Confinement in Polyakov gauge

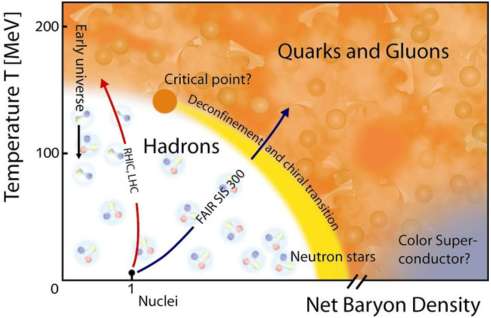
Florian Marhauser

Universität Heidelberg

Quarks and Hadrons in strong QCD

St. Goar, March 17, 2008

QCD Phase Diagram



open questions

- chiral vs. deconfinement phase transition
- finite density
- critical point
- ...

Deconfinement-χSB

Aoki,Fodor,Katz,Szabo: Phys.Lett.B643:46-54,2006

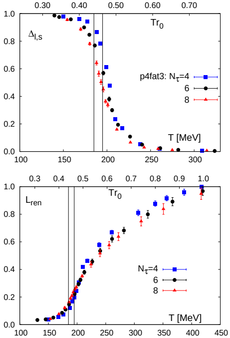
lattice results ($\mu = 0$)

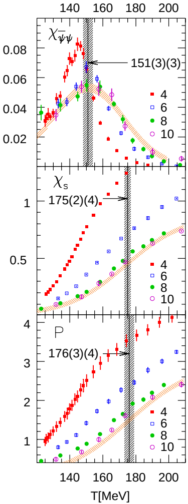
Karsch: $T_{\text{conf}} = T_{\chi\text{SB}}$

Fodor: $T_{\text{conf}} > T_{\chi\text{SB}}$

Note: T_c's do not agree

Karsch: arXiv:0711.0661



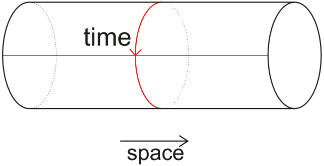


Confinement Order Parameter

$$\phi(\vec{x}) = \langle L(\vec{x}) \rangle = \frac{1}{N_c} Tr_c \left\langle \mathcal{P} \exp \left(ig \int_0^\beta d\tau \, A_0(\vec{x}, \tau) \right) \right\rangle$$

$L(\vec{x})$ puts a static quark into the theory

$$\frac{\partial \Psi}{\partial \tau} = igA_0 \Psi \Rightarrow \Psi(\vec{x}, \tau) = \mathcal{P} \exp \left(ig \int_0^\tau d\tau' \, A_0 \right) \Psi(\vec{x}, 0)$$



source of static quark (δ-fct on the worldline $x(\tau)$):

$$j^\mu = \delta^{\mu 0} \int_0^\beta d\tau \delta(x - x(\tau))$$

Polyakov Loop

free energy:

$$\phi \propto \mathrm{Tr}_c \langle \mathcal{P}\ e^{ig \int d^4x\ A_\mu(x)\ j^\mu(x)} \rangle = \exp\left(-\beta F_q\right)$$

- F_q: energy of an infinitely heavy quark
- confinement ➜ $F_q \to \infty \leftrightarrow \phi = 0$
- deconfinement ➜ $F_q < \infty \leftrightarrow \phi \neq 0$

perturbation theory ➜ expansion around 0 ➜ $F_q < \infty$

phenomenology

$N_c = 2$: YMT in same universality class as Ising model
➜ 2^{nd} order phase transition Z_2
$N_c = 3$ ➜ 1^{st} order phase transition Z_3

Computing $\phi = \langle L(\vec{x}) \rangle$

our approach

F.M., J. Pawlowski, work under completion

- compute $L[\langle A_0 \rangle]$ in Polyakov gauge via $V_{\text{eff}}(\langle A_0 \rangle)$
- Jensen inequality: $L[\langle A_0 \rangle] \geq \langle L[A_0] \rangle$

Polyakov gauge $(SU(2))$

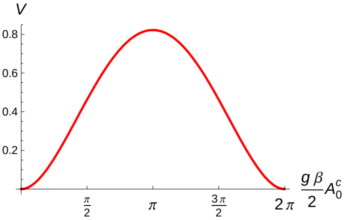
$A_0(x) = A_0^c(\vec{x})\sigma^c$ (rotate A_0 into cartan subalgebra)

