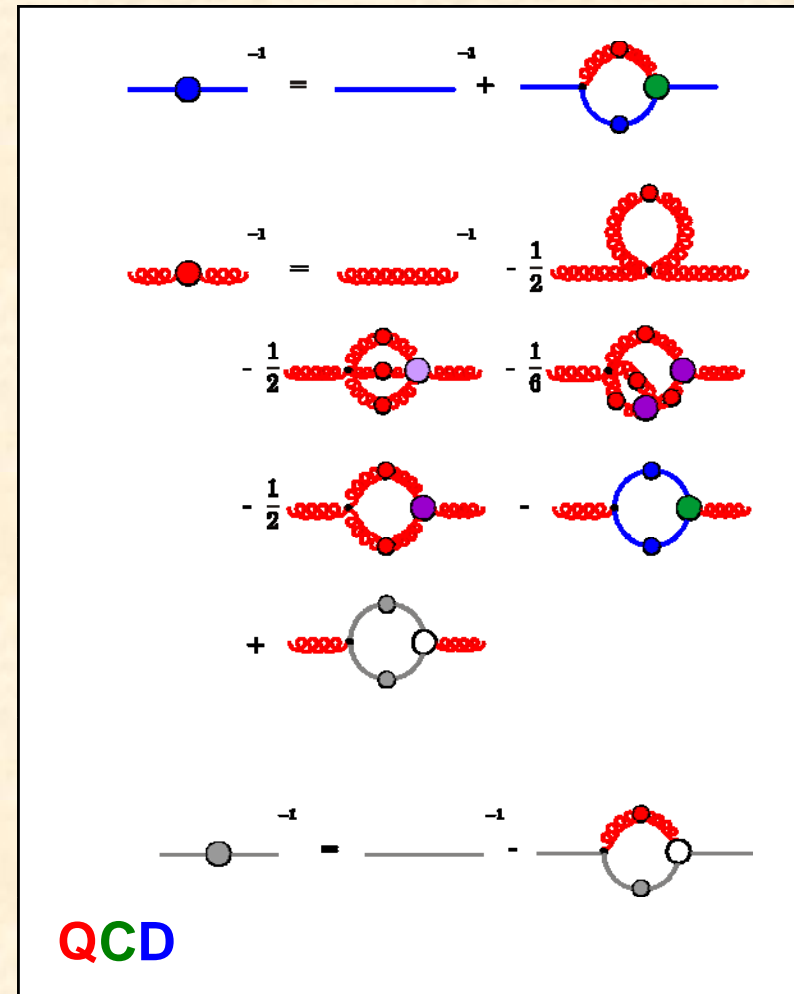


axial gauges

$$k_\lambda \Gamma^{\lambda\mu\nu}(k, p, q) = \Pi^{\mu\nu}(p) - \Pi^{\mu\nu}(q)$$

covariant gauges

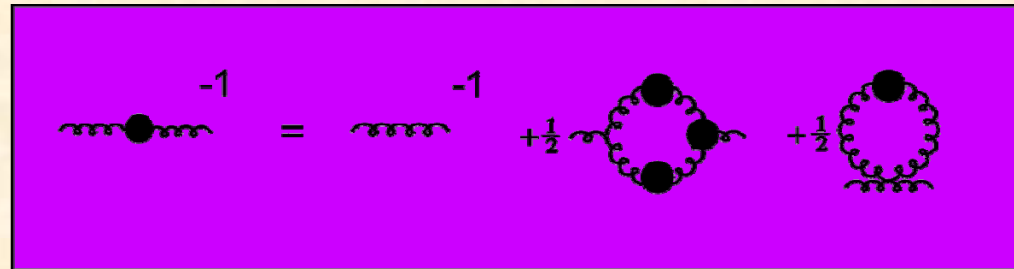
$$k_\lambda \Gamma^{\lambda\mu\nu}(k, p, q) = H(k^2) \left[G_{\mu,\sigma}(q, -k) \Pi_{\sigma,\nu}^T(p) - G_{\nu\sigma}(p, -k) \Pi_{\sigma\mu}^T(q) \right]$$



Studies in covariant gauges

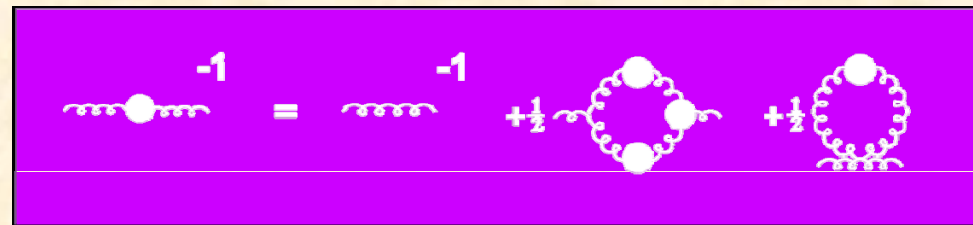
first just gluons

Pagels, Mandelstam, Bar-Gadga



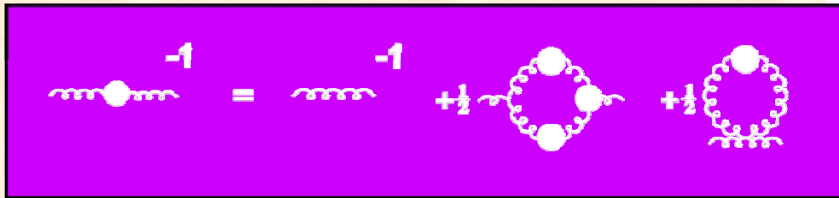
$$\Delta^{\mu\nu}(q) = \frac{\mathcal{G}(q)}{q^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + \xi \frac{q^\mu q^\nu}{q^4}$$

STI $\Rightarrow \Gamma \sim \frac{1}{\mathcal{G}}$

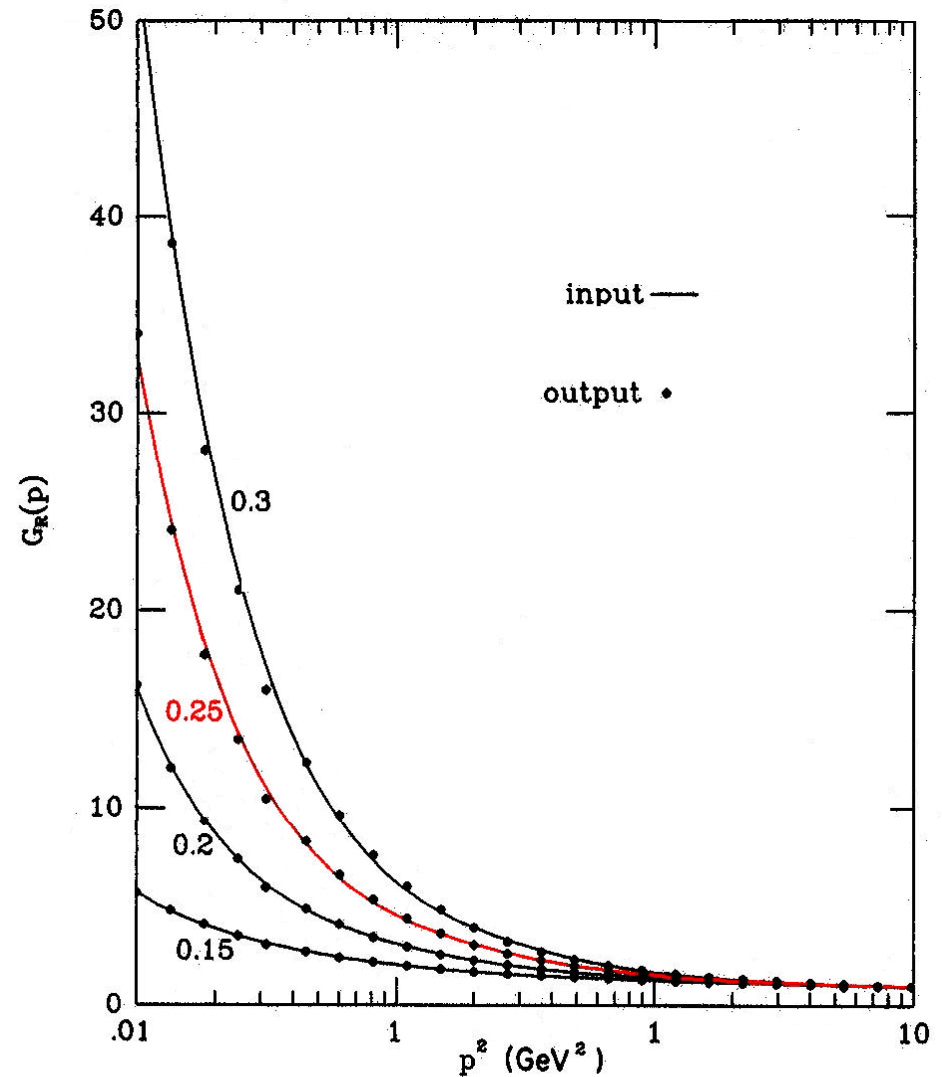


Studies in Landau gauge

$$\Delta^{\mu\nu}(q) = \frac{\mathcal{G}(q)}{q^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right)$$



