Shear viscosity and the r-mode instability window in superfluid neutron stars

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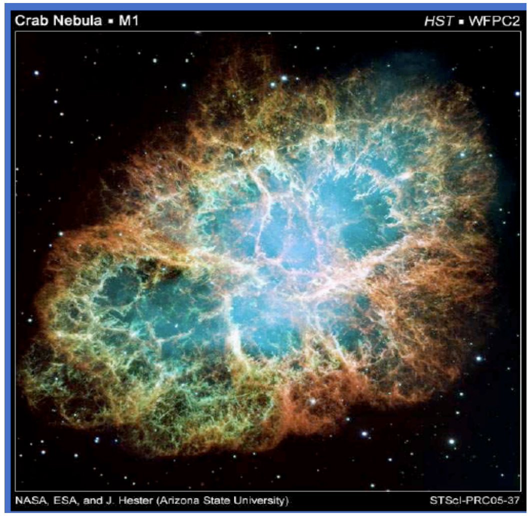
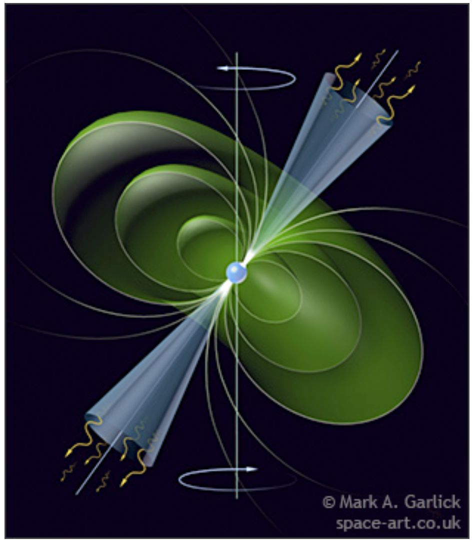
Outline

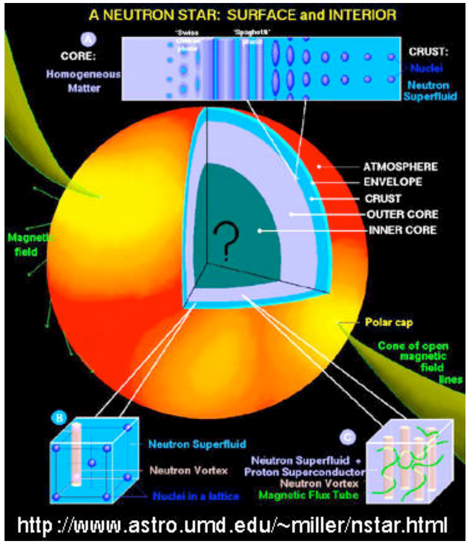
- ✧ Neutron star and r-mode instability window
- ✧ EFT and superfluid phonon
- ✧ EoS for superfluid neutron star matter
- ✧ Shear viscosity due to superfluid phonons
- ✧ r-mode instability window
- ✧ Summary

Manuel and Tolos, Physical Review D 84 (2011) 123007
Manuel and Tolos, arXiv: 1212.2075 [astro-ph.SR]

Neutron star







- produced in core collapse supernova explosions

- encompass not only "normal" stars but also "strange quark" stars

- usually refer to compact objects with $M \approx 1\text{-}2\ M_{\odot}$ and $R \approx 12$ Km

- extreme densities up to 5-10 times nuclear density ρ_0 ($\rho_0 = 0.16\ \mathrm{fm}^{-3} = 3 \bullet 10^{14}\ \mathrm{g/cm}^3$)

- usually observed as pulsars

- magnetic field : $B \sim 10^{8..16}$ G

- temperature: $T \sim 10^{6...11}$ K

Internal Structure and Composition

- Atmosphere

few tens of cm, $\rho \leq 10^4$ g/cm^3 made of atoms

- Outer crust or envelope

few hundred m's, $\rho=10^4-4 \bullet 10^{11}$ g/cm^3 made of free e^- and lattice of nuclei

- Inner crust

1-2 km, $\rho=4 \bullet 10^{11}-10^{14}$ g/cm^3 made of free e^-, neutrons and neutron-rich atomic nuclei

