CHIRAL SYMMETRY AND MESON GASES

Angel Gómez Nicola
Universidad Complutense Madrid, Spain

- **Hot and dense light resonances (ρ, σ) and chiral restoration**
- **Chemical nonequilibrium for interacting pions**

AGN, F.Llanes-Estrada, J.R.Peláez PLB550 (2002) 55; PLB606 (2005) 351.
A.Dobado, AGN, F.Llanes-Estrada, J.R.Peláez PRC66 (2002) 055201.
D.Fernández-Fraile, AGN, E.T.Herruzo PRD76 (2007) 085020.
D.Cabrera, D.Fernández-Fraile, AGN EPJC61 (2009) 879.
D.Fernández-Fraile, AGN PRD (2009).

LIGHT RESONANCES AND CHIRAL RESTORATION

- ρ **(770): Dileptons in RHIC and nuclear matter.**
  Broadening vs mass scaling, ρ-a_1 degeneracy, …

Rapp, Wambach, van-Hees

LIGHT RESONANCES AND CHIRAL RESTORATION

- **ρ (770): Dileptons in RHIC and nuclear matter.**
  Broadening vs mass scaling, ρ-a_1 degeneracy, … Rapp, Wambach, van-Hees

- **f_0 (600) (σ): Vacuum probe for chiral restoration.**

Chiral partner of π $O(3) \rightarrow O(4)$?

Threshold enhancement ?

$f_0(600)$ [i]
or σ

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400–1200)$ MeV
Full width $\Gamma = (600–1000)$ MeV

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- **$M_\sigma \sim \langle\sigma\rangle \downarrow$ (chiral limit) $\Rightarrow \Gamma_\sigma \rightarrow 0$ ($\sigma \rightarrow \pi\pi$) @ $M_\sigma \sim 2m_\pi \Rightarrow \pi\pi$ scattering&production enhanced in I=J=0 channel.**
  assuming $\sigma \approx \bar{q}q$ narrow state! Hatsuda, Kunihiro '85

- **Experimental signals in nuclear matter: $\pi A \rightarrow \pi\pi A'$ (CHAOS,CB), $\gamma A \rightarrow \pi\pi A'$ (MAMI-B).**

- **Compatible with many-body nuclear density analysis.** Davesne et al '00, Roca et al '02

- **Finite *T*? (Heavy Ions).** Patkos et al '02, Hidaka et al '04

OUR APPROACH: UNITARIZED CHIRAL PERTURBATION THEORY

A.Dobado, D.Cabrera, AGN, F.J.Llanes-Estrada, J.R.Peláez, D.Fernández-Fraile, E.Tomás-Herruzo

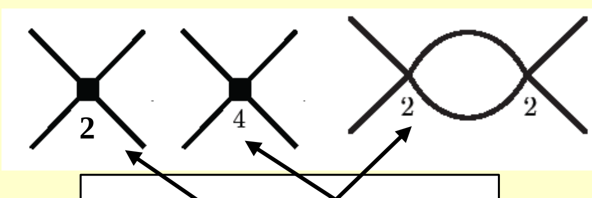
CHIRAL SYMMETRY

$\pi\pi$ scattering and $\pi\pi\gamma$ form factors in $T > 0$

$SU(2)$ one-loop **ChPT** $\rightarrow$ **MODEL INDEPENDENT**

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots$$

$$D = 2 + \sum_n N_n(n-2) + 2L \; .$$



$$t^{IJ} = t_2^{IJ} + t_4^{IJ} + \cdots$$

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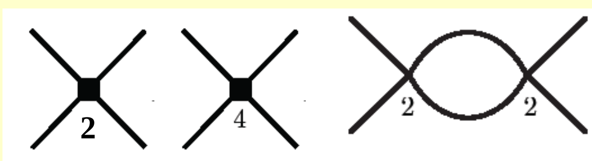


$$t^{IJ} = t_2^{IJ} + t_4^{IJ} + \cdots$$

+

UNITARITY

Inverse Amplitude Method

$$\mathrm{Im}\left[t^{IAM}\right]^{-1} = -\sigma_T$$

$$\sigma_T(E) = \sqrt{1 - \frac{4m_\pi^2}{E^2}}\left[1 + 2n_B(E/2)\right]$$

Thermal phase space (Bose enhanced)

$$t^{IAM}(s;T) = \frac{\left[t_2(s)\right]^2}{t_2(s) - t_4(s;T) + A(s;T)}$$

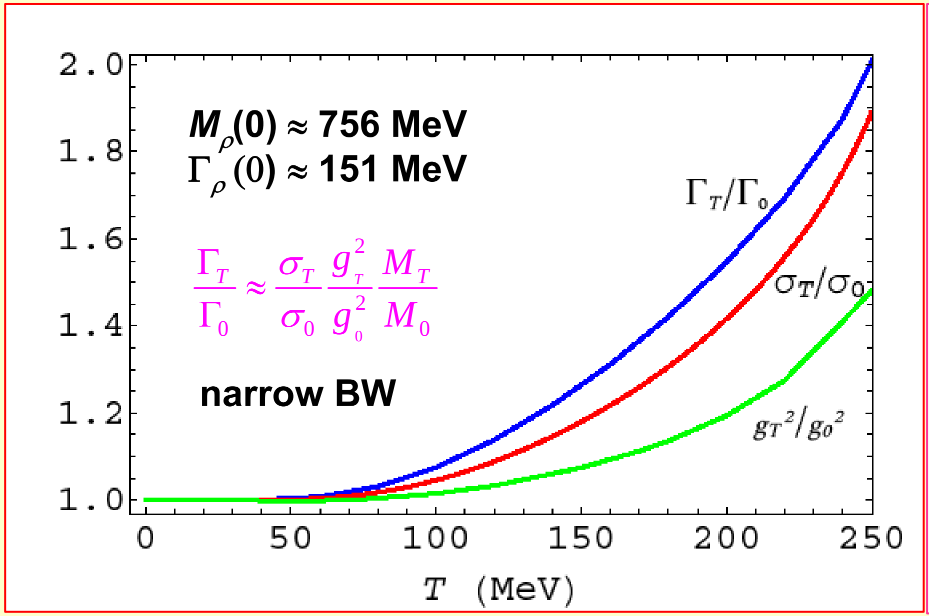
Properties of thermal resonances without initial assumptions about their nature → dynamically generated (at rest).

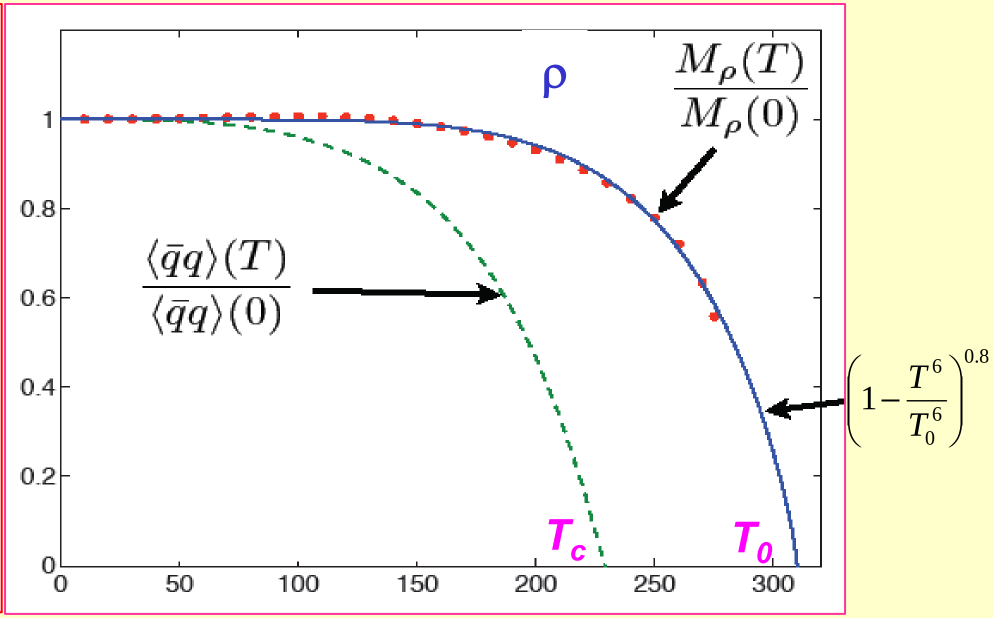
Sucessful at *T*=0 for scattering data up to 1 GeV & low-lying resonance multiplets.

Dobado, Peláez, Oset, Oller, AGN.