Brown & P
1988

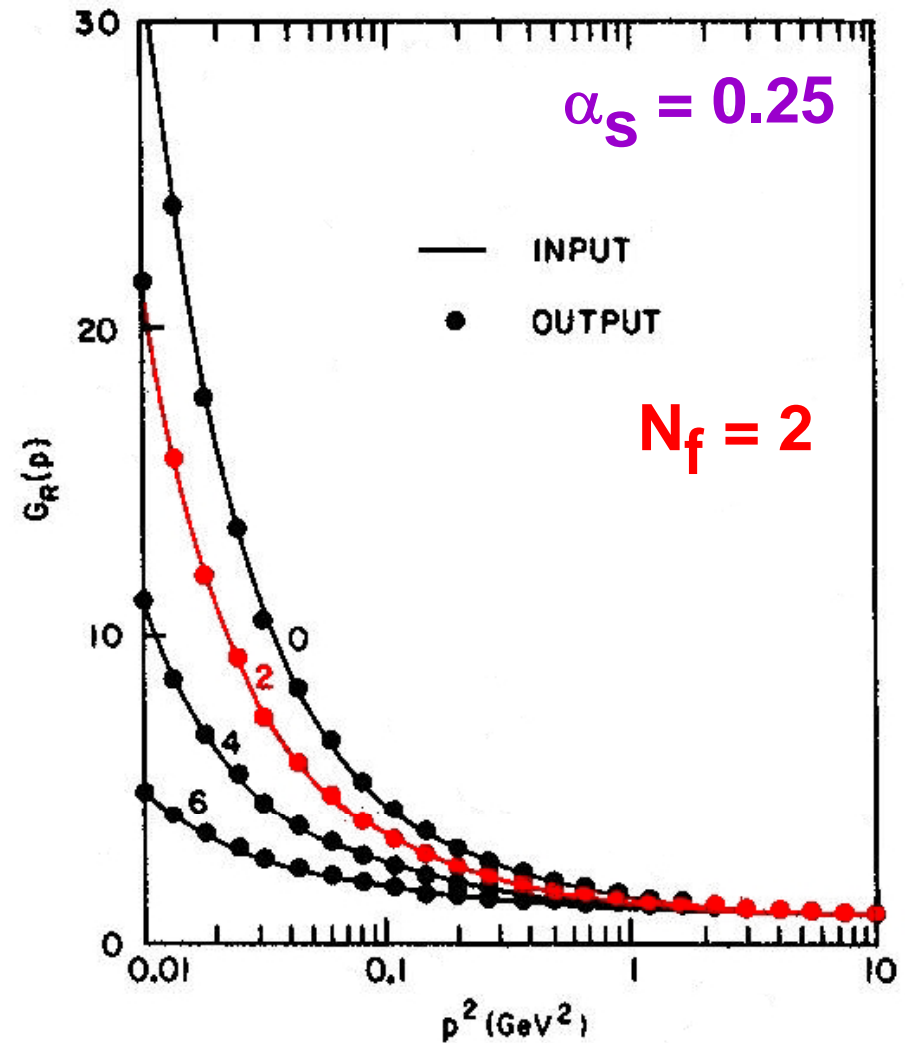


Studies in Landau gauge

$$\Delta^{\mu\nu}(q) = \frac{\mathcal{G}(q)}{q^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right)$$

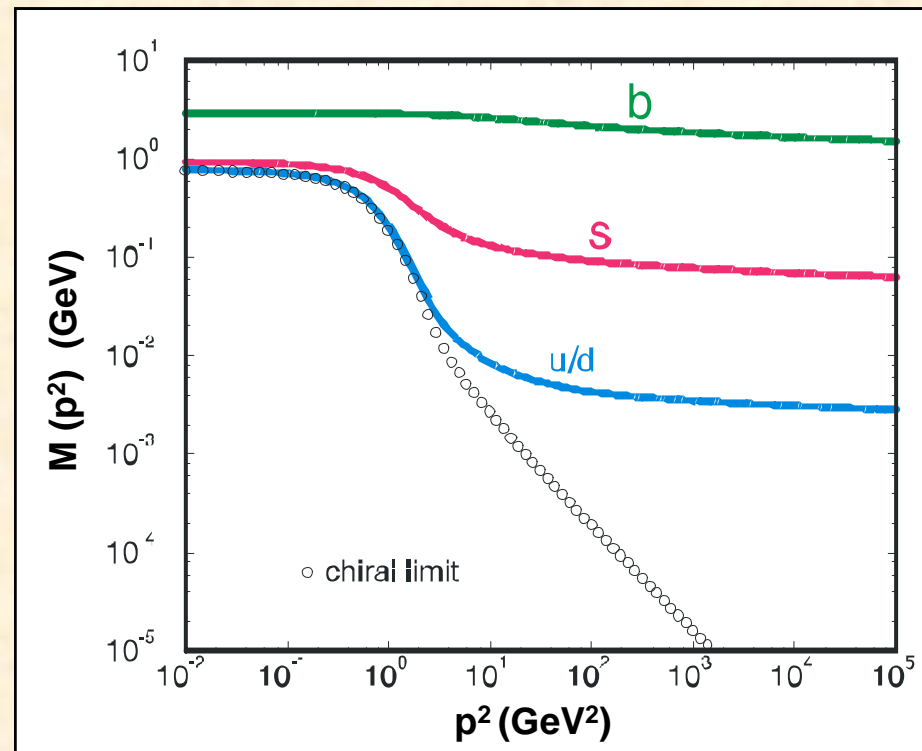
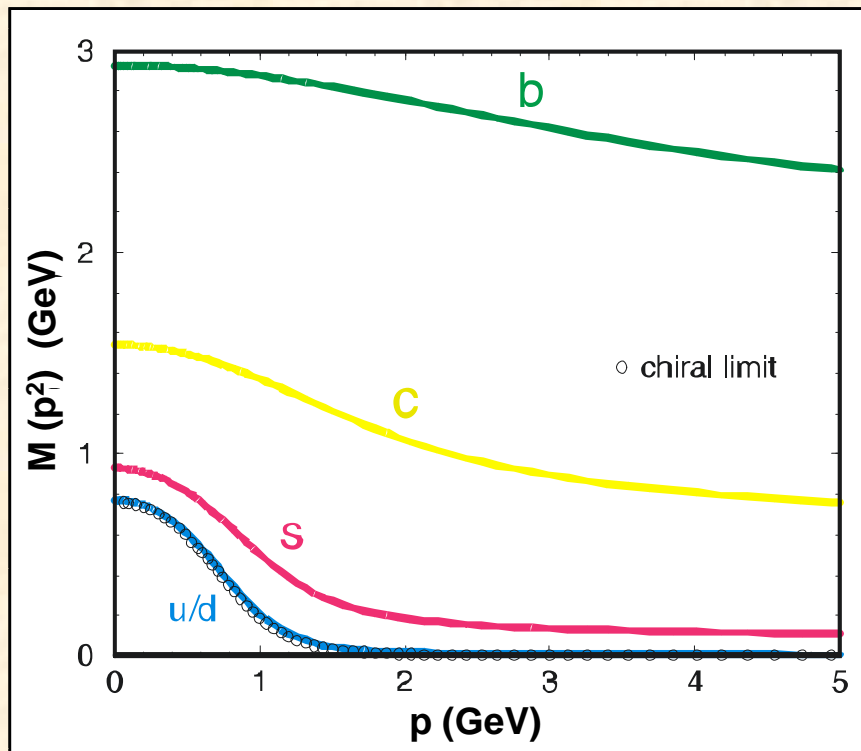
$$\begin{array}{c}
 \text{---} \circ \text{---}^{-1} = \text{---} \circ \text{---}^{-1} + \frac{1}{2} \text{---} \circ \text{---} + \frac{1}{2} \text{---} \circ \text{---} \\
 + N_f \text{---} \circ \text{---}
 \end{array}$$

