

Dual Chiral Condensate and Dressed Polyakov Loops

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The question we want to address:

- At zero temperature QCD shows two characteristic features:
 - Quarks are confined.
 - Chiral symmetry is broken: $\langle \bar{\psi}\psi \rangle \neq 0$.

- QCD has a phase transition at some critical temperature T_c , where:
 - Quarks become deconfined.
 - Chiral symmetry is restored: $\langle \bar{\psi}\psi \rangle = 0$.

Is there an underlying mechanism that links the two key features of QCD?

A possible approach

- Confinement and chiral symmetry breaking both should leave a trace in properties of the Dirac operator D , since D^{-1} describes the propagation of quarks.
- For chiral symmetry breaking the Banks-Casher formula connects the order parameter $\langle \bar{\psi}\psi \rangle$ to IR properties of the Dirac spectrum.
- Concerning confinement it is not even clear where to look in the spectrum, in the UV or the IR part.
- Maybe through analyzing spectral properties of D one can find a link between confinement and chiral symmetry breaking.
- The lattice formulation provides a suitable framework (rigorously defined) which allows for both, analytical and numerical approaches.

Some literature

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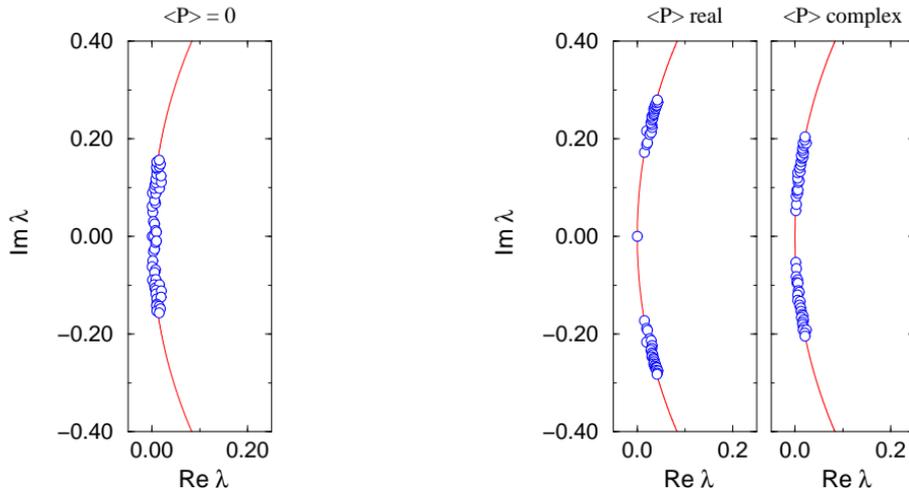
E. Bilgici, C. Gattringer, arXiv:0803.1127 [hep-lat]

Chiral symmetry breaking and Dirac spectrum

- The Banks Casher formula relates the chiral condensate to the spectral density of the Dirac operator at the origin.

$$\langle \bar{\psi} \psi \rangle = -\pi \rho(0)$$

- At the QCD phase transition a gap opens up in the spectrum and the chiral condensate vanishes.



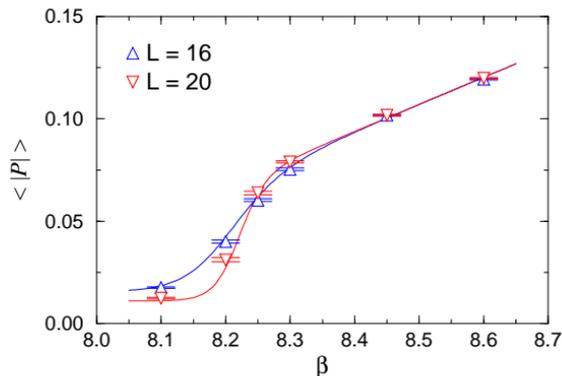
Center symmetry and Polyakov loops

- The gauge action is invariant under center transformations ($z \in Z_3$):

$$U_4(x) \rightarrow z U_4(x) \quad \forall x_4 = t_0$$

- The deconfinement transition of pure gauge theory can be described as spontaneous breaking of the center symmetry.
- The Polyakov loop transforms non-trivially and is an order parameter.