ρ(770) (I=J=1)

$s_{pole} = (M - i\Gamma/2)^2$ **(2nd Riemann sheet)**





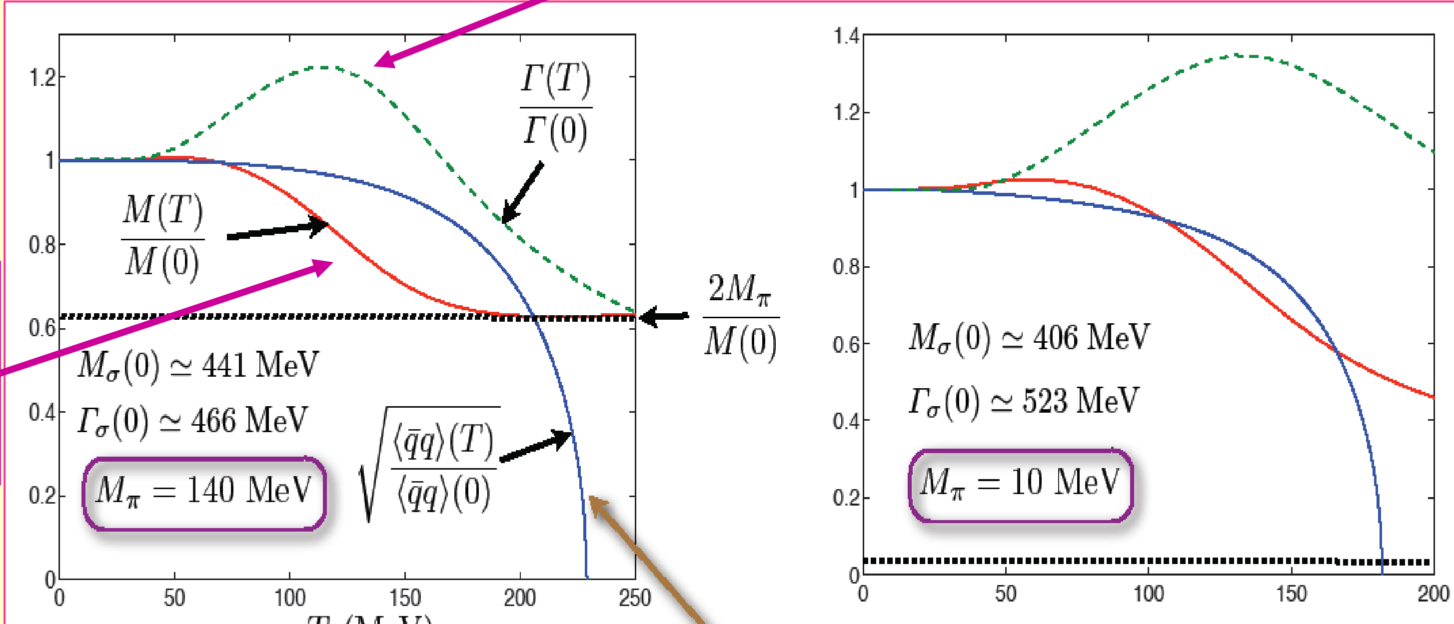
- **Broadening dominant: Bose ph.space** $\frac{\sigma_T}{\sigma_0} = 1 + 2n_B(M_\rho/2)$ **+ $\rho\pi\pi$ vertex g_T^2 .**
- **OK with dileptons (NA60 $\mu^+\mu^-$) & VMD.**
- **Extrap. Mass drops as in BR-HLS models (**Brown&Rho '05, Harada&Sasaki '06**) only near $T_0 > T_c$. No scaling with condensate. Broadening nature no BR-like.**

f_0(600)
(I=J=0)

$$s_{pole} = (M - i\Gamma/2)^2$$

Γ overcomes low-*T* Bose enhancement

Strong pole
mass reduction
(chiral restoration)



For the O(4) model: $m_\sigma \sim \langle\sigma\rangle \sim F_\pi \sim \langle\bar{q}q\rangle^{1/2}$.

f_0(600)
(I=J=0)

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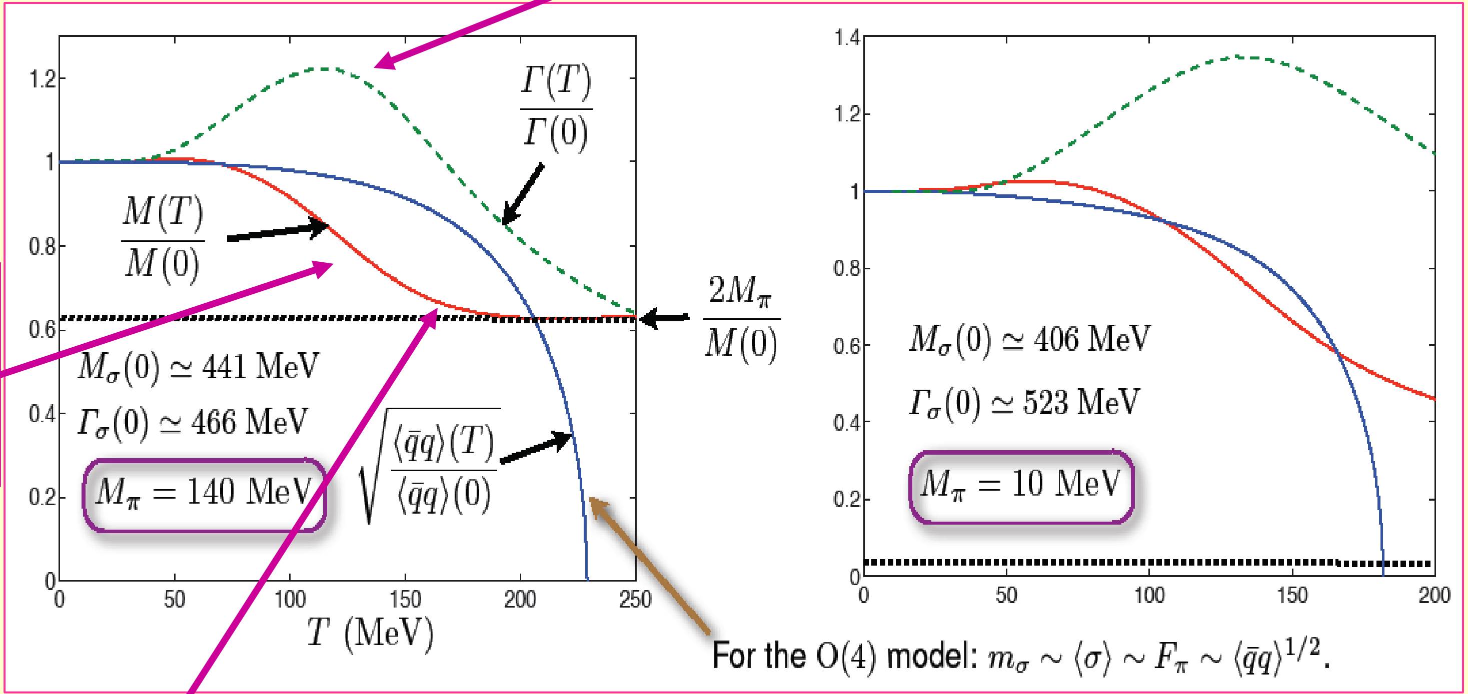


Remains wide for $M \sim 2m_\pi$
Spectral function not peaked around *M*

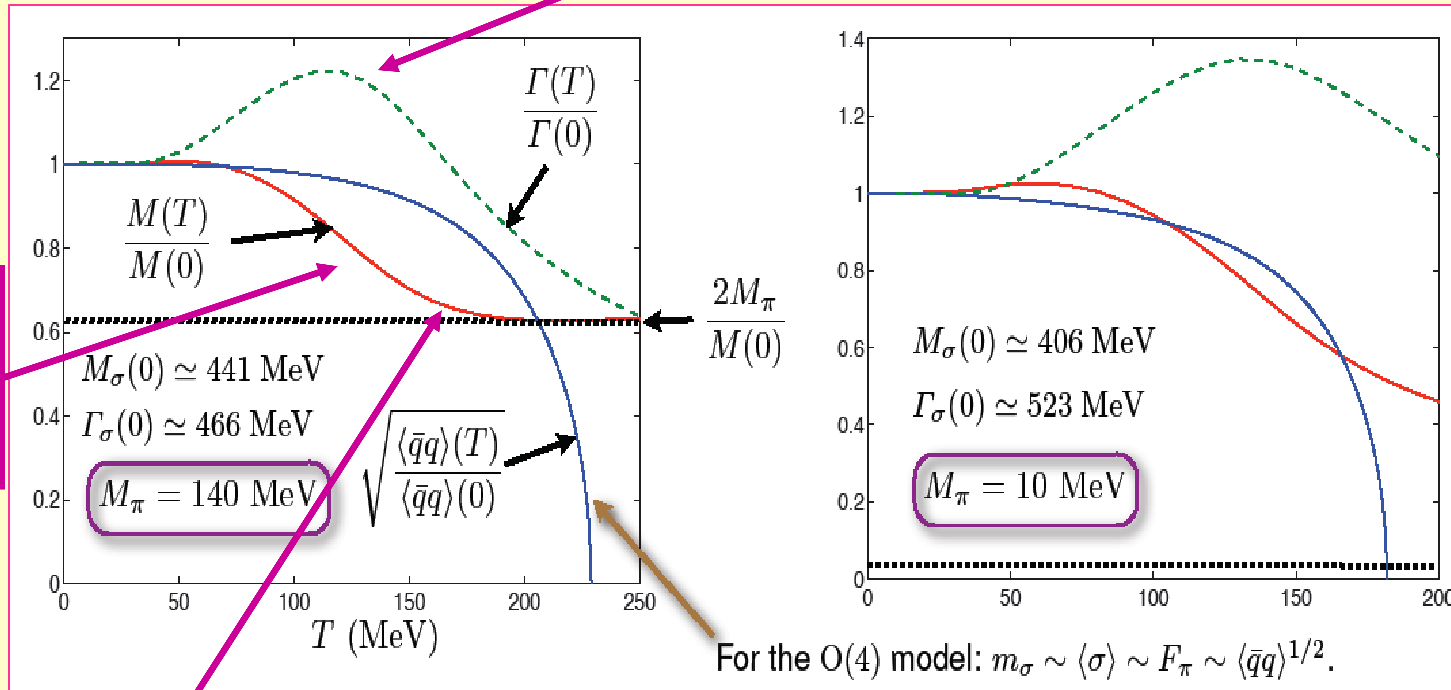
No finite-*T* $\pi\pi$ threshold enhancement