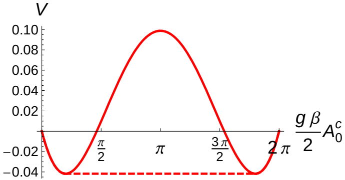
$$L(\vec{x}) = \frac{1}{N_c} Tr_c \left[e^{ig\beta A_0^c(\vec{x})\sigma^c} \right] = \cos \left[\frac{g}{2} \beta A_0^c(\vec{x}) \right]$$

Perturbative Treatment

1-loop perturbation theory

Weiss Potential

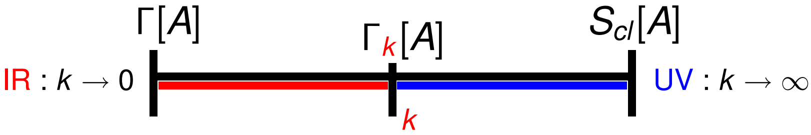




- deconfining $\langle \frac{g\beta}{2} A_0^c \rangle = \{0, 2\pi\} \longrightarrow L = 0$
- periodic (period $= 2\pi$)

Non-Perturbative Treatment

FRG Method



- includes quantum fluctuations by lowering k
- flow equation for effective action Wetterich '93

$$k\partial_k \Gamma_k = \partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k}\partial_t R_k\right]$$

Parameterisation

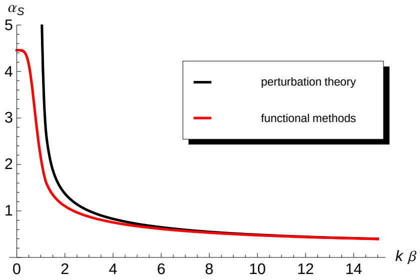
idea: focus on A_0^c since in Polyakov gauge $L(\vec{x}) = \cos\left[\frac{g}{2}\beta A_0^c(\vec{x})\right]$

$$\Gamma_k = \int d\tau \int d^3x \left\{\frac{1}{2} A_0^c \partial^2 A_0^c + V_k(A_0^c)\right\} + \Gamma_\psi$$

- $V_k(A_0^c)$ effective potential of the A_0^c gauge field
- spatial gluons integrated out

Running Coupling

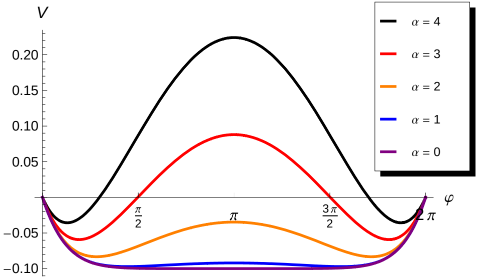
- strong coupling mainly perturbation theory
- ambiguity in non-perturbative definition
- systematic error



Flow

$$\color{red}{\langle A_0^c \rangle}\ \text{from}\ 0 = \left.\frac{\partial V(A_0^c)}{\partial A_0^c}\right|_{A_0^c = \color{red}{\langle A_0^c \rangle}}$$

Effective Potential $V(A_0^c)$

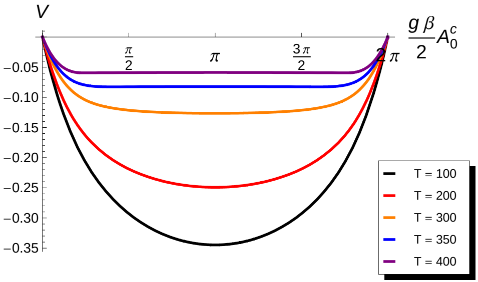


scale $\alpha = k\beta$; $\varphi = g\beta A_0^c$

Effective Potential

remember: $L[\langle A_0 \rangle] = \cos[\frac{g}{2}\beta\langle A_0 \rangle]$

phase transition ($T_c = 325^{+10}_{-30}$ MeV) numerical error < 1 MeV

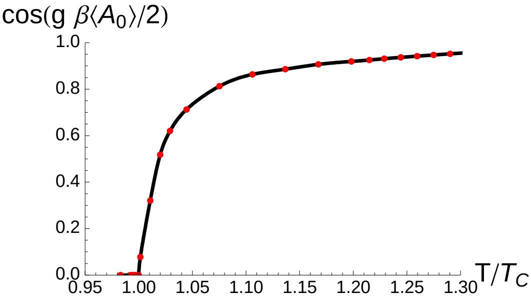


$\frac{T_c}{\sqrt{\sigma}} = 0.7386^{+0.0227}_{-0.0682}$ lattice QCD: $\frac{T_c}{\sqrt{\sigma}} = 0.7091$