$$L(\vec{x}) = \text{tr}_c \prod_{t=1}^{N_t} U_4(\vec{x}, t)$$
$$L(\vec{x}) \rightarrow z L(\vec{x})$$



The Dirac operator on the lattice

- Discretized Dirac operator on the lattice

$$D = \frac{1}{2a} \sum_{\mu=1}^4 \gamma_{\mu}(x) \left[U_{\mu}(x) \delta_{x+\hat{\mu}, y} - U_{\mu}(x - \hat{\mu})^{\dagger} \delta_{x-\hat{\mu}, y} \right]$$

- The gauge links

$$U_{\mu}(x) = e^{i a A_{\mu}(x)}$$

- are the objects we need for the Polyakov loop

$$L(\vec{x}) = \text{tr}_c \prod_{t=1}^{N_t} U_4(\vec{x}, t)$$

- The gauge links appear in hopping terms that connect nearest neighbors on the lattice.

Fermion propagators and loops

- The chiral condensate has an expansion in terms of loops:

$$\begin{aligned}\langle \bar{\psi}\psi \rangle &= -\frac{1}{V} \text{Tr}[m + D]^{-1} = -\frac{1}{mV} \sum_{k=0}^{\infty} \frac{(-1)^k}{m^k} \text{Tr} [D^k] \\ &= -\frac{1}{mV} \sum_{l \in \mathcal{L}} \frac{s(l)}{(2am)^{|l|}} \text{Tr}_c \prod_{(x,\mu) \in l} U_\mu(x)\end{aligned}$$

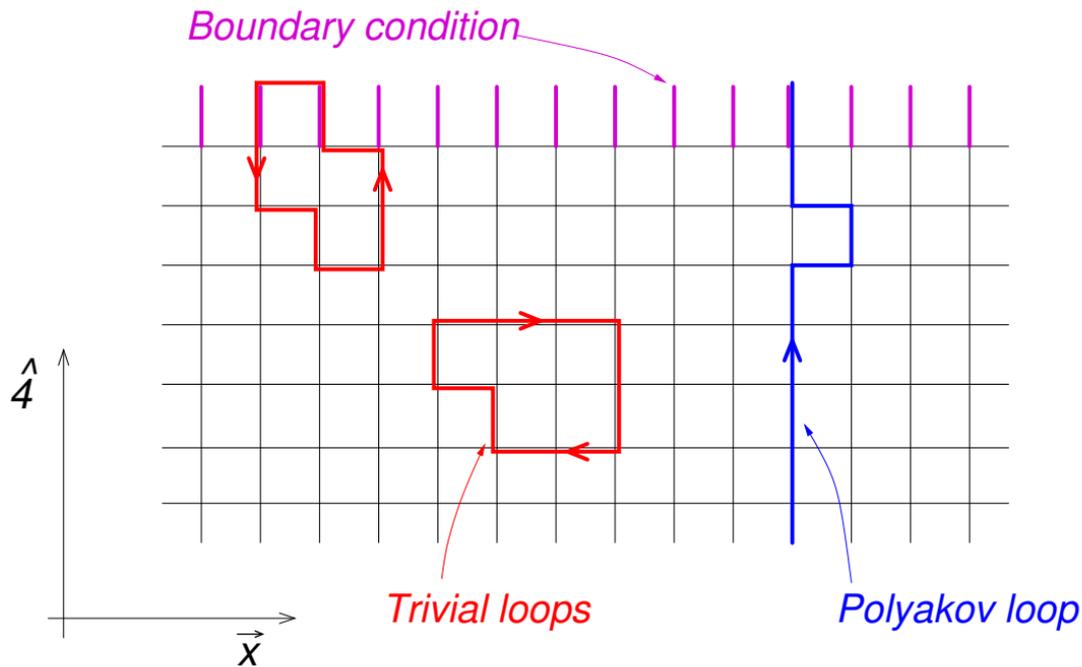
- A change of the temporal boundary conditions

$$U_4(\vec{x}, N_t) \longrightarrow z U_4(\vec{x}, N_t) \quad , \quad z = e^{i\varphi} \in \text{U}(1)$$

affects only loops that wind non-trivially around compact time.