Brown & P
1988



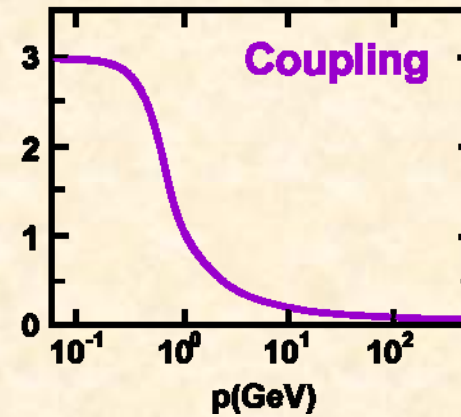
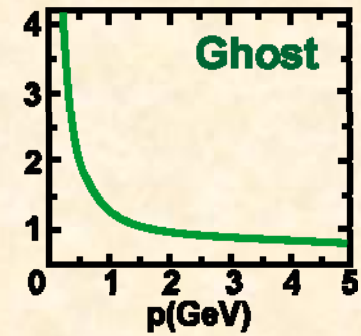
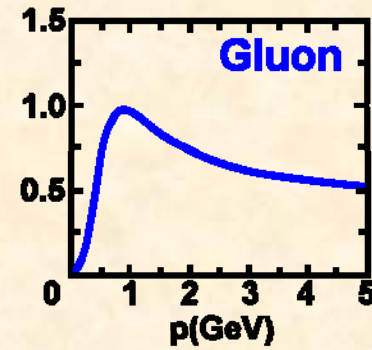
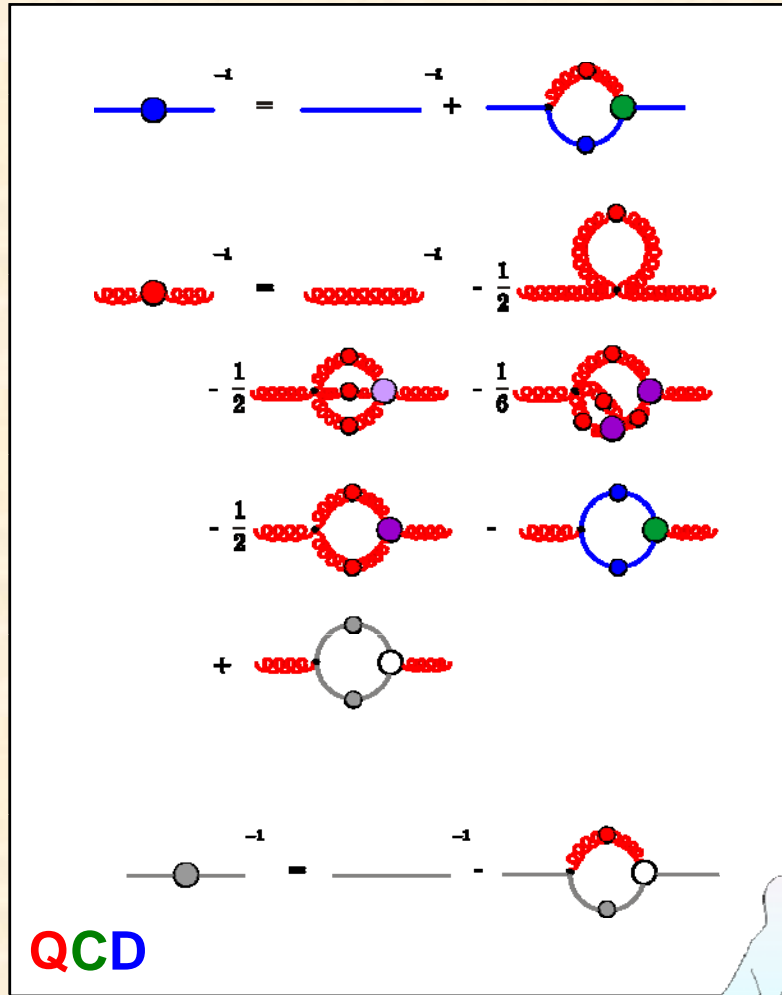
QCD running coupling