f_0(600) (I=J=0)

$s_{pole} = (M - i\Gamma/2)^2$

Γ overcomes low-T Bose enhancement

Strong pole mass reduction (chiral restoration)



Remains wide for M~$2m_\pi$

Spectral function not peaked around M

No finite-T $\pi\pi$ threshold enhancement

- **Does not behave as a pure (thermal) $\bar{q}q$ state, not even near the chiral limit.**
- **Expected to be a not-$\bar{q}q$ scalar "molecule" nonet member.**

Alford,R.L.Jaffe '00, Peláez '04

Nuclear density: chiral restoring

At T=0, $\rho_N \neq 0$ approximately encoded in f_π

$$\frac{f_\pi^2(\rho_N)}{f_\pi^2(0)} \approx \frac{\langle \bar{q}q \rangle(\rho_N)}{\langle \bar{q}q \rangle(0)} \approx 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2(0)} \rho_N = 1 - 0.35 \frac{\rho_N}{\rho_0}$$

Thorsson,Wirzba '95
Meissner,Oller,Wirzba '02

$$\mathrm{GOR} \rightarrow m_\pi^2 f_\pi^2 \approx -m_q \langle \bar{q}q \rangle \ \ (m_\pi \mathrm{const})$$

$$\sigma_{\pi N} \approx 45\ \mathrm{MeV}$$

$$\rho_0 \approx 0.17\ \mathrm{fm}^{-3}$$

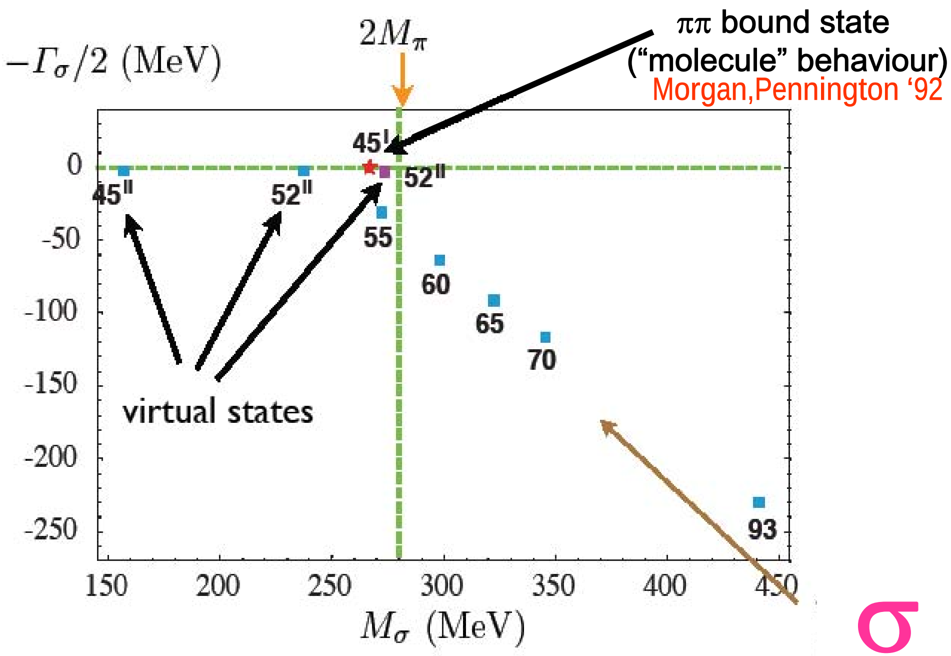
- **Important in the σ-channel as density approaches chiral restoration**

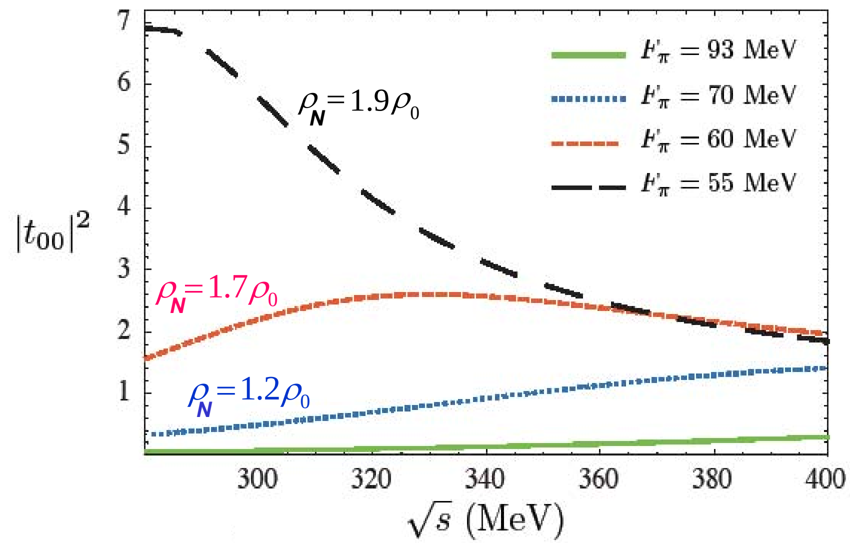
No broadening to compete with now !

- **Many-body effects not included (p-h, Δ-h, *p*-wave π self-energy, …)**

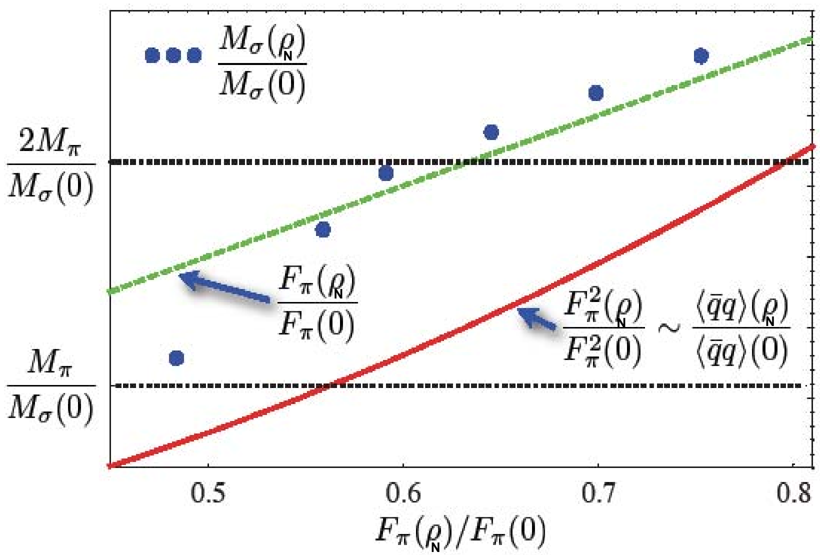
Chanfray et al '91
Chiang et al '98
Cabrera et al '05