- Fourier transformation of φ allows one to project to the equivalence class of loops that wind exactly once: *Dressed Polyakov Loops*

Graphical representation



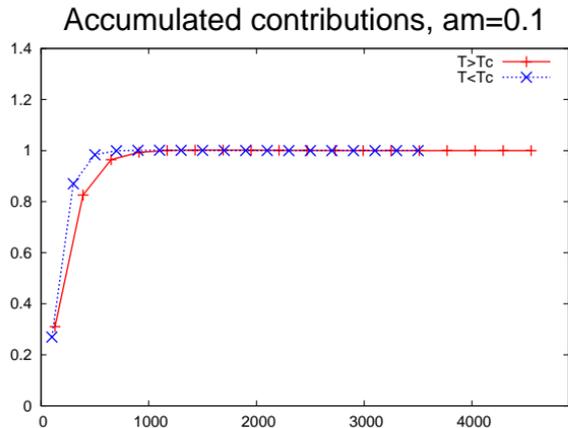
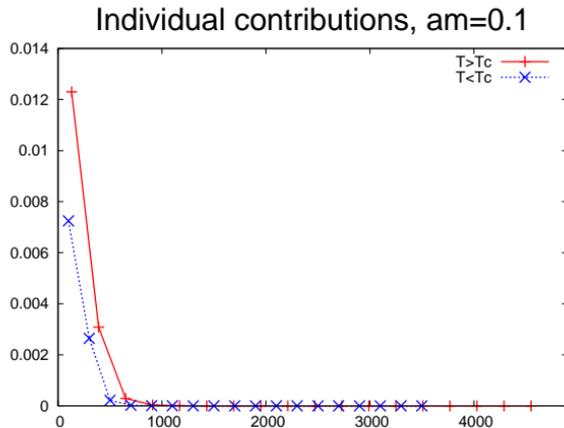
Dual chiral condensate = dressed Polykov loop

- Fourier transformation with respect to the boundary condition connects the order parameters for confinement and for chiral symmetry breaking:

$$\begin{aligned}\widehat{\langle \bar{\psi}\psi \rangle}_{\mathbf{1}} &= \int_0^{2\pi} \frac{d\varphi e^{-i\varphi}}{2\pi} \langle \bar{\psi}\psi \rangle_{\varphi} = \frac{1}{mV} \sum_{l \in \mathcal{L}_1} \frac{s(l)}{(2am)^{|l|}} \left\langle \text{Tr}_c \prod_{(x,\mu) \in l} U_{\mu}(x) \right\rangle \\ &= - \int_0^{2\pi} \frac{d\varphi e^{-i\varphi}}{2\pi V} \sum_k \left\langle \frac{1}{m + \lambda_{\varphi}^{(k)}} \right\rangle_{\varphi}\end{aligned}$$

