Confinement and Parton-Nucleon Scattering Amplitude

Felipe J. Llanes-Estrada[1], Adam P. Szczepaniak[2], Tim Londergan[2] & Stanley J. Brodsky[3]

fllanes@fis.ucm.es

[1]U. Complutense Madrid, [2]Indiana U. & [3]Stanford Linear Accelerator Center

Positivity for the propagator

$\langle A|A\rangle \geq 0$ in a Hilbert space;

$$\langle 0|\left[\phi(x),\phi(y)\right]|0\rangle = i\int_0^\infty dm^2\rho(m^2)\Delta(x-y,m)$$

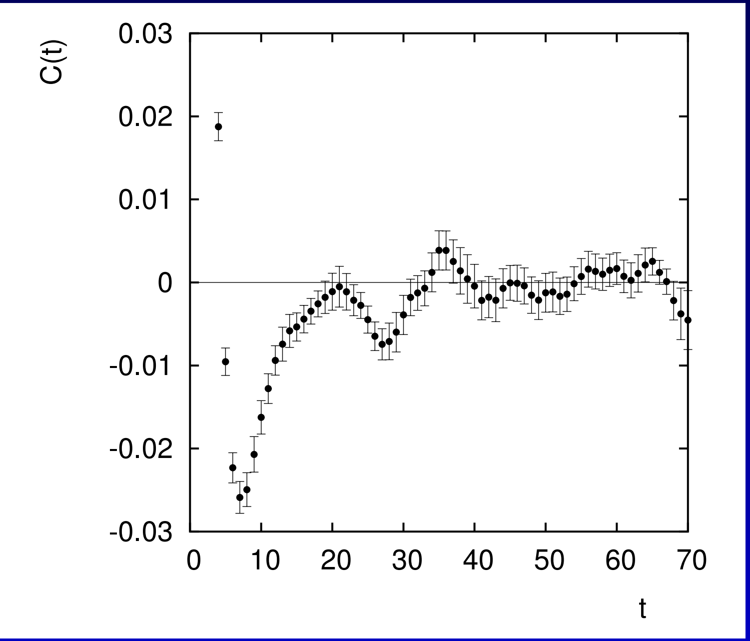
where (See Itzykson & Zuber)

$$\rho(q^2)\theta(q^0) = (2\pi)^3\sum_\alpha \delta^4(q-p_\alpha)|\langle 0|\phi(0)|\alpha\rangle|^2 \geq 0$$

$1 = Z + \int_{m_1^2}^\infty \rho(m^2)$ for a finite theory, but
$Z_3^{-1} = Z + \int_{m_1^2}^\infty \rho(m^2)$ for gluons in QCD (See Alkofer & Von Smekal...)

Positivity violations in QCD

In Euclidean space, DSE's establish positivity violations (see C. Fischer's thesis) in $C(t) = \int_0^\infty dm \rho_E(m^2) e^{-mt}$ also in the lattice

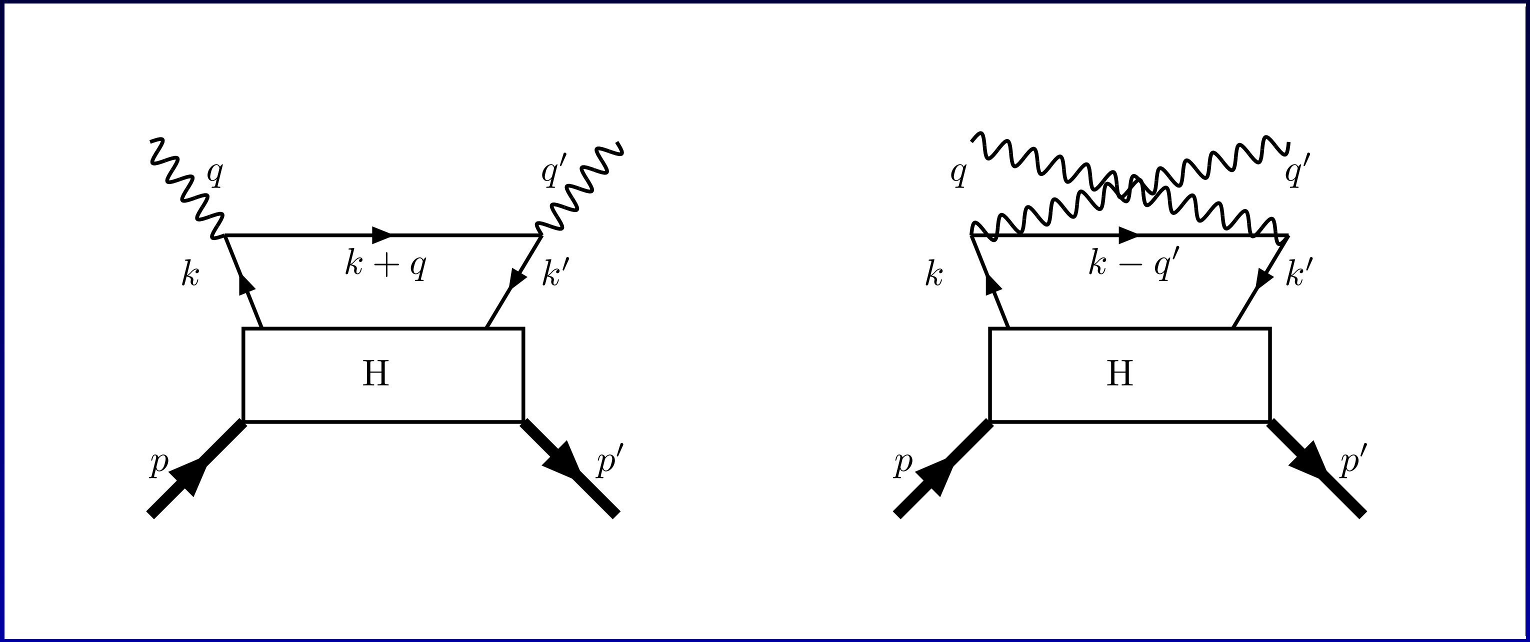


(Cucchieri, Mendes, Taurines, hep-lat/0406020)

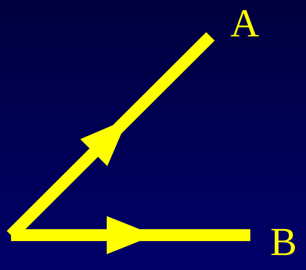
Also for quarks, positivity violations can indicate confinement (review C.D.Roberts, 0712.0633).