$\langle \bar{q}q \rangle = 0$ at $\rho_N \approx 2.8\rho_0$





σ

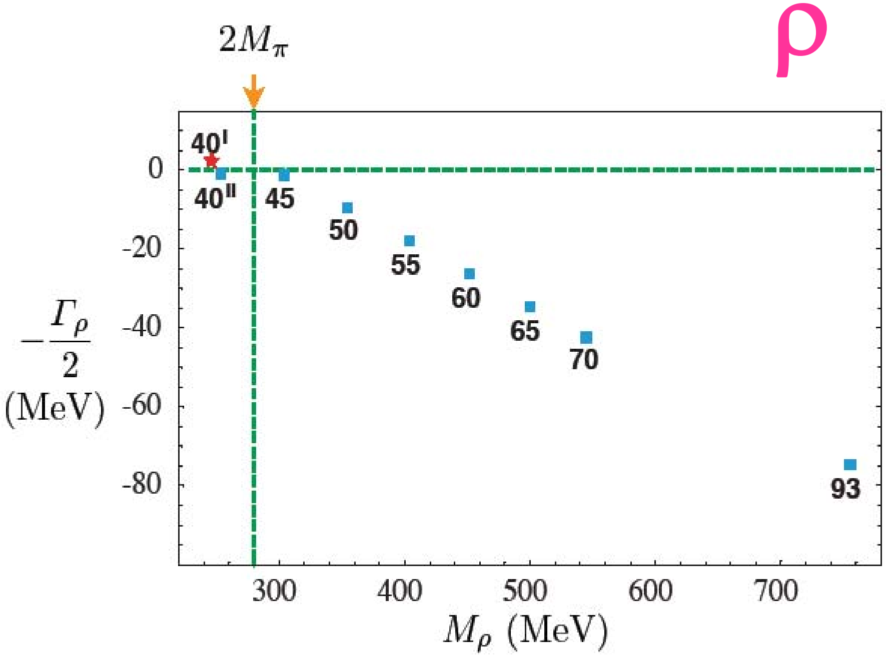


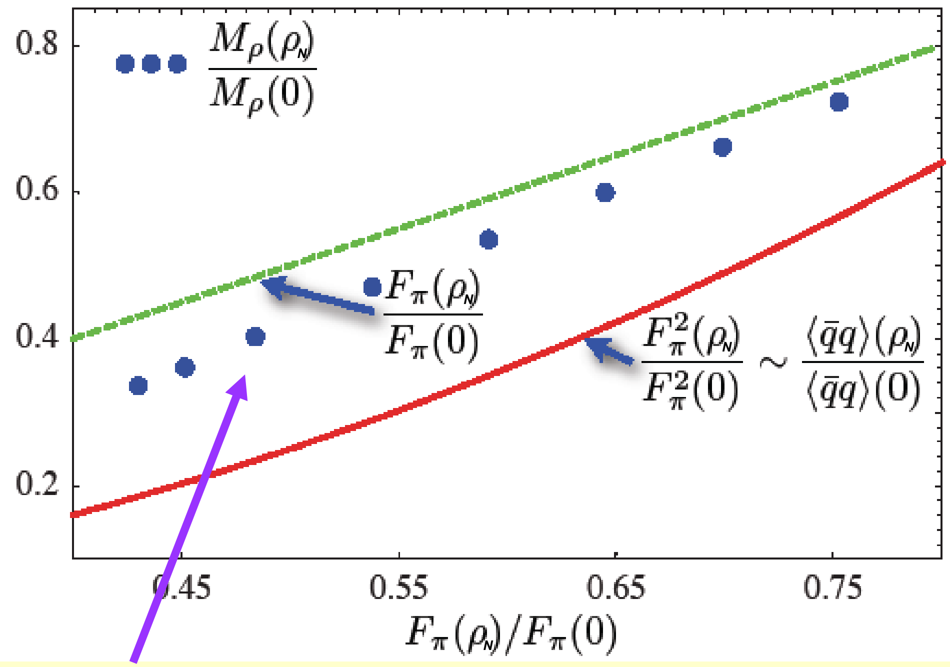
Threshold enhancement qualitatively compatible with experimental results of reactions $\pi A \to \pi\pi A'$ and $\gamma A \to \pi\pi A'$.

The σ pole approximately follows the condensate curve.

For high densities, a virtual $\bar{q}q$-like state coexists with a $\pi\pi$ bound state (compatible with other analyses).

ρ





Compatible with BR-like scaling Brown,Rho '04

Mass linear fit up to $\rho_N \sim \rho_0$:

$$\frac{M(\rho_N)}{M(0)} \approx 1 - \alpha \frac{\rho_N}{\rho_0} \quad \longrightarrow \quad \alpha \approx 0.2$$

Dileptons in nuclear matter:

KEK-E325 (C,Fe-Ti): α **= 0.092±0.002**

Jlab-CLAS (C,Cu): α **= 0.02±0.02**

Scaling (ch.sym.rest) &QCD sum rules $\longrightarrow$ $\alpha \approx 0.1 - 0.2$ Brown&Rho '91, Hatsuda&Lee '92

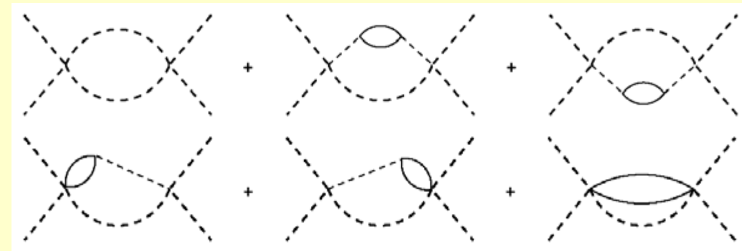
Many-body analysis (broadening) $\longrightarrow$ $\alpha \approx 0$

Peters et al '98, Herrman et al '92, Cabrera et al '02

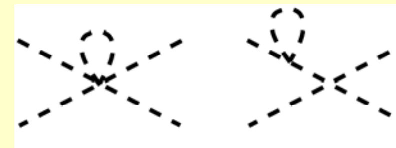
Nuclear density: Many body approach (σ chann)

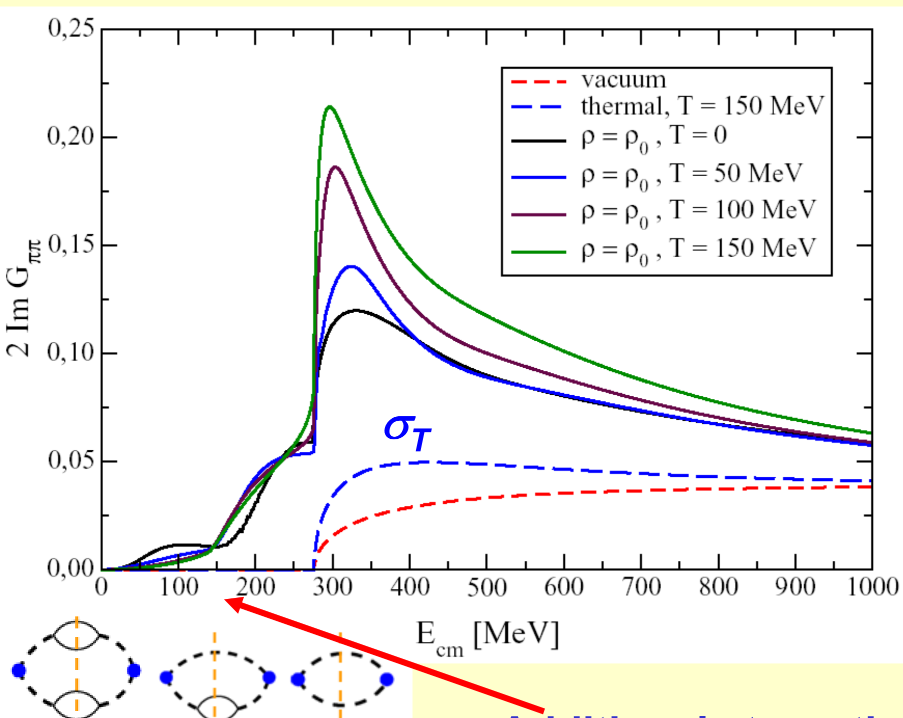
BS $\pi\pi$ scattering + π *SE in nuclear medium*

E. Oset et al.



- *p-wave ph*, Δh + short-range corr. + vertex corrections from ch.sym.
- *Finite-T tadpoles.*





$O(\rho_N^2)$ $O(\rho_N)$ $O(1)$

Additional strength below threshold

- Finite-*T* and f_π scaling at $\rho_N = 0$ OK with IAM.
- π dynamics in medium accelerates migration of σ pole to 2π threshold.
- Important threshold enhancement but sizable Γ_σ at 2π threshold

(Bose + in-med baryon σ-decay channels)

$M_\sigma \approx 2m_\pi$, $\Gamma_\sigma \approx 300$ MeV @ $\rho_N = \rho_0$ $T = 100$ MeV

CHEMICAL NONEQUILIBRIUM FOR INTERACTING PIONS

D.Fernández-Fraile,AGN PRD '09

- Inelastic processes $\pi\pi \leftrightarrow \pi\pi\pi\pi$ strongly suppressed

Bebie et al, '92
Song, Koch, '97
Braun-Munzinger et al '03

for T_{TFO}~100-120 MeV < T < T_{CFO} ~ 160-180 MeV:

• Pion gas in thermal but not chemical equilibrium

• TOTAL pion number $N_{\pi 0}+N_{\pi +}+N_{\pi -}$ approximately conserved $\Rightarrow \mu_\pi \neq 0$

