Bose ghost propagators

& Long-range force in QCD

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Gribov scenario leads to infrared suppression of gluon propagator.

Recently proposed as origin of "magnetic mass" that enters finite-T calculations at order g^6.

But then what is origin of long-range confining force?

Local action with bose & fermi ghosts

1. Introduction

Simplest analysis of cut-off

at Gribov horizon in Landau gauge $\Rightarrow$

i) IR suppression of gluon propagator

ii) IR enhancement of ghost propagator

(But what confines quarks?

need long-range force??)

a) Supported by numerical study in

$D=2$ dimension Maas, arXiv 0704.0722

b) Not supported by numerical

studies in $D=3$ & 4 dimensions

c) May be consistent with numerical

studies in $D=3$ dimensions if

one can go to absolute minimum

of gauge-fixing functional (FMR).

Maas arXiv: 0808.3047

What to do?

a) Modify model to account for dynamics of auxiliary ghosts
Dudal, Sorella, Vandersickel
Verschelde PRD 77 (2008) 071501
" + Gracey PRD 78 (2008) 065047

b) Persist with original model
DZ NPB 399 (1993) 477

→ Derive exact properties for comparison with numerical studies.

Presumably valid at least for D=2

Problems in Landau gauge

If gluon propagator is suppressed in Landau gauge in infrared what is origin of long-range force needed to confine quarks?

What is origin of "magnetic mass" that is introduced to control Linde infrared divergences of finite-T QCD?

Fundamental modular region

Λ = set of absolute minima of minimizing function

$$F_A(g) = \| {}^g A \|^2$$

$${}^g A_\mu = g^{-1} A_\mu g + g^{-1} \partial_\mu g$$

Gribov region : Ω

Ω = set of local minima of minimizing function

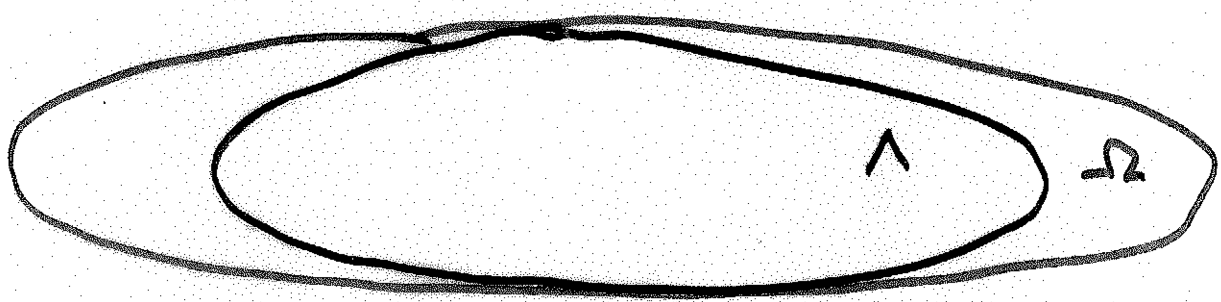
Ω : $\partial_\mu A = 0$ $\qquad -\partial \cdot D(A) \geq 0$

$\lambda_n(A) \geq 0$

$$\Lambda \subset \Omega$$

Both regions are convex and bounded in every direction

A^t



We don't know how to integrate over Λ. We define model by integrating over Ω instead and investigate consequences.

Parts of boundary are common

$D(A)\omega = 0$

$\partial\Omega: \; -\partial \cdot D(A)\omega = 0$

Proximity of Gribov horizon in infrared directions suppresses suppresses gluon propagator in Infrared.

Landau gauge $D(k) = \dfrac{k^2}{k^4 + \gamma}$

Coulomb gauge $D(k) = \dfrac{1}{k_0^2 + \vec{k}^2 + \dfrac{\gamma}{\vec{k}^2}}$

This is proposed origin of "magnetic mass" (Enters finite-T calculations at order g^6.)