Exclusive processes

Are nucleon+parton on-shell states normalizable?



Positivity with two states



$$|\langle A|B\rangle| \leq \sqrt{\langle A|A\rangle\langle B|B\rangle}$$

Or algebraically

$$\langle \alpha^* A + \beta^* B|\alpha A + \beta B\rangle \geq 0$$

$$det \begin{pmatrix} \langle A|A\rangle & \langle A|B\rangle \\ \langle B|A\rangle & \langle B|B\rangle \end{pmatrix} \geq 0$$

Positivity for GPD H

$$|\sum_{k=1}^{2} b_\lambda^k \int \frac{dy^-}{2\pi} e^{iy^- x_k P^+} \Psi_\alpha(y^-)|P_k,\lambda\rangle|^2 \geq 0$$

with $y^+ = z + t = 0$, $\mathbf{y}^\perp = 0$

$$|H(x,\zeta,t)| \leq \sqrt{f(x) f(\frac{x-\zeta}{1-\zeta}) \left(\frac{(1-\zeta/2)^2}{(1-\zeta)(1-t_0/t)} \right)}$$

P. Pobylitsa, PRD65, 077504

Wavefunction representation

(Brodsky, Diehl, Hwang)

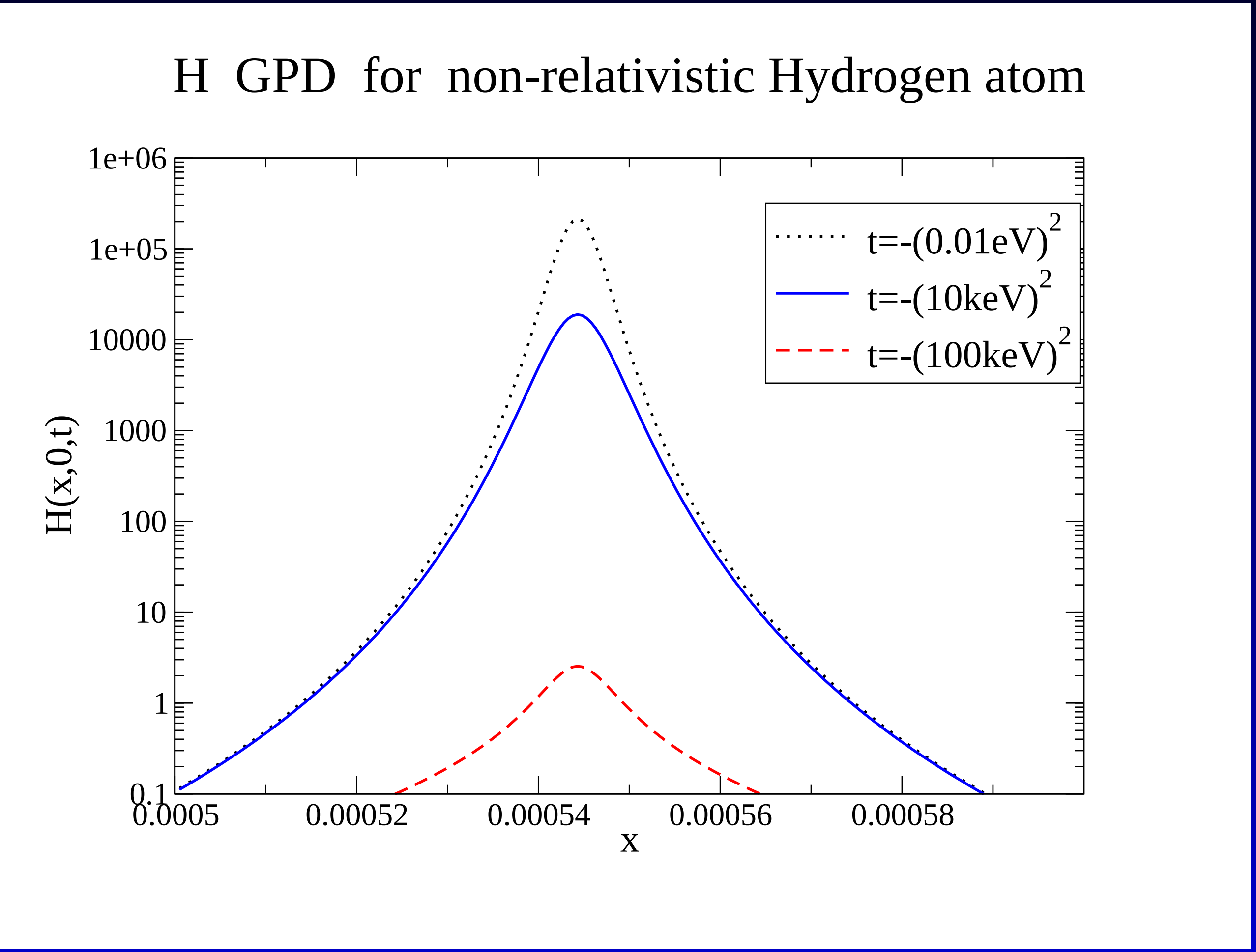
$$H(x,\zeta,t)=\theta(x-\zeta)\frac{1}{(1-\zeta)(1-\zeta/2)}\sum_{\lambda_i}$$

$$\int\prod_{i=1}^{3}\left(\frac{dx_i d^2\mathbf{k}_{\perp\ i}}{16\pi^3}\right)16\pi^3\delta(1-x_1-x_2-x_3)\delta^{(2)}\left(\sum\mathbf{k}_{\perp\ i}\right)$$