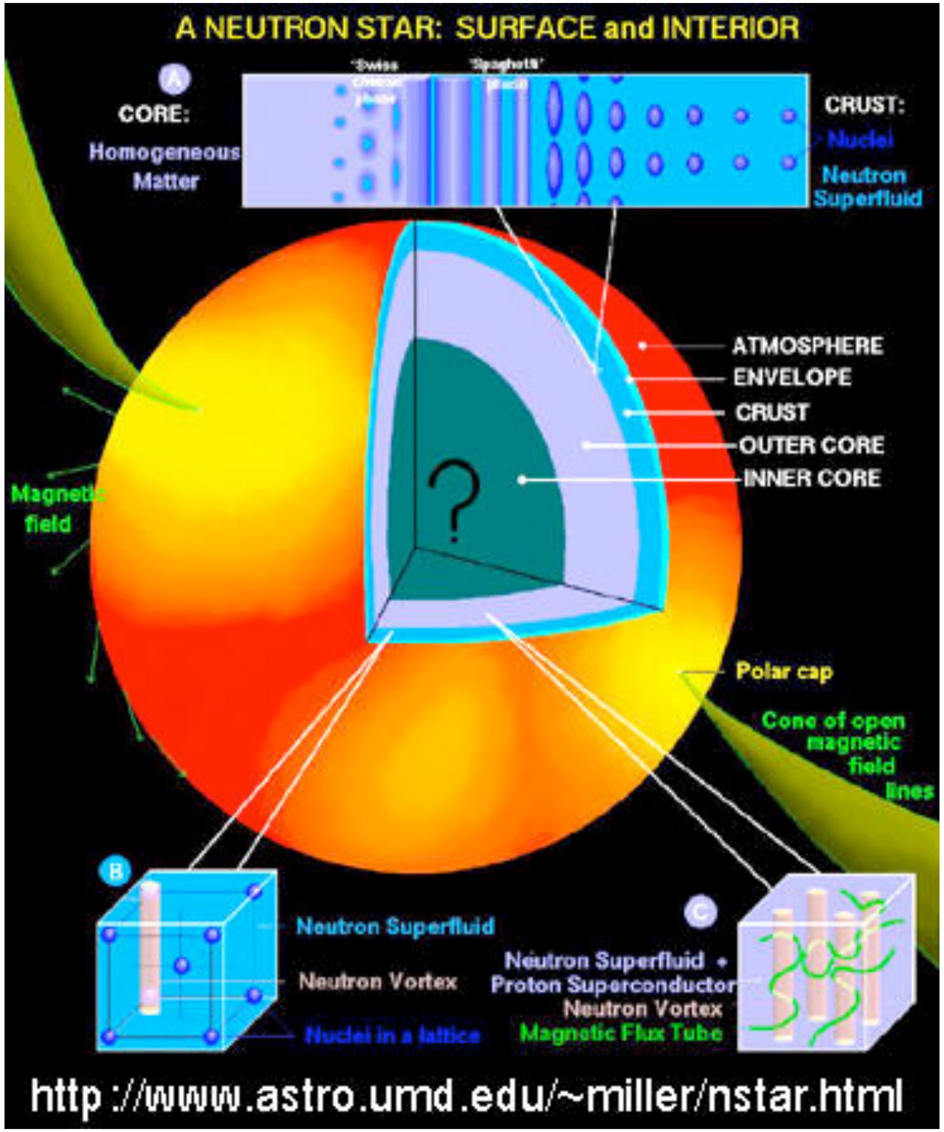
$\rho_0/2$: uniform fluid of n,p,e^-

- Outer core

$\rho_0/2-2\rho_0$ is a soup of n,e^-,μ and possible neutron $^3P_2$ superfluid or proton $^1S_0$ superconductor

