The Phase Diagram of Two Color QCD

Simon Hands (Swansea University)

- Why two colors?
- Bulk thermodynamics for $\mu, T \neq 0$: number/energy densities, pressure, trace anomaly, quark number susceptibility
- Superfluidity & deconfinement
- chiral condensate
- Phase diagram
- (if time) Topology, quarkonia

Related talks: Schaefer, von Smekal, Maas, Yamamoto

Collaborators: Phil Kenny, **Seyong Kim**, **Jon-Ivar Skullerud**, Pietro Giudice, Seamus Cotter

QGHMX St. Goar 20th March 2013

Why Two Colors?

(PDG only recognises 3)

- Chance to explore systematics of lattice simulations at $\mu \neq 0$

Good news: cutoff fixed as μ varies, no quantum corrections to $n_q = -\partial f / \partial \mu$

Bad news: UV/IR artifacts are complicated

- Chance to explore "deconfinement" in a new physical régime

Why Two Colors?

(PDG only recognises 3)

- Chance to explore systematics of lattice simulations at $\mu \neq 0$

Good news: cutoff fixed as μ varies, no quantum corrections to $n_q = -\partial f / \partial \mu$

Bad news: UV/IR artifacts are complicated

- Chance to explore "deconfinement" in a new physical régime

- No sign problem stupid!

QC$_2$D - the large N_c^{-1} limit

QCD with gauge group SU(2) and an even N_f of fundamental quarks has a real positive functional measure even once $\mu\neq 0$. It is the simplest system of dense matter with long-ranged interactions amenable to LGT simulation.

Hadron multiplets contain both $q\bar{q}$ mesons and $qq, \bar{q}\bar{q}$ (anti-)baryons. For $m_\pi \ll m_\rho$ the μ-dependence can be studied using chiral effective theory.

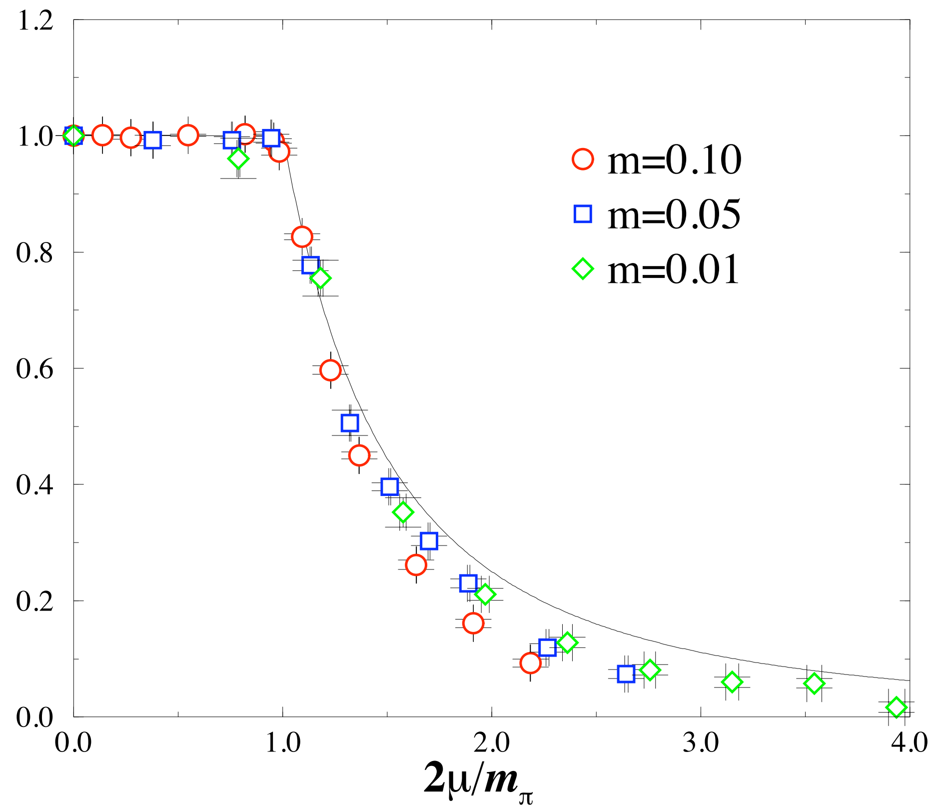
Key result: for $\mu \geq \mu_o = \frac{1}{2} m_\pi$ a baryon charge density $n_q > 0$ develops, along with a gauge-invariant **superfluid** condensate $\langle qq \rangle \neq 0$. For $\mu \gtrsim \mu_o$ the system is a BEC consisting of dilute weakly-interacting 0^+ qq diquarks.

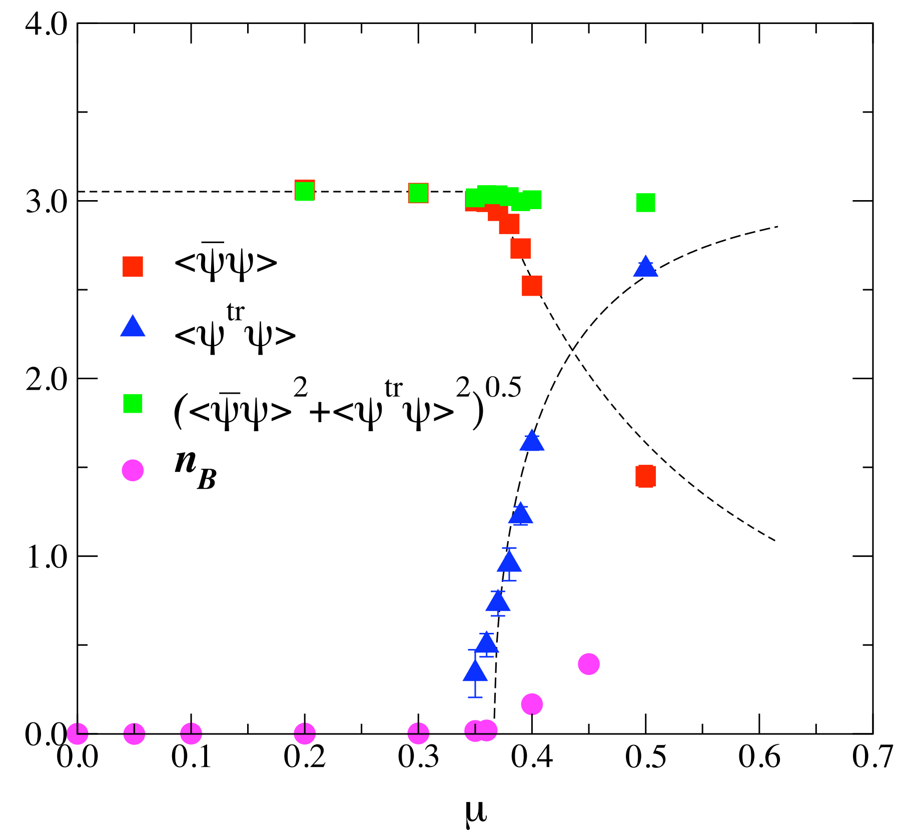
Quantitatively, for $\mu \gtrsim \mu_o$ χPT predicts

$$\frac{\langle\bar\psi\psi\rangle}{\langle\bar\psi\psi\rangle_0} = \left(\frac{\mu_o}{\mu}\right)^2; \quad n_q = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right); \quad \frac{\langle qq\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1 - \left(\frac{\mu_o}{\mu}\right)^4}$$

[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]

confirmed by QC$_2$D simulations with staggered fermions





[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud, Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]

Simulation Details ($N_f = 2$ Wilson flavors)

SJH, S. Kim & J.I Skullerud, EPJC48 (2006) 193; PRD81 (2010) 091502(R)
S. Cotter, P. Giudice, SJH & J.I Skullerud, PRD87 034507 (2013)
Boz, Cotter, Fister, Mehta & Skullerud, arXiv:1303.3223

Machines range from u/g lab PCs to IBM BlueGene

		a(fm)	$m_\pi a$	m_π/m_ρ	T(MeV)
coarse	8^3x16	0.229(3)	0.78(1)	0.804(10)	55(1)
fine	12^3x24	0.178(6)	0.645(8)	0.805(9)	47(2)

also have μ-scans on 12^3x16, 16^3x20,....,4 $\Rightarrow T$ = 56,70,...,282 MeV

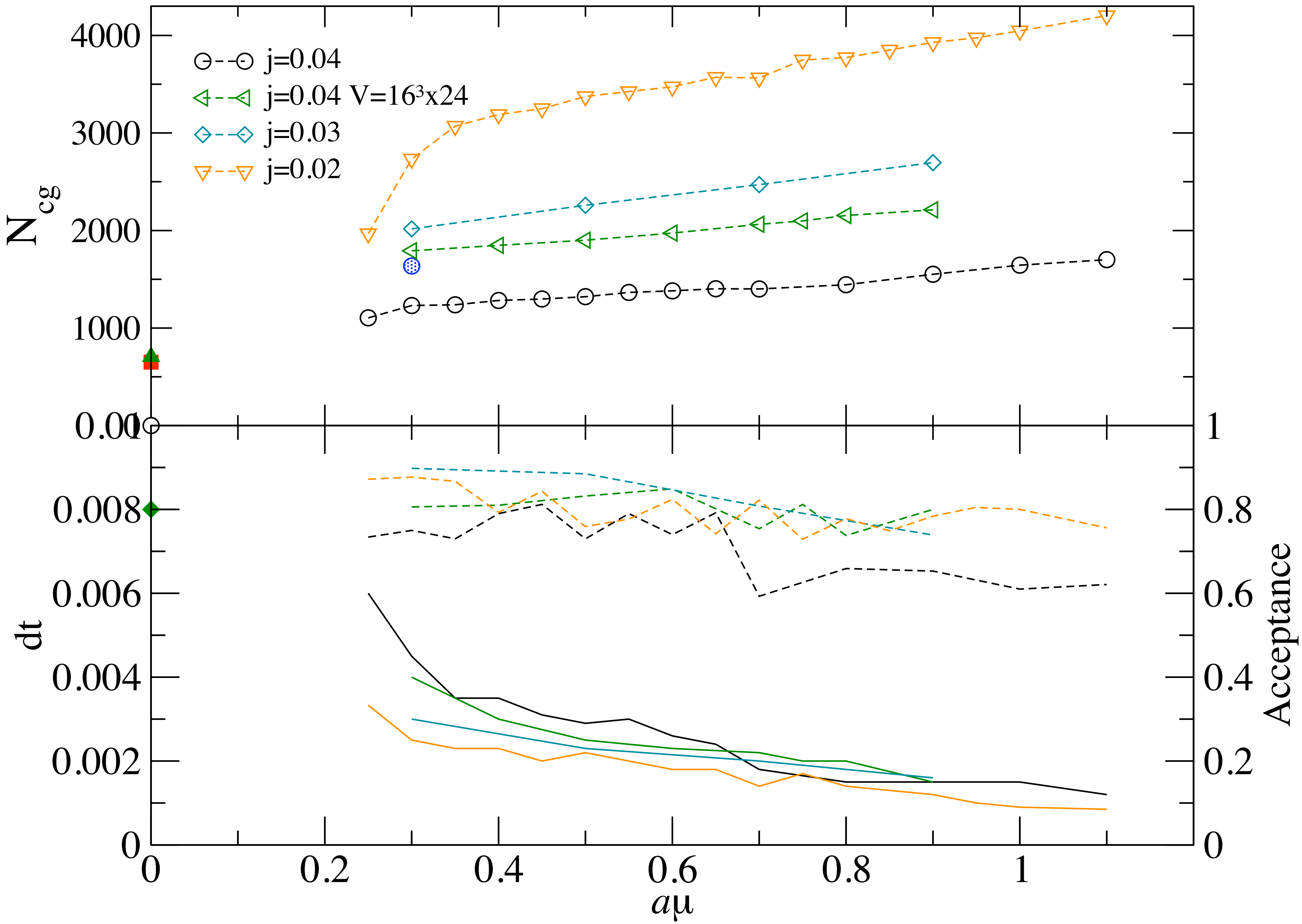
To counter IR fluctuations and maintain HMC ergodocity,
we introduce a diquark source term $j\kappa(\psi_2^{tr} C\gamma_5\tau_2\psi_1 - \bar\psi_1 C\gamma_5\tau_2\bar\psi_2^{tr})$

Have results for ja=0.04 everywhere

to enable $j\rightarrow 0$ have ja=0.02, 0.03 at selected points

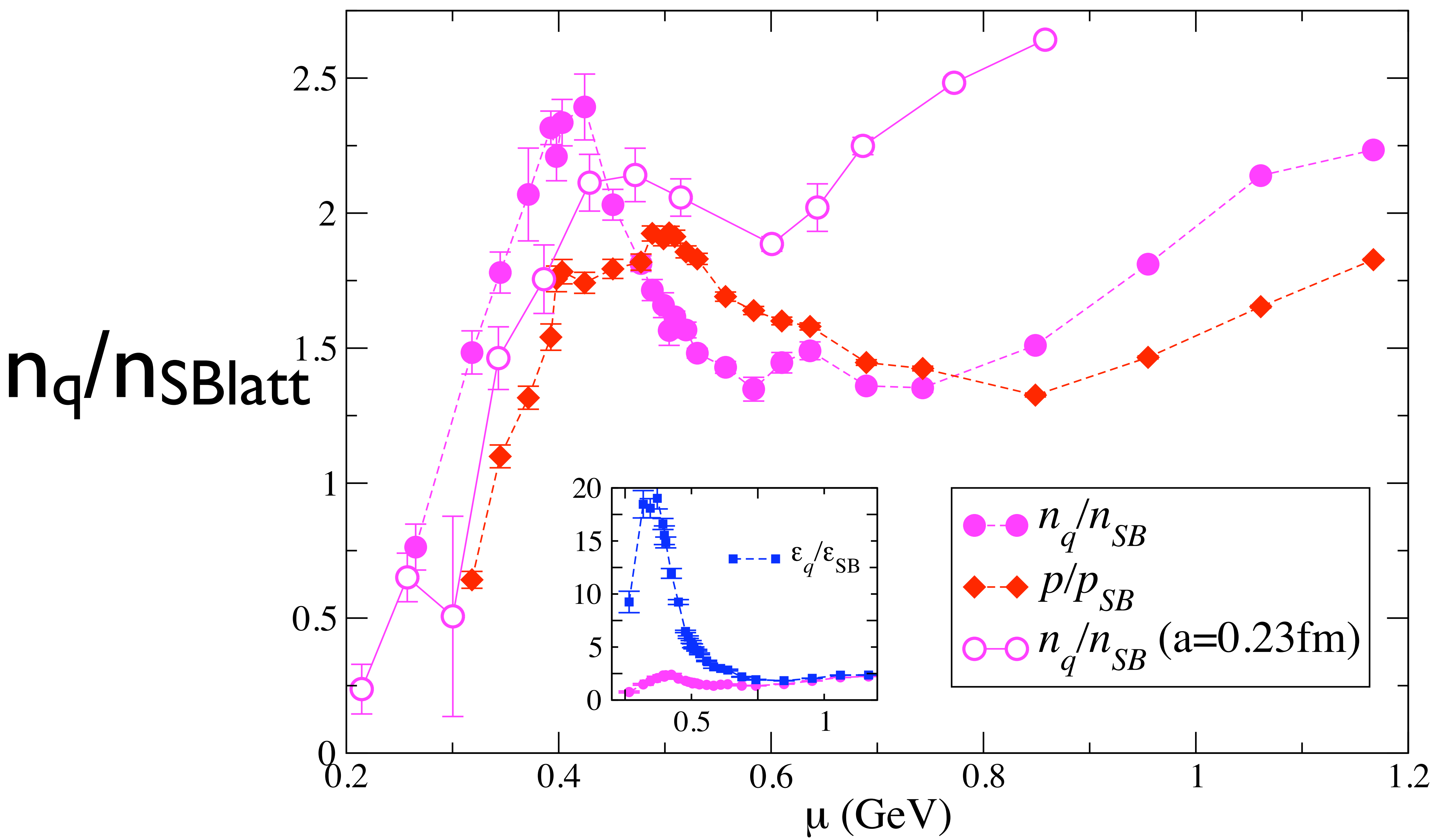
The $j\rightarrow 0$ limit resembles the chiral limit in the vacuum

Computer Effort (*sans* Sign Problem!)



The number of **`congrad`** iterations required for convergence during HMC guidance rises with μ ⇔ accumulation of small eigenvalues of M?

Equation of State on Fine Lattice (12^3x24, ja=0.04)



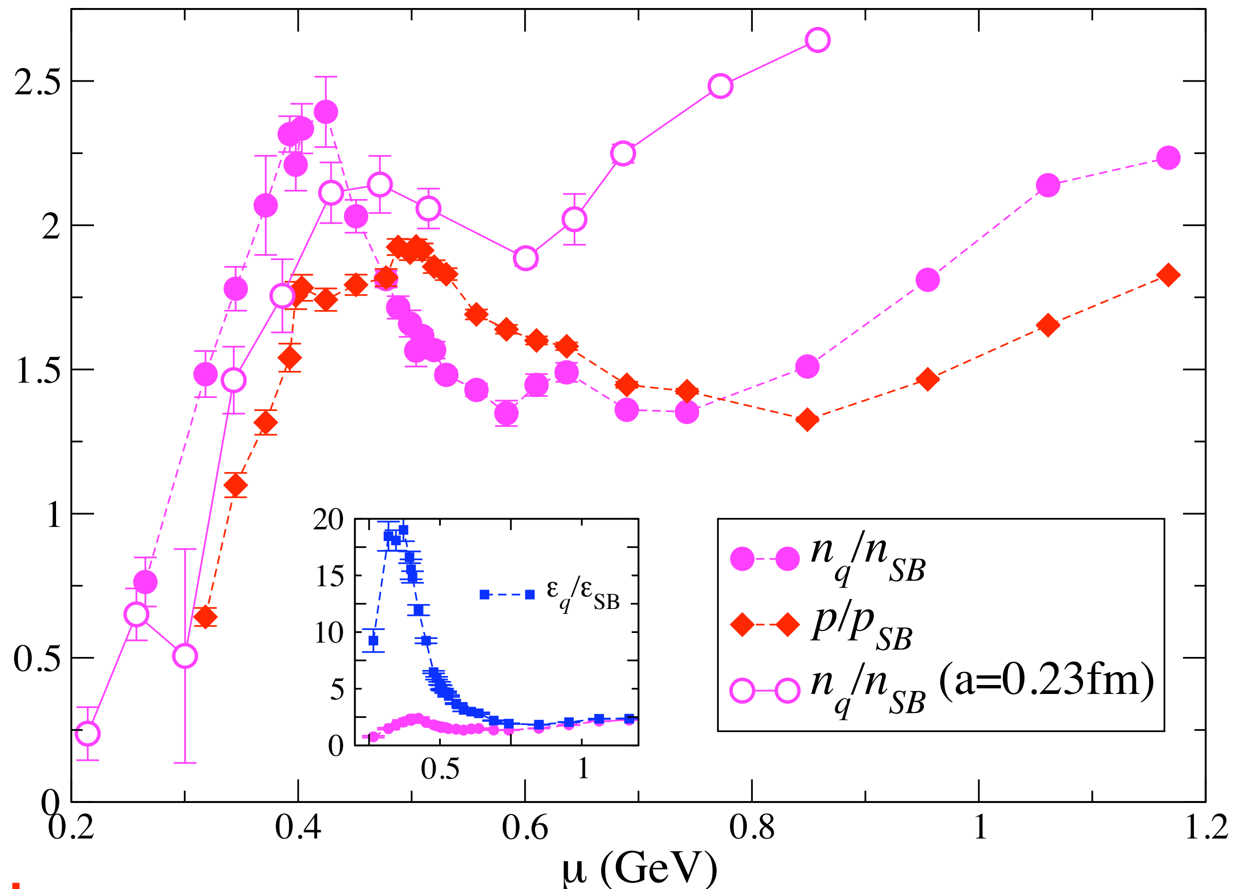
Identify:

onset $\mu_o \approx$ 360MeV

crossover to "quarkyonic phase" $\mu_Q \approx$ 530MeV $n_q \approx$ 4 - 5 fm^{-3}