- $\mu_\pi \sim$ 70-100 MeV at TFO well supported at SPS and RHIC energies:

Fits of low-p_T π spectrum — Gavin&Ruuskanen '91, Hung&Shuryak, '98, Kolb&Rapp, '03

Total particle yields and yield ratios — Letessier&Rafelski '08

- A **real-time** formalism (Keldysh-like) can be constructed to describe the ***interacting*** pion gas at chemical non-equilibrium in ChPT:

$$[H, N_\pi] \simeq 0$$

$$\mu_\pi < m_\pi$$

Holomorphic PI

$$E_p = \sqrt{|\vec{p}|^2 + m_\pi^2}$$

$$\begin{aligned} G_{11}(p) &= \frac{i}{p_0^2 - E_p^2 + i\epsilon} + 2\pi\delta(p_0^2 - E_p^2) n_B(|p_0| - \mu_\pi) \\ G_{22}(p) &= \frac{-i}{p_0^2 - E_p^2 + i\epsilon} + 2\pi\delta(p_0^2 - E_p^2) n_B(|p_0| - \mu_\pi) \\ G_{12}(p) &= 2\pi\delta(p_0^2 - E_p^2)\left[\theta(-p_0) + n_B(|p_0| - \mu_\pi)\right] = G_{21}(-p) \end{aligned}$$

- **Imaginary-time** ill-defined. Nonequilibrium loss of KMS:

$$\Delta_T(\tau + \tilde{\beta}_p, p) = \Delta_T(\tau, p) \neq \Delta_T(\tau + \beta, p)$$

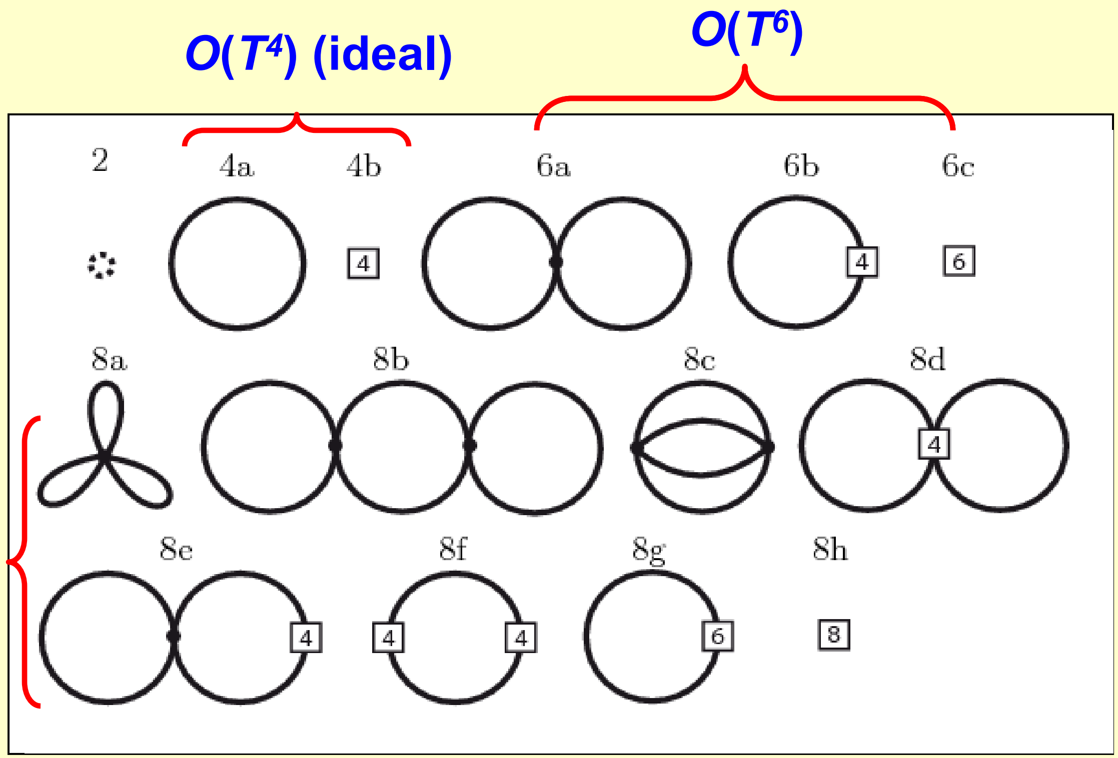
$$\tilde{\beta}_p \equiv \beta\left(1 - \frac{\mu_\pi}{E_p}\right)$$

Matsubara rep. only for $\tau \in [-\tilde{\beta}_p, \tilde{\beta}_p] < [-\beta, \beta]$

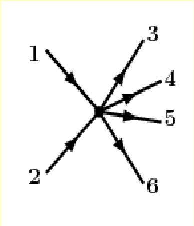
$P(T, \mu_\pi(T))$ **in this phase calculable in RT identifying external vertices as type 1**

Matsumoto,Nakano,Umezawa '85

ChPT

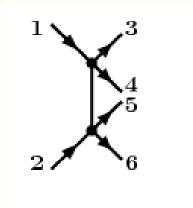


At $O(T^8)$ particle number changing becomes effective:



1↔2
3↔4
5↔6

8a



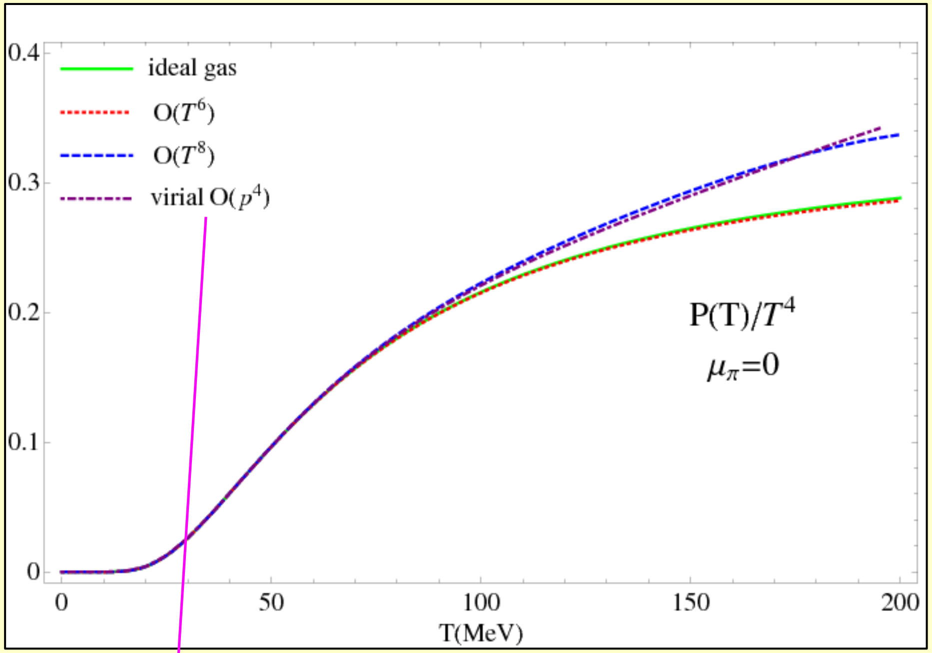
1↔2
3↔6
4↔5

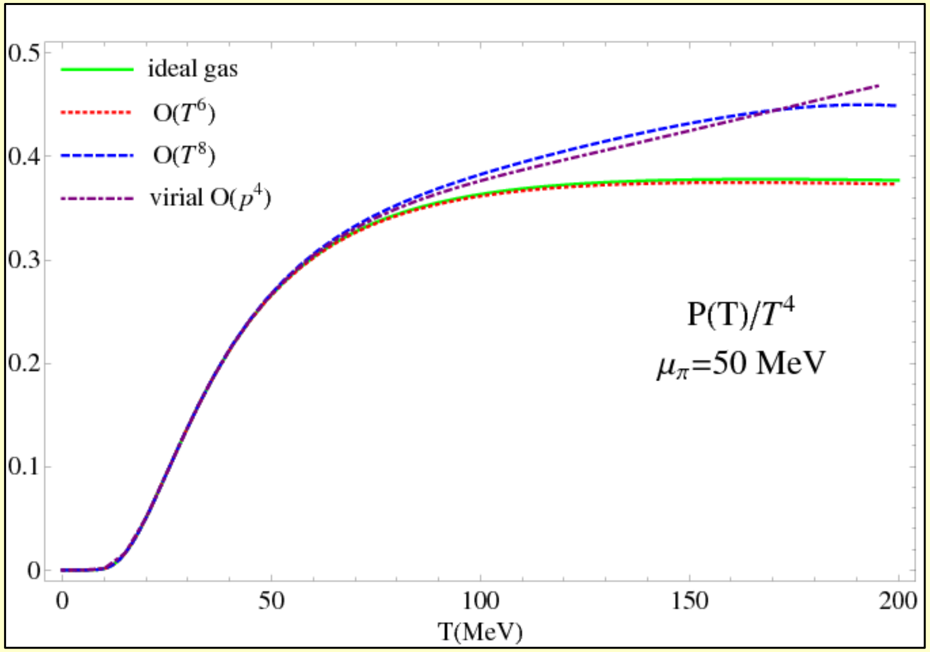
8c

Pert. Scheme consistent if:

$$\mu_\pi(T) \to 0$$

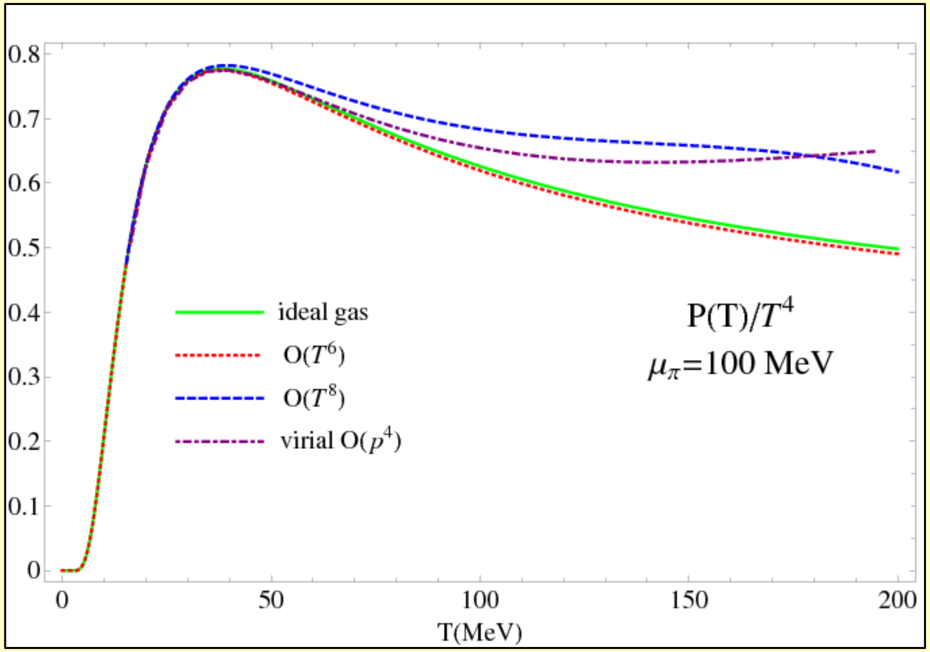
$T \to T_{CFO}$



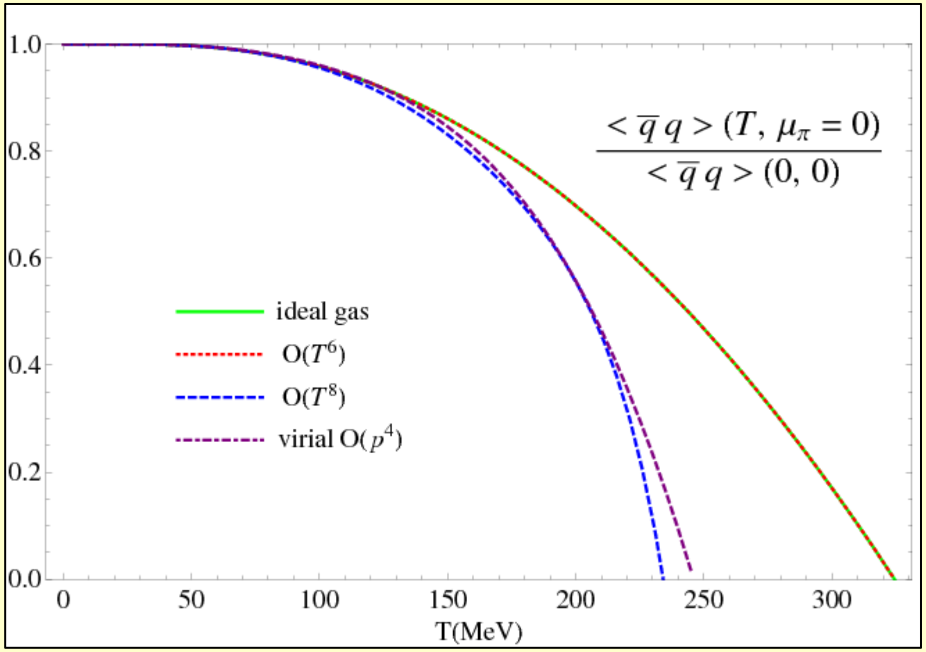


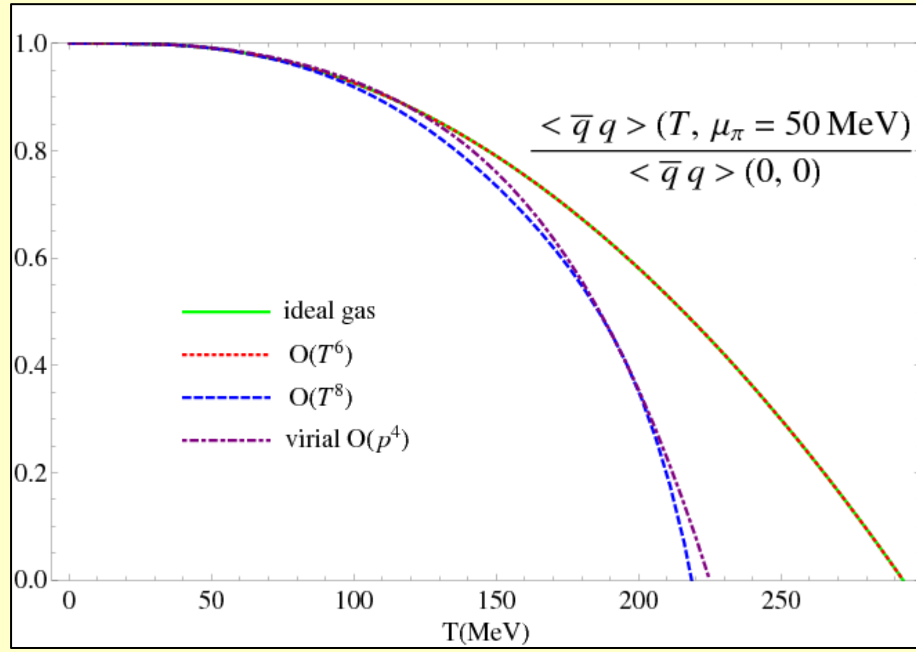
P **in dilute regime in terms of ππ phase shifts**

Dobado, Peláez '99

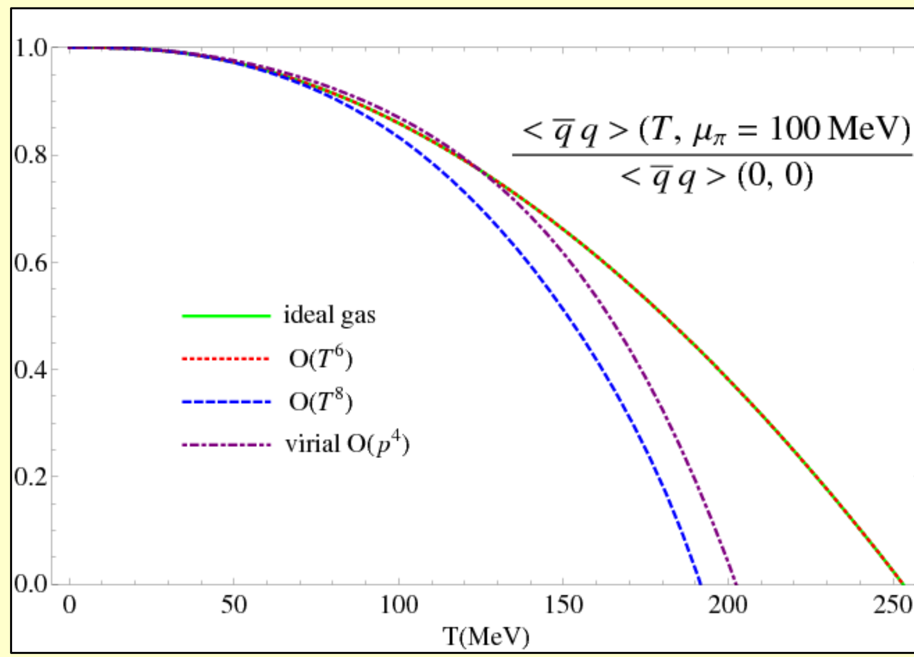


Interactions and μ_π enhance thermal effects (higher *P*, lower quark condensate)





$$\langle\bar{q}q\rangle_T = \langle\bar{q}q\rangle_0 + \frac{\partial P}{\partial m_q}$$