- Inner core

2-10 ρ_0 with unknown interior made of hadronic, exotic or deconfined matter

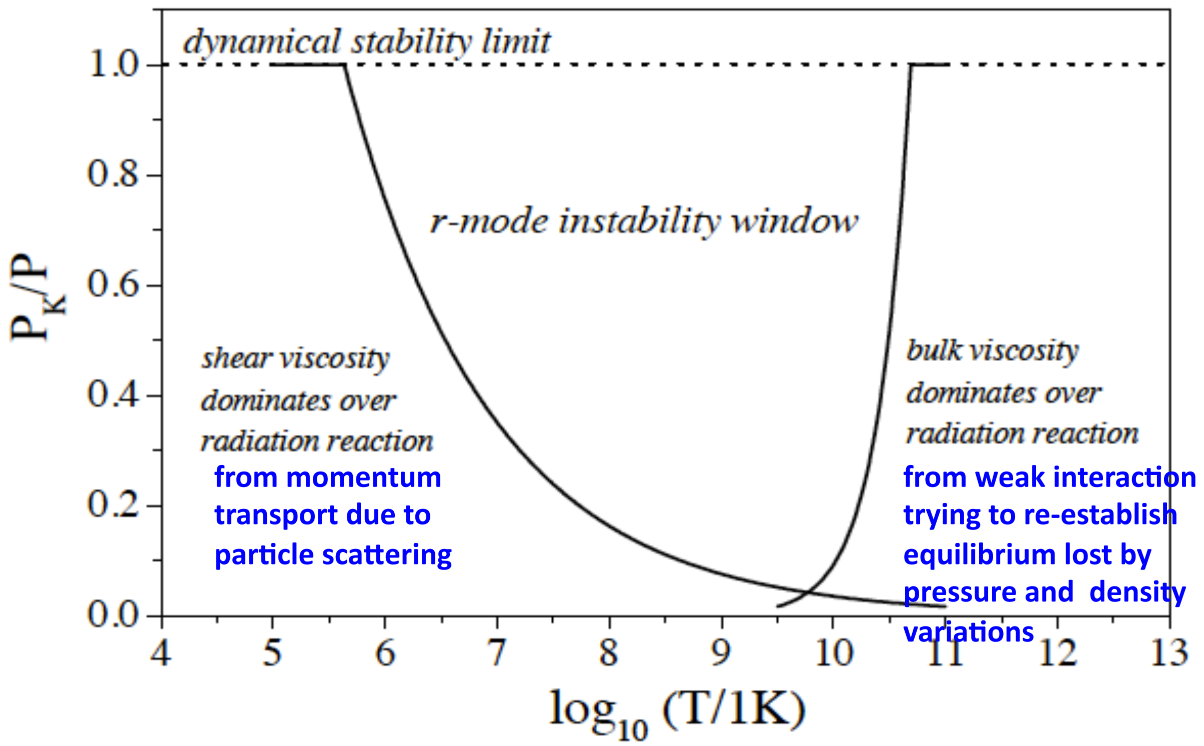


r-mode instability in rotating neutron stars

The low-frequency toroidal r-modes is one of the families of pulsation modes in rotating neutron stars.

r-modes are unstable via emission of gravitational waves. However, there are damping mechanisms (viscous processes) that may counteract the growth of an unstable r-mode.

r-modes can help to constrain the neutron star internal structure



if $\tau_{vp} << \tau_{gw}$, then r-modes are damped

Andersson and Kokkotas '00
Lindblom '01

EFT and superfluid phonon

Exploit the universal character of EFT at leading order by obtaining the effective Lagrangian associated to a superfluid phonon and implement the particular features of the system, associated to the coefficients of the Lagrangian, via the EoS

Son '02
Son and Wingate '06

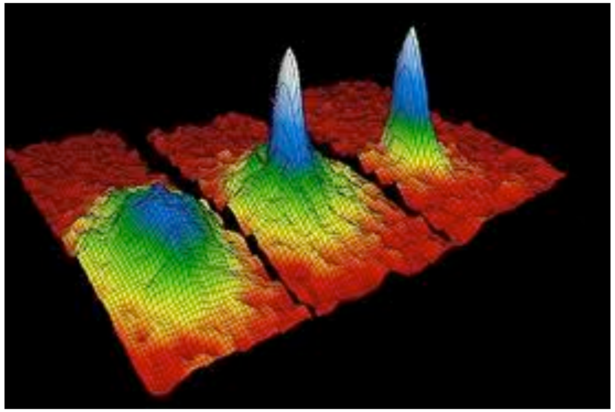
non-relativistic case

$$\mathcal{L}_{\rm LO} = P(X)$$

$$X = \mu - \partial_t \varphi - \frac{(\nabla\varphi)^2}{2m}$$

$P(\mu)$	pressure
μ	chemical potential
φ	phonon field
m	mass condense particles

Applicable in superfluid systems such as cold Fermi gas at unitary, ^{4}He or neutron stars



Effective Lagrangian for superfluid phonon

$$\mathcal{L}_{\rm LO} = \frac{1}{2}((\partial_t\phi)^2 - v_{\rm ph}^2(\nabla\phi)^2) - g((\partial_t\phi)^3 - 3\eta_g\partial_t\phi(\nabla\phi)^2) + \lambda((\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4) + \cdots .$$

with Φ the rescaled phonon field, and where the different phonon self-couplings can be expressed in terms of the speed of sound at T=0

$$v_{ph} = \sqrt{\frac{\frac{\partial P}{\partial \mu}}{m\frac{\partial^2 P}{\partial \mu^2}}} = \sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$

and derivatives with respect to mass density:

Escobedo and Manuel '10

$$u = \frac{\rho}{c_s}\frac{\partial c_s}{\partial \rho}, \quad w = \frac{\rho}{c_s}\frac{\partial^2 c_s}{\partial \rho^2},$$

$$g = \frac{1-2u}{6c_s\sqrt{\rho}}, \qquad \eta_g = \frac{c_s^2}{1-2u}, \qquad \lambda = \frac{1-2u(4-5u)-2w\rho}{24c_s^2\rho},$$

$$\eta_{\lambda,1} = \frac{6c_s^2(1-2u)}{1-2u(4-5u)-2w\rho}, \qquad \eta_{\lambda,2} = \frac{3c_s^4}{1-2u(4-5u)-2w\rho}$$

Results valid for neutrons pairing in $^1S_0$ channel and also valid for $^3P_2$ neutron pairing if corrections $\bar{\Delta}(^3P_2)^2/\mu_n^2$ are ignored Bedaque, Rupak and Savage '03

EoS for superfluid neutron star matter

In order to obtain the speed of sound at T=0 and the different phonon self-couplings one has to determine the EoS for neutron matter in neutron stars.