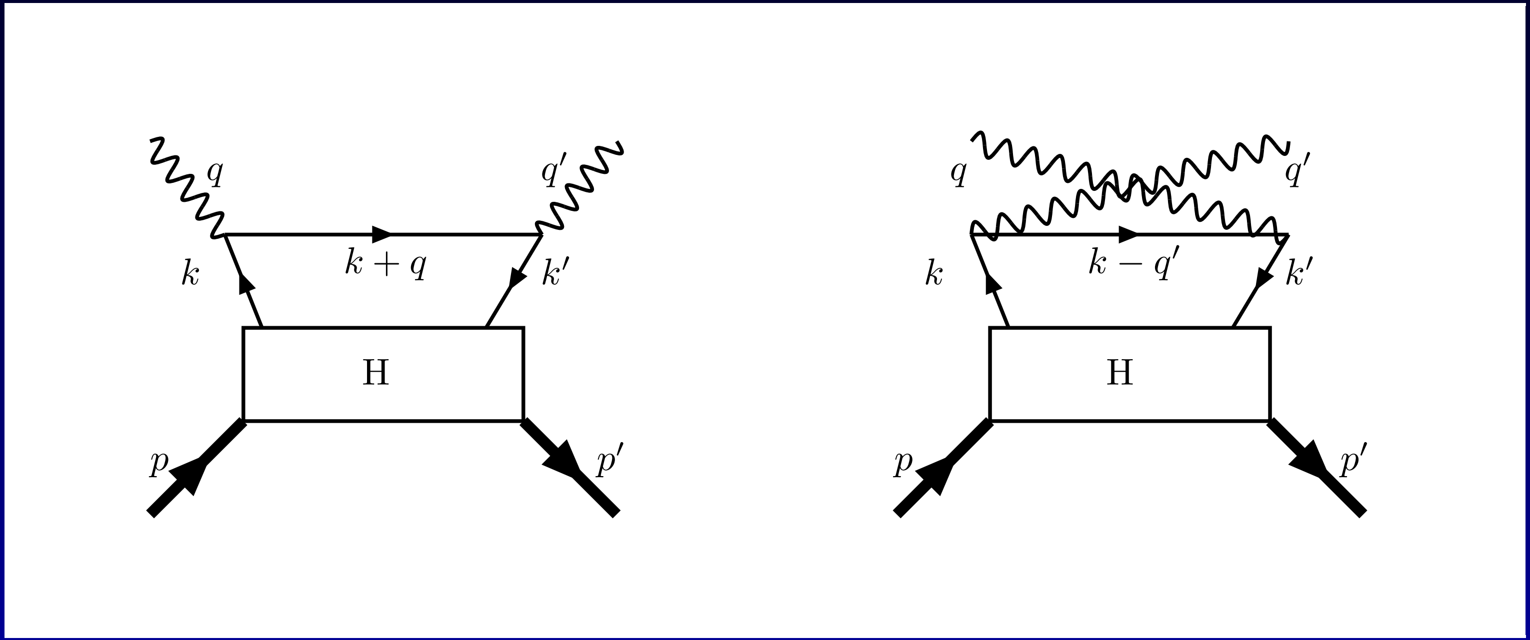
$$\times\delta(x-x_1)\psi^*_{\lambda'=+}(x'_i,\mathbf{k}'_{\perp\ i},\lambda'_i)\psi_{\lambda=+}(x_i,\mathbf{k}_{\perp i},\lambda_i)\ .$$

- $x>\zeta$, quark wavefunction in the two protons
- $x<0$, antiquark wavefunctions
- $0<x<\zeta$, quark-antiquark wf in one of them

GPD for Hydrogen atom



Factorization theorems



The hard propagator brings-in $1/x$

$$\mathcal{A}^{++}(s,t,Q^2) = \frac{-e_q^2}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}$$

$$\int_{\zeta-1}^{1} dx \left[\frac{1}{x-i\epsilon} + \frac{1}{x-\zeta+i\epsilon}\right] H(x,\zeta,t)$$

Regge behavior

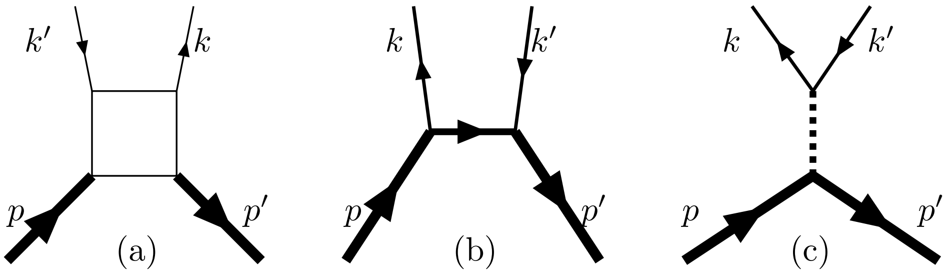
$$\mathcal{A}_{\gamma^* p \to \gamma p} \propto s^{\alpha(t)}$$

$\alpha(0) = 1$ (pomeron)
$\alpha(0) \simeq 1/2$ (non-singlet Reggeons, $p - n$).

When $\alpha(t) = J$, an integer,
resonance exchanged in t-channel

Parton-Nucleon scattering

$$H(x,\zeta,t) = xP^+(1-\zeta/2)\int\frac{d^4k}{(2\pi)^4}\delta(P^+x-k^+)T[s,t,u,k^2]$$



(Landshoff, Polkinghorne, Short)

Regge subtraction

Does the parton-nucleon scattering amplitude have the same analytical structure of hadron-hadron amplitudes?
In particular,

$$T_s = \sum_n c_n (2\pi)^4 \int dm^2 \left[\frac{\rho_n}{s_{pp} - m^2 + i\epsilon} - \frac{\rho_R^n}{-m^2 + i\epsilon} \right]$$

$$I_n \left(\frac{1}{k^2 - m_q^2 + i\epsilon} \frac{1}{k'^2 - m_q^2 + i\epsilon} \right)$$

$\rho^R \propto (m^2)^{\alpha(t)}$

(Brodsky, Close and Gunion)

Regge behavior in GPD's?