$\alpha(Q^2) > 1$ for $Q^2 < Q_0^2 \approx \Lambda^2$ \Rightarrow χ SB



Maris & Roberts

Studies in covariant gauges



Alkofer et al



Lattice QCD



model QCD

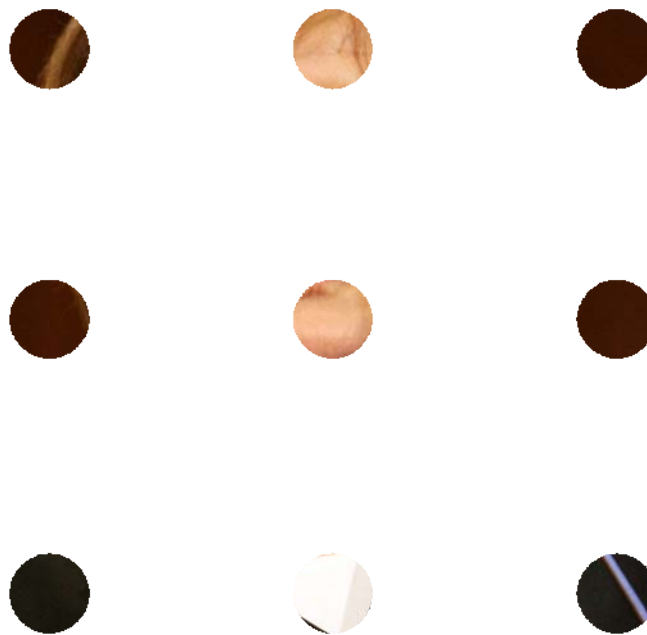
$$a \longrightarrow 0$$

$$V \longrightarrow \infty$$

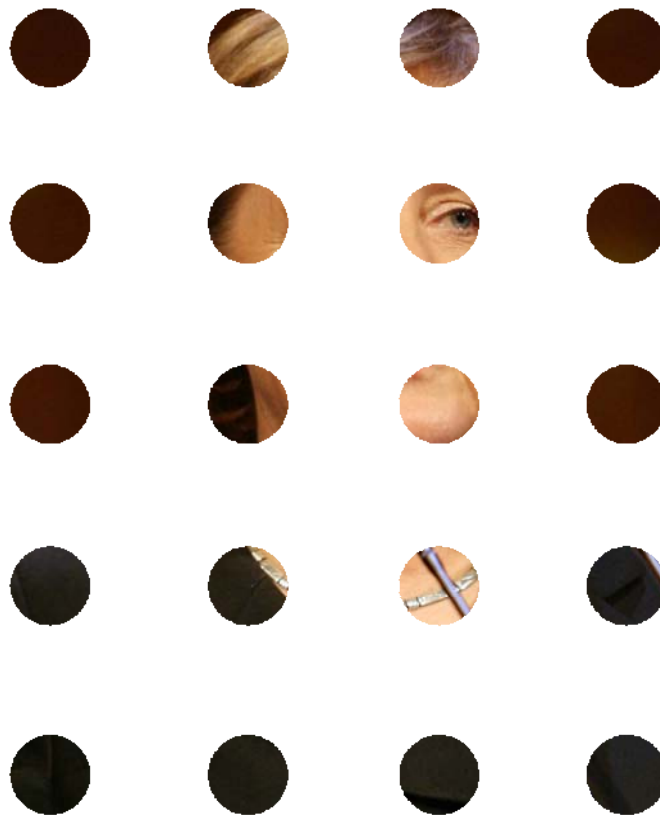
$$m \longrightarrow m_q$$



Lattice QCD



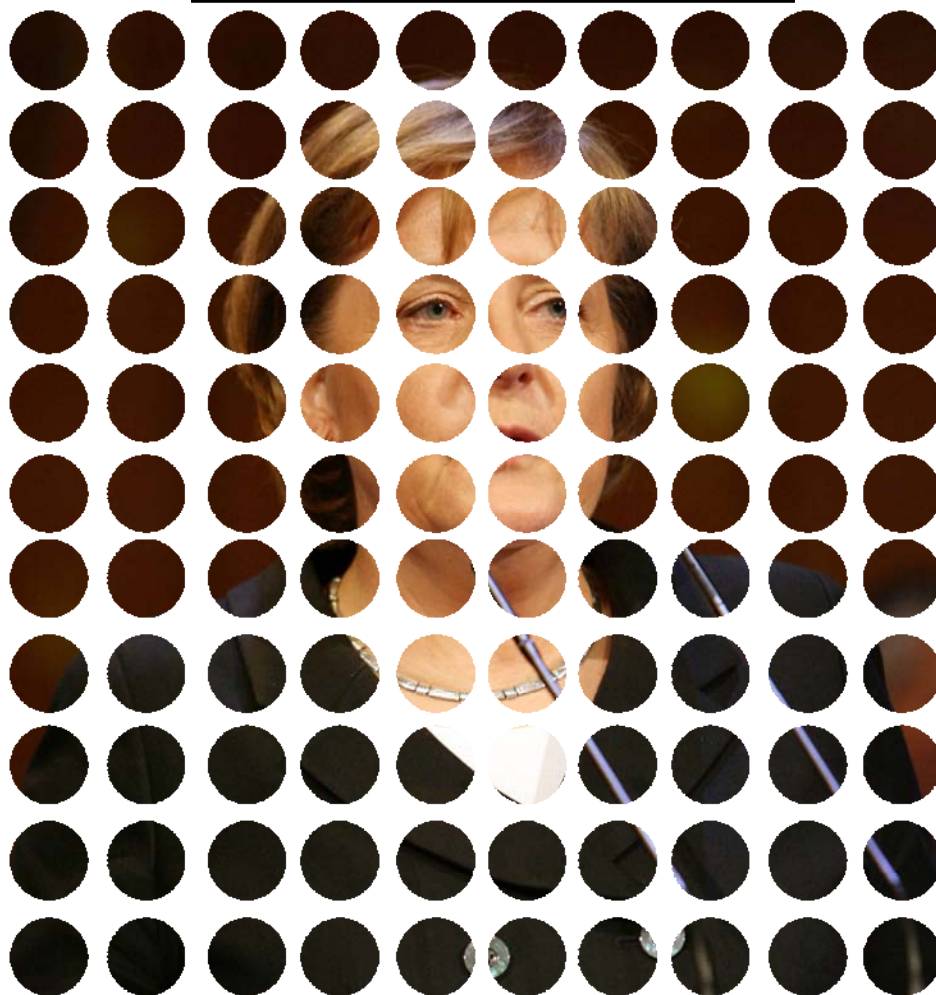
Lattice QCD



Lattice QCD



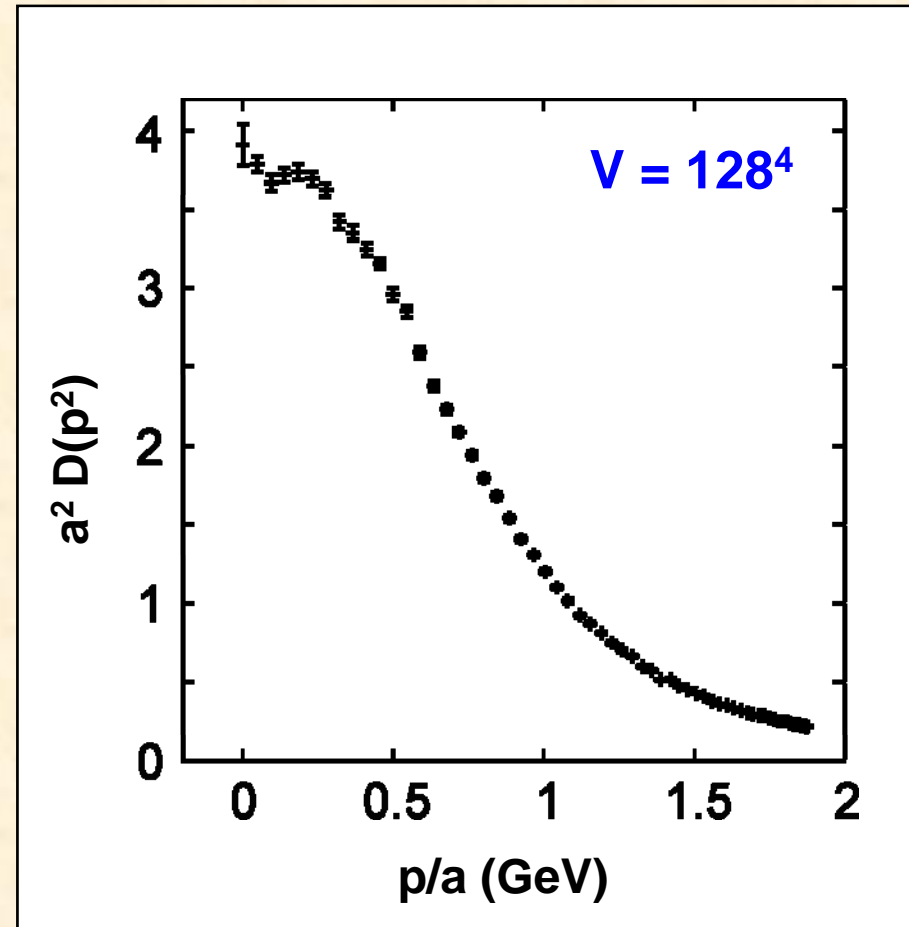
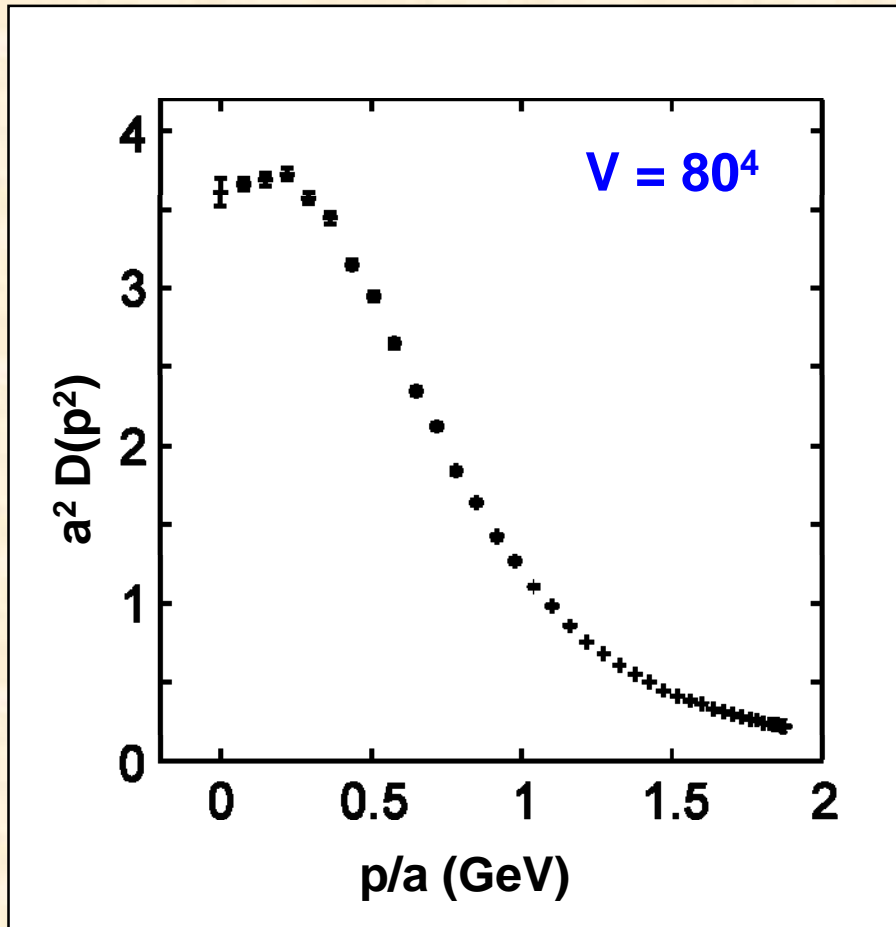
$a \rightarrow 0$