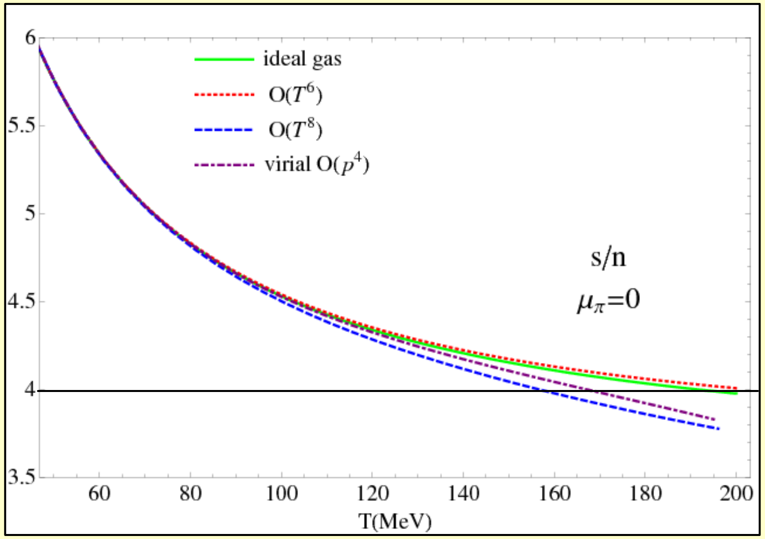


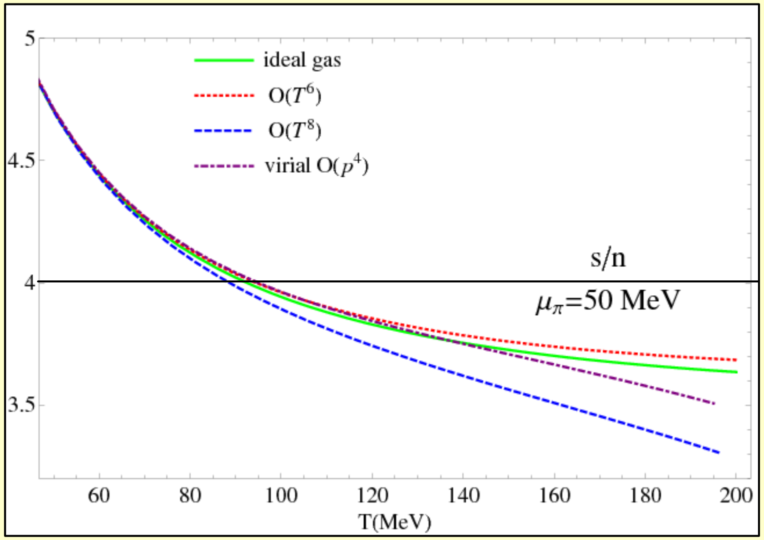
T_c↓ significant only if $T_{CFO} > T_{\chi SR}$

Isentropic approximation ⇒ $s/n \sim$ const for $T_{TFO} < T < T_{CFO}$

Bebie et al, '92

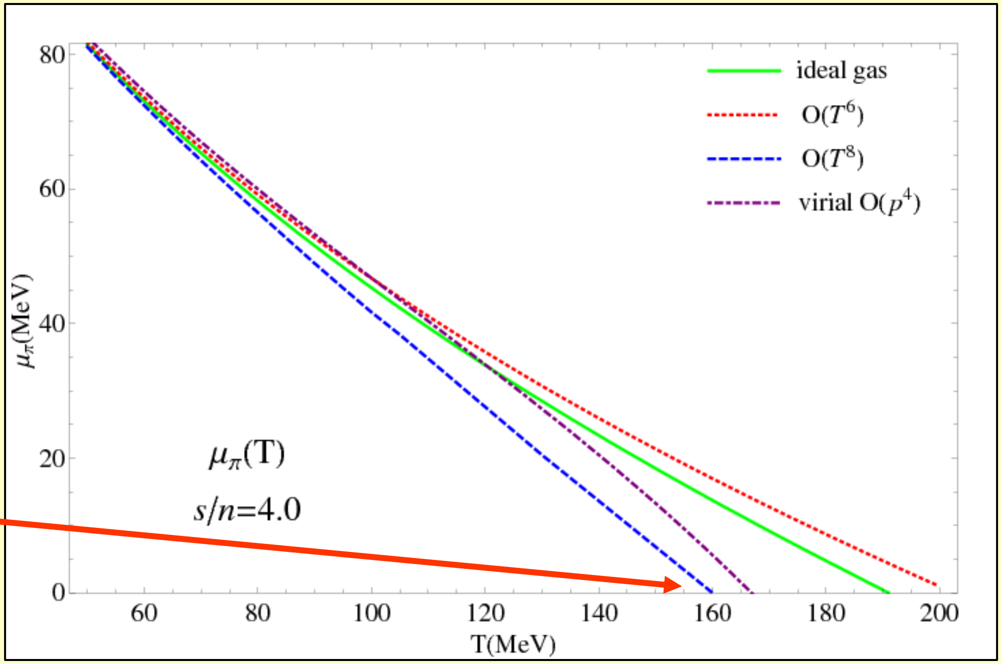
Adiabatic local thermal eq. w/o dissipation (hydro) + particle number conservation

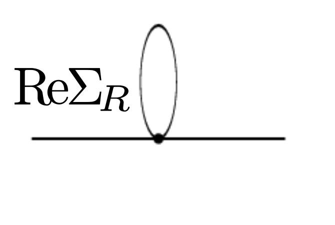




$\mu_\pi(T)$ buils up to maintain s/n constant (only π component)

$\Delta T_{CFO} \sim -25$ MeV by π interactions





Self-energy (thermal mass)

$$m_\pi^2(T,\mu_\pi) - m_\pi^2 = -\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{2E_p} n_B(E_p - \mu_\pi)\mathrm{Re}\left[T^f_{\pi\pi}(s=(E_p+m_\pi)^2 - |\vec{p}|^2)\right]$$

Luscher-type formula (dilute regime)

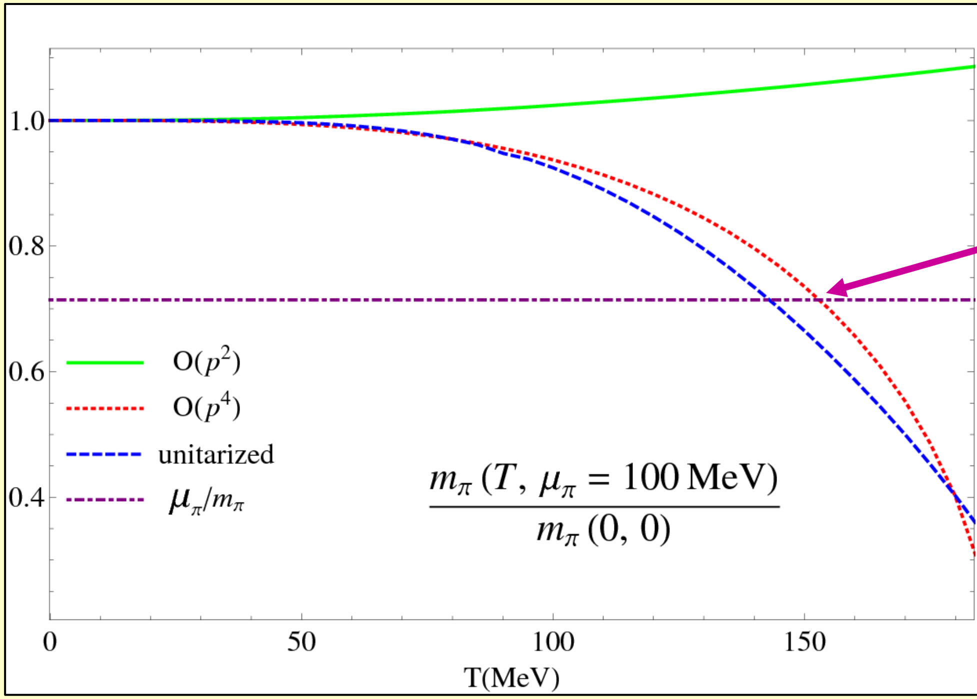
Luscher '86

$$T^f_{\pi\pi}(s) = \frac{32\pi}{3}\sum_{I=0}^{2}\sum_J (2I+1)(2J+1)\, t_{IJ}(s)$$ → **allows h.o. or unitarized extension**

forward scattering amplitude (*T*=0)