Assume quark-hadron scattering has the same Regge behavior as hadron-hadron scattering

$$T^R(u,t) = \beta(t)\frac{1-e^{i\pi\alpha(t)}}{\sin\pi\alpha(t)}\left(\frac{u}{u_0}\right)^{\alpha(t)}$$

At low x,

$$f(x) \propto x^{-\alpha}$$

(in agreement with HERA data) but also

$$H(x\to\zeta^+,\zeta,t) \propto (x-\zeta)^{-\alpha}$$

The GPD has a divergence at the break point

Failure of collinear factorization

$$\mathcal{A}^{++}(s,t,Q^2) = \frac{-e_q^2}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}$$

$$\int_{\zeta-1}^{1} dx \left[\frac{1}{x-i\epsilon} + \frac{1}{x-\zeta+i\epsilon}\right] H(x,\zeta,t) = \infty$$

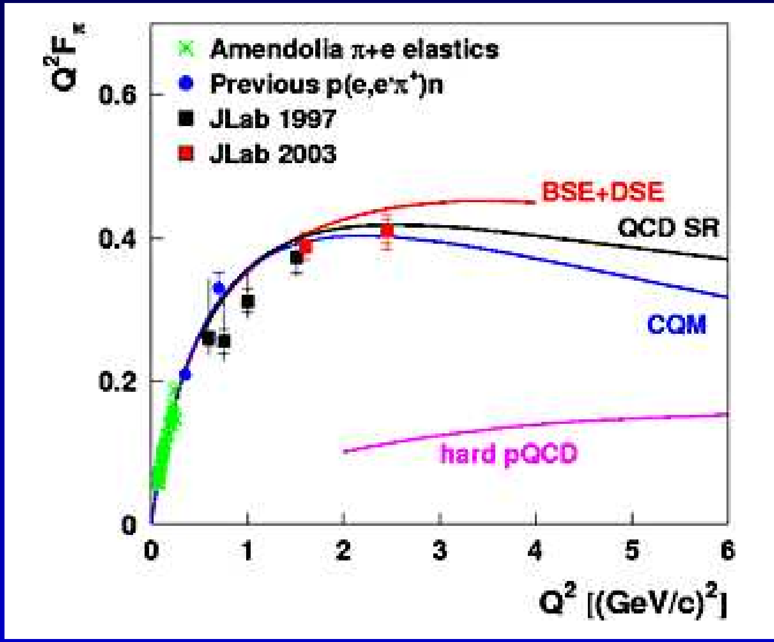
Diverges if $\alpha < 0$! (Problem of this formula, not of Compton amplitude)

Not the first time

Farrar & Jackson, PRL43 246, 1979:

$$F_\pi(Q^2) = 16\pi f_\pi^2 \frac{\alpha_s(Q^2)}{Q^2}$$

Jlab data for pion form-factor gives $\alpha_s = 0.96$!!!

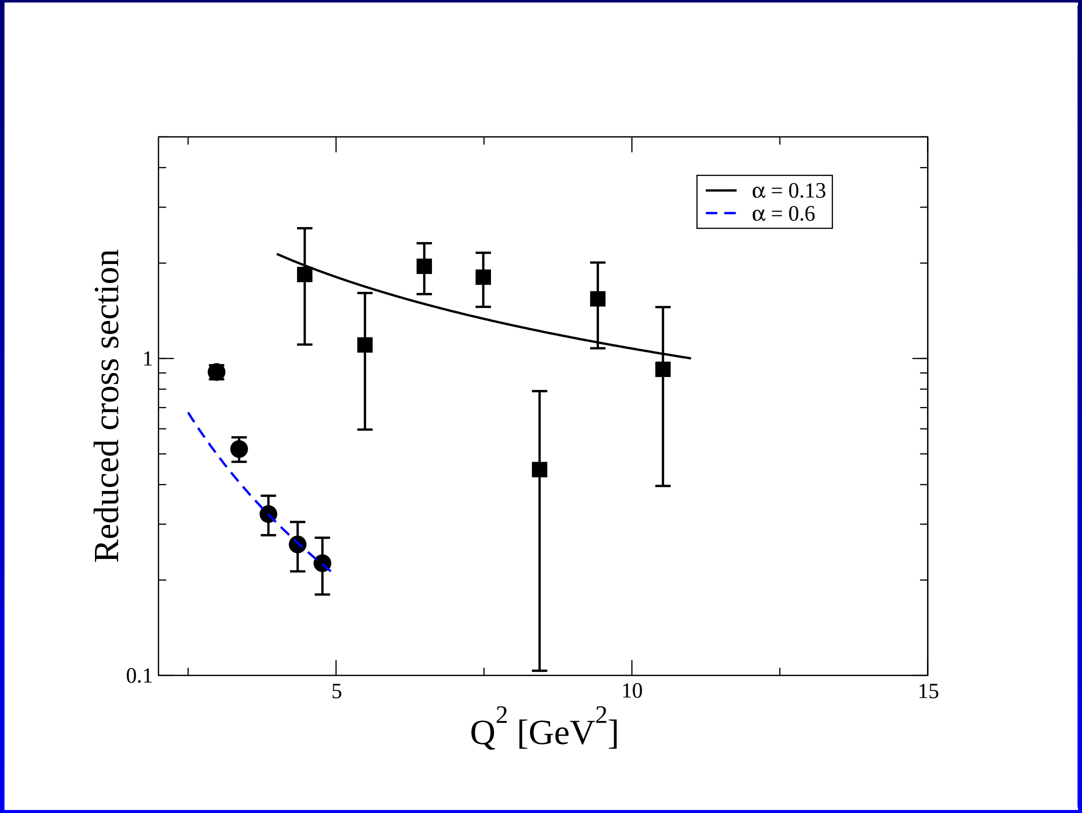


What went wrong with pQCD? Non-factorizable contribution at the end-point, see P. Hoyer *et al*, PRD70, 014001 (2004)

Test for scaling

$$\mathcal{A}_{\gamma * p \to \pi p} \propto \left(\frac{Q^2}{\zeta \mu^2} \right)^{\alpha} F(\zeta)$$