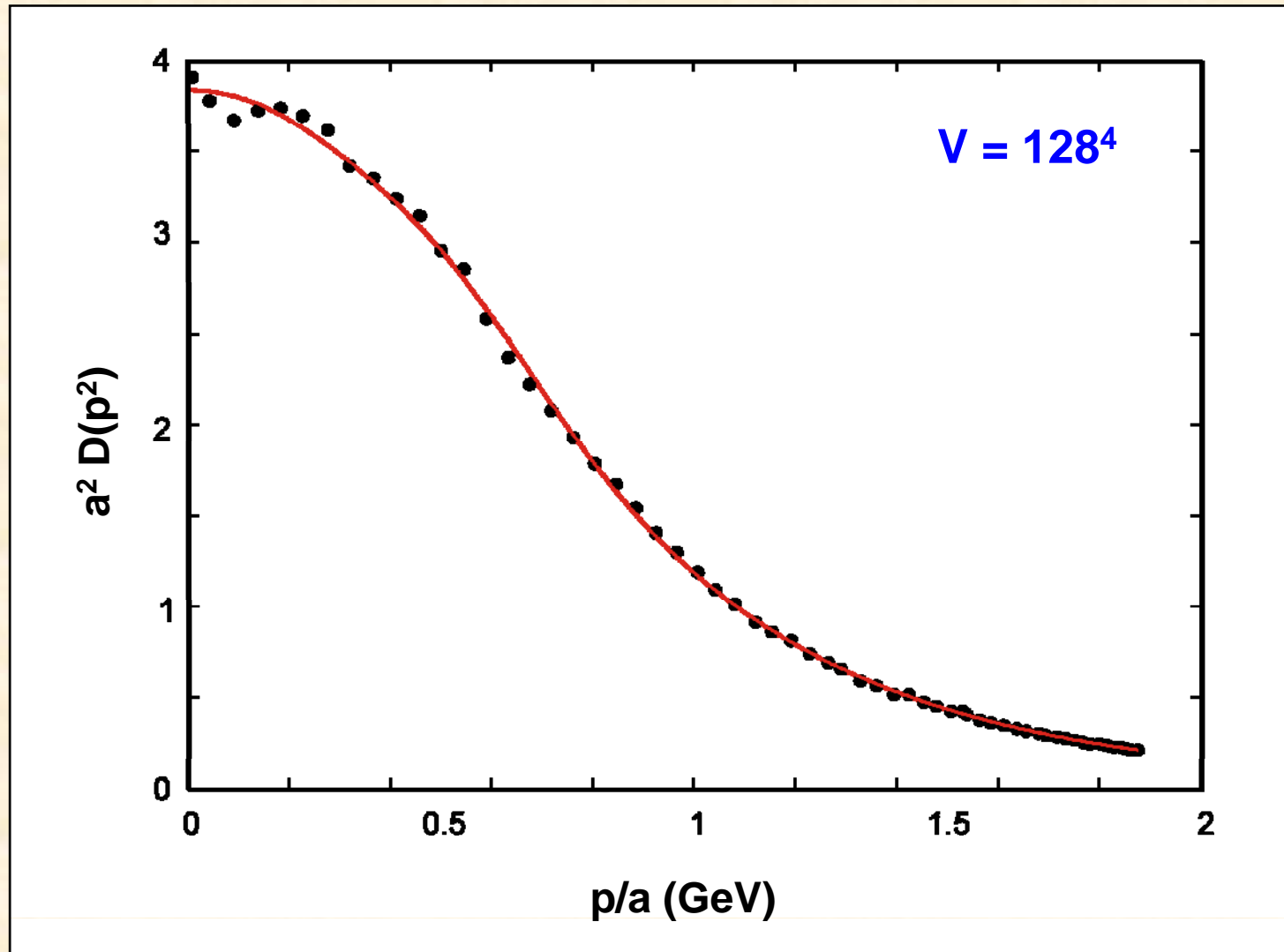
$V \rightarrow \infty$



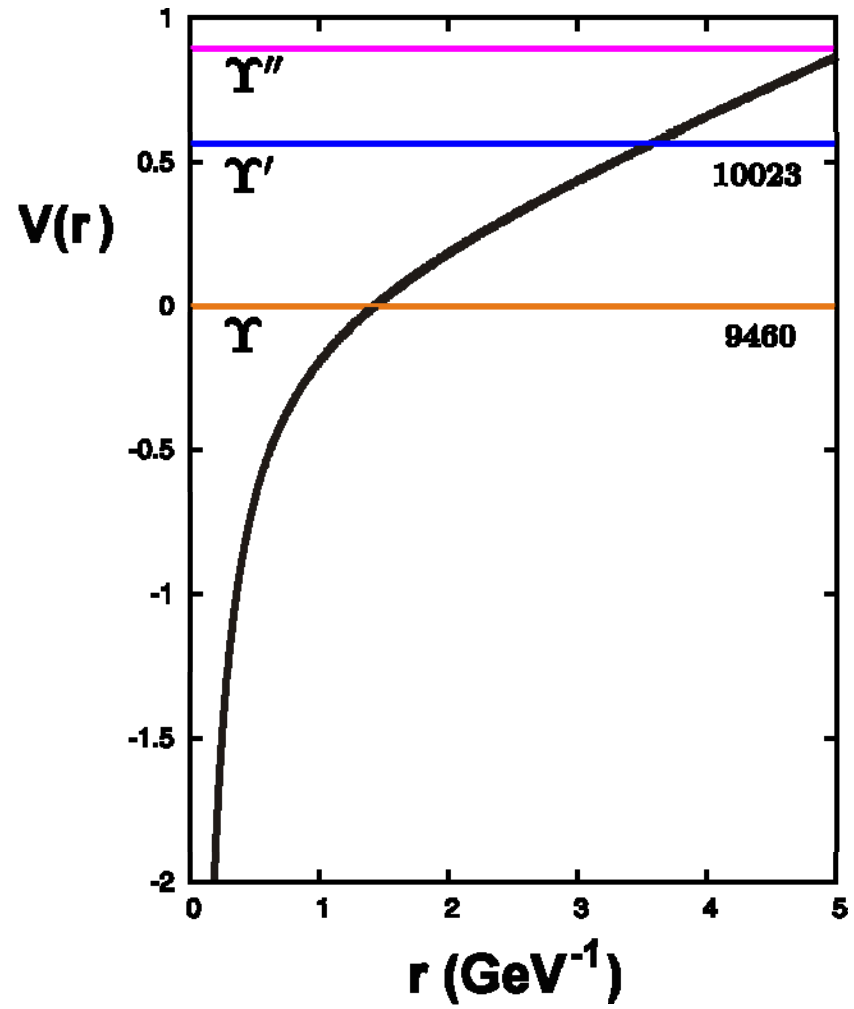
Lattice Results: Cucchieri, Mendes



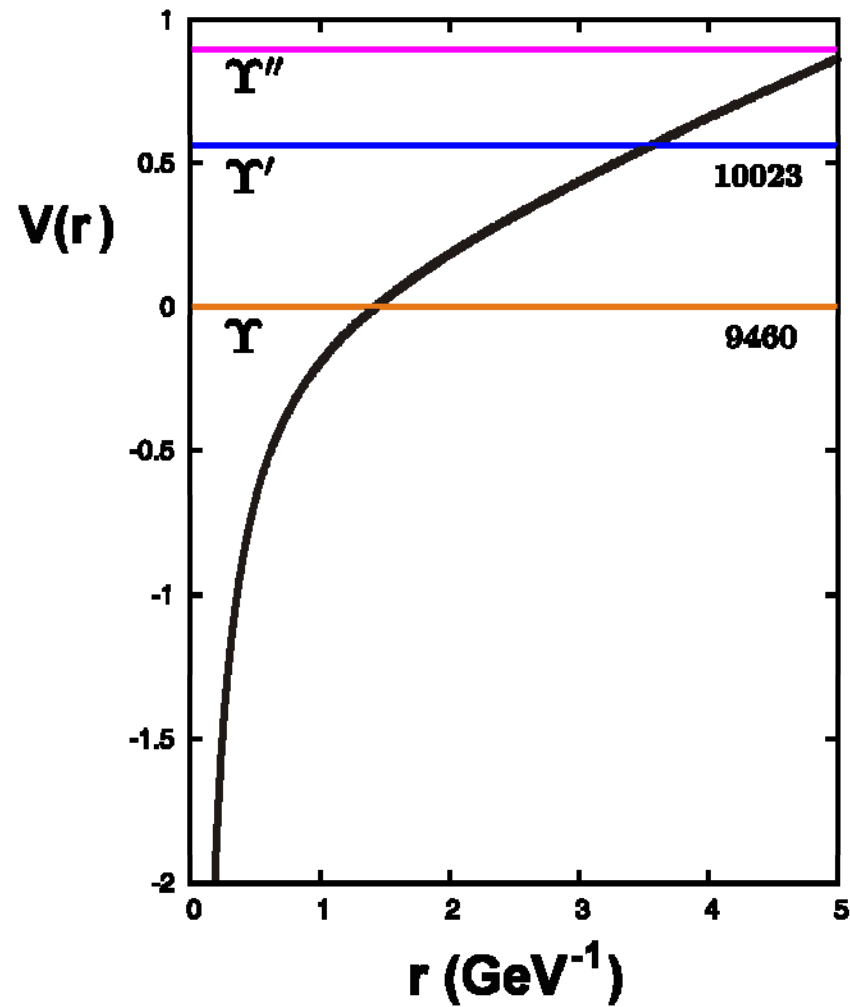
Lattice Results: Cucchieri, Mendes



Richardson Potential