A common benchmark for nucleonic EoS is APR98
Akmal, Pandharipande and Ravenhall '98

which was later parametrized in a causal form   Heiselberg and Hjorth-Jensen '00

$$E/A = \mathcal{E}_0 y \frac{y-2-\delta}{1+\delta y} + S_0 y^{\beta}(1-2x_p)^2$$

$$y = n/n_0 \qquad x_p = n/n_0$$

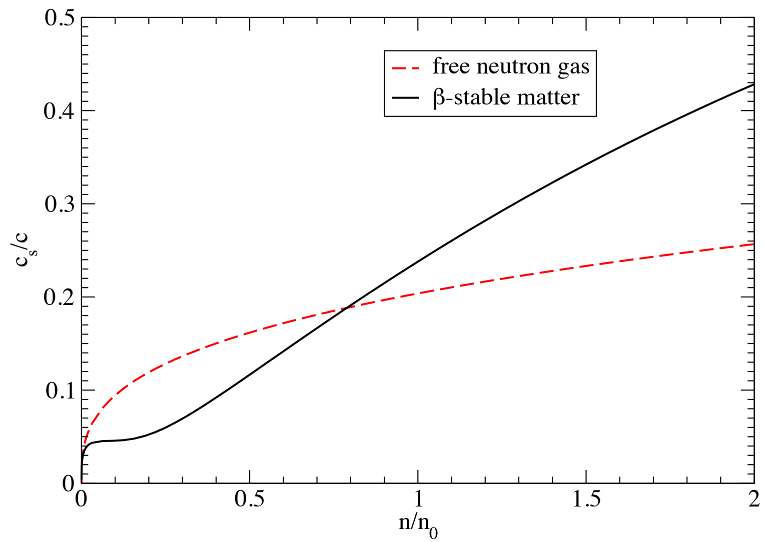
$$n_0 = 0.16\,\mathrm{fm}^{-3}$$

$$\mathcal{E}_0 = 15.8\,\mathrm{MeV} \qquad \delta = 0.2$$

$$S_0 = 32\,\mathrm{MeV} \qquad \beta = 0.6$$

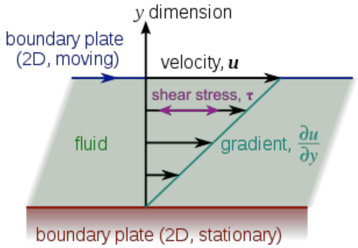
For β-stable matter made up of neutrons, protons and electrons, the speed of sound at T=0 is