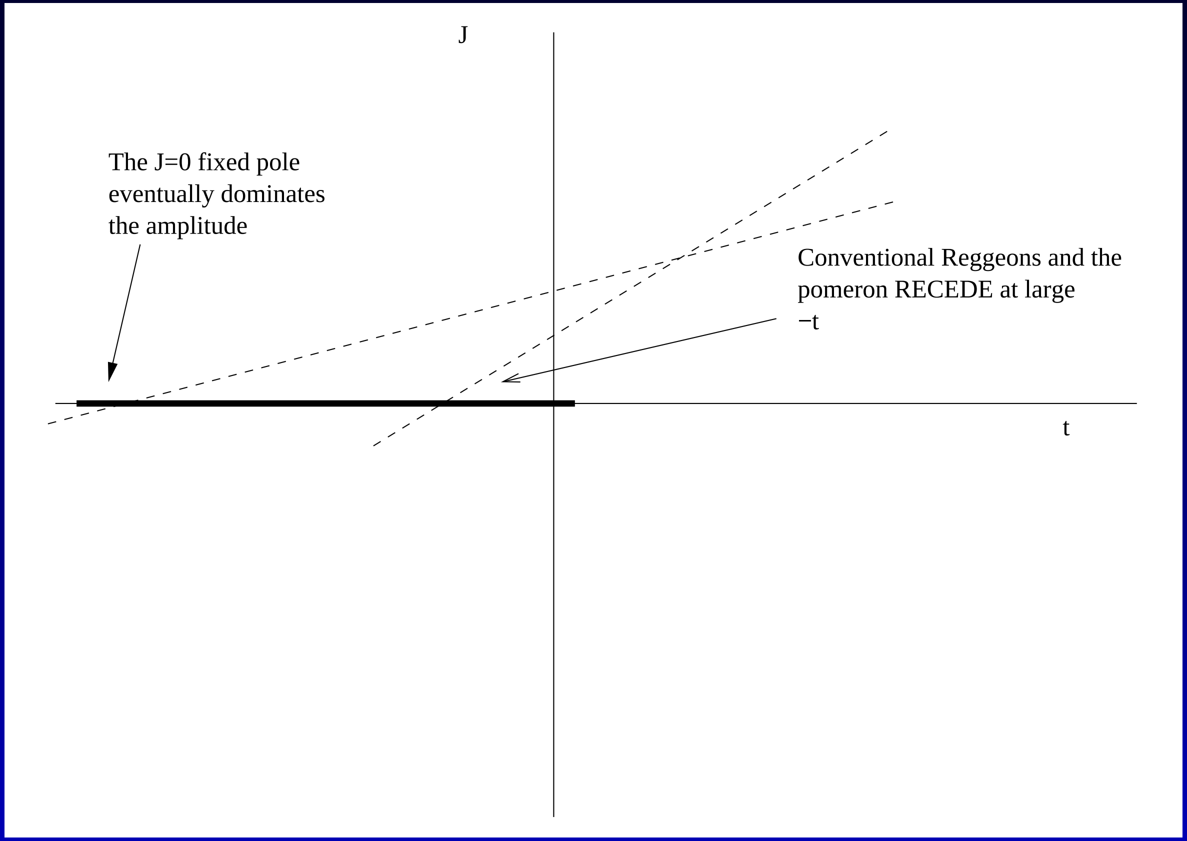
Additional Q^2-dependence!



Conclusions

- Regge behavior of pdf's and GPD's obtainable from Parton-Nucleon Scattering amplitude
- This is a colored amplitude. What can we assume about it?
- Violation of factorization theorems
- Violations of positivity

Reggeons recede $t < -0.7\, GeV^2$



$$\alpha(t) \to -1$$

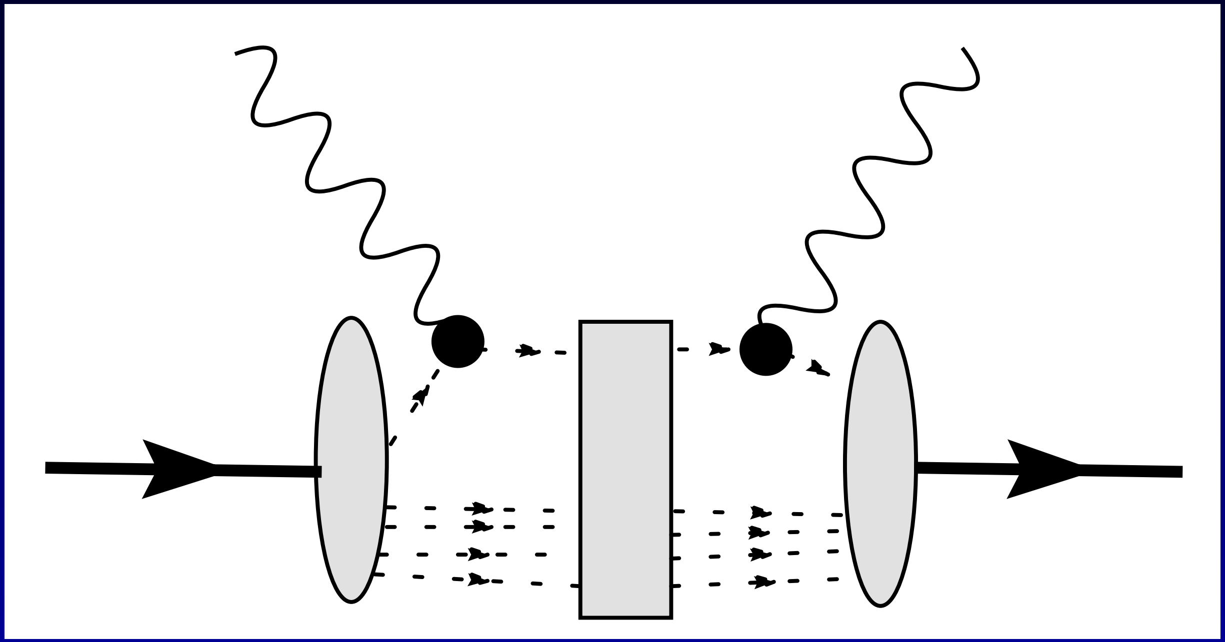
J=0 fixed pole

$$\mathcal{A}_{\gamma^* p \to \gamma p} \longrightarrow s^0$$

At High Energy, the Compton amplitude becomes a constant
(Creutz, Drell, Paschos 1969)

Why?