But what is origin of long range confining force

Cut-off Functional Integral at Gribov Horizon

$$Z = \int_{\Omega} dA \; \delta(\partial\cdot A) \; \det(-\partial\cdot D) \, e^{-S_{YM}}$$

$\Omega: \; -\partial\cdot D(A) > 0$

$\lambda_n(A) > 0$

$\Omega: \quad H(A) < 0$

$H(A)$ = "horizon function"

$$H(A) = \int d^D x \, f^{abc} g A^b_\mu(x) \times \left[\int d^D y \; M^{-1\,ce}(x,y;A) \, f^{ade} g A^d_\mu(y) - D(N^2-1) \right]$$

M = Faddeev-Popov operator

$= -\partial\cdot D(A)$

$$Z = \int dA \; \Theta(H(A)) \; \delta(\partial \cdot A) \; \det(-\partial \cdot D) \times e^{-S_{YM}}$$

Recall equivalence of microcanonical & canonical ensembles

$$\delta(H - E) \longleftrightarrow e^{-\beta H}$$

$$\beta: \quad \langle H \rangle = 0$$

Also equivalence of Θ-ensemble and canonical

$$\Theta(H) \longleftrightarrow e^{-\gamma H}$$

$$\gamma: \quad \langle H \rangle = 0$$

γ = Gribov parameter

$$Z = \int dA \; \delta(\partial \cdot A) \; \det(-\partial \cdot D) \times e^{-S_{YM} - \gamma H}$$

Why cut-off works

$$e^{-\gamma H} \sim e^{-\gamma (A, M^{-1} A)}$$

$$M^{-1}(x,y) = \sum_n \frac{\psi_n(x)\, \psi_n^*(y)}{\lambda_n(A)}$$

$$e^{-\gamma H} \sim e^{-\gamma \sum_n \frac{(A, \psi_n)^2}{\lambda_n(A)}}$$

Blows up exponentially when any eigenvalue $\lambda_n(A)$ of Faddeev-Popov operator $-\partial \cdot D(A)$ approaches 0.

We want local action!

$$\delta(\partial\cdot A) = \int db\, e^{i\int b\,\partial\cdot A}$$

$$\det(-\partial\cdot D) = \int dc\, d\bar{c}\, e^{-\int \partial_\mu \bar{c}\, D_\mu c}$$

$$\exp[-\gamma(A, M^{-1} A)]$$

$$= (\det M)^{q} \int d\varphi\, d\bar{\varphi}\, e^{-\int \bar{\varphi} M \varphi + \gamma^{1/2}(\varphi - \bar{\varphi}) A}$$

$$= \int d\varphi\, d\bar{\varphi}\, d\omega\, d\bar{\omega} \times e^{-\int [\bar{\varphi} M \varphi - \bar{\omega} M \omega + \gamma^{1/2}(\varphi - \bar{\varphi}) A]\, d^D x}$$

We used Gaussian identity to get local action. Auxiliary fields

$b, c, \bar{c}, \varphi\ \bar{\varphi}$ auxiliary bose ghosts

$\omega, \bar{\omega}$ auxiliary fermi ghosts

$$\mathcal{L} = \mathcal{L}_{FP} + \mathcal{L}_{aux} + \mathcal{L}_\gamma$$

$$\mathcal{L}_{FP} = \frac{1}{4} F_{\mu\nu}^2 + s\,\partial_\mu \bar{c}\, A_\mu$$

$$= \frac{1}{4} F_{\mu\nu}^2 + i\,\partial_\mu b\, A_\mu - \partial_\mu \bar{c}\, D_\mu c$$

$$\mathcal{L}_{aux} = s\,\partial_\lambda \bar{\omega}_\mu\, D_\lambda \varphi$$

$$= \partial_\lambda \bar{\varphi}\, D_\lambda \varphi - \partial_\lambda \bar{\omega}\, D_\lambda \omega +$$

$$s\varphi_\mu^{ab} = \omega_\mu^{ab} \qquad s\omega = 0$$

$$s\bar{\omega}_\mu^{ab} = \bar{\varphi}_\mu^{ab} \qquad s\bar{\varphi} = 0$$

BRST spontaneously broken by

$$\mathcal{L}_\gamma = \gamma^{1/2} D_\mu (\varphi - \bar{\varphi})_\mu^{aa} - \gamma(N^2-1)D$$

$$\frac{\partial \Gamma}{\partial \gamma} = 0 \qquad \text{" horizon condition"}$$

This model describes restriction to interior of Gribov region

LOCAL RENORMALIZABLE

Could bose ghost be origin of long-range force in Landau-gauge QCD?