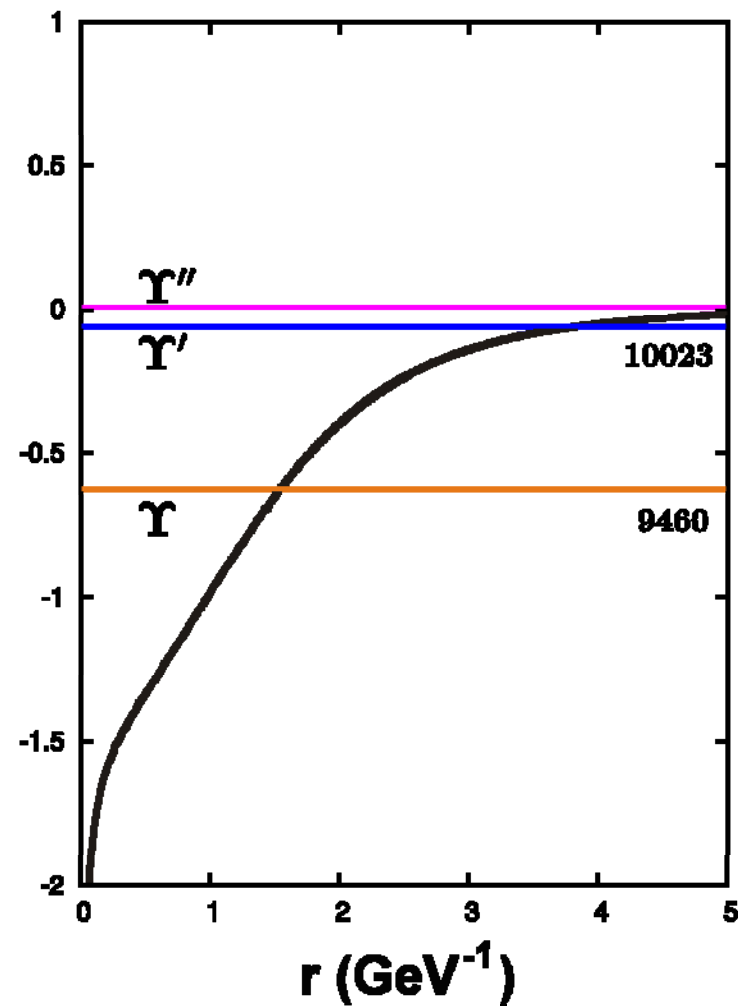


Richardson Potential



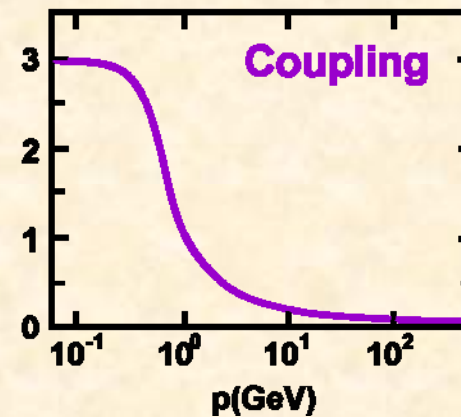
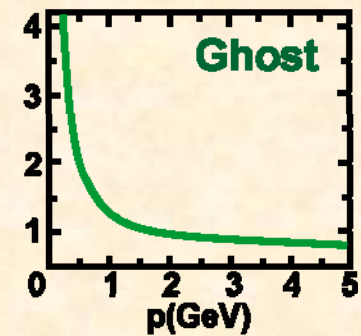
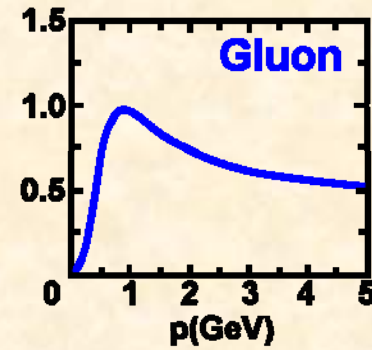
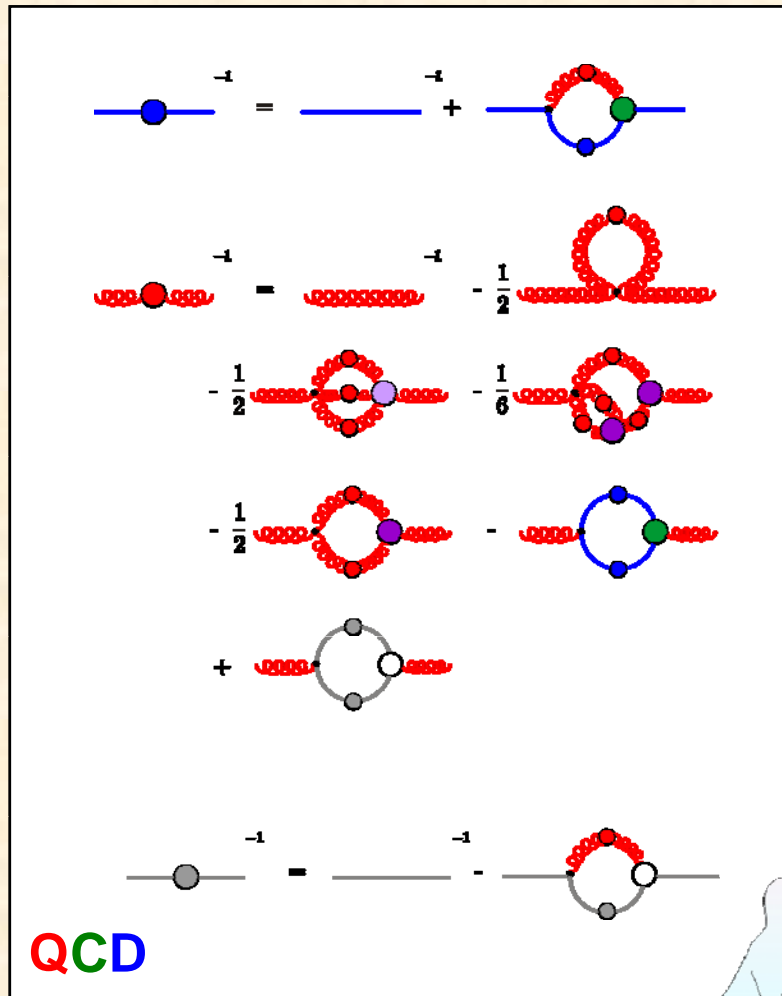
$$2m_b = 10.08 \text{ GeV}$$