"deconfinement" $\mu_d \approx$ 850MeV $n_q \approx$ 16 - 32 fm^{-3}

Artifacts



However:

(a) the j→0 extrapolation gives large corrections at small μ, so plateau closer to non-interacting value

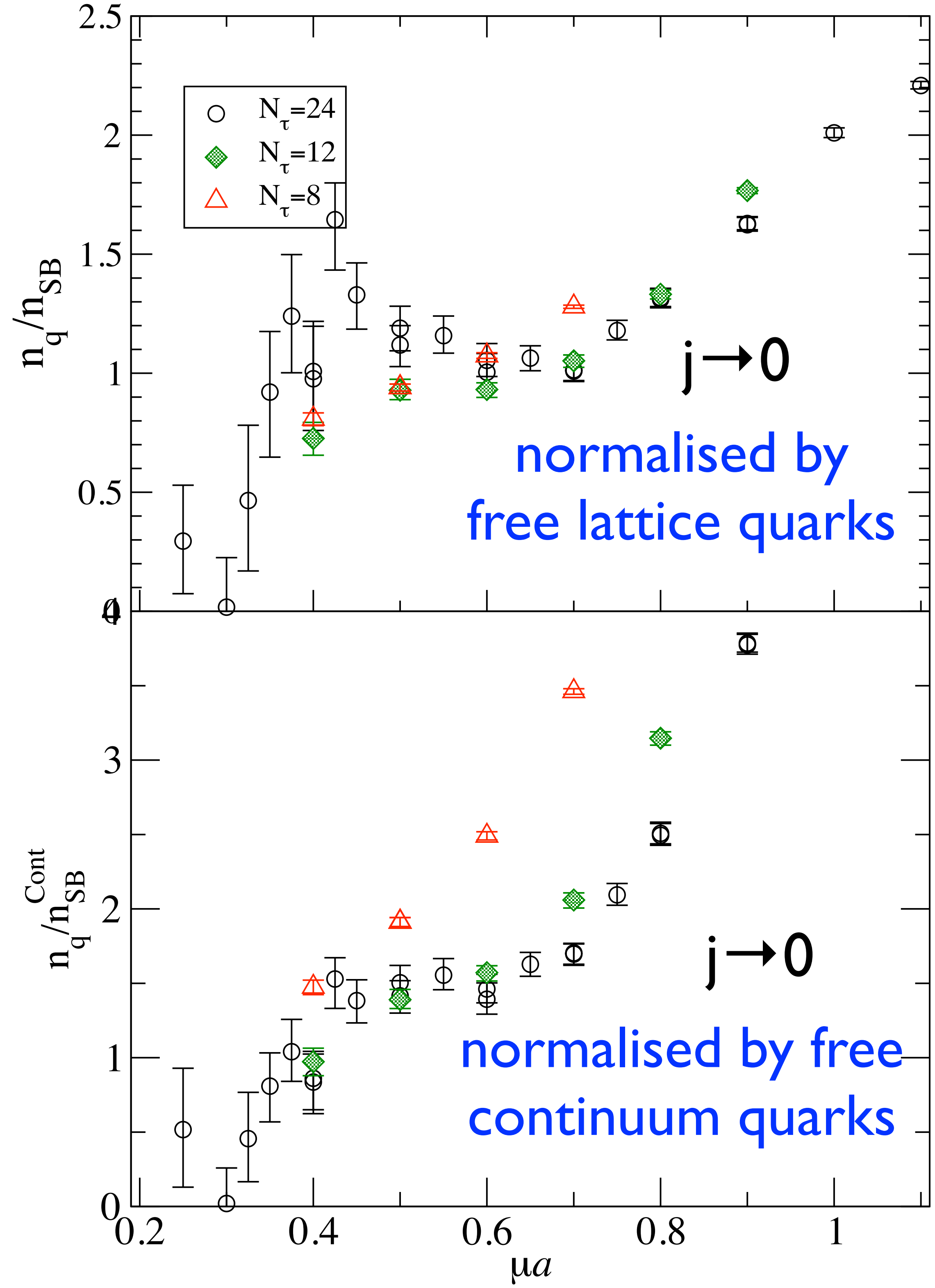
j≠0 promotes diquark pairing

significant correction for interacting quarks

(c) UV artifacts are present at larger μ

free *lattice* quark correction

more reliable here

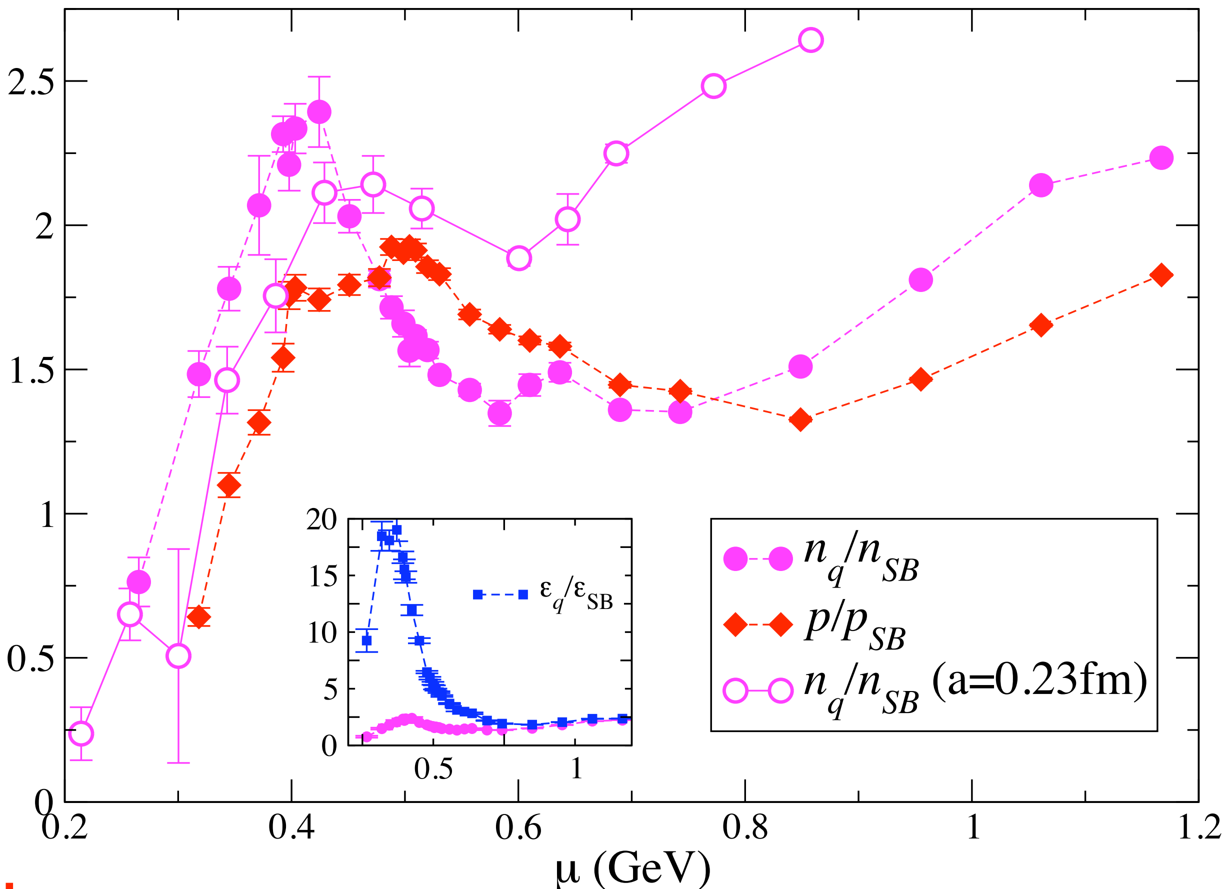


(b) the peak above onset at low T is very sensitive to IR artifacts (non-sphericity of Fermi surface)

$T \ll \Delta k = 2\pi/L_s$

significant correction for free *lattice* quarks

Artifacts



However:

(a) the j→0 extrapolation gives large corrections at small μ, so plateau closer to non-interacting value

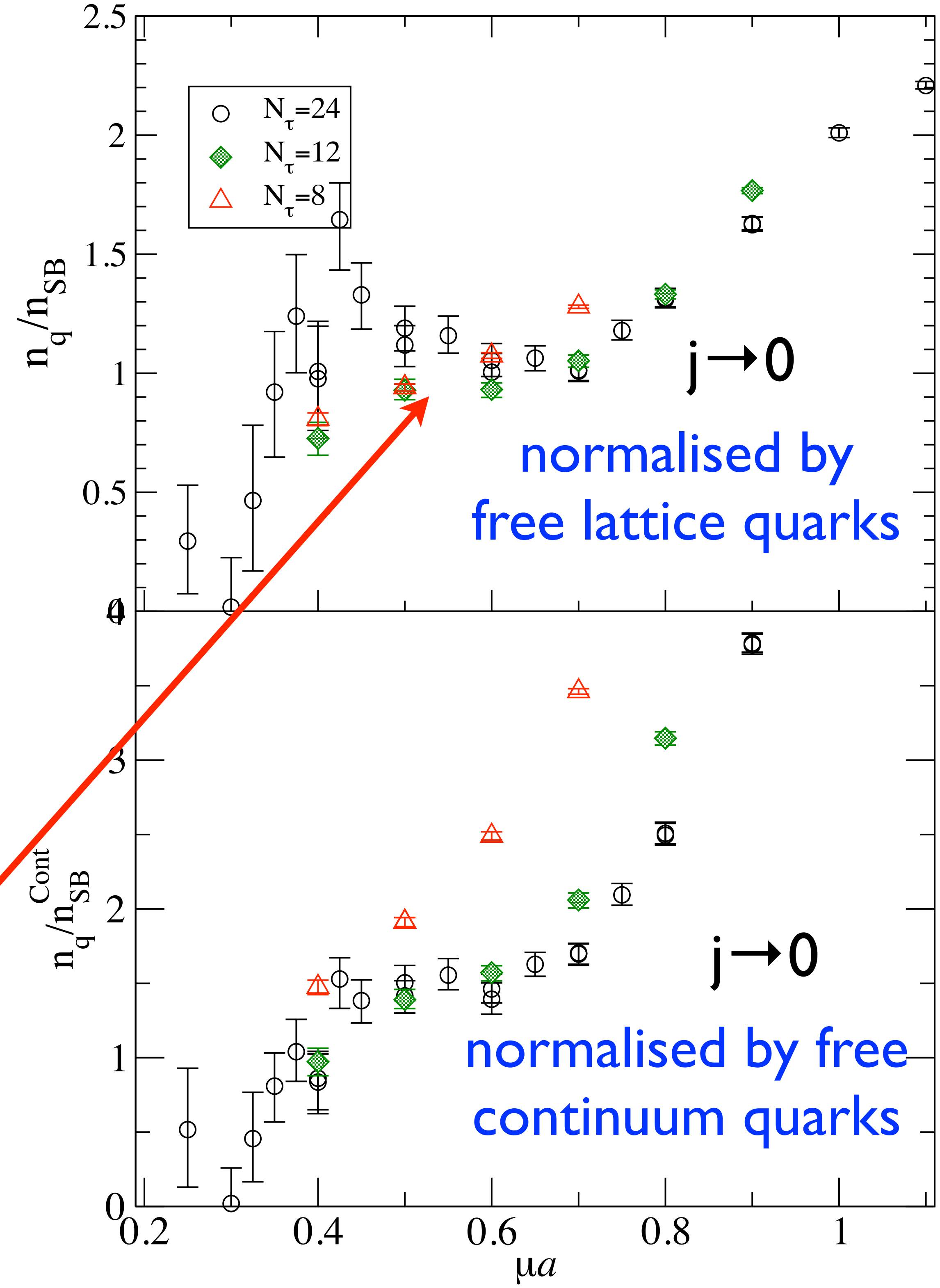
j≠0 promotes diquark pairing

significant correction for interacting quarks

(c) UV artifacts are present at larger μ

free *lattice* quark correction

more reliable here

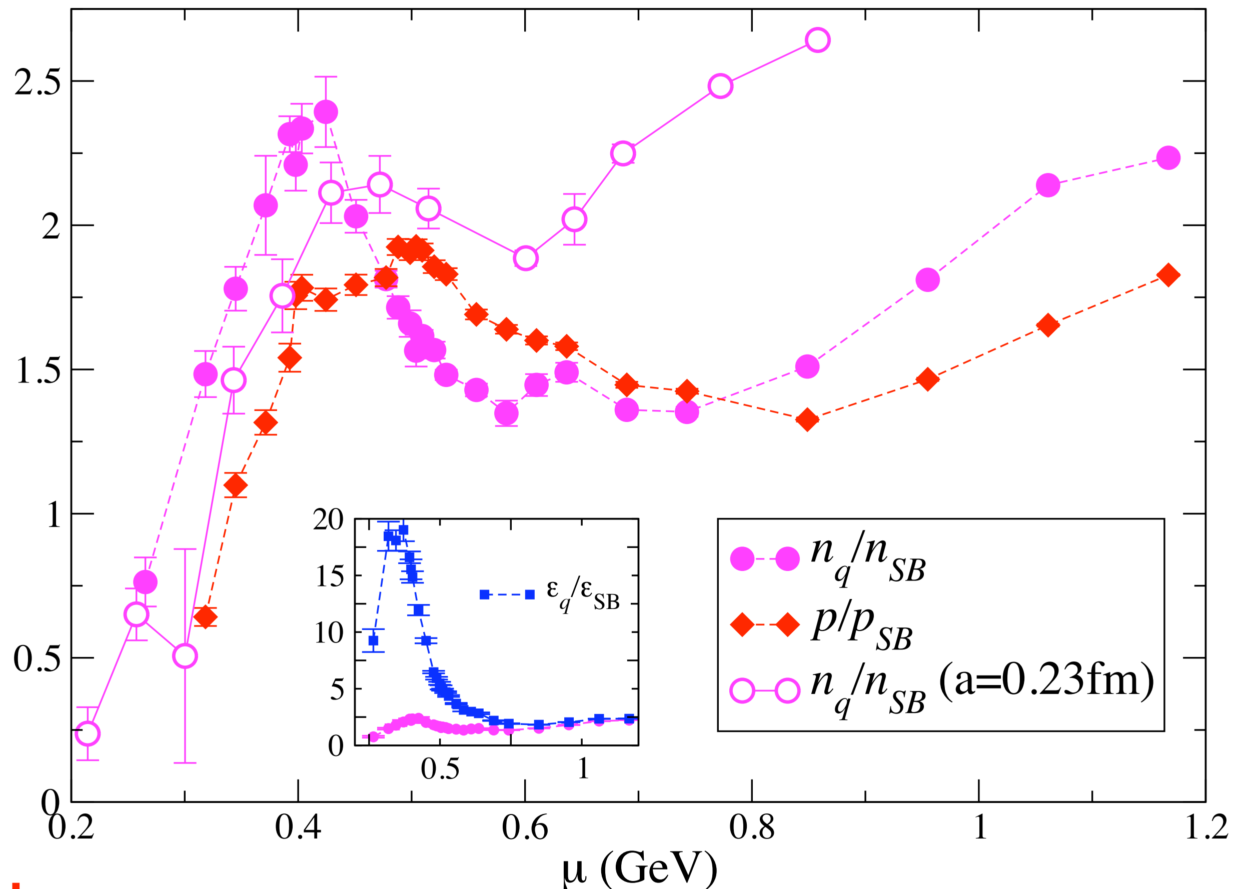


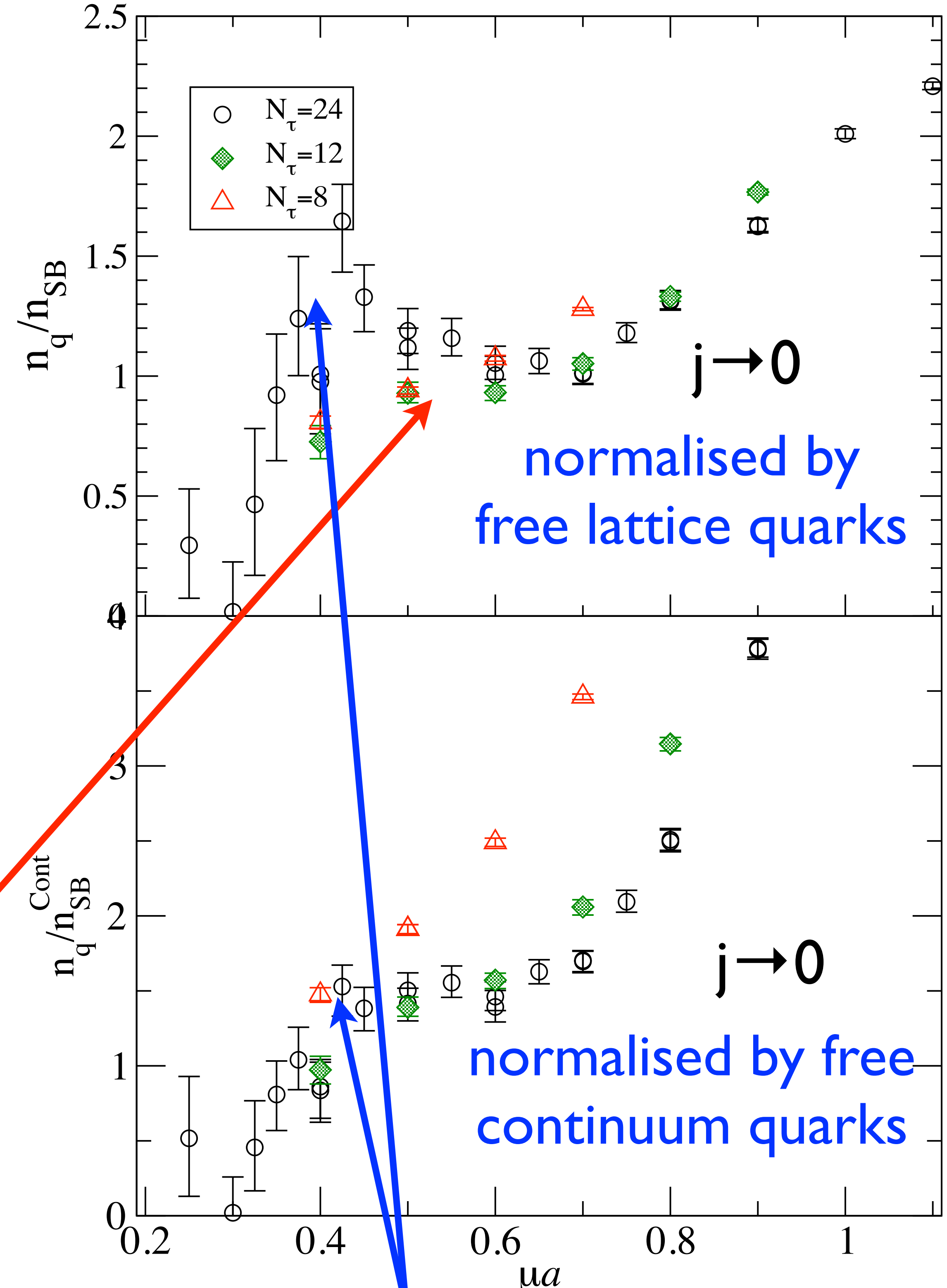
(b) the peak above onset at low T is very sensitive to IR artifacts (non-sphericity of Fermi surface)