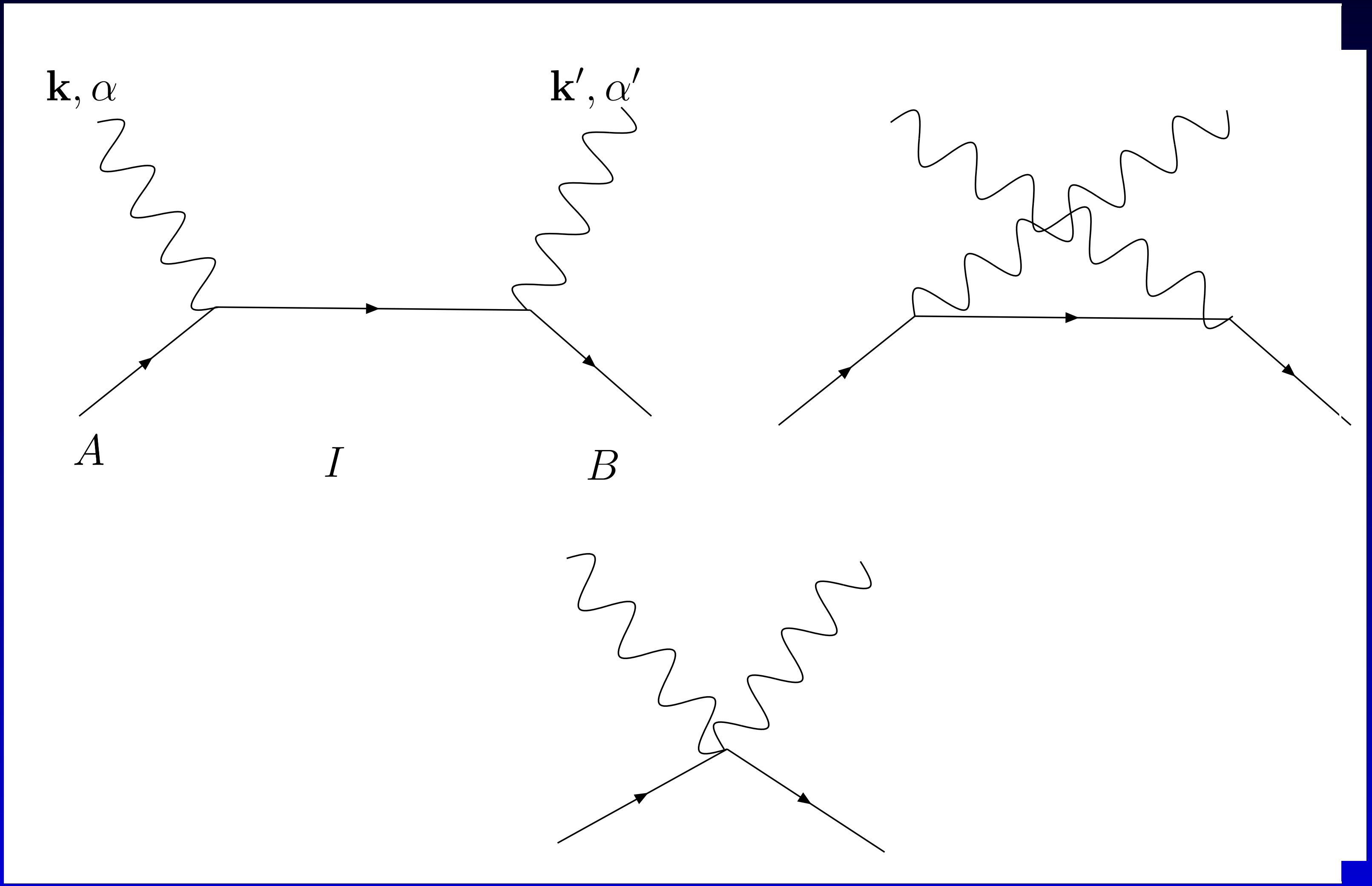
Point-like constituents



Intermediate states bring denominators $1/s$

However pointlike (seagull coupling) is a constant

Kramers-Heisenberg

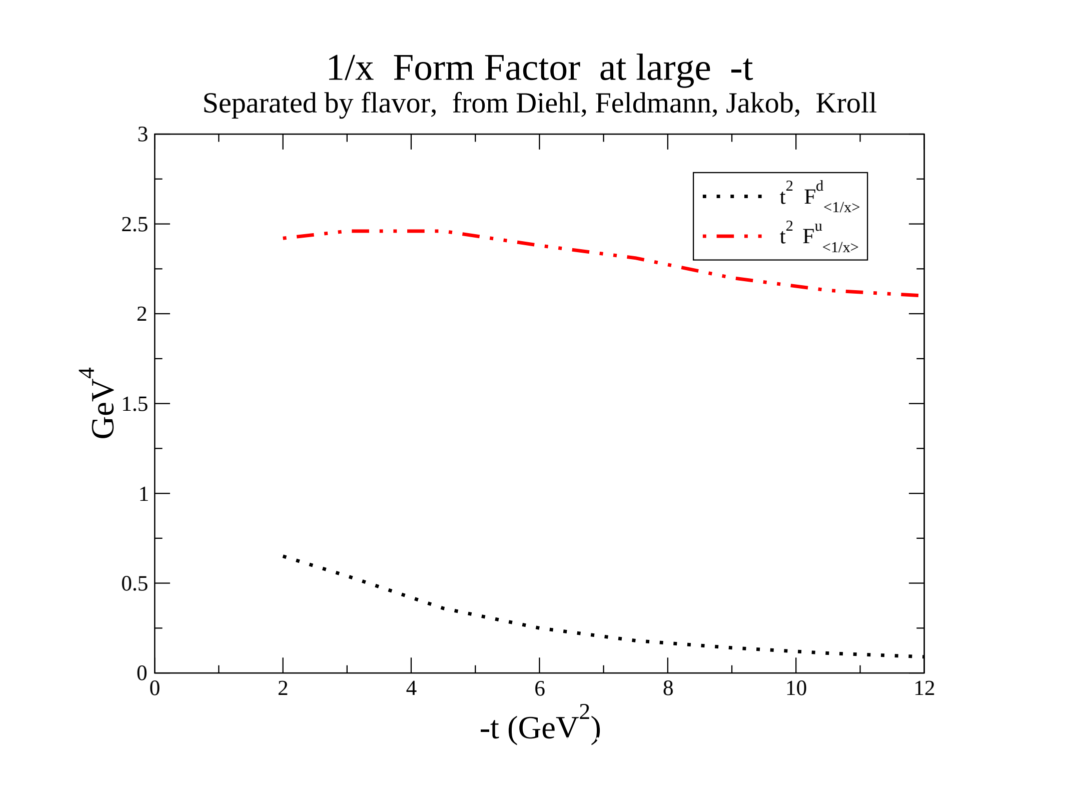


Thomson high-energy cross section, a constant

The Nucleon's $F_{1/x}$ Form factor