$$\sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$



Shear viscosity due to superfluid phonons

Shear viscosity

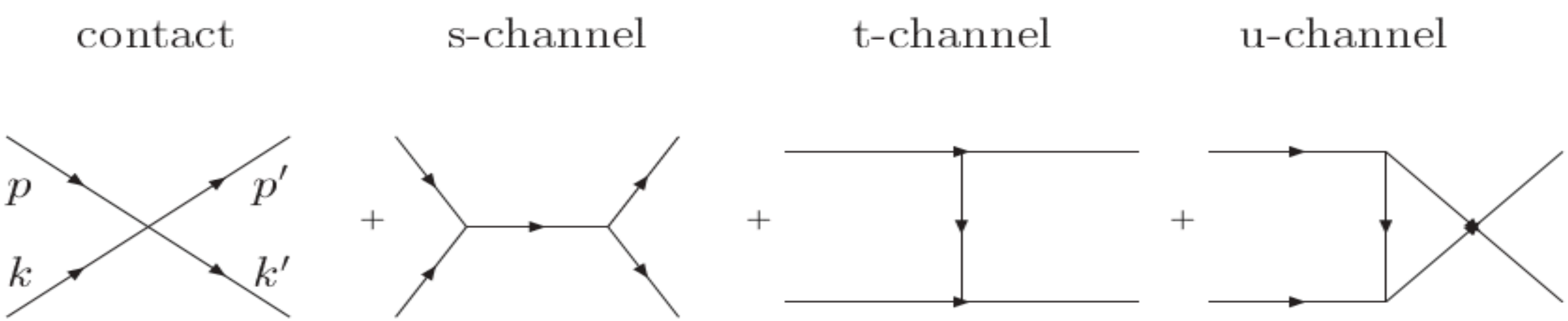


wikipedia

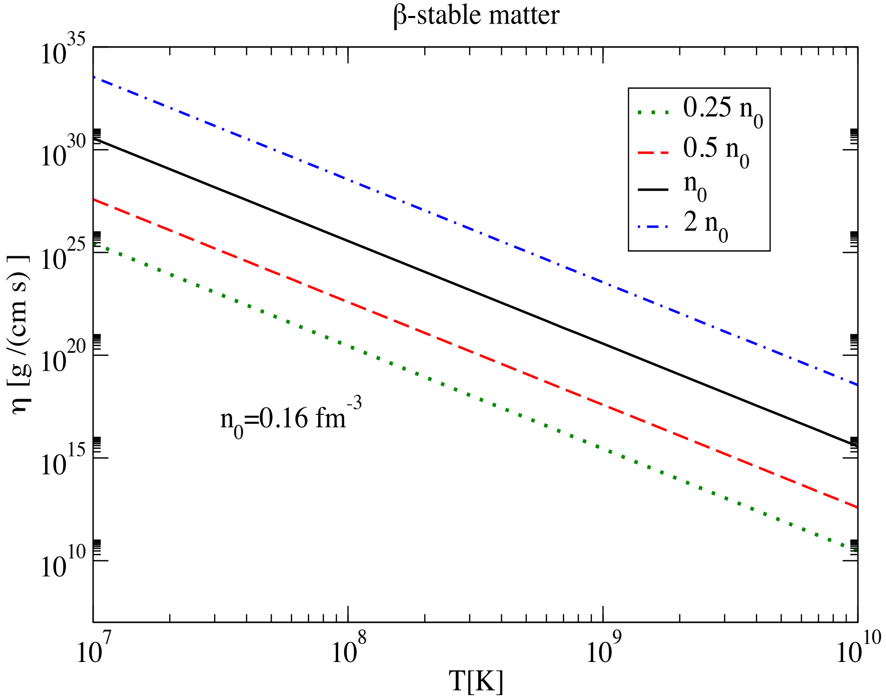
The shear viscosity is calculated using variational methods for solving the transport equation as

$$\eta = \left(\frac{2\pi}{15}\right)^4 \frac{T^8}{c_s^8} \frac{1}{M}$$

where M is the scattering cross section of the collisions responsible for dissipation. For superfluid neutron matter within neutron stars, the dominant dissipation processes are binary collisions (by analyzing dispersion relation at NLO)



Shear viscosity due to binary collisions of phonons scales as $\eta \propto 1/T^5$ (also for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS.