$T \ll \Delta k = 2\pi/L_s$

significant correction for free *lattice* quarks

Artifacts





However:

(a) the j→0 extrapolation gives large corrections at small μ, so plateau closer to non-interacting value

j≠0 promotes diquark pairing

significant correction for interacting quarks

(b) the peak above onset at low T is very sensitive to IR artifacts (non-sphericity of Fermi surface)

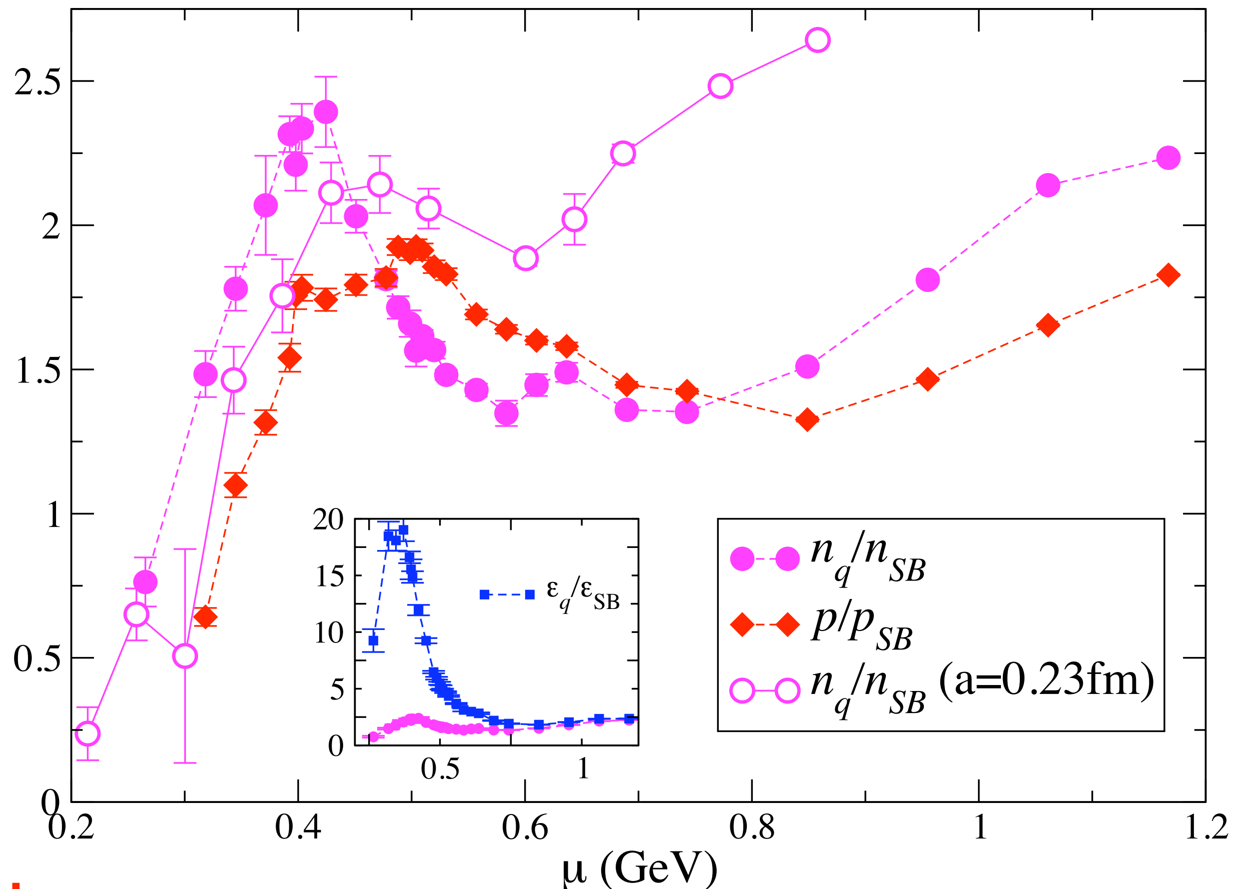
$T \ll \Delta k = 2\pi/L_s$

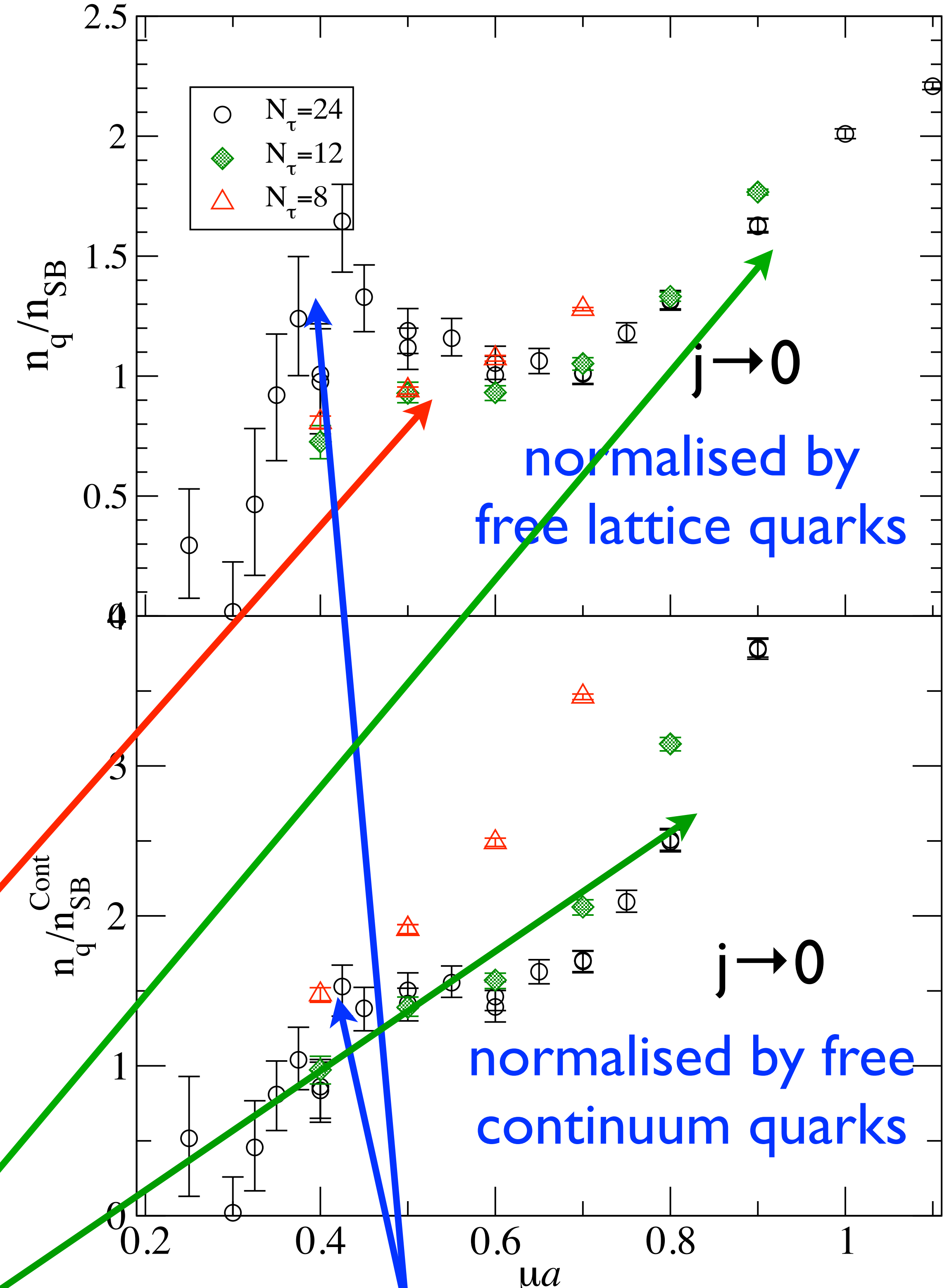
significant correction for free *lattice* quarks

(c) UV artifacts are present at larger μ

free *lattice* quark correction more reliable here

Artifacts





However:

(a) the j→0 extrapolation gives large corrections at small μ, so plateau closer to non-interacting value

j≠0 promotes diquark pairing

significant correction for interacting quarks

(b) the peak above onset at low T is very sensitive to IR artifacts (non-sphericity of Fermi surface)

$T \ll \Delta k = 2\pi/L_s$

significant correction for free *lattice* quarks

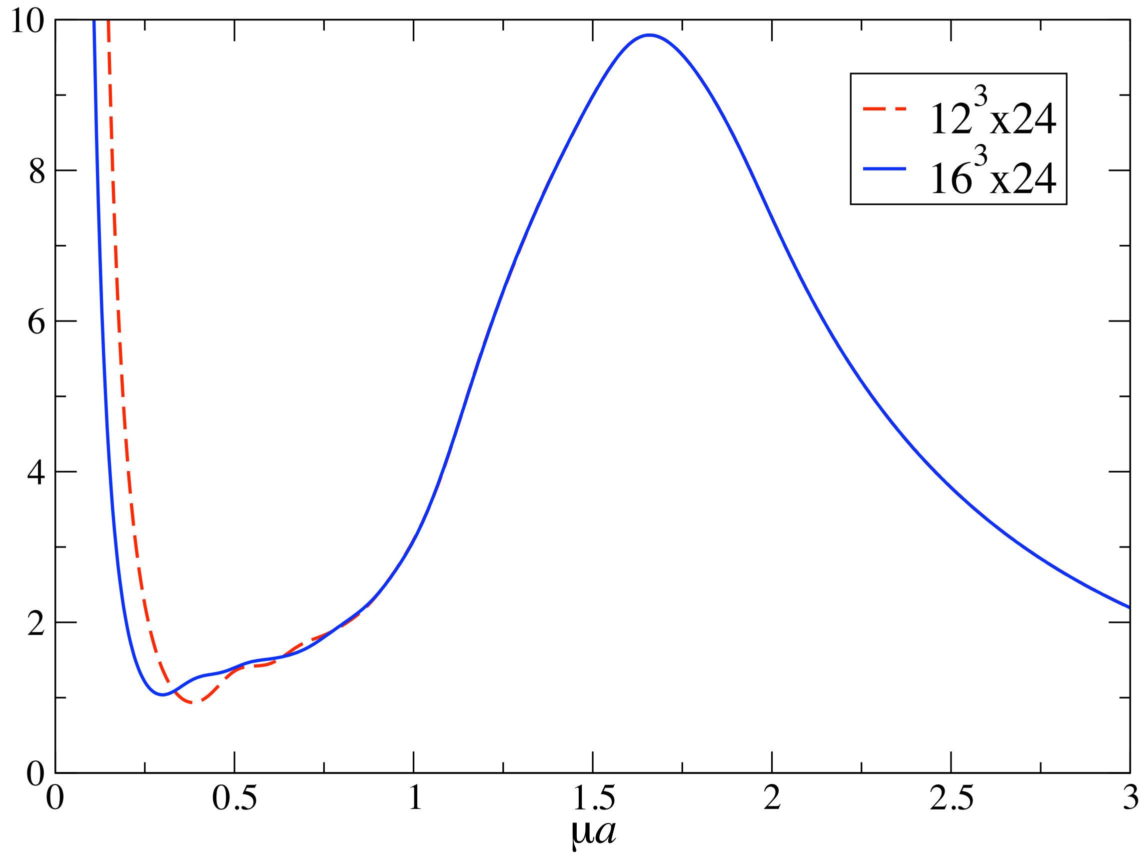
(c) UV artifacts are present at larger μ

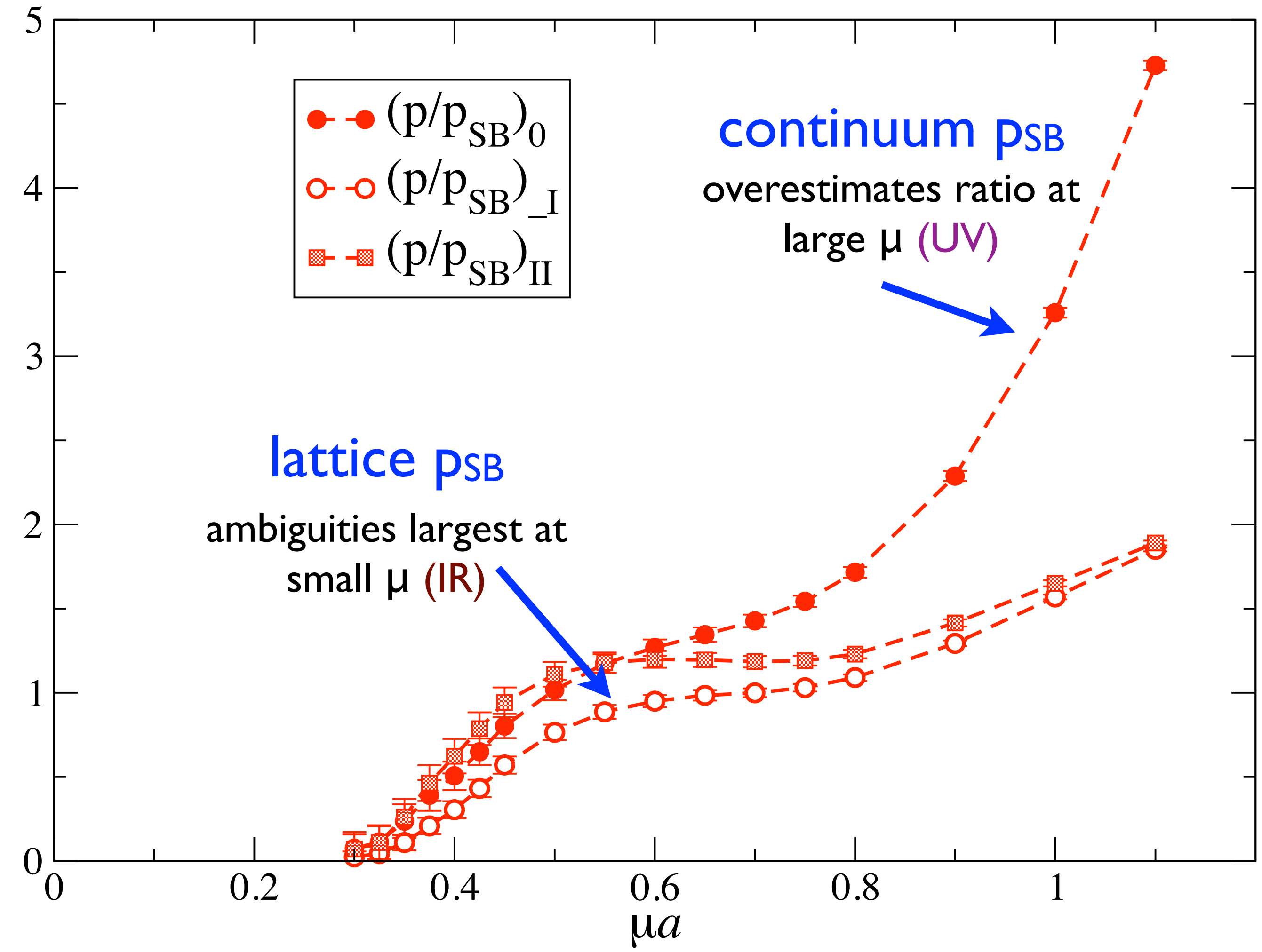
free *lattice* quark correction more reliable here

Pressure for $j \to 0$ on $12^3 \times 24$

$$p = \int^{\mu} n_q \, d\mu$$

n_{SB}(latt)/n_{SB}(continuum)





Robust: Still see onset at $\mu_o \approx$ 360MeV

Transition to "quark matter" at $\mu_Q \approx$ 530MeV $E_F \approx k_F$

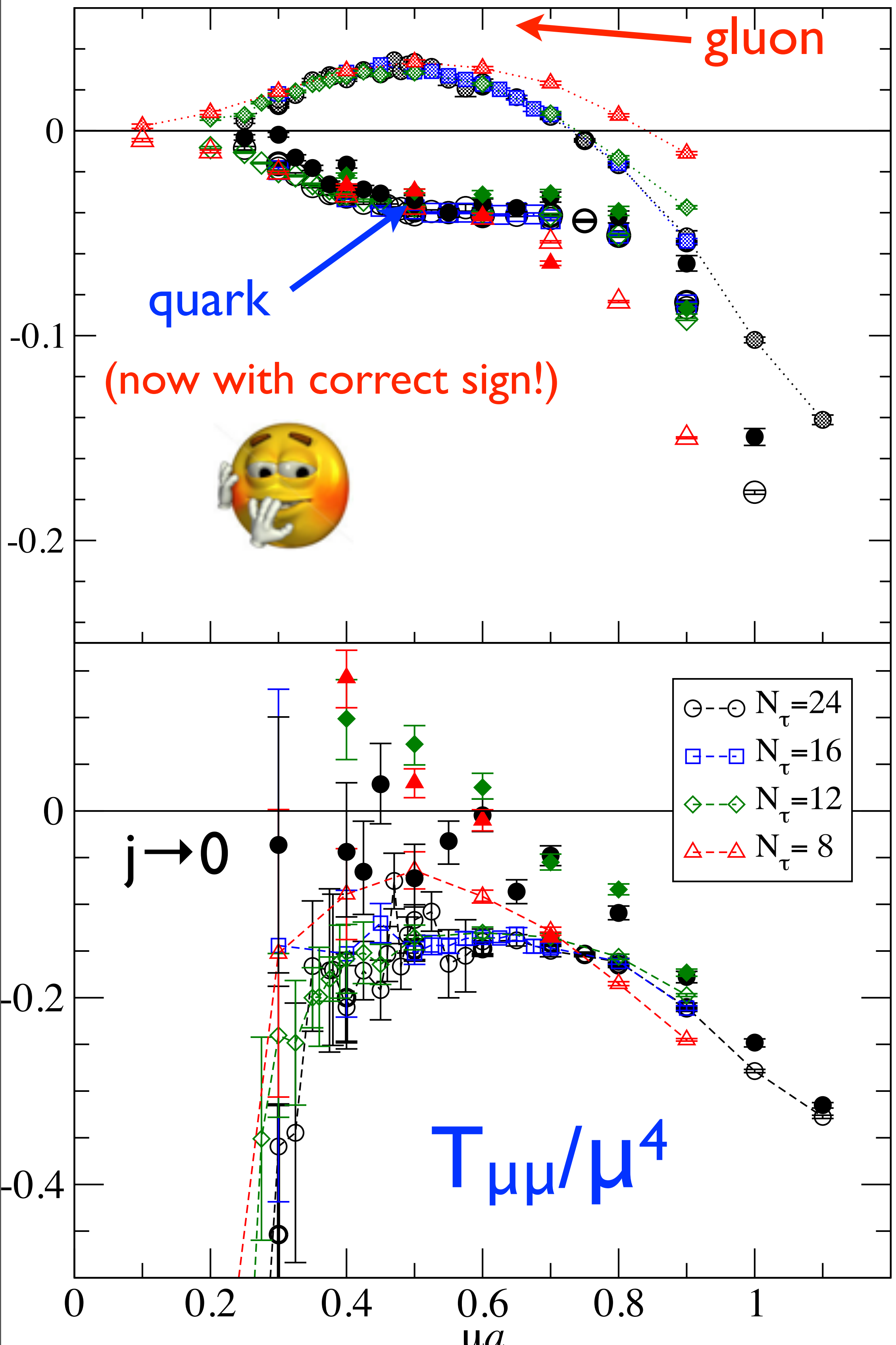
"Deconfinement" sets in at $\mu_d \approx$ 850MeV $E_F < k_F$