$$\mathcal{A} \to -\sum_q e_q^2 F_{1/x}^q(t)$$

$$F_{1/x}(t) = \int \frac{dx}{x} H(x,0,t)$$

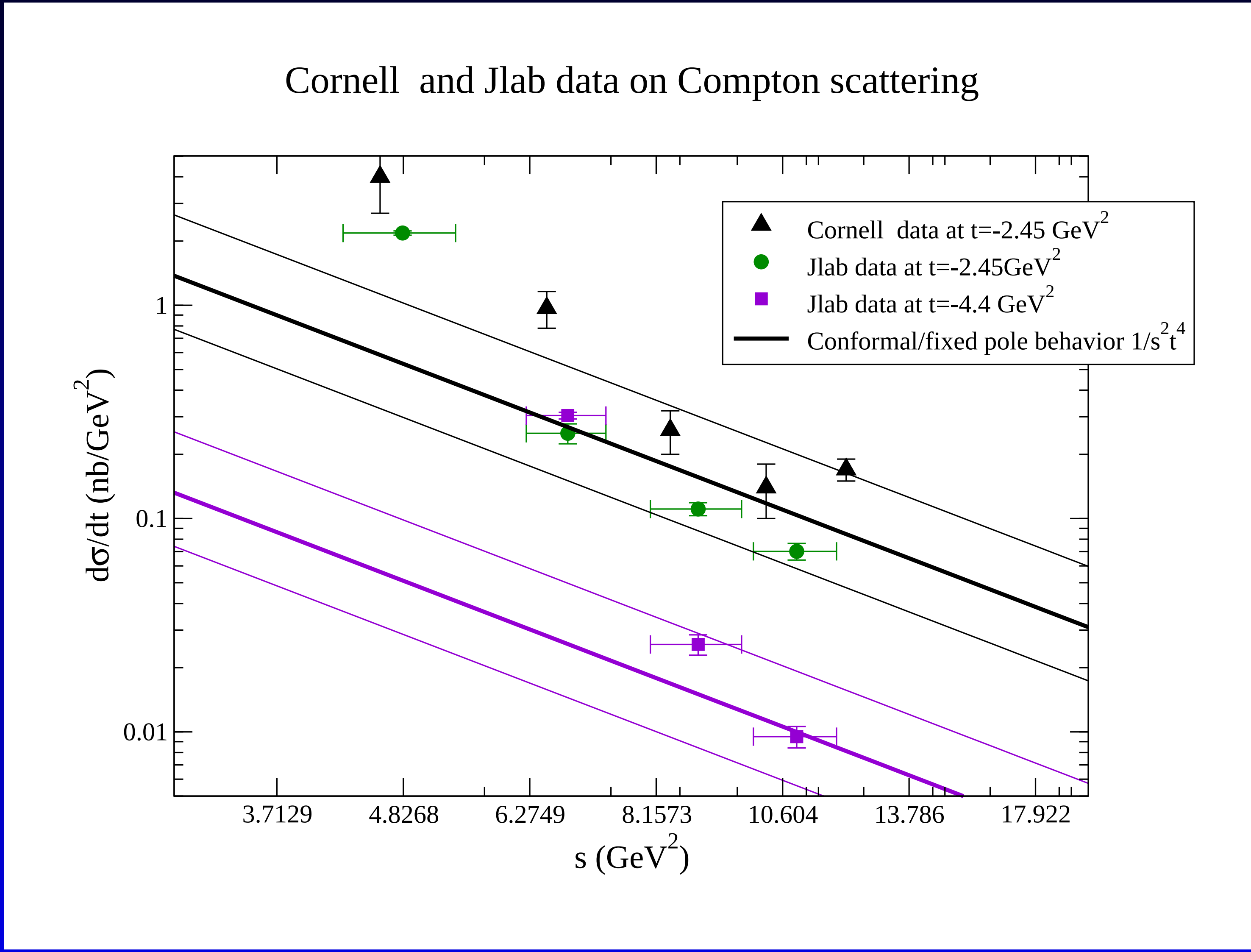


The Weisberger relation

$$\lim_{t \to 0} H(x,0,t) = f(x)$$

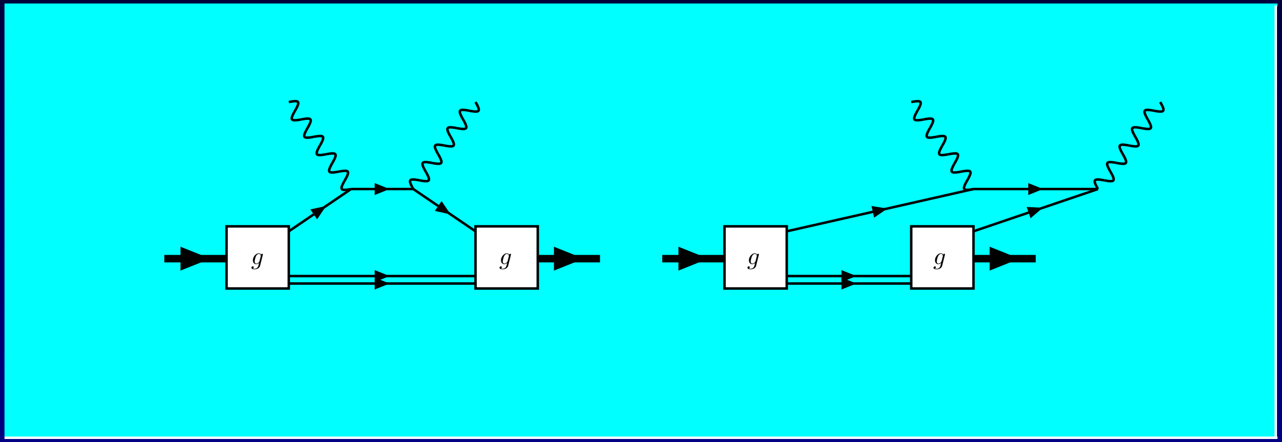
$$\frac{\delta M_N^2}{\delta m_i^2(\mu)} = \int_0^1 \frac{dx}{x} \left(f_i(x)_\mu + \bar{f}_i(x)_\mu \right) \ .$$