F.M., J. Pawlowski, in preparation

Phase Diagram

$T_c = 325^{+10}_{-30}$ MeV

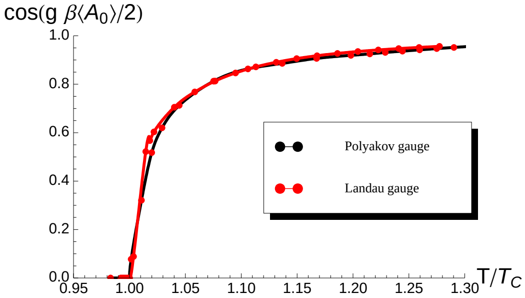


Formulation in $\langle L[A_0]\rangle$ should flatten curve

Phase Diagram

Polyakov gauge vs. Landau gauge

Landau gauge: Braun, Gies, Pawlowski, arXiv:0708.2413



Polyakov loop gauge invariant ➜ results should agree

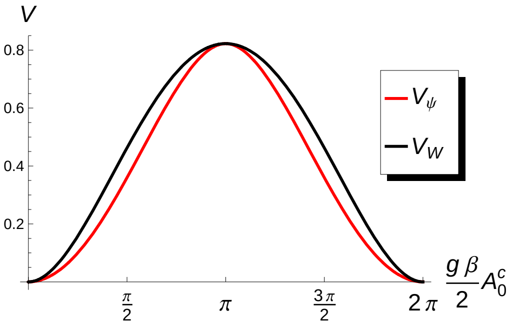
Dynamical Quarks

F.M., J.Pawlowski, L. Speyer, work in progress

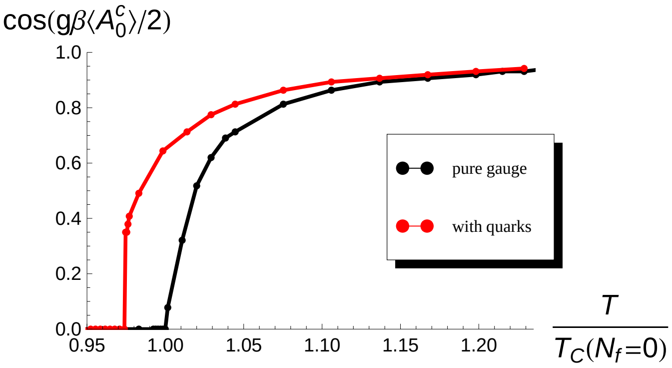
perturbation theory ($m = 0$):

$$V_\psi(g\beta A_0^c/2) = -V_{\text{Weiss}}(g\beta A_0^c/2 + \pi)$$

potentials



Dynamical Quarks



phase transition with quarks not crossover, truncation not sufficient

summary

- calculation of ϕ for $SU(2)$ in Polyakov gauge
- 2nd order phase transition
- $T_c = 325^{+10}_{-30}$ MeV
- dynamical quarks

outlook

- $SU(3)$
- improved treatment of spatial gluons
- momentum dependent ϕ ➜ string tension
- non-perturbative quarks
- QCD phase diagram (i.e. finite density)

appendix

spatial gluons ...

... are treated perturbatively ➜ $\Gamma_{\mathrm{YM},\mathrm{A}_i} = S_{\mathrm{YM}}$

... can be integrated out

effective potential V_k generated by the flow

$$\partial_t V_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{S^{(2)}_{\mathrm{YM}} + R_k}\partial_t R_k\right] = \frac{1}{2}\partial_t \mathrm{Tr}\,\mathrm{Ln}\left[S^{(2)}_{\mathrm{YM}} + R_k\right]$$

S_{YM} is the Yang-Mills action, $\alpha = k\beta$

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- $V_k(A_0)$ is treated as external input (flow of spatial gluons)
- $V_{k=0} = V_{\mathrm{Weiss}}$
- V_k is periodic in $\varphi = g\beta A_0$, period 2π

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