But: no longer any firm evidence for a BEC "peak" just above onset

Quark matter is strongly **self-bound** at high density

Conformal Anomaly

$T_{\mu\mu} = \varepsilon\text{-}3p$



$$(T_{\mu\mu})_g = -a\frac{\partial\beta}{\partial a}\bigg|_{LCP} \times \frac{3\beta}{N_c}\mathrm{Tr}\langle\Box_t + \Box_s\rangle;$$

$$(T_{\mu\mu})_q = -a\frac{\partial\kappa}{\partial a}\bigg|_{LCP} \times \kappa^{-1}(4N_fN_c - \langle\bar\psi\psi\rangle)$$

Quark and gluon contributions:

almost cancel for $\mu<\mu_Q$: conformal?

differ for $\mu>\mu_Q$

$T_{\mu\mu} < 0$ for $\mu \gtrsim \mu_Q$

$(T_{\mu\mu})_q$ changes sharply at $\mu_d \approx$ 850MeV

$\Rightarrow \varepsilon<3p$ in limit $\mu\to\infty$

consistent with self-binding

Calculation of Energy Density

$$\varepsilon = -\frac{1}{V}\frac{\partial Z}{\partial T^{-1}}\bigg|_V = -\frac{\xi}{N_s^3 N_\tau a_s^3 a_\tau}\left\langle \frac{\partial S}{\partial \xi}\bigg|_{a_s}\right\rangle$$ with $$\xi \equiv \frac{a_s}{a_\tau}$$ physical anisotropy

anisotropic action

$$\mathcal{L} = -\frac{\beta}{N_c}\left[\frac{1}{\gamma_g}\Box_s + \gamma_g \Box_\tau\right] + \bar{\psi}\left[1 + \gamma_q \kappa D_0[\mu] + \kappa \sum_i D_i\right]\psi$$

$$\Rightarrow \quad \frac{\varepsilon_g}{T^4} = \frac{3N_\tau^4}{\xi^2 N_c}\left[\langle \Box_s \rangle \left(\gamma_g^{-1}\frac{\partial \beta}{\partial \xi} + \beta \frac{\partial \gamma_g^{-1}}{\partial \xi}\right) + \langle \Box_\tau \rangle \left(\gamma_g \frac{\partial \beta}{\partial \xi} + \beta \frac{\partial \gamma_g}{\partial \xi}\right)\right]$$

$$\Rightarrow \quad \frac{\varepsilon_q}{T^4} = -\frac{N_\tau^4}{\xi^2}\left[\sum_i \langle \bar{\psi} D_i \psi \rangle \frac{\partial \kappa}{\partial \xi} + \langle \bar{\psi} D_0 \psi \rangle \left(\gamma_q \frac{\partial \kappa}{\partial \xi} + \kappa \frac{\partial \gamma_q}{\partial \xi}\right)\right]$$

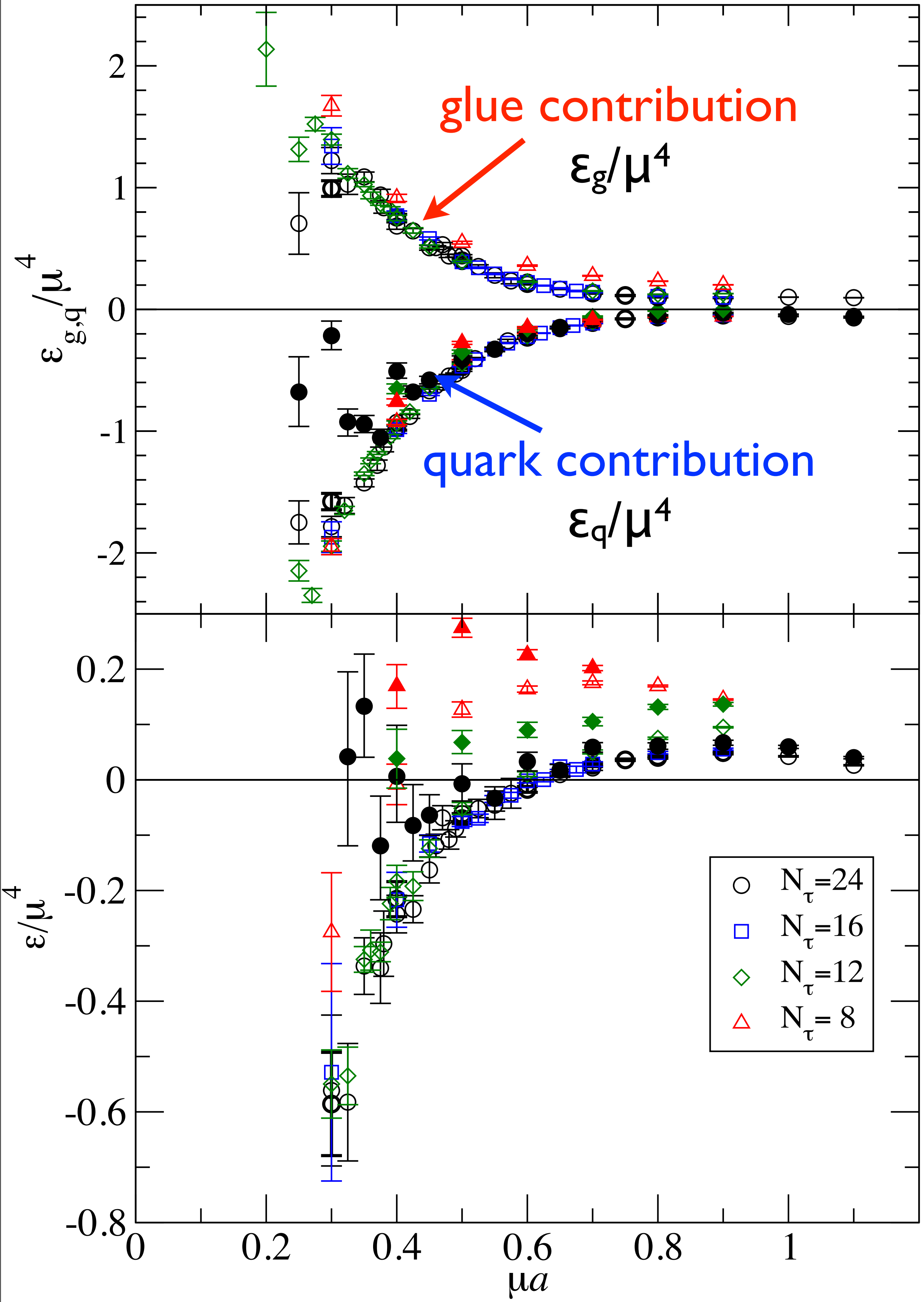
Karsch coefficients $$\frac{\partial \beta}{\partial \xi};\ \frac{\partial \gamma_g}{\partial \xi};\ \frac{\partial \kappa}{\partial \xi};\ \frac{\partial \gamma_q}{\partial \xi}$$

estimated at $\xi=1$, $\mu=T=0$
by simulating with
$\gamma_g=1\pm\delta\gamma_g$, $\gamma_q=1\pm\delta\gamma_q$
and assuming linear response

ξ_g from sideways potential, ξ_q from pion dispersion

Levkova, Manke & Mawhinney, PRD73 (2006) 074504; R. Morrin (TCD thesis)

Energy densities



ε_q/μ^4 now negative for all μ -

no more peak!

again, consistent with self-binding.

(indeed ε only barely positive for smaller μ)

Results very sensitive to values of Karsch coefficients (particularly $\frac{\partial\kappa}{\partial\xi}; \frac{\partial\gamma_q}{\partial\xi}$) $\Rightarrow$ systematic error O(100%)?

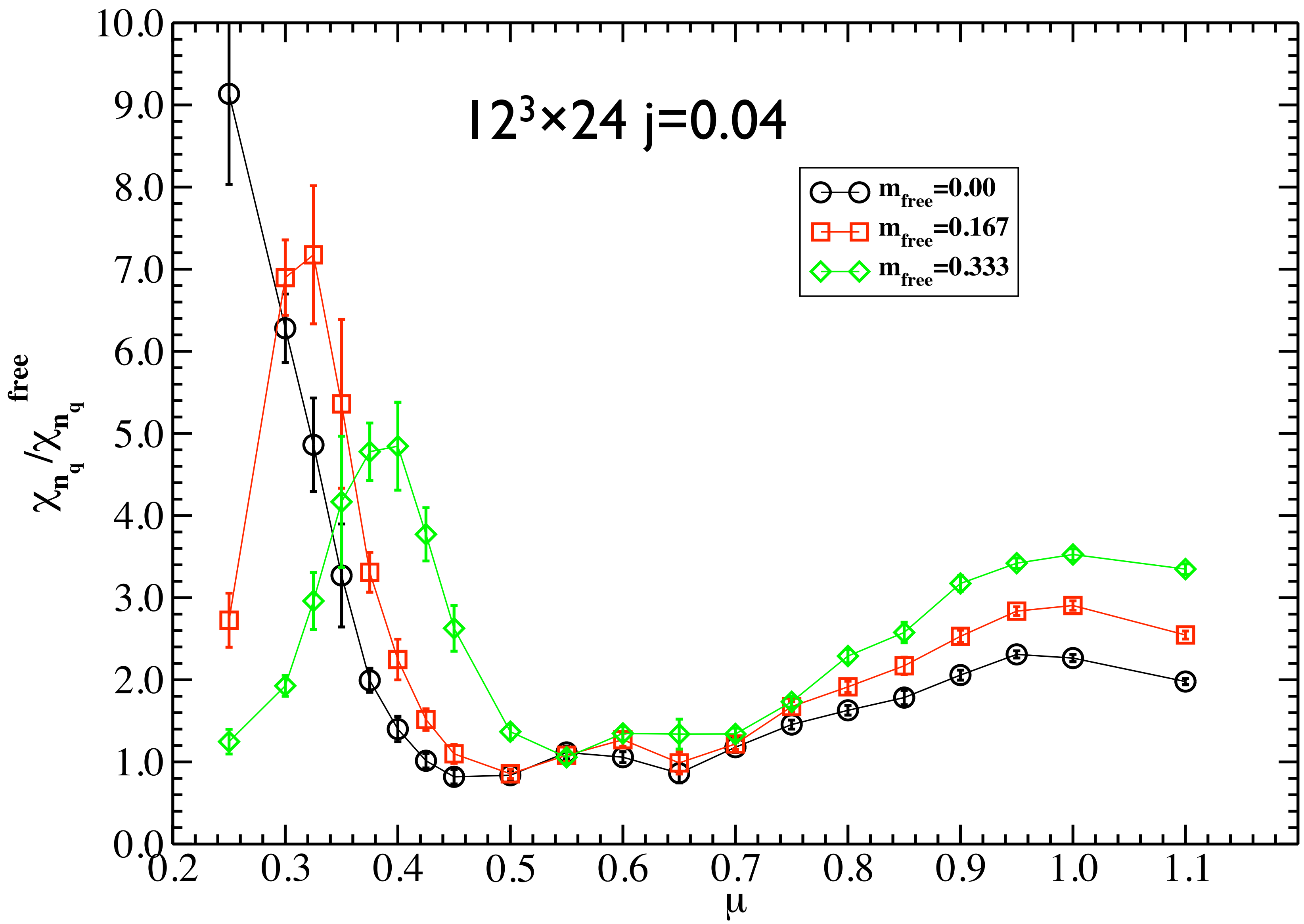
BUT qualitatively similar to bare ε found for $N_f = 4$

Note $a^{N_f=4} \approx ⅓\, a^{N_f=2}$

SJH, P. Kenny and J.I. Skullerud, EPJA 47 (2011) 60

Quark Number Susceptibility

P. Giudice, SJH, & J.I Skullerud POS(LATT2011)193



$$\chi_q = \frac{T}{V_s}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu^2}$$

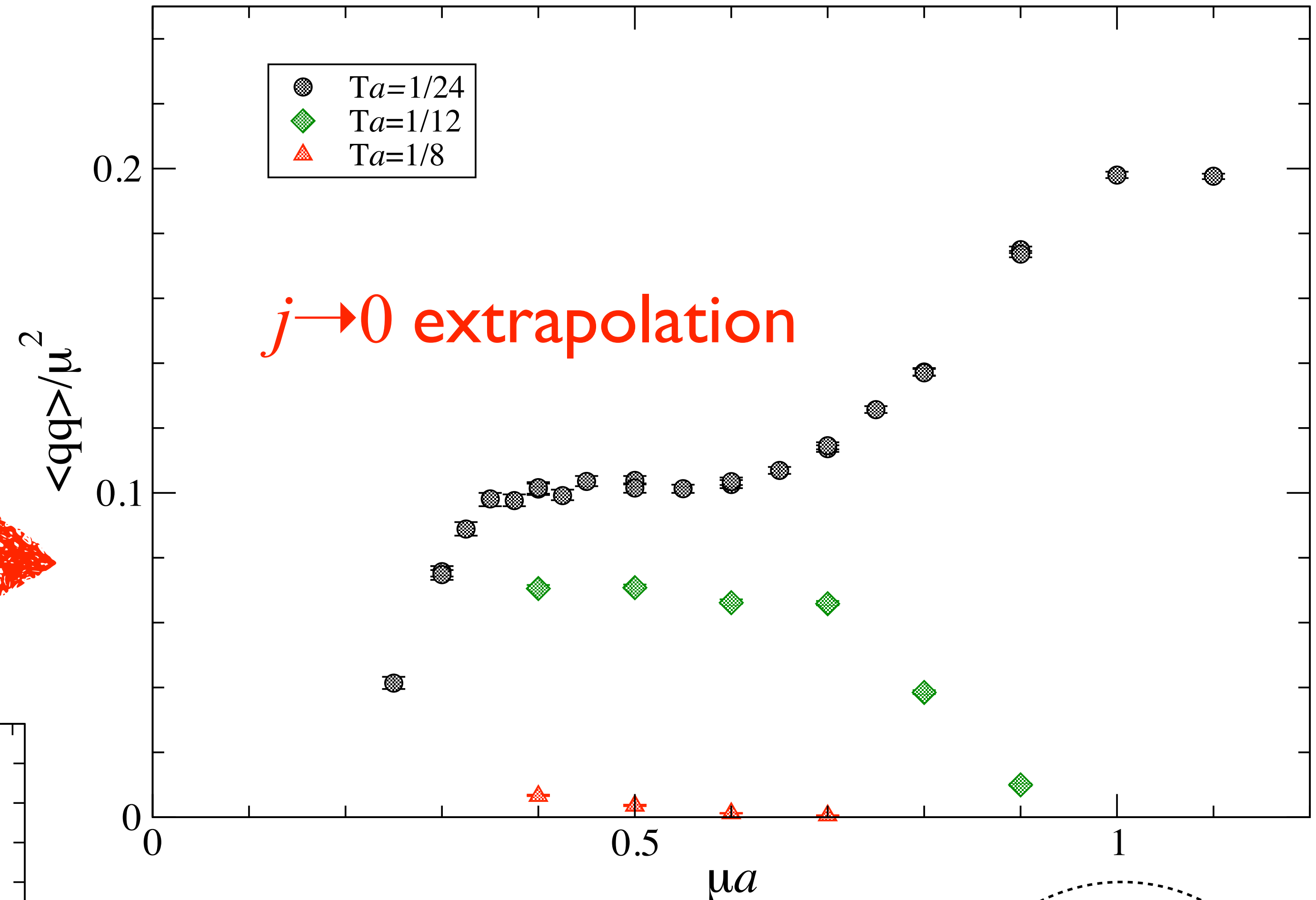
Dominant contribution from hairpin diagram: phase space ∝ area of Fermi surface ∝ μ^2

- ★ consistent with free degenerate quarks for $\mu_Q<\mu<\mu_d$
- ★ Sensitivity to value of m_{free}

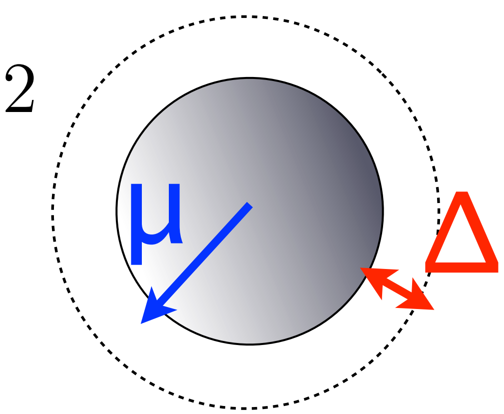
Order parameters

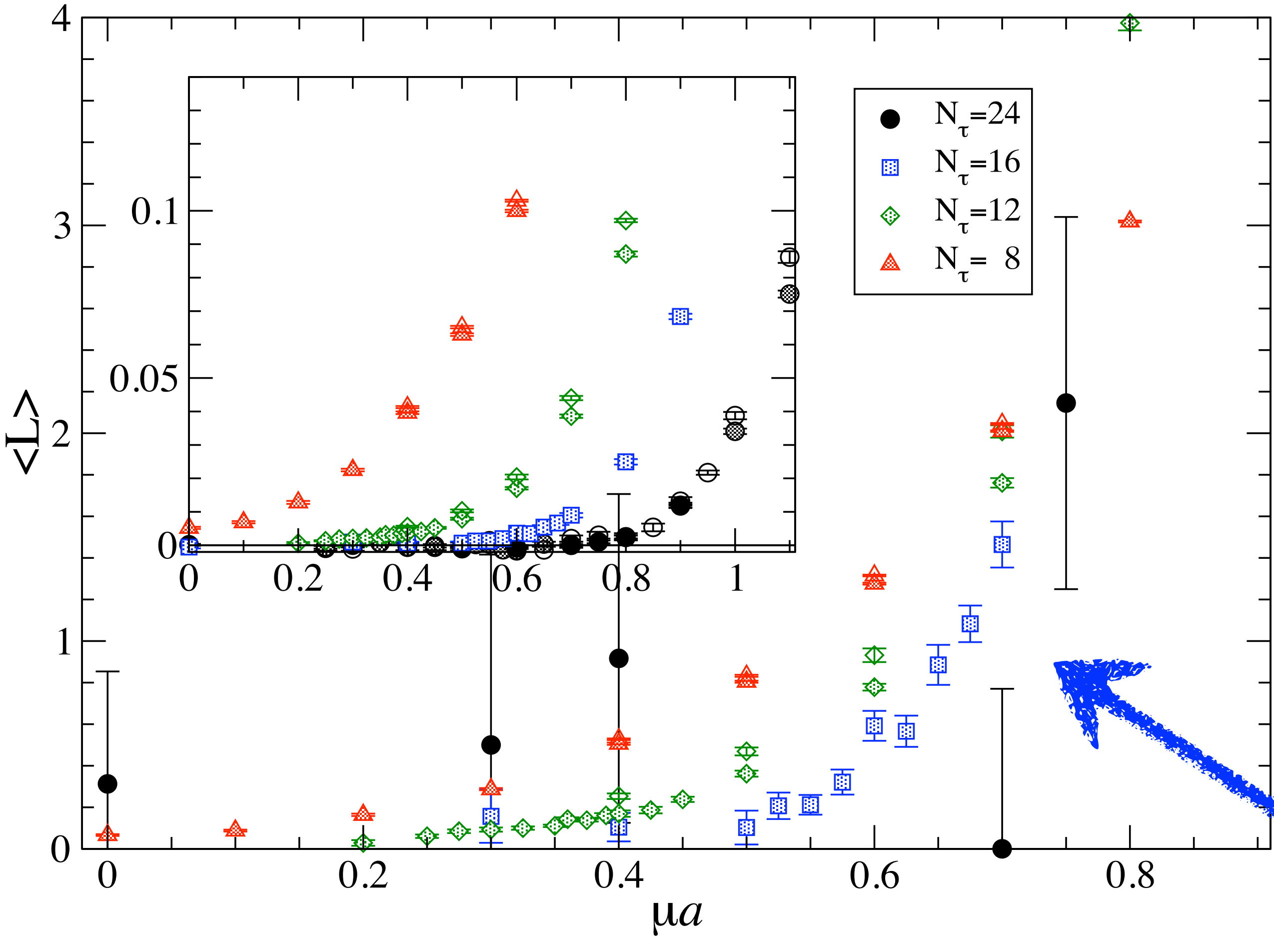
Superfluid condensate $\langle qq \rangle$
scales à la BCS (ie $\propto \mu^2$)
for $\mu_Q \leq \mu \leq \mu_d$