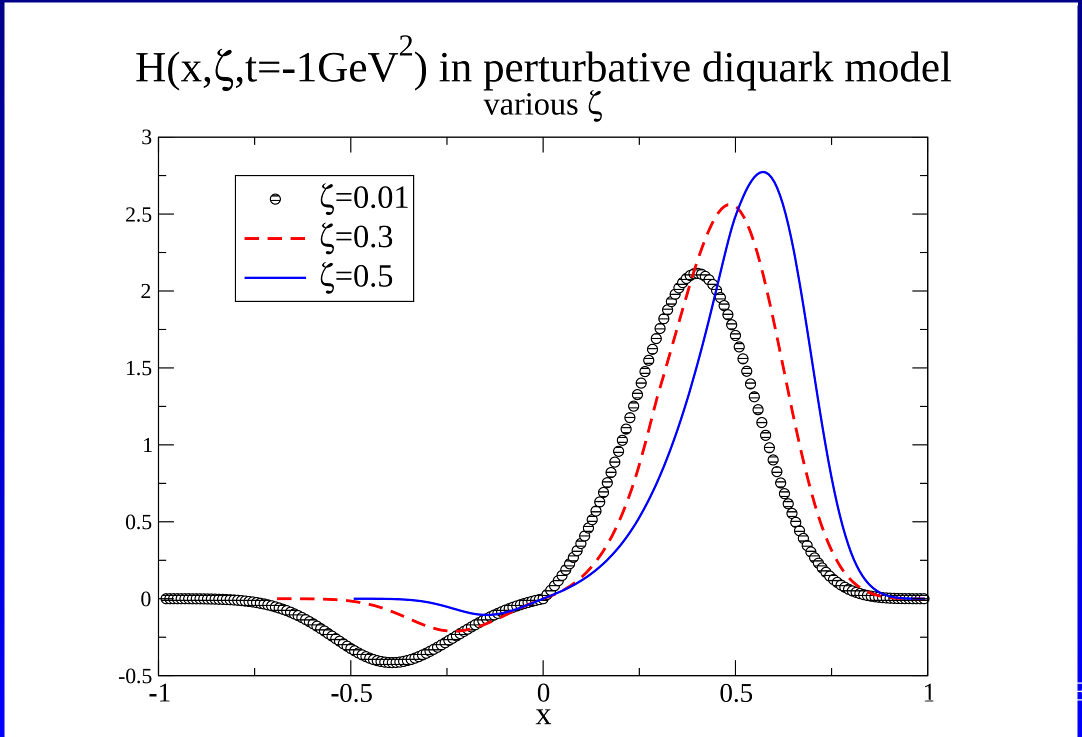
Real Compton scattering



Increase $-t$

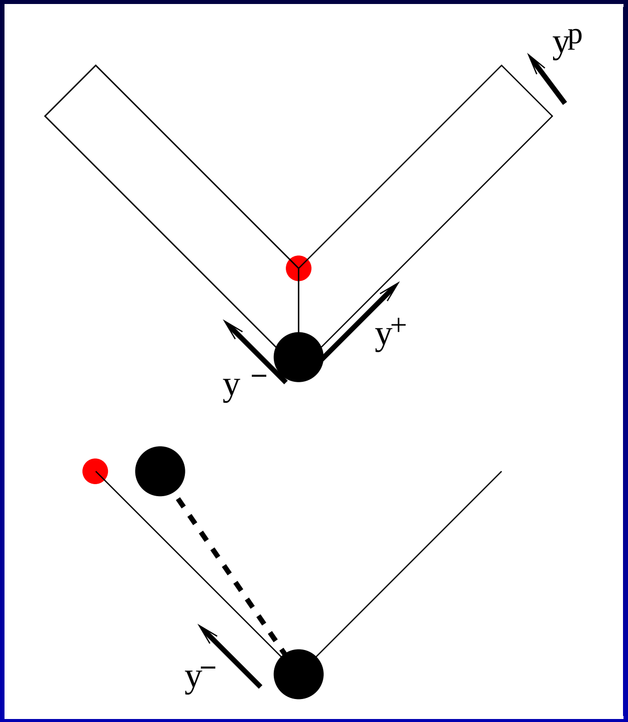
Perturbative diquark model relevant





Positivity violations in GPD's?

Equal light-front time $y^+ = z + t = 0$



$$y^- \geq \frac{2R_{pv}}{\gamma(1-\beta)} \to 4R_{pv}E/M_N\ .$$

(Distance at which we might expect positivity to fail)

Positivity violations in GPD's?

$H(x, \zeta, t)$ is defined through Fourier transform, large y^- is probed at low x and low $x - \zeta$. In particular, if

$$x - \zeta \leq \frac{M_N}{4E^2 R_{PV}}$$

Problem for theory:
Are there positivity violations near $x = 0$, $x = \zeta$?

Conclusions

- Studies of GPD's at low t run into difficulties with Regge behavior
- The $J = 0$ fixed-pole reveals quark structure of nucleon
- Independent of s; t-dependence $F_{1/x}$ form factor
- To isolate it, $s >> Q^2 >> -t \simeq 1\ GeV^2$ (Jlab upgrade, e-RHIC)
- Extrapolation to $t = 0$ gives $\delta M_N^2 / \delta m_q^2$

Generalized Parton Distribution

$$\frac{1}{2P^+}\bar{U}(P')_{\lambda'}\left[H(x,\zeta,t)\gamma^+ + E(x,\zeta,t)\frac{i}{2M_N}\sigma^{+\alpha}(-\Delta_\alpha)\right]U(P)_\lambda =$$

$$\int\frac{dy^-}{8\pi}e^{ixP^+y^-/2}\langle P',\lambda'|\bar{\Psi}(0)\gamma^+\Psi(y)|P,\lambda\rangle|_{y_+=0,\ y_\perp=0}$$

(Mueller, Ji, Radyushkin, Diehl...)

x, $x-\zeta$ parton momentum fractions; t momentum transfered to the proton.