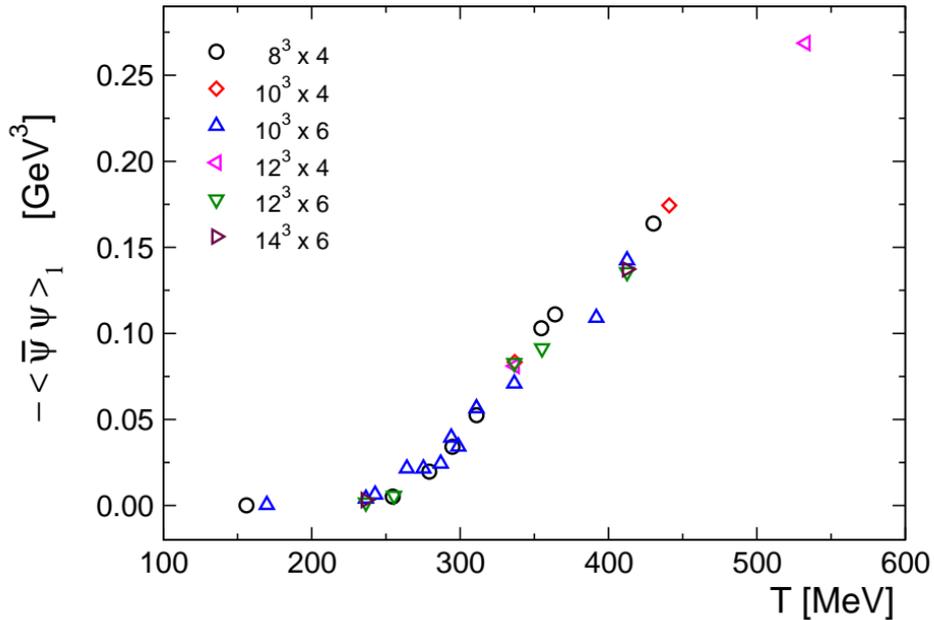
- The representation as a spectral sum of Dirac eigenvalues allows one to study the role of IR and UV eigenmodes for the mechanisms of confinement and chiral symmetry breaking.

The Dressed Polyakov Loop is dominated by IR modes



$$-\widehat{\langle \bar{\psi}\psi \rangle}_1 = \sum_k \frac{1}{2\pi V} \int_0^{2\pi} d\varphi e^{-i\varphi} \left\langle \frac{1}{m + \lambda_\varphi^{(k)}} \right\rangle_\varphi$$

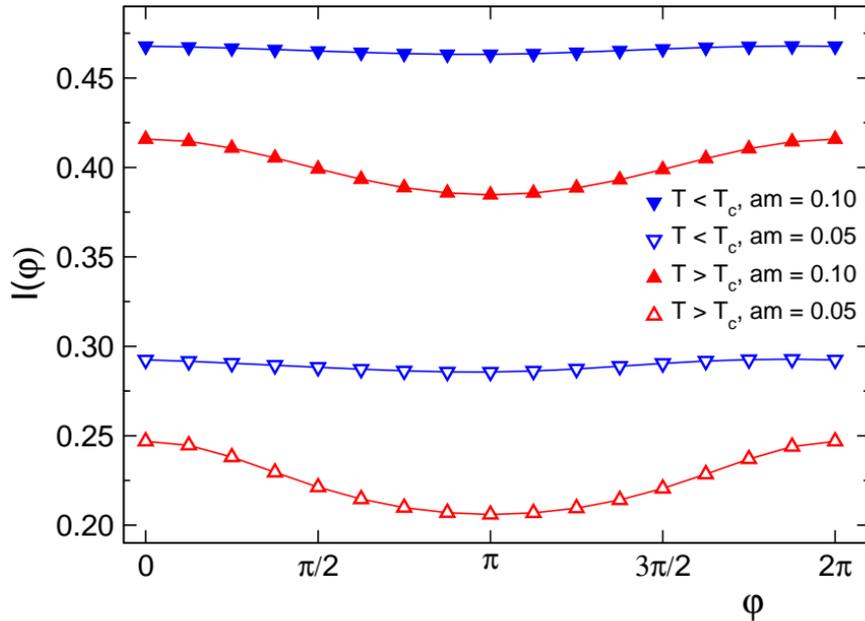
The Dressed Polyakov Loop is an order parameter



Results from different lattices fall on a universal curve.

→ Good scaling and renormalization properties.

Spectral properties at the phase transition



The confined and deconfined phases give rise to a different response of the IR part of the Dirac spectrum to changing boundary conditions.