Vanishes as T increases



$$\langle qq(\mu)\rangle \propto \Delta(\mu)\mu^2$$





$\mu \lesssim \mu_d \quad \Delta \propto \Lambda_{QCD}$

$\mu \gtrsim \mu_d \quad \Delta \propto \mu$

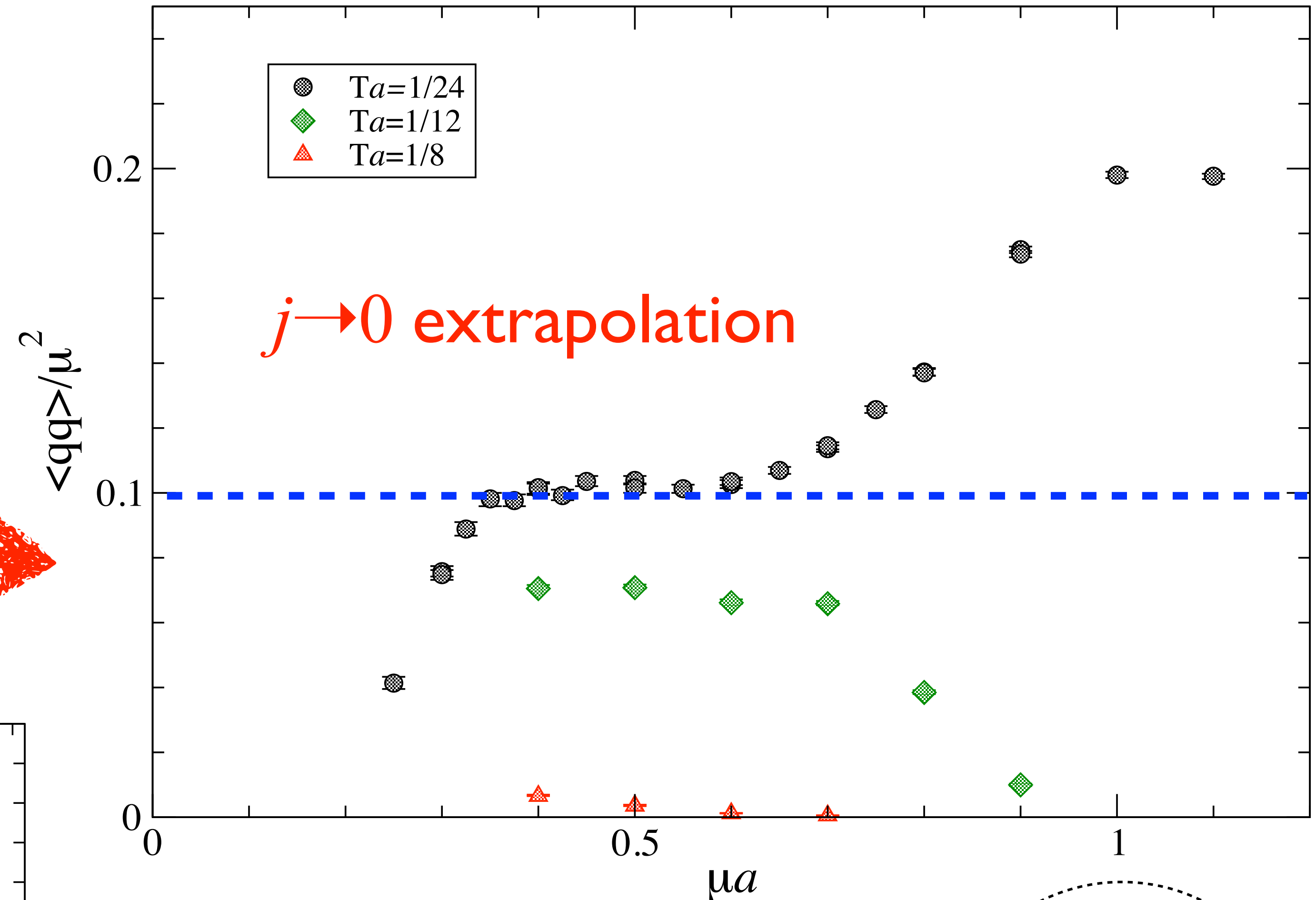
(renormalised) Polyakov line rises from zero at $\mu \approx \mu_d$

$\Rightarrow$ **Deconfinement** at $\mu_d \approx$ 850MeV $n_q \approx$ 16 - 32 fm^{-3}

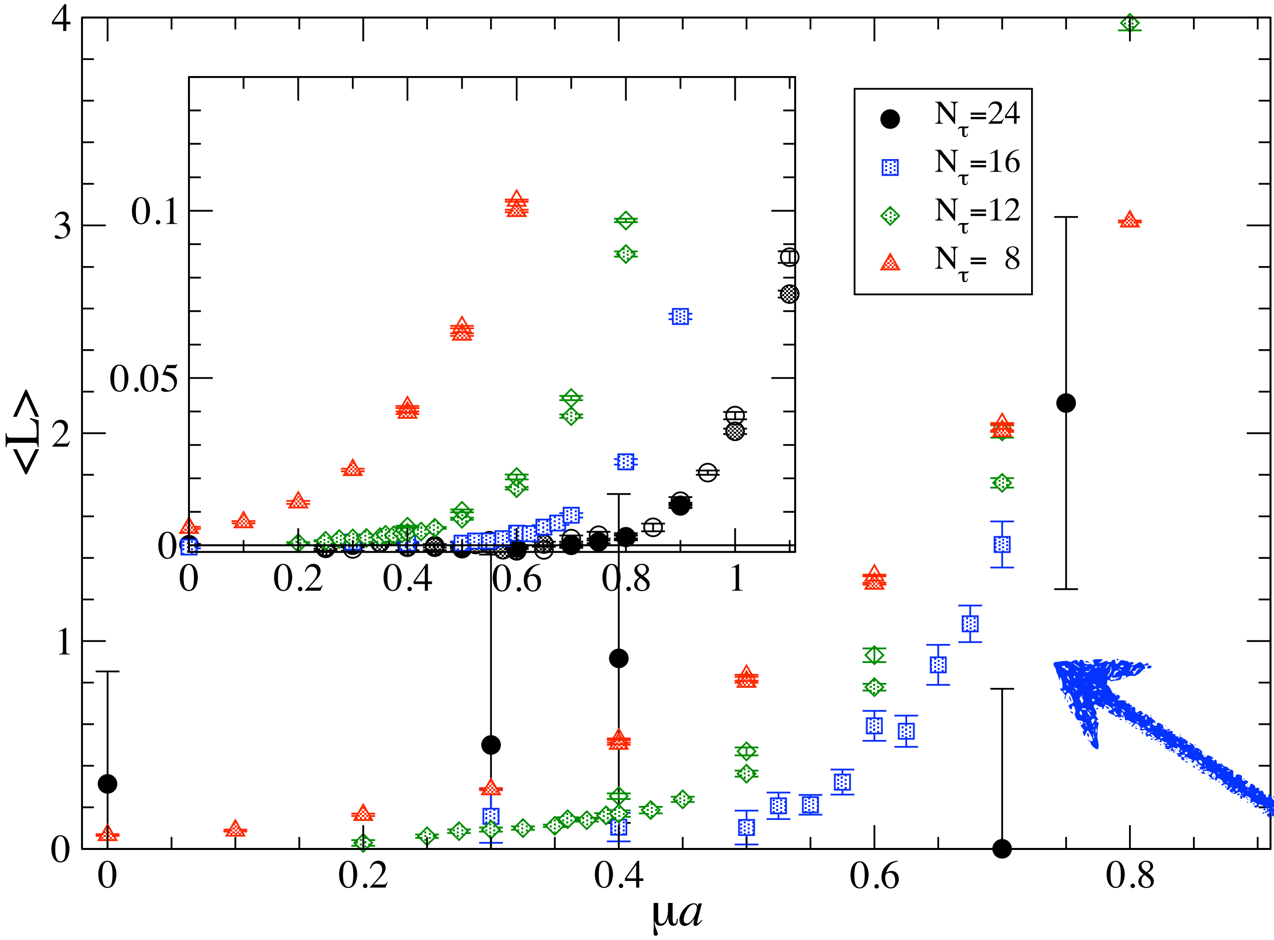
Order parameters

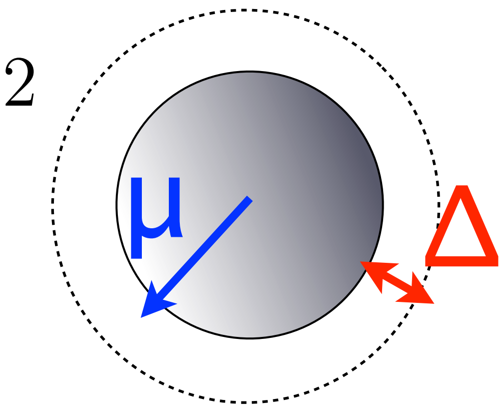
Superfluid condensate $\langle qq \rangle$
scales à la BCS (ie $\propto \mu^2$)
for $\mu_Q \leq \mu \leq \mu_d$

Vanishes as T increases



$$\langle qq(\mu) \rangle \propto \Delta(\mu)\mu^2$$





$\mu \lesssim \mu_d \quad \Delta \propto \Lambda_{QCD}$

$\mu \gtrsim \mu_d \quad \Delta \propto \mu$

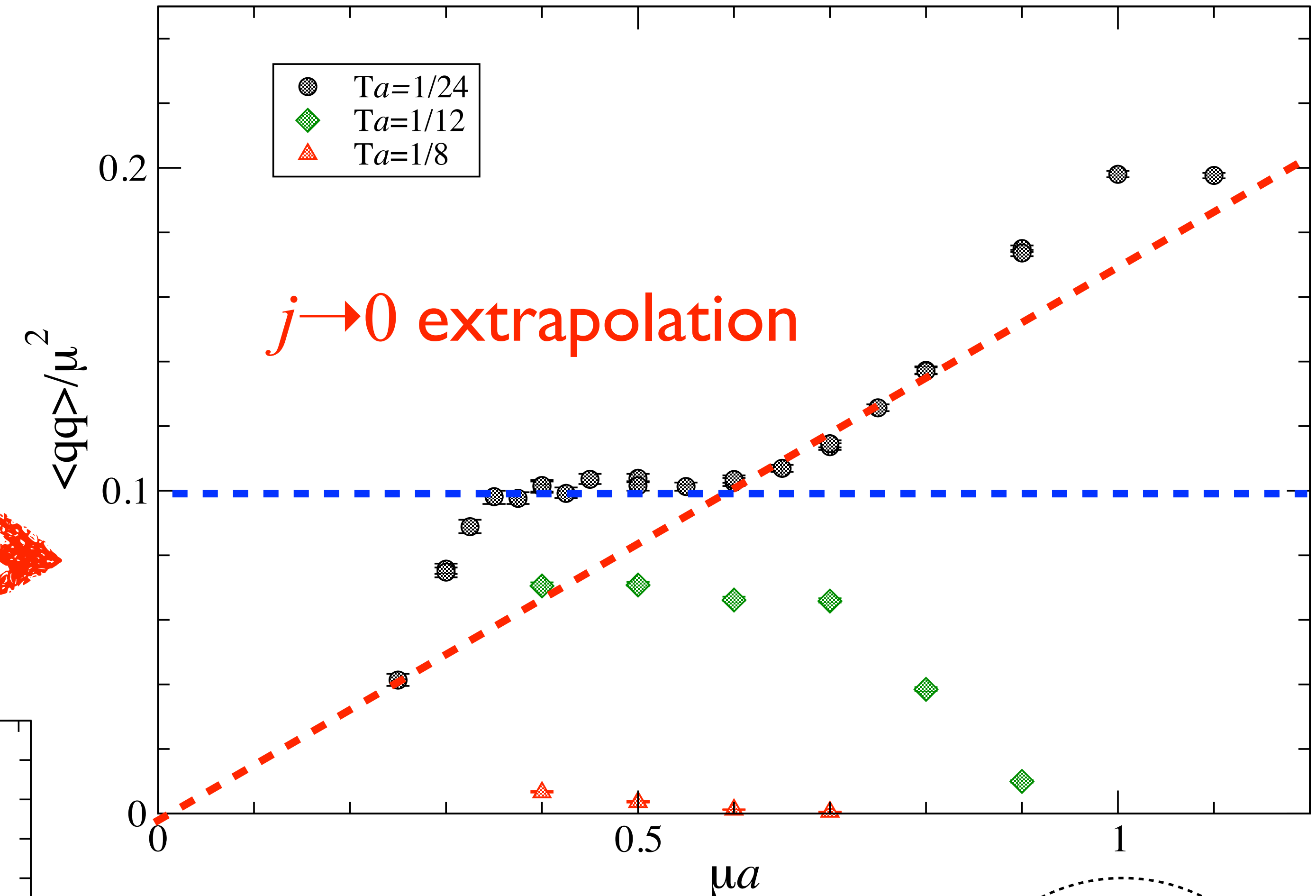
(renormalised) Polyakov line rises from zero at $\mu \approx \mu_d$

$\Rightarrow$ **Deconfinement** at $\mu_d \approx$ 850MeV $n_q \approx$ 16 - 32 fm^{-3}

Order parameters

Superfluid condensate $\langle qq \rangle$ scales à la BCS (ie $\propto \mu^2$) for $\mu_Q \leq \mu \leq \mu_d$