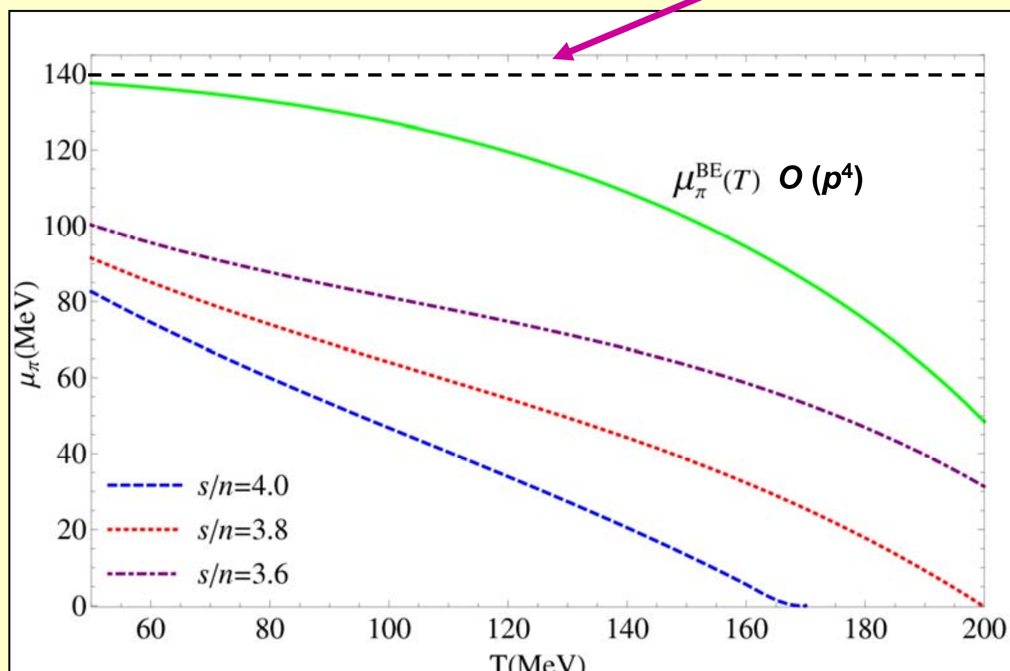
$$\frac{f_\pi^2(T,\mu_\pi)m_\pi^2(T,\mu_\pi)}{\langle\bar{q}q\rangle(T,\mu_\pi)} = \frac{f_\pi^2(0,0)m_\pi^2(0,0)}{\langle\bar{q}q\rangle(0,0)} = -m_q$$

GOR (one-loop ChPT)

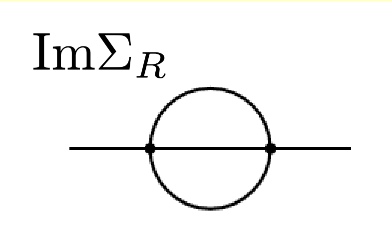


BE condensation of quasi-pions π^0 & $\pi^\pm$ by dropping of thermal mass driven by interactions

Ideal gas



Pions closer to BEC for realistic CFO conditions



Self-energy (thermal width)

$$\Gamma_p = -\frac{\mathrm{Im}\,\Sigma_R(E_p,|\vec{p}|)}{2E_p} = \frac{i}{4E_p}\left[\Sigma_{21}(E_p,|\vec{p}|) - \Sigma_{12}(E_p,|\vec{p}|)\right]$$

Kobes '90

$$\Gamma_p(T,\mu_\pi) = \frac{1}{8E_p}\frac{1}{1+n_B(E_p-\mu_\pi)}\int\prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3 2E_i} n_B(E_1-\mu_\pi)\left[1+n_B(E_2-\mu_\pi)\right]\left[1+n_B(E_3-\mu_\pi)\right]$$
$$\times\ |T_{\pi\pi}(s,t)|^2(2\pi)^4\delta(E_p+E_1-E_2-E_3)\delta^{(3)}(\vec{p}+\vec{k}_1-\vec{k}_2-\vec{k}_3)$$

$$s = (E_p+E_1)^2 - |\vec{p}+\vec{k}_1|^2$$
$$t = (E_p-E_2)^2 - |\vec{p}-\vec{k}_2|^2$$

as expected from kinetic theory (Goity&Leutwyler '89)

Dilute Gas

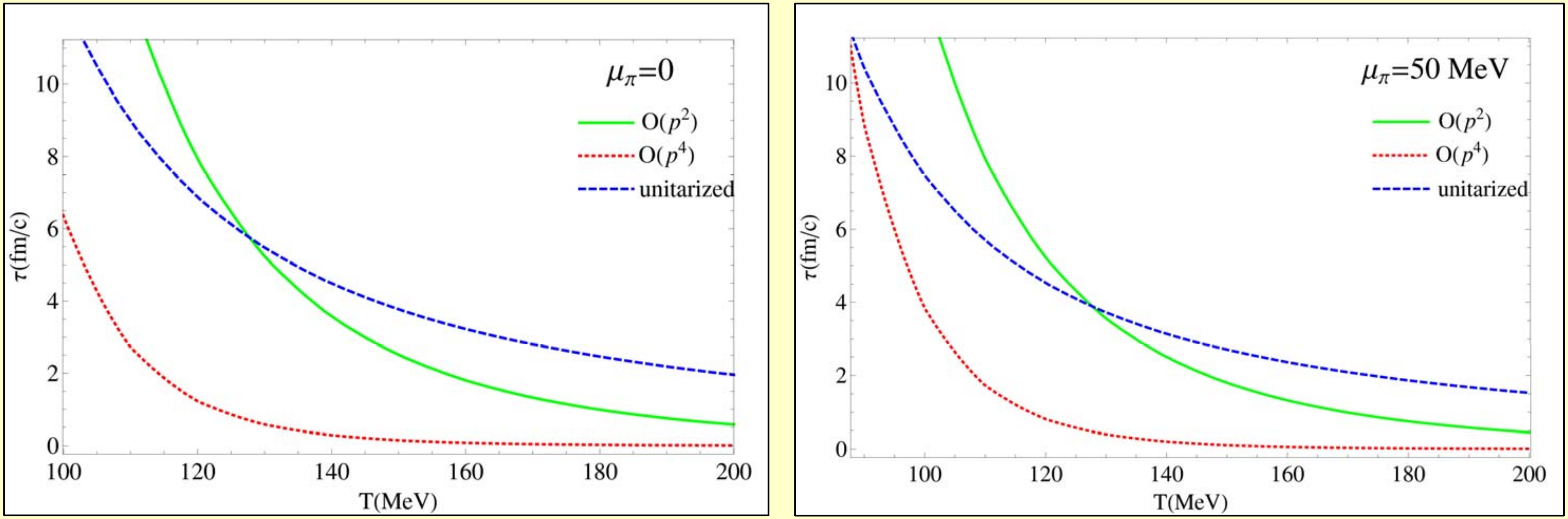
$n_B \ll 1$

$$\Gamma_p^{DG}(T,\mu_\pi) = \frac{1}{2E_p}\int\frac{d^3\vec{k}_1}{(2\pi)^3} n_B(E_1-\mu_\pi)\frac{\sqrt{s(s-4m^2)}}{2E_1}\sigma_{\pi\pi}(s)$$
$$= \frac{1}{2E_p}\int\frac{d^3\vec{k}_1}{(2\pi)^3 2E_1} n_B(E_1-\mu_\pi)\mathrm{Im}\,T^f_{\pi\pi}(s)$$

Luscher-type

Mean collision time $\tau = 1/(2\bar{\Gamma})$

$$\bar{\Gamma}(T,\mu_\pi) = \frac{\int d^3\vec{p}\ \ \Gamma_p(T,\mu_\pi) n(E_p-\mu_\pi)}{\int d^3\vec{p}\, n(E_p-\mu_\pi)}$$



- **Im $T_{\pi\pi}$ more sensitive to unitariz. $\Rightarrow$ crucial for transport coeff. $\sim 1/\Gamma$**

D.Fernández-Fraile&AGN

- $\tau\downarrow$ **as** $\mu_\pi\uparrow \Rightarrow \Delta T_{TFO} \sim -20$ **MeV with** $\tau(T_{TFO}) \sim$ **10 fm/c and isentropic** $\mu_\pi(T)$**.**

CONCLUSIONS

★ **Scattering poles in Unitarized ChPT provide chiral symmetry predictions for the spectral properties and nature of in-medium light meson resonances :**

Finite *T*: f_0(600)/σ migrates to 2π threshold (chiral restoration) but remains wide, not-$\bar{q}q$ $\Rightarrow$ no threshold enhancement. $\rho(770)$ dominated by thermal broadening in agreement with dilepton data. Mass dropping does not scale with the condensate.

T=0 **$f_\pi(\rho_N)$ scaling *(chiral-restoring)* drives poles to real axis $\Rightarrow$ thres. enhancement, "molecular" σ *gives* $\pi\pi$ bound state and *coexists with* π*-partner*. BR-like ρ.**

Many-body analysis provides additional decay channels for π and σ: strength below threshold and sizable σ width.

★ **Chemical nonequilibrium phase can be described within $\mu_\pi \neq 0$ ChPT:**

π interactions at $\mu_\pi \neq 0$ reduce T_{CFO} *and* T_{TFO}.

π self-energy obbeys GOR and Luscher-like extensions.

BEC accesible via π-mass dropping