Potential : Cucchieri-Mendes gluon

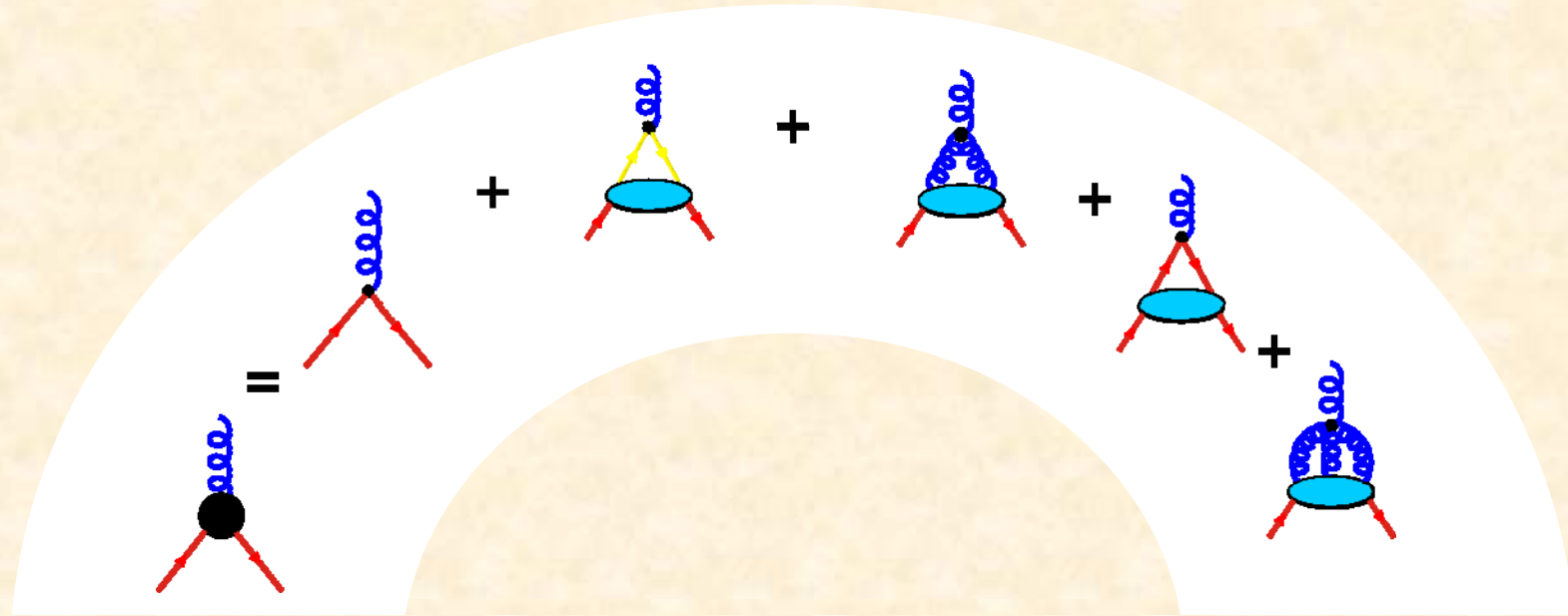


$$2m_b = 9.46 \text{ GeV}$$

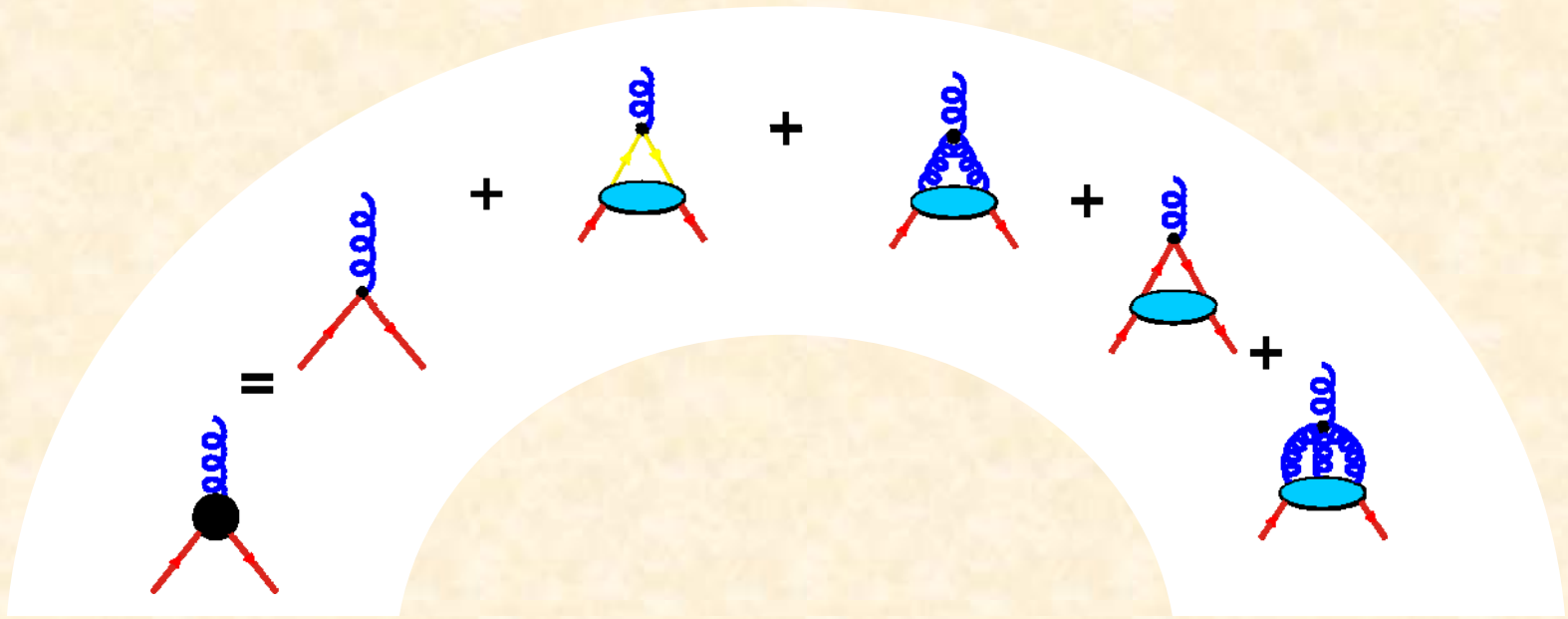
Confinement potential



Alkofer et al

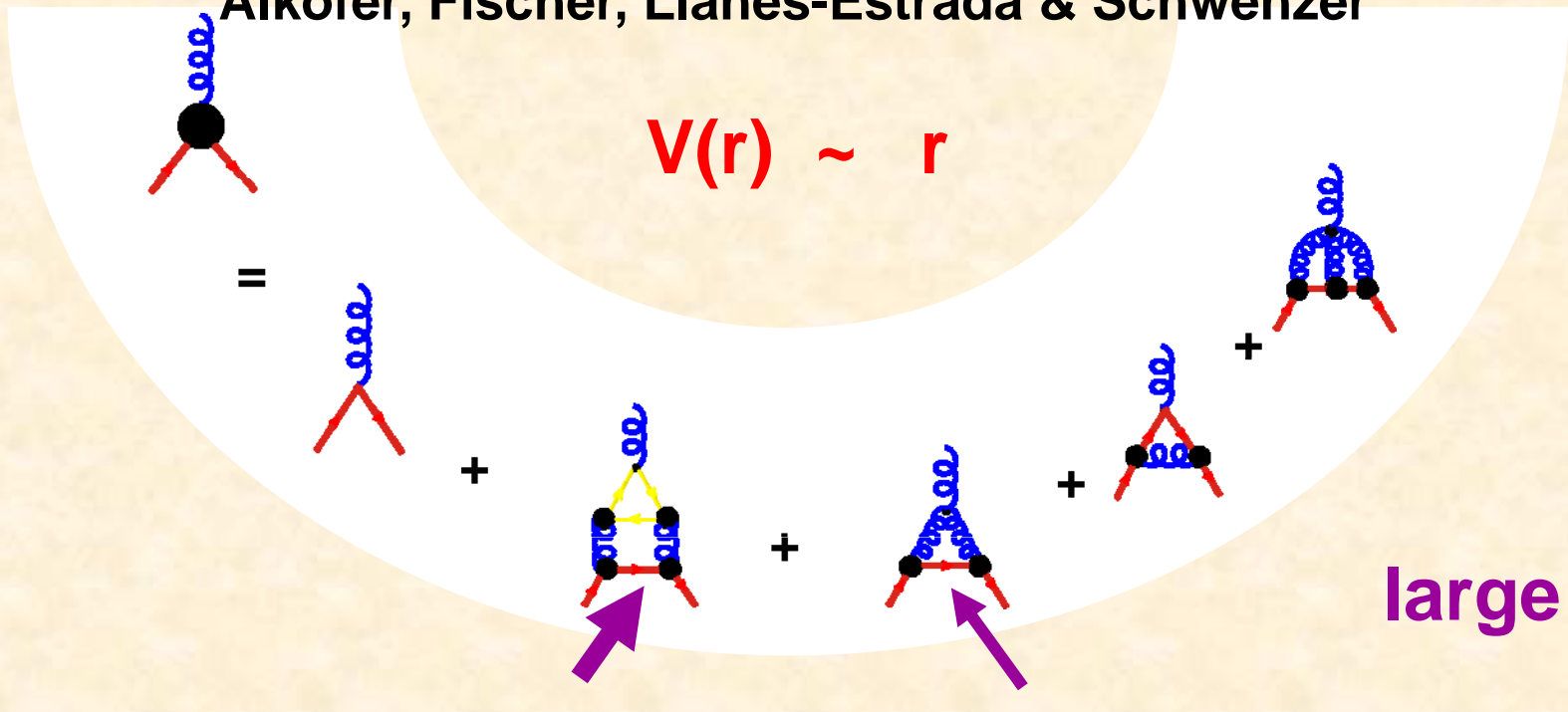


Alkofer, Fischer, Llanes-Estrada & Schwenzer

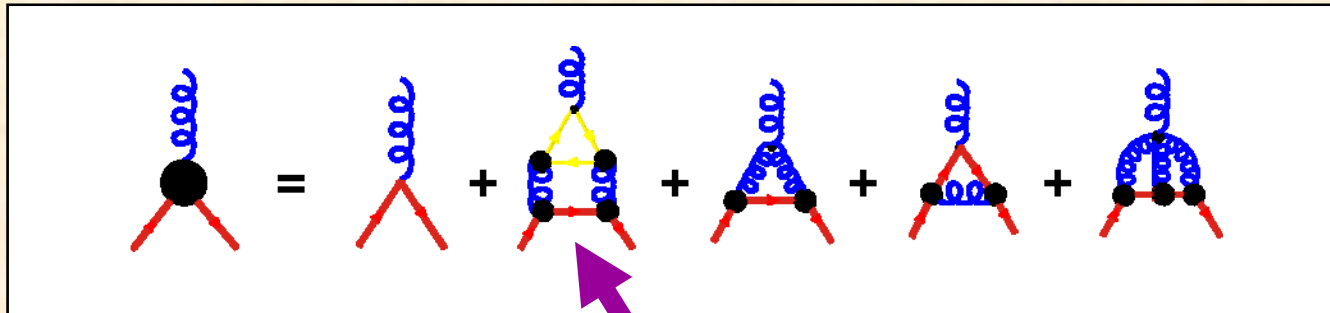


Alkofer, Fischer, Llanes-Estrada & Schwenzer

$$V(r) \sim r$$

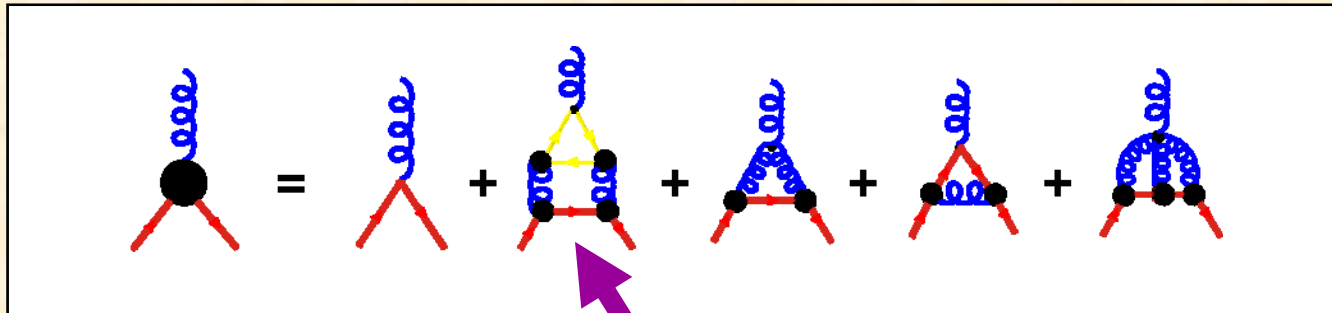


large M



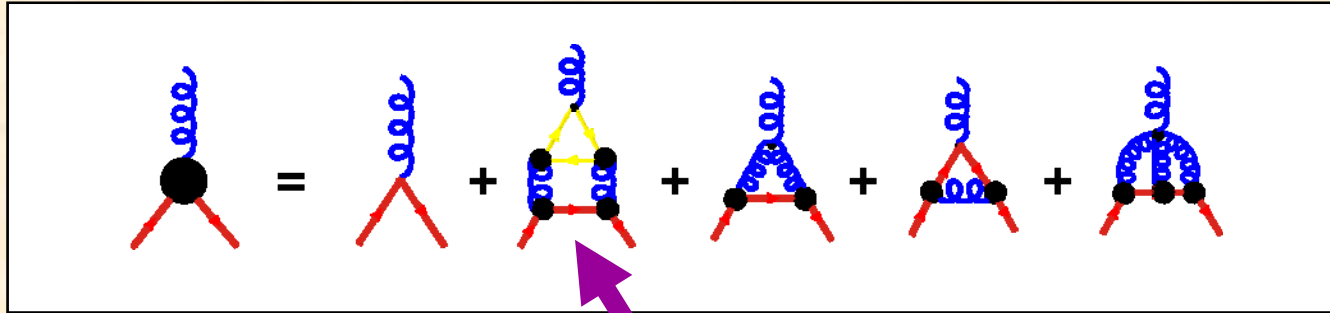
scalar component in vertex

$$\Gamma^\mu(p, p', q) \sim \int d^4k \dots (M(p-k) - M(p)) \frac{\dots}{k^2 + M(k)^2} (M(p'-k) - M(p'))$$



$$\Gamma^\mu(p, p', q) \sim \int d^4k \dots (M(p-k) - M(p)) \frac{\dots}{k^2 + M(k)^2} (M(p'-k) - M(p'))$$

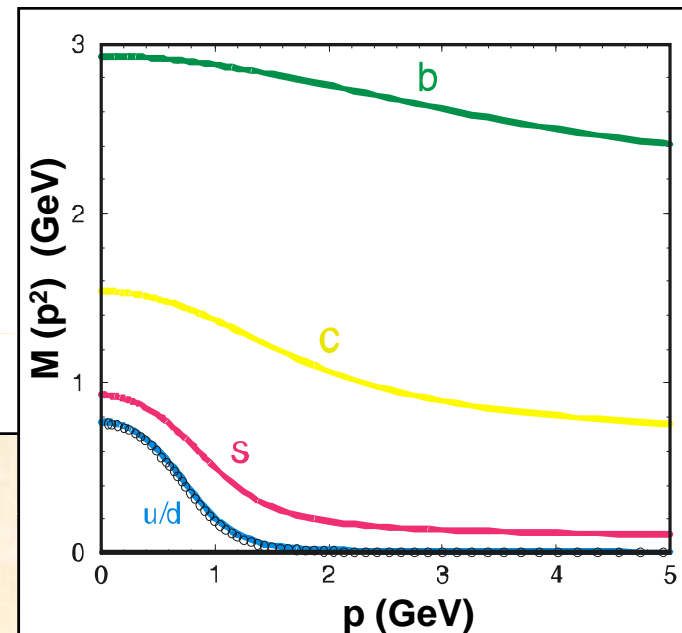