Vanishes as T increases

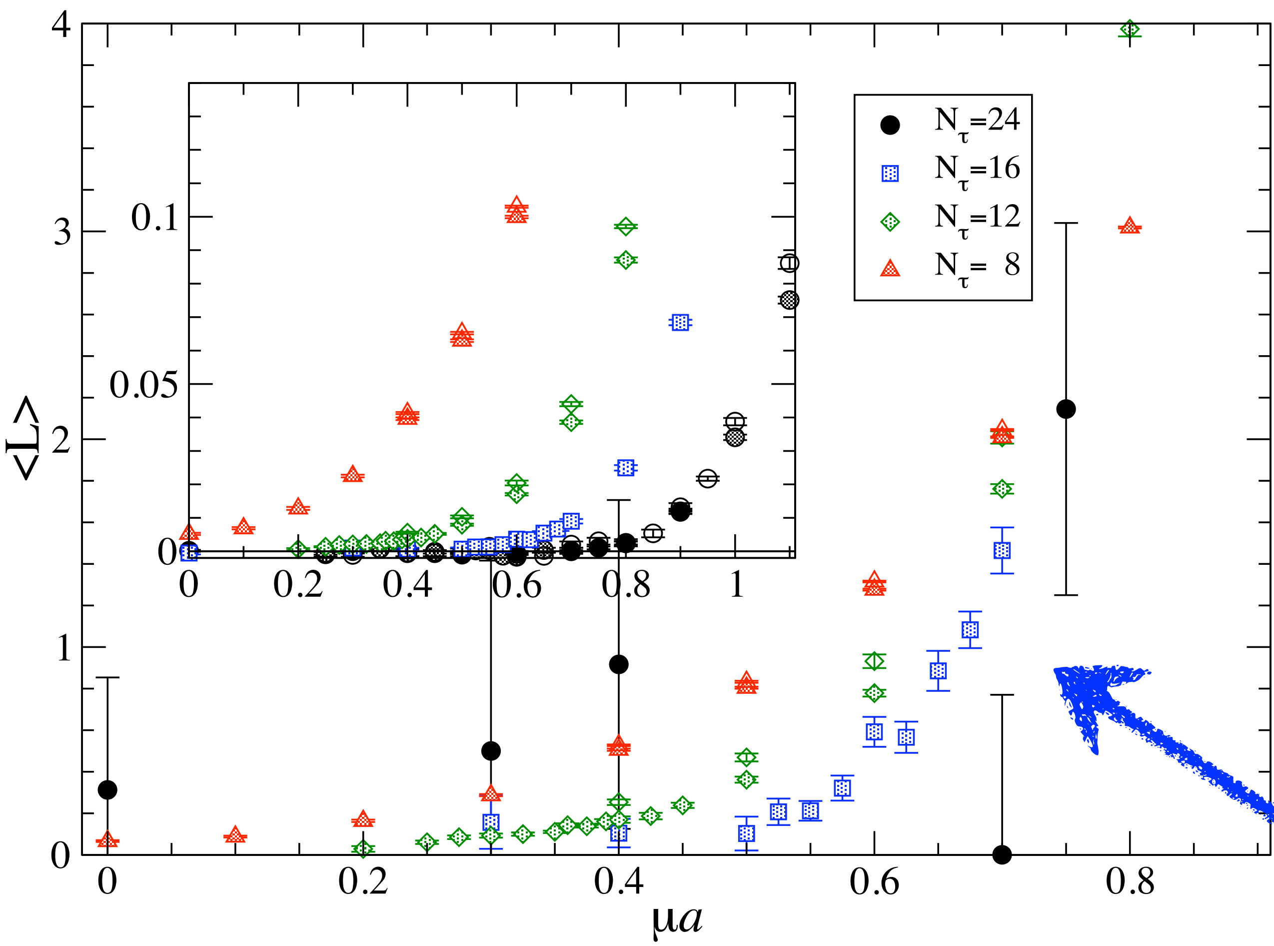


$$\langle qq(\mu) \rangle \propto \Delta(\mu)\mu^2$$

μ
Δ

μ≲μd   Δ∝ΛQCD

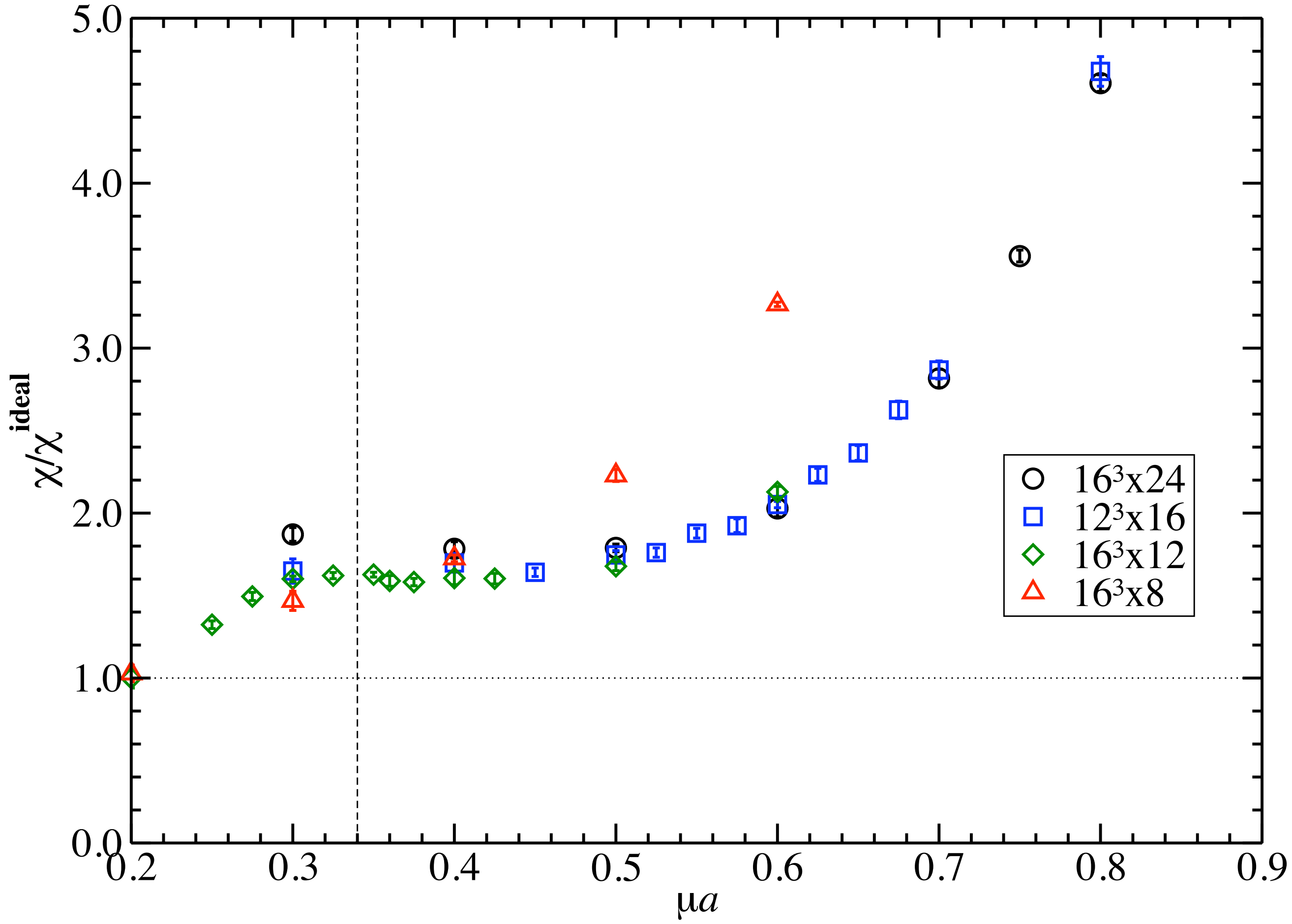
μ≳μd   Δ∝μ

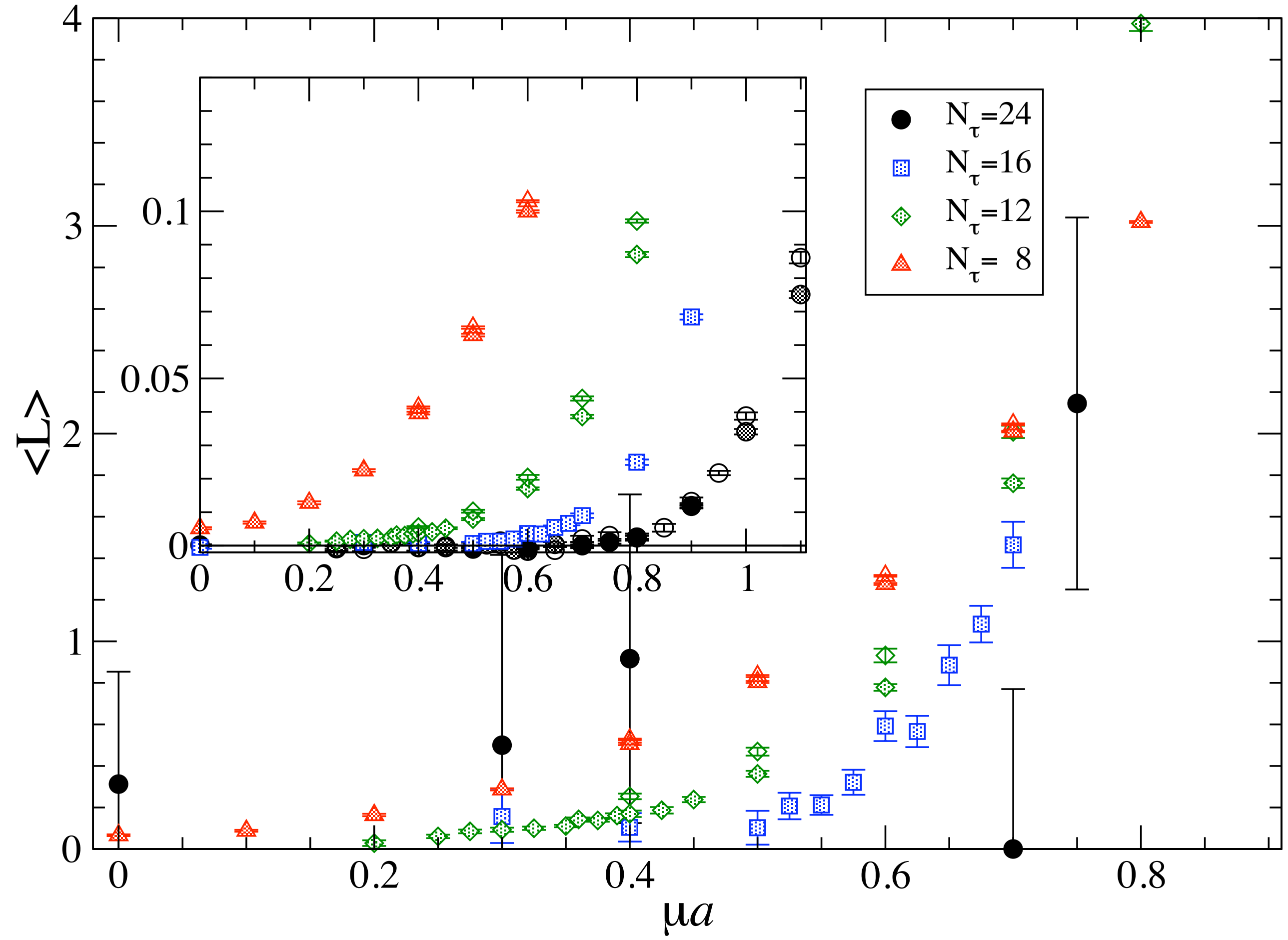


(renormalised) Polyakov line rises from zero at $\mu \approx \mu_d$

⇒ **Deconfinement** at $\mu_d \approx$ 850MeV $n_q \approx$ 16 - 32 fm^{-3}

$\chi_q(\mu)$ does not show same T-dependence as the Polyakov loop





The increase in χ_q is **not** associated with "deconfinement"

So χ_q is ***not*** a proxy for L when $\mu/T >> 1$

Qualitatively different from:

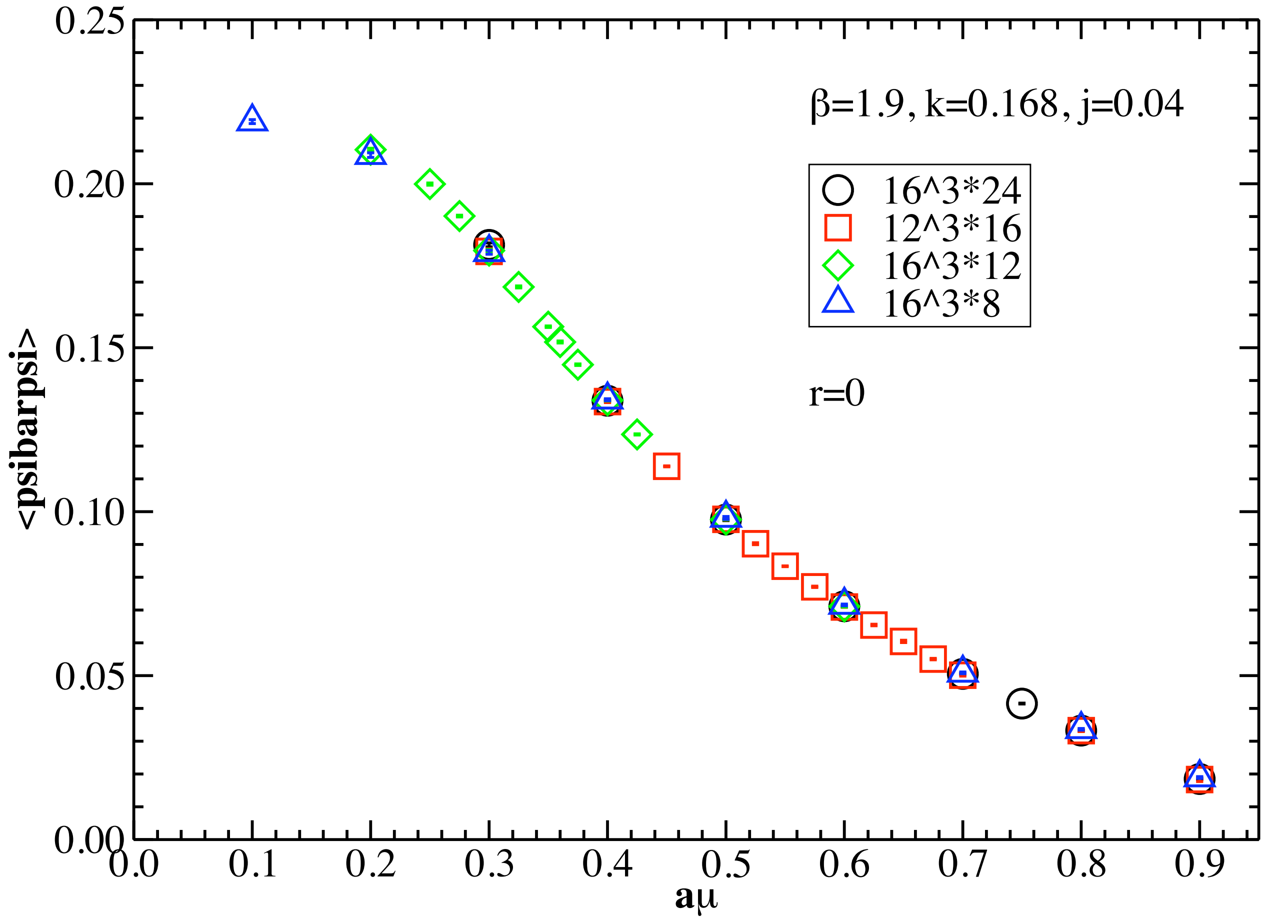
(a) the thermal QCD phase transition

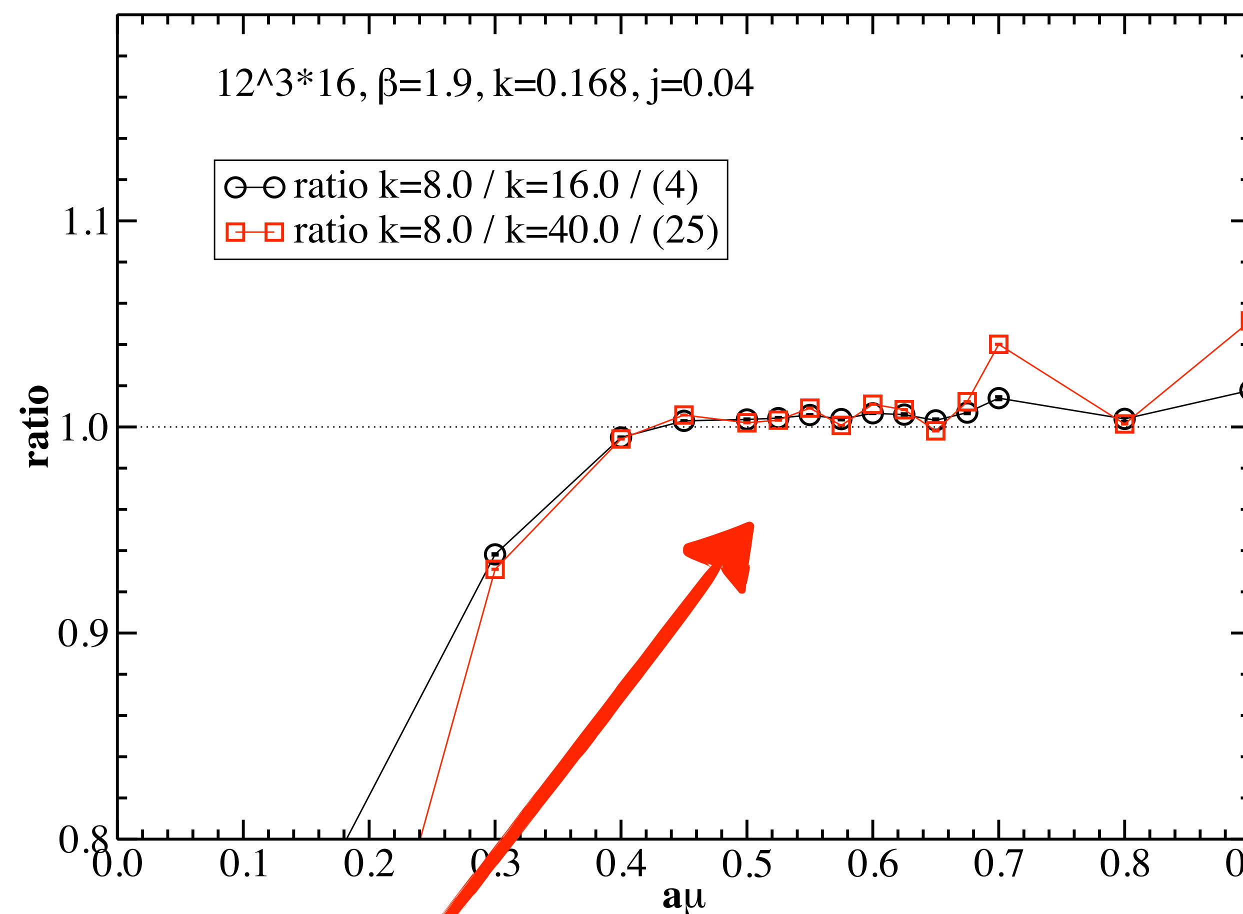
(b) strong coupling with heavy quarks Fromm, Langelage, Lottini, Neuman, Philipsen arXiv/1207.3005

(c) analytic/numerical studies on small, cold volumes (the "attoworld")

SJH, J. Myers, T.J. Hollowood, JHEP 1007 (2010) 086, 1012 (2010) 057

And chiral symmetry?....





χPT prediction: $\langle\bar\psi\psi\rangle \propto \dfrac{m}{\mu^2}$

$$\frac{\kappa_1^2\langle\bar\psi\psi\rangle_1}{\kappa_2^2\langle\bar\psi\psi\rangle_2} = \frac{m_2\langle\bar qq\rangle_1}{m_1\langle\bar qq\rangle_2} \begin{cases} = 1 & \text{chirally symmetric;} \\ < 1 & \chi\text{SB with } m_2 < m_1. \end{cases}$$

interrogate configurations using "naive" fermions with $r = 0, ja = 0.04$
and $\kappa = 8.0, 16.0, 40.0$

Chiral symmetry restored for μa>0.4 ?

The degenerate, confined & “chirally symmetric” system observed for $\mu_Q < \mu < \mu_d$ resembles the quarkyonic phase postulated by McLerran and Pisarski based on large-N_c considerations. [L. McLerran and R.D. Pisarski Nucl. Phys. A796 (2007) 83]

The degenerate, confined & "chirally symmetric" system observed for $\mu_Q < \mu < \mu_d$ resembles the quarkyonic phase postulated by McLerran and Pisarski based on large-N_c considerations. [L. McLerran and R.D. Pisarski Nucl. Phys. A796 (2007) 83]

Conjecture: The change in behaviour of bulk quantities $(n_q,\, p,\, \chi_q, T_{\mu\mu})$ observed at μ_d is a transition from short-ranged (binary?) to longer-ranged inter-quark interactions (ie. from weak to strong self-binding) *within* the medium.

Only weakly dependent on T.

Relevant for bulk thermodynamics

The degenerate, confined & "chirally symmetric" system observed for $\mu_Q<\mu<\mu_d$ resembles the quarkyonic phase postulated by McLerran and Pisarski based on large-N_c considerations. [L. McLerran and R.D. Pisarski Nucl. Phys. A796 (2007) 83]

Conjecture: The change in behaviour of bulk quantities $(n_q, p, \chi_q, T_{\mu\mu})$ observed at μ_d is a transition from short-ranged (binary?) to longer-ranged inter-quark interactions (ie. from weak to strong self-binding) *within* the medium.

Only weakly dependent on T.

Relevant for bulk thermodynamics

The T-dependent behaviour of the Polyakov loop is due to degrees of freedom close to the Fermi surface.

Relevant for transport properties.

The degenerate, confined & "chirally symmetric" system observed for $\mu_Q<\mu<\mu_d$ resembles the quarkyonic phase postulated by McLerran and Pisarski based on large-N_c considerations.

[L. McLerran and R.D. Pisarski Nucl. Phys. A796 (2007) 83]

Conjecture: The change in behaviour of bulk quantities $(n_q, p, \chi_q, T_{\mu\mu})$ observed at μ_d is a transition from short-ranged (binary?) to longer-ranged inter-quark interactions (ie. from weak to strong self-binding) *within* the medium.

Only weakly dependent on T.