Generalization of the Banks-Casher formula

- Having identified the connection between spectral properties and the dressed Polyakov loops, we can now formulate the physical picture in terms of a generalized Banks-Casher relation.
- Performing $\lim_{m \rightarrow 0} \lim_{V \rightarrow \infty}$ we find:

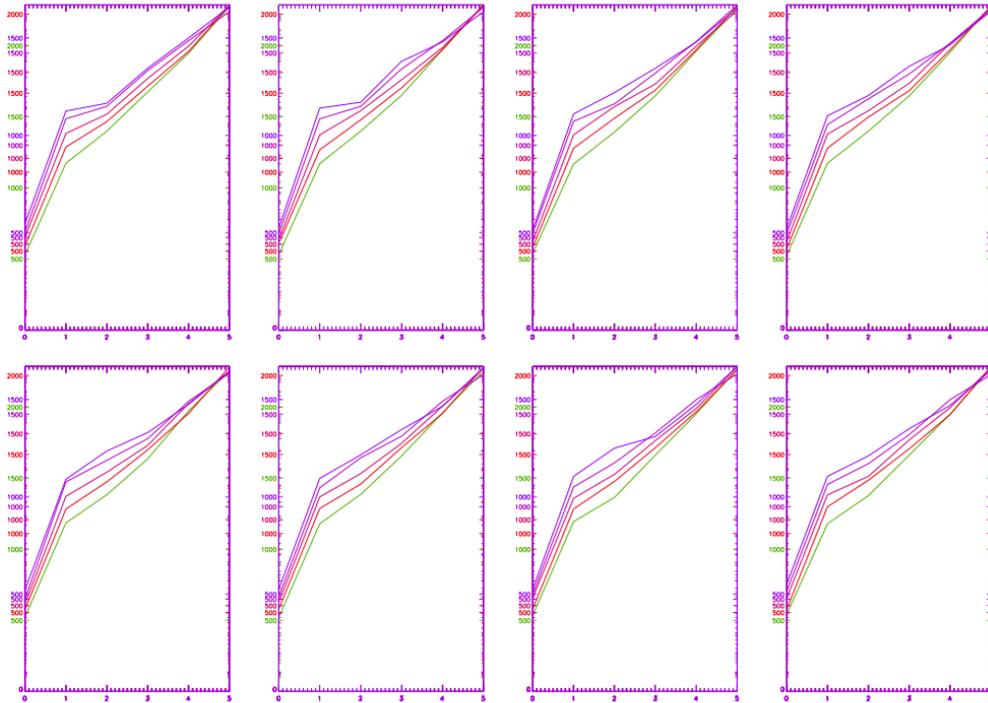
$$-\widehat{\langle \bar{\psi} \psi \rangle}_{\mathbf{1}} = \frac{1}{2} \int_0^{2\pi} d\varphi e^{-i\varphi} \rho(0)_\varphi$$

- How does the spectral density $\rho(0)_\varphi$ at the origin have to behave as a function of φ such that:

$$-\widehat{\langle \bar{\psi} \psi \rangle}_{\mathbf{1}} = 0 \quad \text{below } T_c$$