$$\sim \frac{1}{p^2} (p + p')^\mu \quad p \rightarrow 0$$



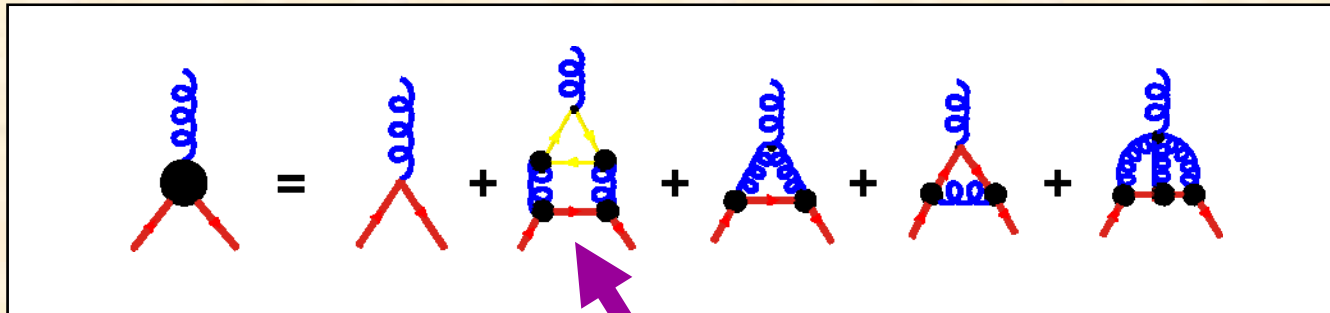
$$\Gamma^\mu(p, p', q) \sim \int d^4k \dots (M(p-k) - M(p)) \frac{\dots}{k^2 + M(k)^2} (M(p'-k) - M(p'))$$

$$\sim \frac{1}{p^2} (p + p')^\mu \quad p \rightarrow 0$$

$$M(k) \approx M_0 + c \frac{\Lambda_{\text{QCD}}^3}{k^2 + \Lambda_{\text{QCD}}^2}$$



Maris & Roberts



$$\Gamma^\mu(p, p', q) \sim \int d^4k \dots (M(p-k) - M(p)) \frac{\dots}{k^2 + M(k)^2} (M(p'-k) - M(p'))$$

$$\sim \frac{1}{p^2} (p + p')^\mu \quad p \rightarrow 0$$

$$M(k) \simeq M_0 + c \frac{\Lambda_{\text{QCD}}^3}{k^2 + \Lambda_{\text{QCD}}^2}$$

$$\Gamma^\mu(p, p', q) \simeq \gamma^\mu + \frac{1}{p^2} \frac{\Lambda_{\text{QCD}}^3}{p^2 + M_0^2} (p + p')^\mu$$

Quarks & Hadrons in Strong QCD