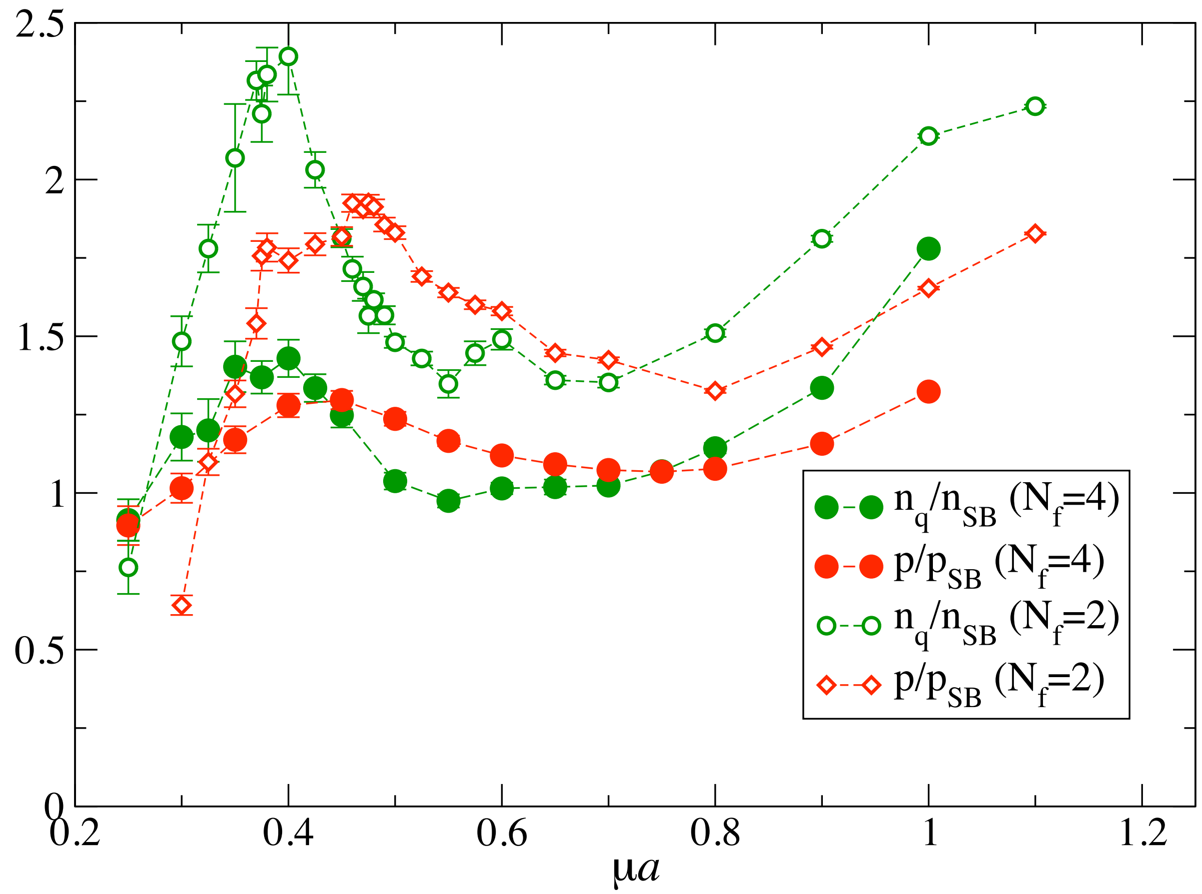
Relevant for bulk thermodynamics

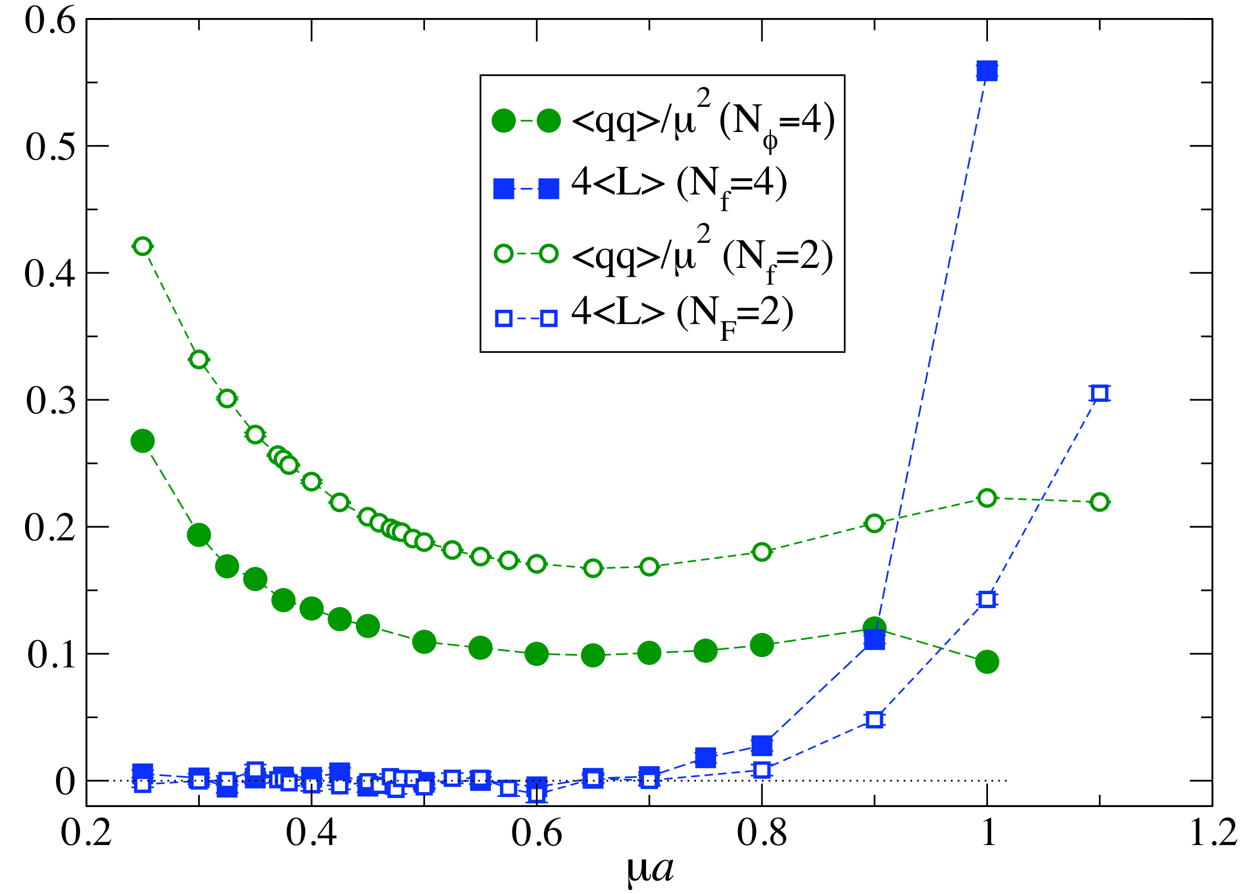
The T-dependent behaviour of the Polyakov loop is due to degrees of freedom close to the Fermi surface.

Relevant for transport properties.

Caveat: UV artifacts may dominate at large μ

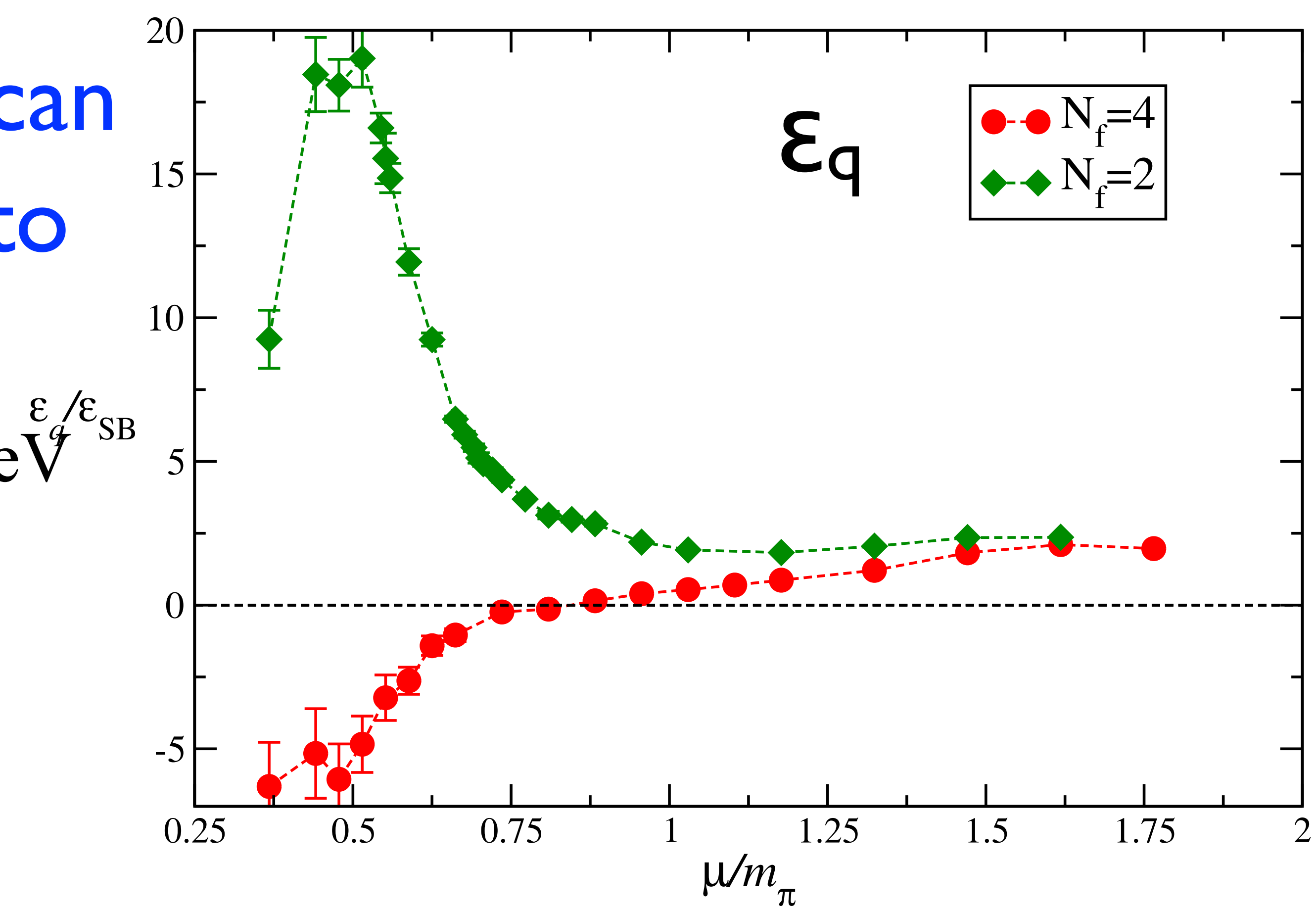
And N_f=4 ? SJH, P. Kenny, S. Kim & J.I. Skullerud, EPJA**47** (2011) 60





Same distinct physical regimes can be identified but *much* closer to continuum: a=0.062(2)fm

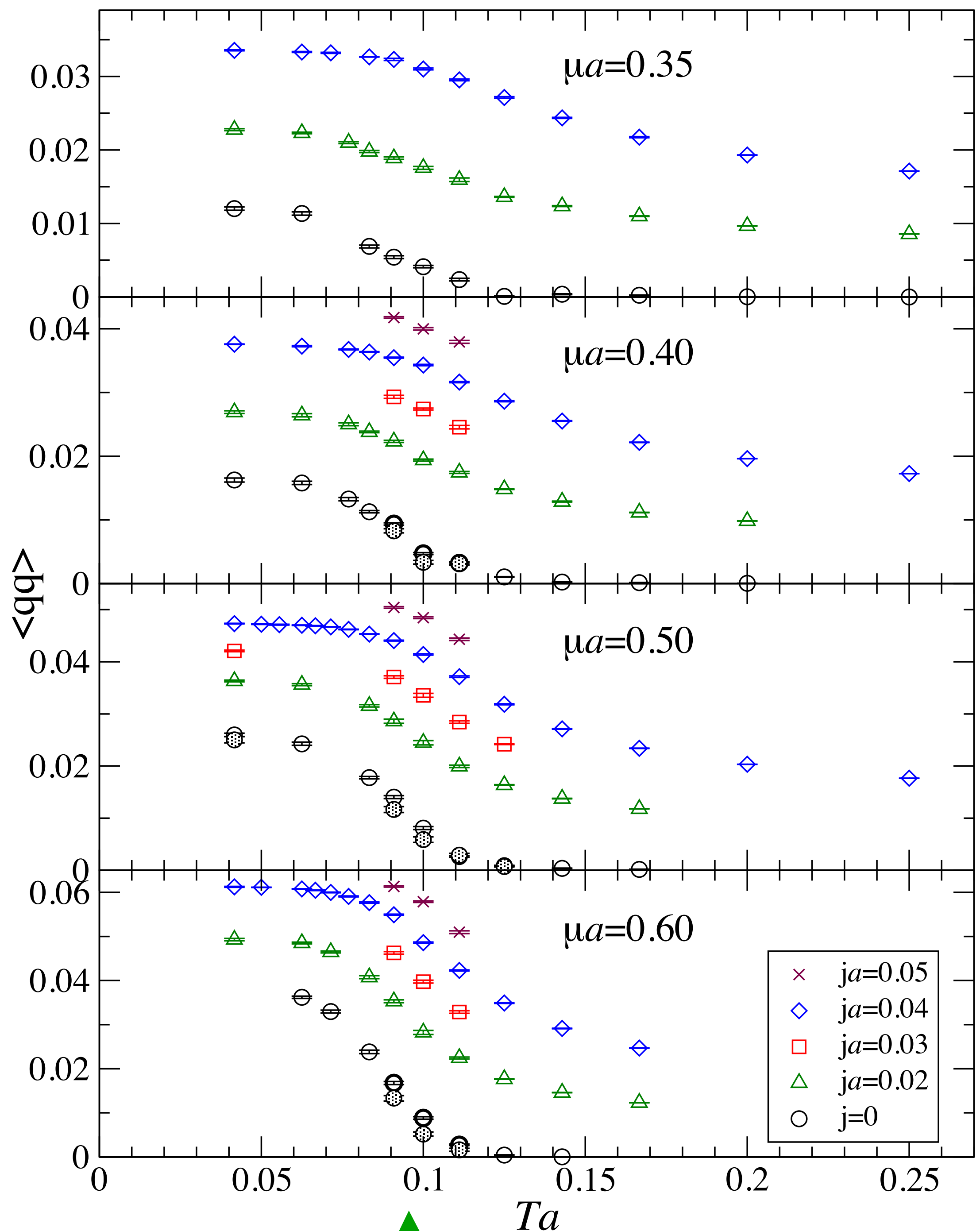
⟹ μ_Q≈1.5GeV, μ_d≈2.5GeV, T=133(4)MeV

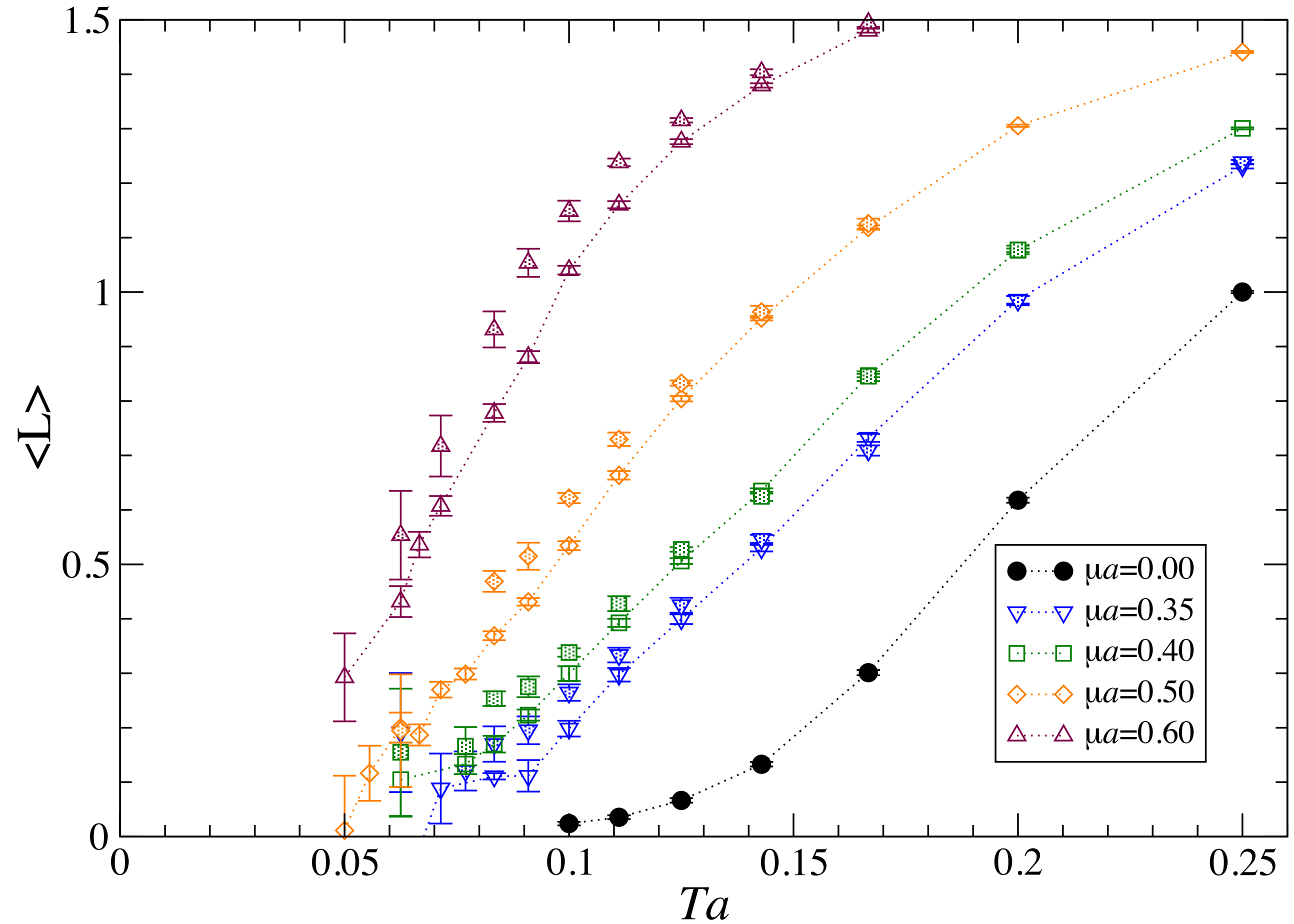


Negative ε_q consistent with renormalised result at $N_f=2$

Recent simulations on $16^3 \times N_T = 4,\ldots,20$ have sketched out the picture at higher T, intermediate μ