Could Gribov parameter γ be origin of "magnetic mass"

go to p. 9

Dynamics of gluon - bose ghost

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^{2}$$ gluon

$$+ \frac{1}{2} \partial_\lambda V_\mu^{ab} (D_\lambda V_\mu)^{ab}$$ bose ghost

$$+ i\sqrt{2}\, \gamma^{1/2} f^{abc}\, g A_\lambda^{b} V_\lambda^{ca}$$ mixing term

3. What is long range behavior of bose ghost?

Of interest because bose ghost mixes with gluon

$$\mathcal{L}_\gamma = \gamma^{1/2} f^{abc} A^b_\lambda (\varphi - \bar{\varphi})^{ca}_\lambda + \cdots$$

$$\varphi = \frac{1}{\sqrt{2}} (U + iV)$$

$$\bar{\varphi} = \frac{1}{\sqrt{2}} (U - iV)$$

$$\mathcal{L}_V = \frac{1}{2} \partial_\lambda V^{ab}_\mu (D_\lambda V_\mu)^{ab}$$
$$+ i\sqrt{2}\, \gamma^{1/2} f^{abc} A^b_\lambda V^{ca}_\lambda$$

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5. Bose ghost propagator $\langle VV \rangle$

Define

$D_{VV}(k) \equiv$ F. T. $\langle V(x)\, V(y) \rangle$

The bose ghost V appears at most quadratically in action.

Integrate out V by Gaussian integration and use

$$\langle M^{-1}(x,y;A) \rangle = \langle c(x)\, \bar{c}(y) \rangle_A .$$

get local formula

Gaussian integration

$$\mathcal{L}_V = \tfrac{1}{2}\,\partial_\lambda V_\mu^{ab}\,(D_\lambda V_\mu)^{ab} + i\sqrt{2}\, f^{abc}\, g\, A_\lambda^b\, V_\mu^{ca}.$$

Shift V_λ^{ab} by

$$i\sqrt{2}\,\gamma^{1/2}\,(M^{-1} A_\lambda)^{ab}$$

$M = -\partial\cdot D$ = Faddeev-Popov

$$V = V' - i\sqrt{2}\,\gamma^{1/2}\,(M^{-1}A)$$

$$\langle V_x V_y \rangle = \langle V'_x V'_y \rangle - 2\gamma g^2 \langle (M^{-1}A)_x (M^{-1}A)_y \rangle$$

$$= \langle c(x)\,\bar{c}(y) \rangle - 2\gamma g^2 \int dz\, dw \times \langle c_x \bar{c}_z A_z \; c_y \bar{c}_w A_w \rangle$$

used $M^{-1}(x,y;A) = \langle c_x \bar{c}_y \rangle_A$

Cluster expansion

$$\langle c_x \bar{c}_z A_z \quad c_y \bar{c}_w A_w \rangle$$

$$= \langle c_x \bar{c}_z \rangle \langle c_y \bar{c}_w \rangle \langle A_z A_w \rangle$$

$$+ \cdots$$

Assume that leading infrared behavior is dominated by leading term in cluster expansion. Get