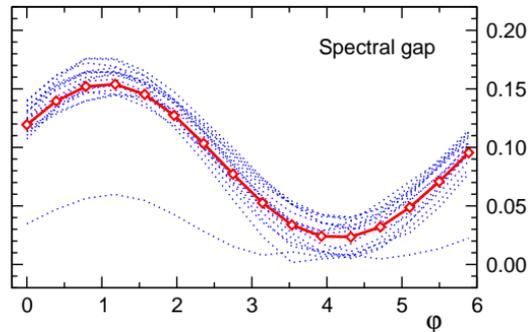
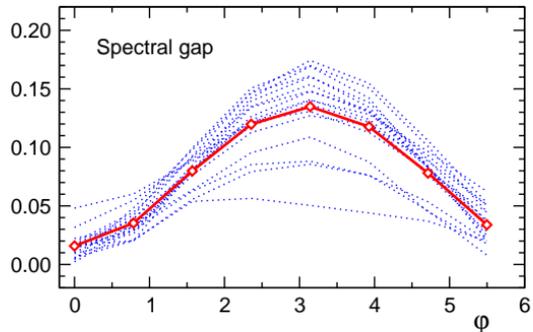
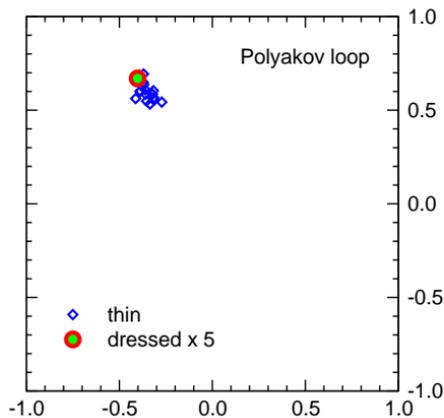
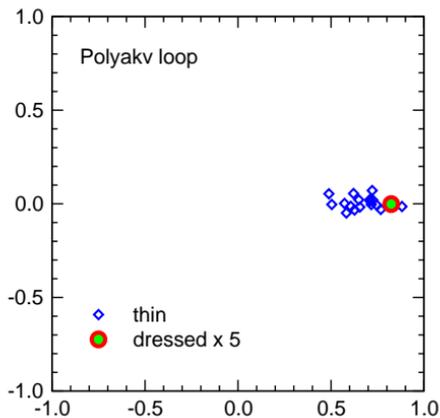
$$-\widehat{\langle \bar{\psi} \psi \rangle}_{\mathbf{1}} > 0 \quad \text{above } T_c$$

Spectral density below T_c



Below T_c the spectral density $\rho(0)_\varphi$ is independent of the boundary angle φ .

Spectral gap above T_c



Spectral gap depends on the relative phase between b.c. and Polyakov loop.

Emerging picture for the generalized Banks-Casher formula

- The spectral density at the origin, $\rho(0)_\varphi$, behaves as (θ denotes the phase of the Polyakov loop):

$$\rho(0)_\varphi = \text{const} \quad \text{below } T_c$$

$$\rho(0)_\varphi \propto \delta(\varphi + \theta) \quad \text{above } T_c$$

- The dual chiral condensate is given by:

$$-\widehat{\langle \bar{\psi}\psi \rangle}_{\mathbf{1}} = \frac{1}{2} \int_0^{2\pi} d\varphi e^{-i\varphi} \rho(0)_\varphi$$

- And behaves correctly as:

$$-\widehat{\langle \bar{\psi}\psi \rangle}_{\mathbf{1}} = 0 \quad \text{below } T_c$$

$$-\widehat{\langle \bar{\psi}\psi \rangle}_{\mathbf{1}} > 0 \quad \text{above } T_c$$

Summary

- Fourier transforming the chiral condensate with respect to the fermionic boundary condition we define the *Dual Chiral Condensate*.
- The dual chiral condensate is an order parameter for center symmetry, interpreted as *Dressed Polyakov Loops*.
- The dual condensate can be represented as a spectral sum of Dirac eigenvalues which is dominated by the IR modes.
- At the phase transition the behavior of the low-lying eigenvalues changes:
 1. The chiral transition is signalled by a change from a non-zero to a vanishing density (Banks-Casher).
 2. The deconfinement transition is manifest in a different response of the eigenvalues to a change in the temporal boundary conditions.

Summary (continued)

- Most elegantly the results are expressed as a generalized Banks-Casher formula for the dual condensate:

$$-\widehat{\langle \bar{\psi}\psi \rangle}_1 = \frac{1}{2} \int_0^{2\pi} d\varphi e^{-i\varphi} \rho(0)_\varphi$$

1. In the confined phase we have a non-vanishing spectral density $\rho(0)_\varphi$ at the origin which is independent of the boundary conditions.
2. Above T_c the spectral gap has a non-trivial dependence on the phase between boundary condition and Polyakov loop and $\rho(0)_\varphi \propto \delta(\varphi+\theta)$.

Chiral symmetry breaking and confinement are, via a duality transformation, connected to closely related spectral properties of the IR Dirac spectrum.

Link between confinement and chiral symmetry breaking?