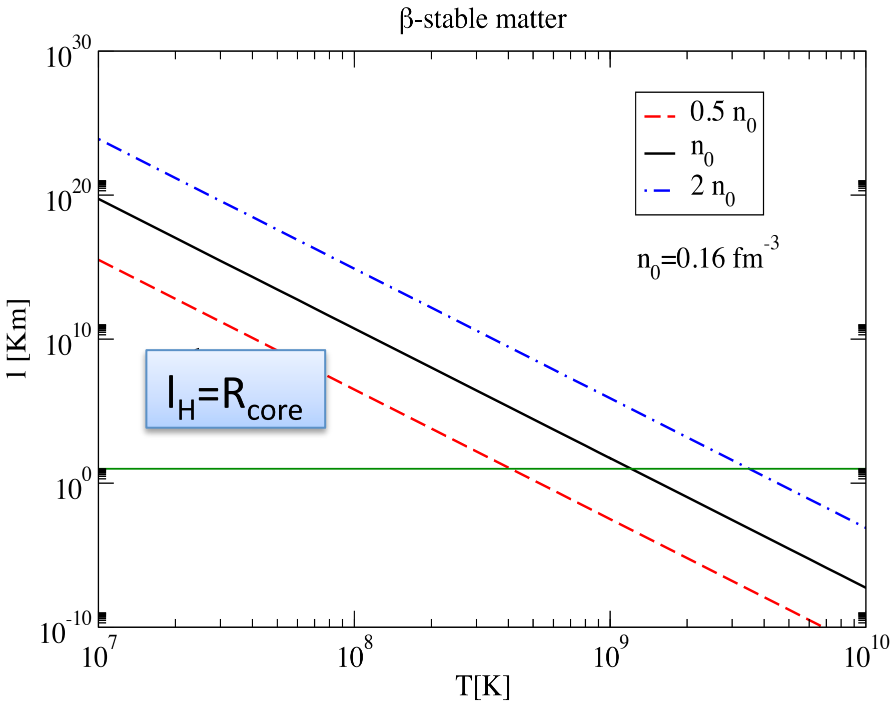


Mean free path of phonons: establish when phonons become hydrodynamic

$$l = \frac{\eta}{n < p>}$$

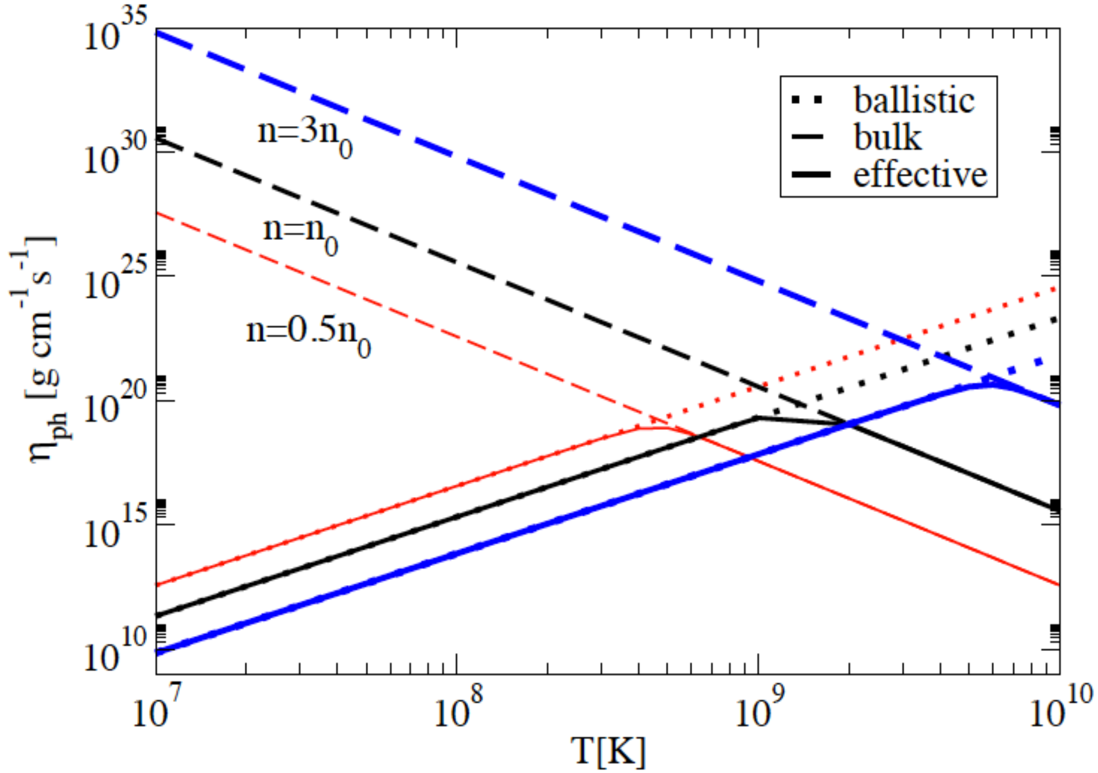
$< p >$: thermal average

n : phonon density



Alford, Braby, and Mahmoodifar '10

Taking into account finite size effects…



For a Knudsen number $K_n = l/R_{core} >> 1$, ballistic regime (as in ^{4}He for low T):

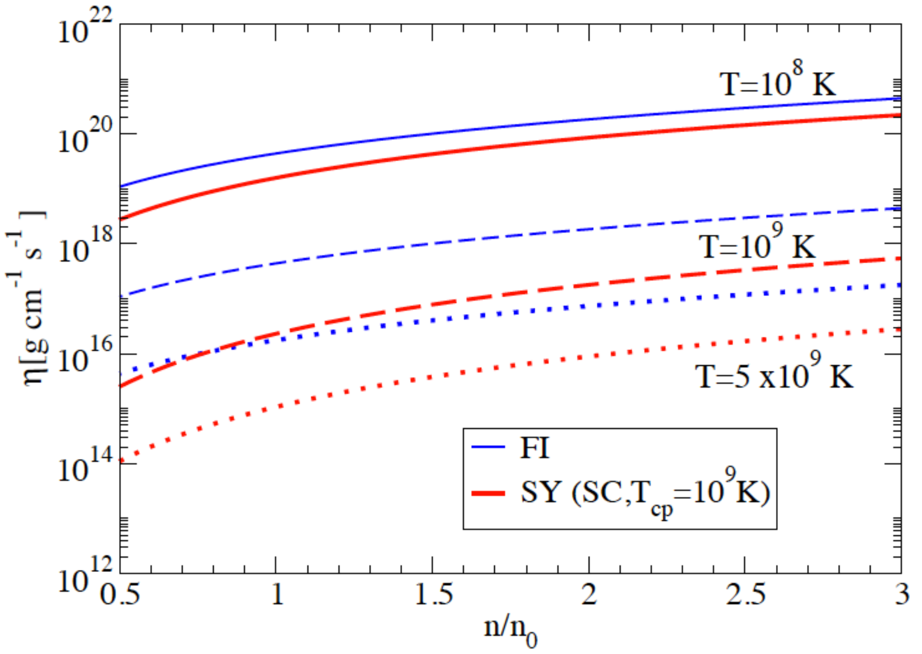
$$\eta_{\rm ball} \equiv \frac{1}{5}\rho_{\rm ph} c_s a \qquad \rho_{\rm ph} = \frac{2\pi^2 T^4}{45 c_s^5}$$

$$a = R_{core}$$

For $K_n = l/R_{core} << 1$, hydrodynamical behaviour:

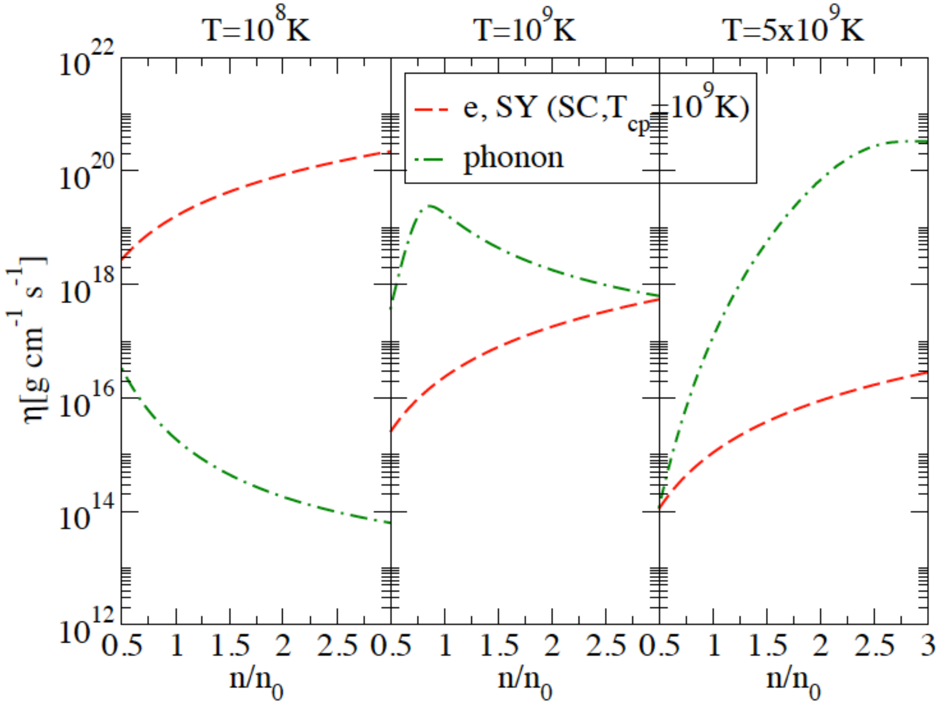
$$\eta_{\rm bulk}$$

In the intermediate region, $$\eta_{\rm eff} = \left(\eta_{\rm bulk}^{-1} + \eta_{\rm ball}^{-1}\right)^{-1}$$



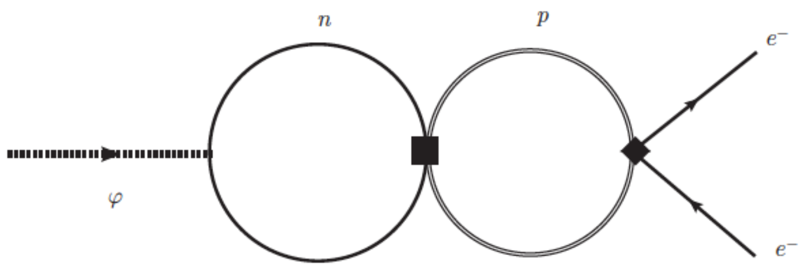
Shear viscosity has to be compared to other processes such as viscosity due to electron scattering

FI: Flowers and Itoh '76
SY: Shternin and Yakovlev '08



Shear viscosity due to phonons becomes important for T≈10^9 K when the hydrodynamical behaviour takes over the ballistic description

Interaction among superfluid phonons and electrons in the core of neutron stars



$$\mathcal{L}_{\rm ph-el} = g\bar{\psi}_e \gamma^\mu \psi_e \tilde{A}_\mu$$
$$\tilde{A}_\mu = (\partial_0 \varphi, a_s \partial_i \varphi)$$

low density limit: $g \approx \frac{2\,p_n m\, c_s}{\pi \sqrt{\rho_n}} a_{np}$

The interaction of phonons with electrons inside the Fermi sea are suppressed:

$\mu_e \approx$ 100-200 MeV while T<<1 MeV

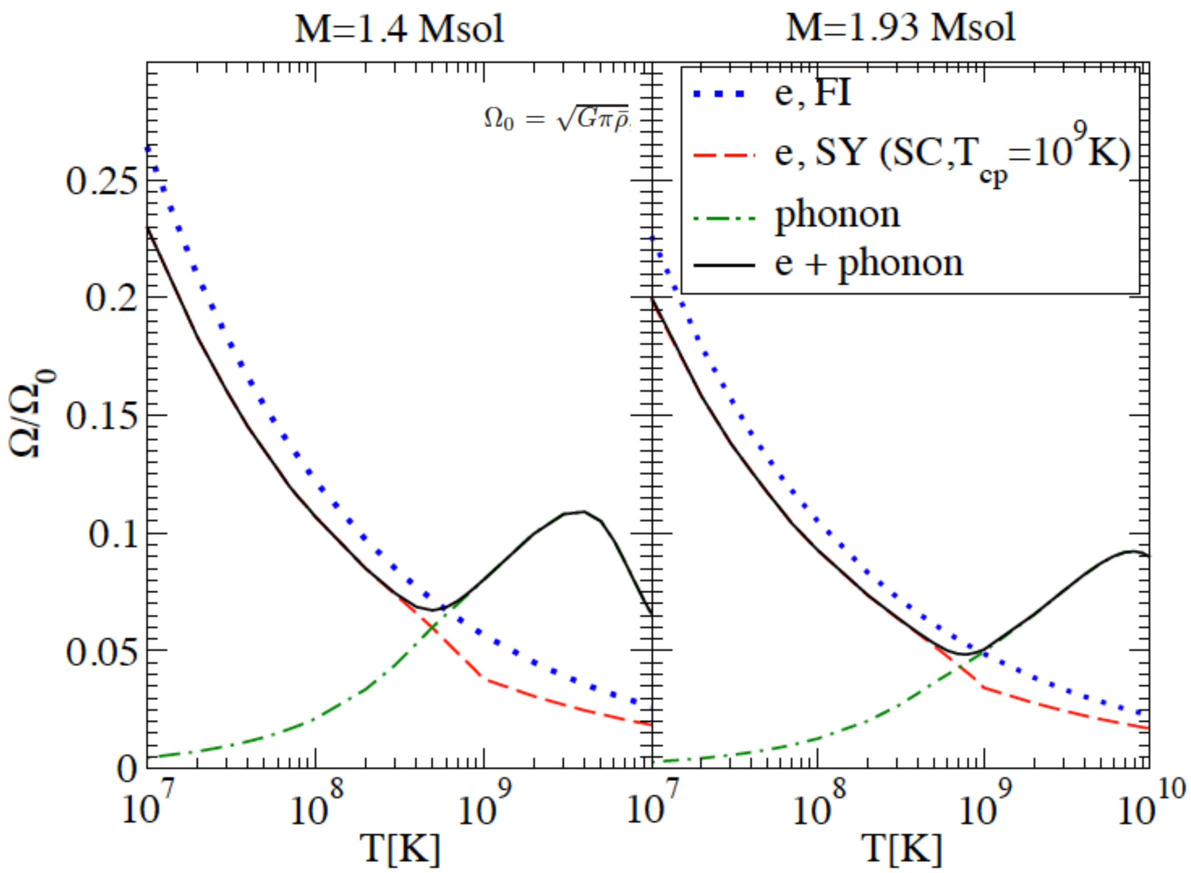
What about collisions among electrons lying on their Fermi surface mediated by one-phonon exchange?

$$\mathcal{M} \sim e^2 J^e_\mu D^{\mu\nu}(k) J^{e'}_\nu$$
$$\mathcal{M} \sim g^2 J^e_\mu \tilde{k}^\mu \tilde{k}^\nu G(k) J^{e'}_\nu$$

$$\frac{e^2}{g^2 T^2} > 1$$
$$T \lesssim (10^8 - 10^9)\mathrm{K}$$

For T≤ $(10^8$-$10^9)$ K, the electron collisions mediated by one-photon exchange dominate

r-mode instability window



$$-\frac{1}{|\tau_{\rm GR}(\Omega)|} + \frac{1}{\tau_{\eta_{\rm eff}}(T)} = 0$$

Dissipation due to superfluid phonons start to be relevant at T≈5 x10^8 K for 1.4 $M_\odot$ and T≈10^9 K for 1.93 $M_\odot$

$$\frac{1}{|\tau_{\rm GR}(\Omega)|} = \frac{32\,\pi\,G\,\Omega^{2l+2}}{c^{2l+3}}\,\frac{(l-1)^{2l}}{((2l+1)!!)^2}\left(\frac{l+2}{l+1}\right)^{2l+2}\int_0^{R_c}\rho r^{2l+2}dr$$

$$\frac{1}{\tau_{\eta_{\rm eff}}(T)} = (l-1)(2l+1)\int_0^R \eta_{\rm eff} r^{2l}dr\left(\int_0^{R_c}\rho r^{2l+2}dr\right)^{-1}$$

l=2 (dominant)

Summary

We have studied the phonon contribution to the shear viscosity in superfluid neutron stars and the effect on the r-mode instability window

Starting from a general formulation for the collisions of superfluid phonons, we compute the associated shear viscosity in terms of the EoS of the system

- Binary collisions of phonons produce a shear viscosity that scales with $1/T^5$ (universal feature seen for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS (microscopic theory)

- “Effective” shear viscosity includes finite size effects

r-mode instability window modified for $T \geq (10^8\text{-}10^9)$ K due to phonon processes

Future: r-mode instability window considering other dissipative processes, such as bulk viscosity or rubbing core-crust, …

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