$$D_{vv}(k) = -2\gamma g^2 G^2(k) D(k) + \cdots$$

$$G(k) \sim \frac{1}{(k^2)^{1+\alpha_G}} \qquad \alpha_G > 0.$$

$$D(k) \sim \frac{1}{(k^2)^{1+\alpha_D}}$$

α_D, α_G infrared critical exponent of gluon and ghost

$$D_{vv}(k) \sim G^3(k)\, D(k)$$

$$= \frac{1}{(k^2)^{3+\alpha_D+2\alpha_G}}$$

From DSE get exact result

$$\alpha_D + 2\alpha_G = \frac{1}{2}(D-4)$$

$$D_{vv}(k) \sim \frac{1}{(k^2)^{1+D/2}}$$

$\alpha_V = D/2$ Exact critical exponent

$D=4$

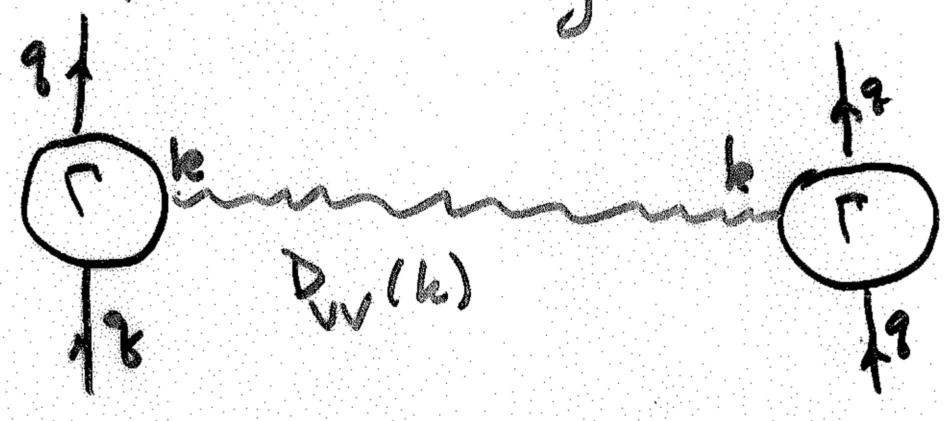
$$D_{vv}(k) \sim \frac{1}{(k^2)^6}$$

Linearly rising $\frac{1}{(k^2)^4}$ $\qquad \frac{1}{k^D} = \frac{1}{k^{5+1}}$

Too singular by $\frac{1}{k^2}$!

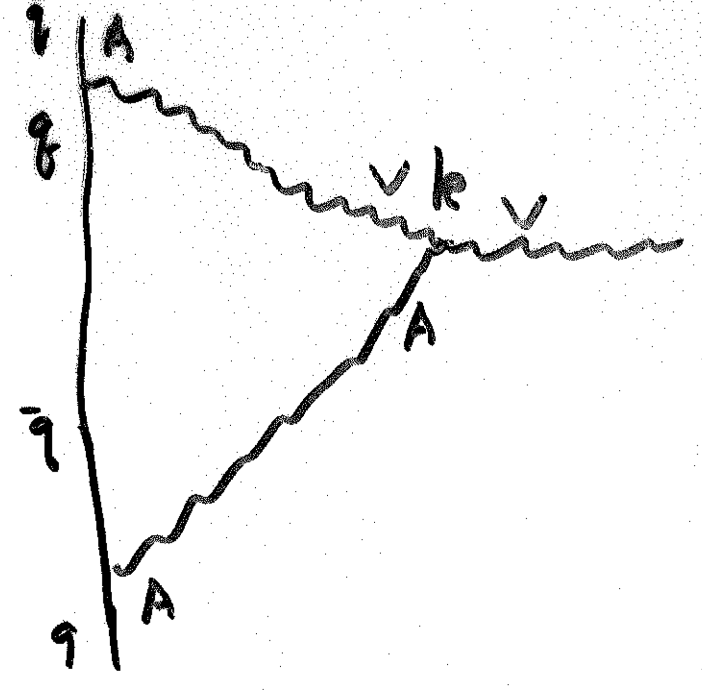
But ghost does not couple directly to quarks.

Couples indirectly



Quarks exchange V boson in adjoint representation

Example / q-V vertex



Theorem: factorization of external ghost momentum

$\Gamma_{qV\bar{q}}(k) \sim k\, X(k)$

Effective exchange $k\, k\, D_{VV}(k)$

$$\sim \frac{1}{k^{D}} = \frac{1}{k^{S+1}}$$

linearly rising in dimension D.

But we don't know $X(k)$ non-perturbatively
so V ghost is candidate
for confining force.