Boz, Cotter, Fister, Mehta & Skullerud, arXiv:1303.3223





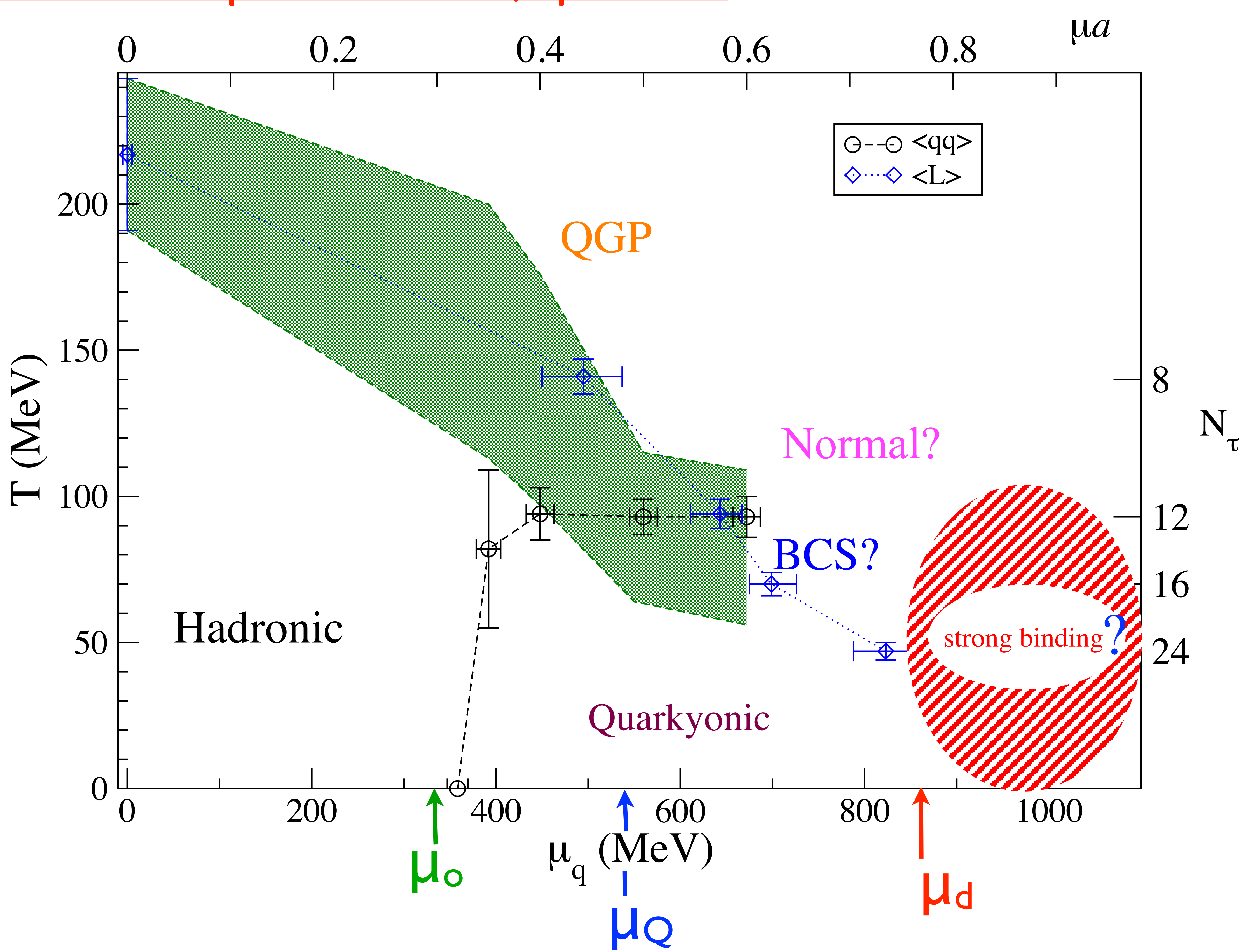
T_s is strikingly μ-independent

Identify:

superfluid → normal transition via point of inflection of <qq(T)>

deconfining crossover via linear regime of <L(T)>

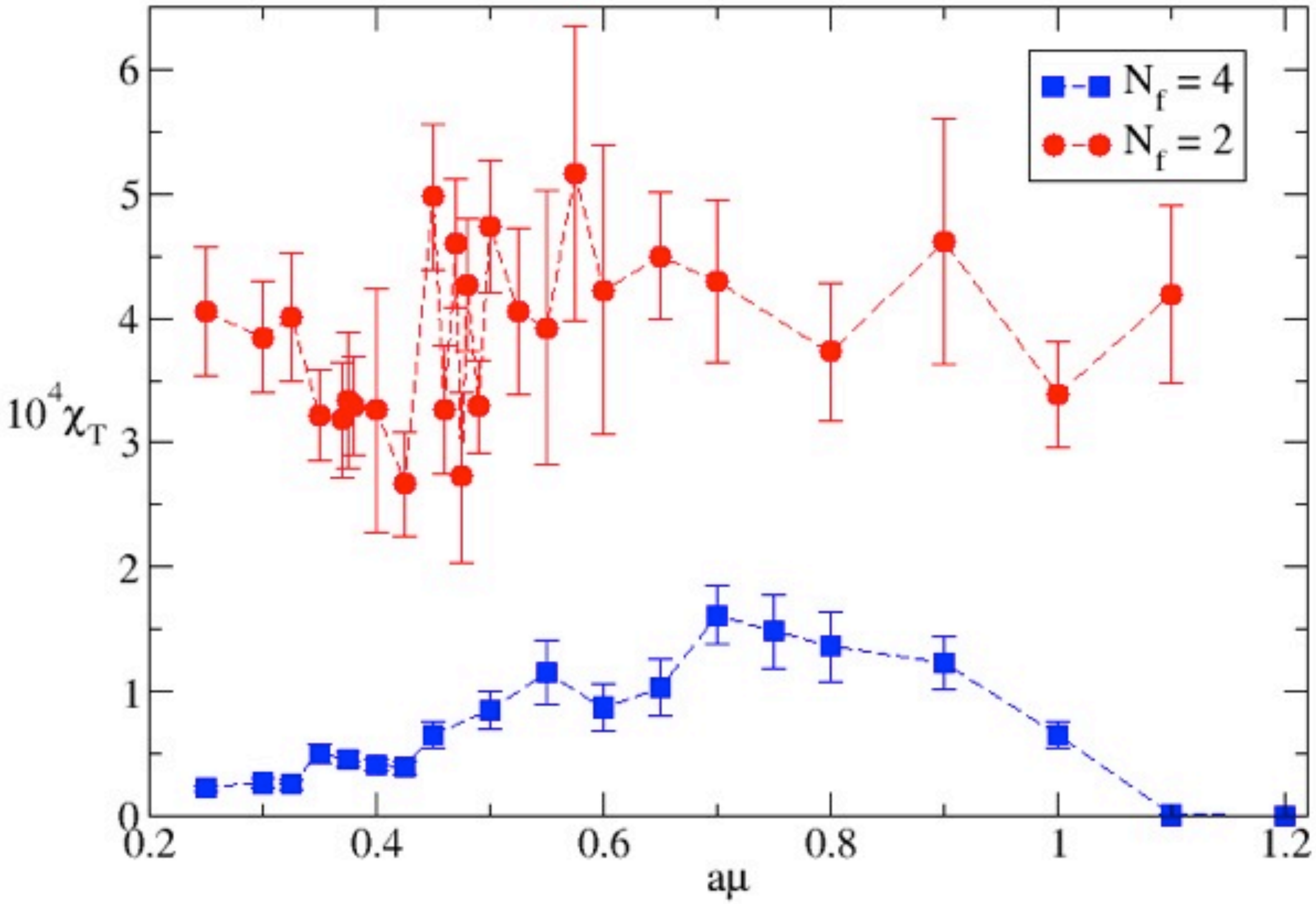
Crude map of the T-μ plane...

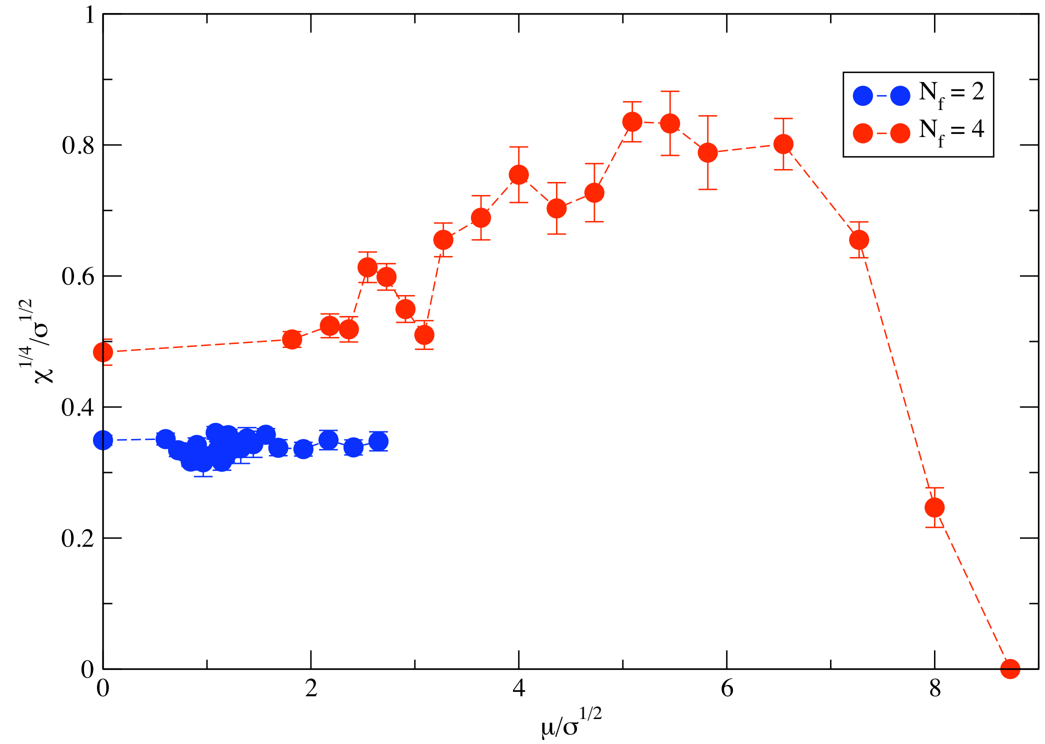


Topological Susceptibility

SJH, P. Kenny, PLB701 (2011) 373

We have investigated instanton distributions and sizes using **cooling**

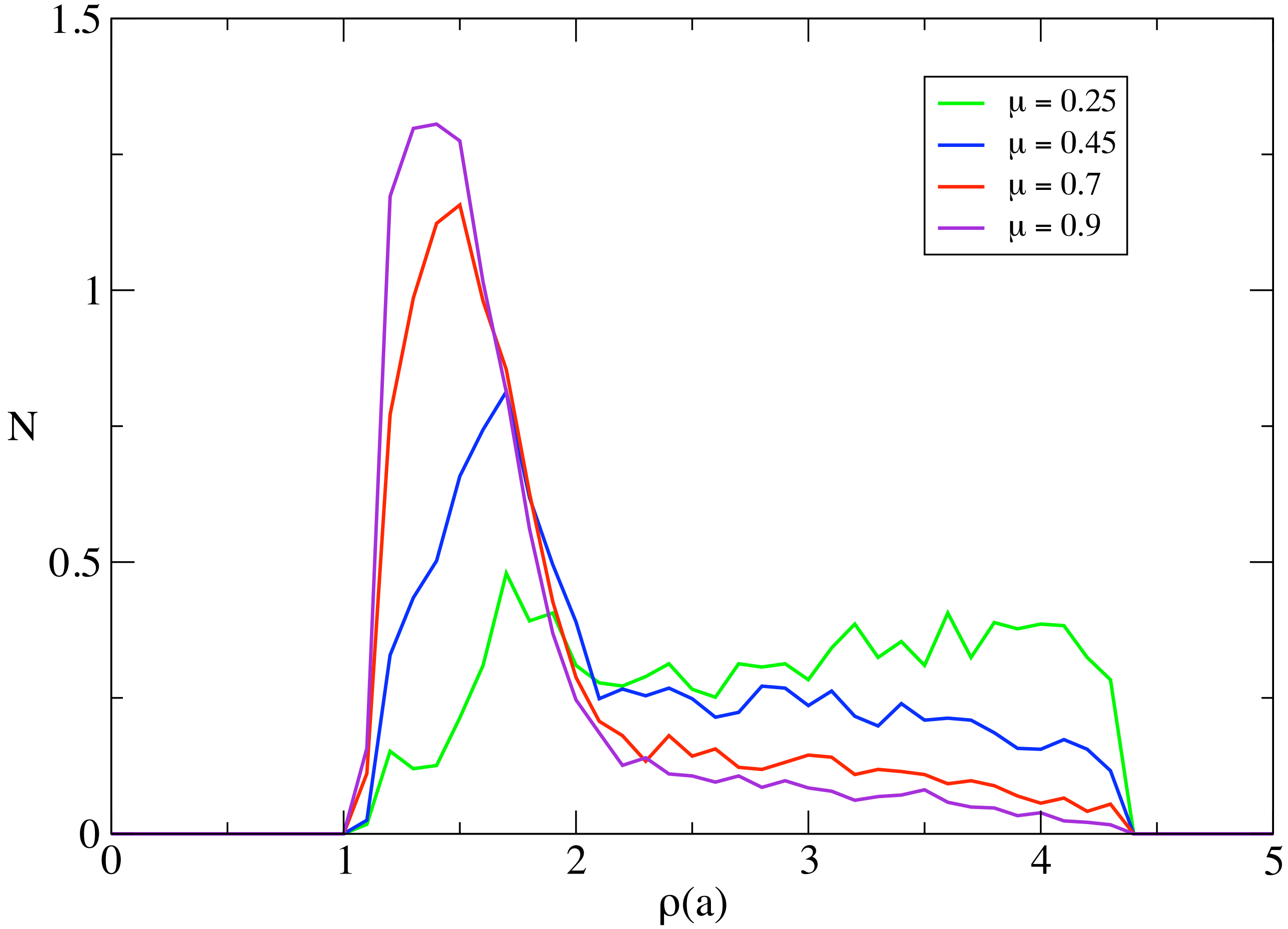


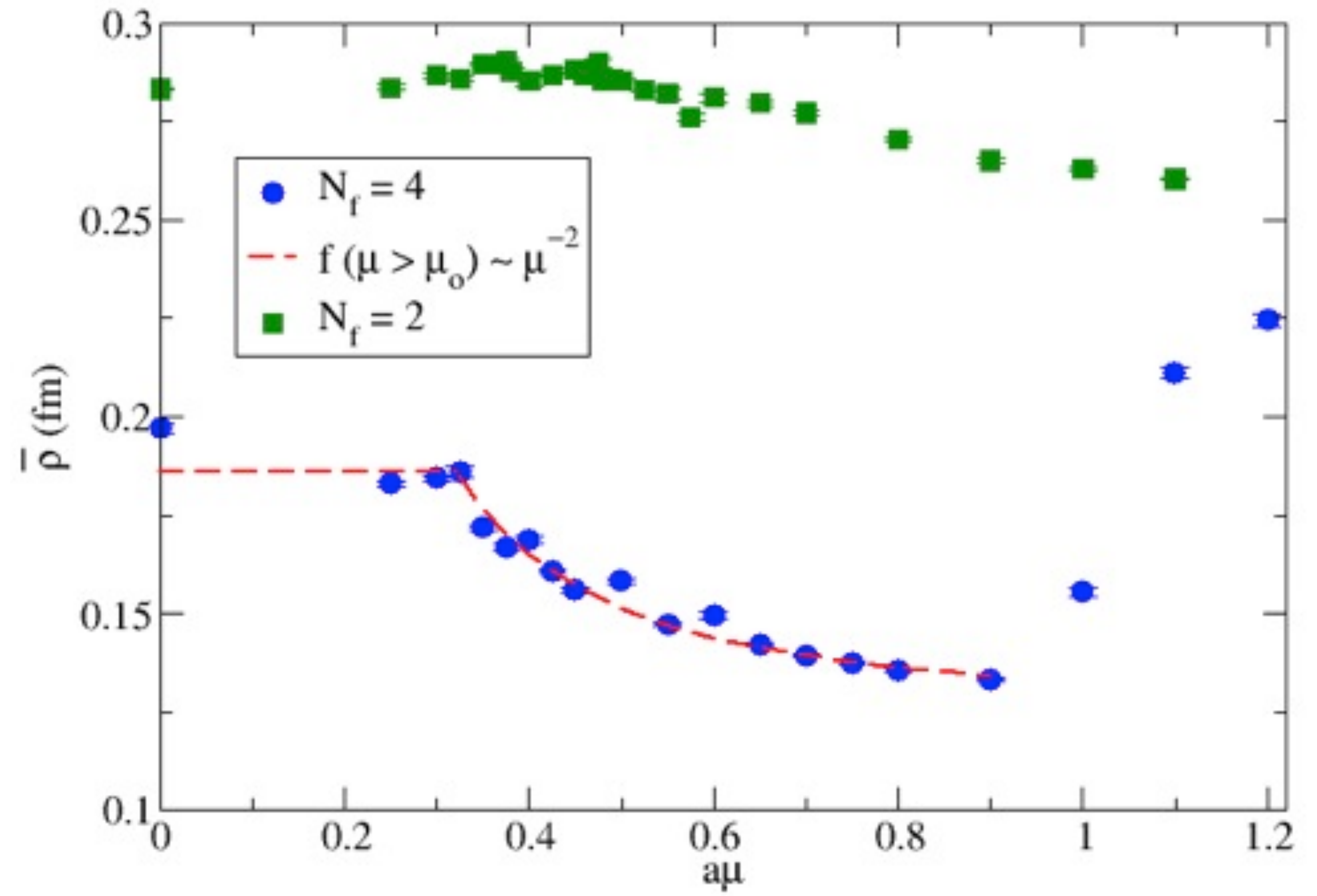


dimensionless plot
$\chi^{0.25}/\sigma^{0.5}$ vs. $\mu/\sigma^{0.5}$

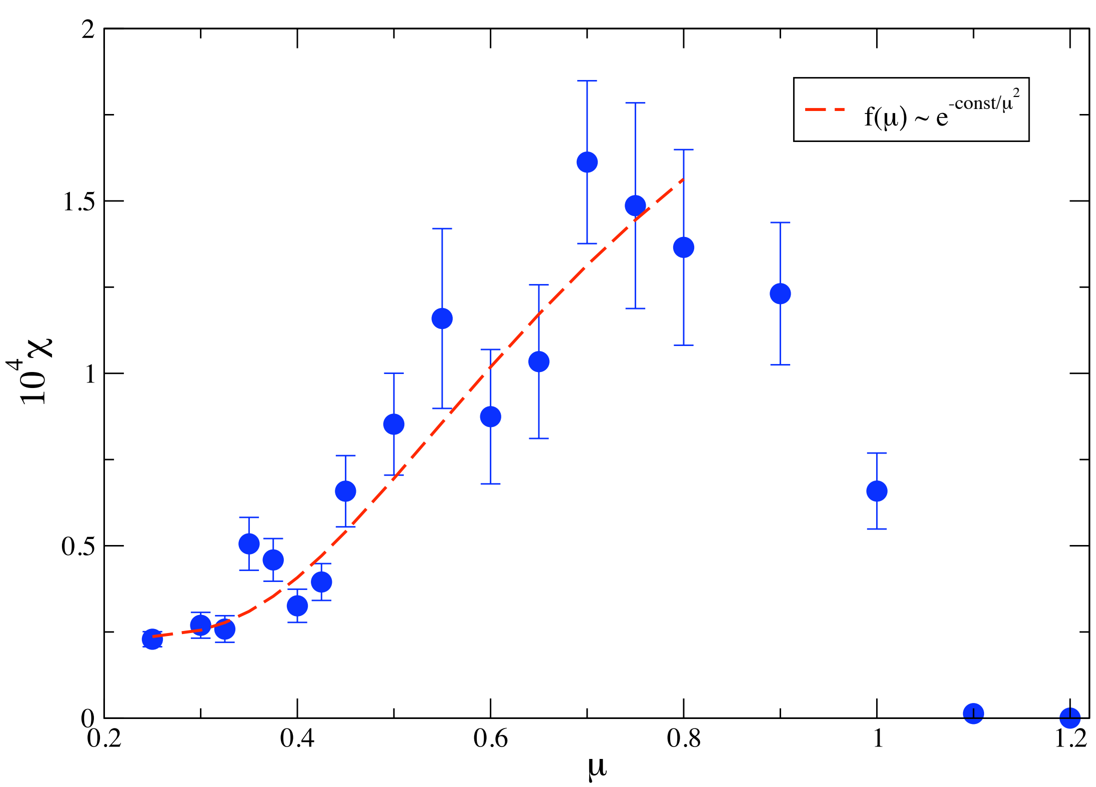
Topological susceptibility shows no structure for $N_f=2$
(maybe lattice too coarse?)
but appears **enhanced** in quarkyonic region for $N_f=4$

Cf. suppression in superfluid phase for $N_f=8$
B. Alles, M. D'Elia & M.P. Lombardo, NPB752(2006)124





For $\mu_o<\mu<\mu_d$ the mean instanton size ρ_I decreases



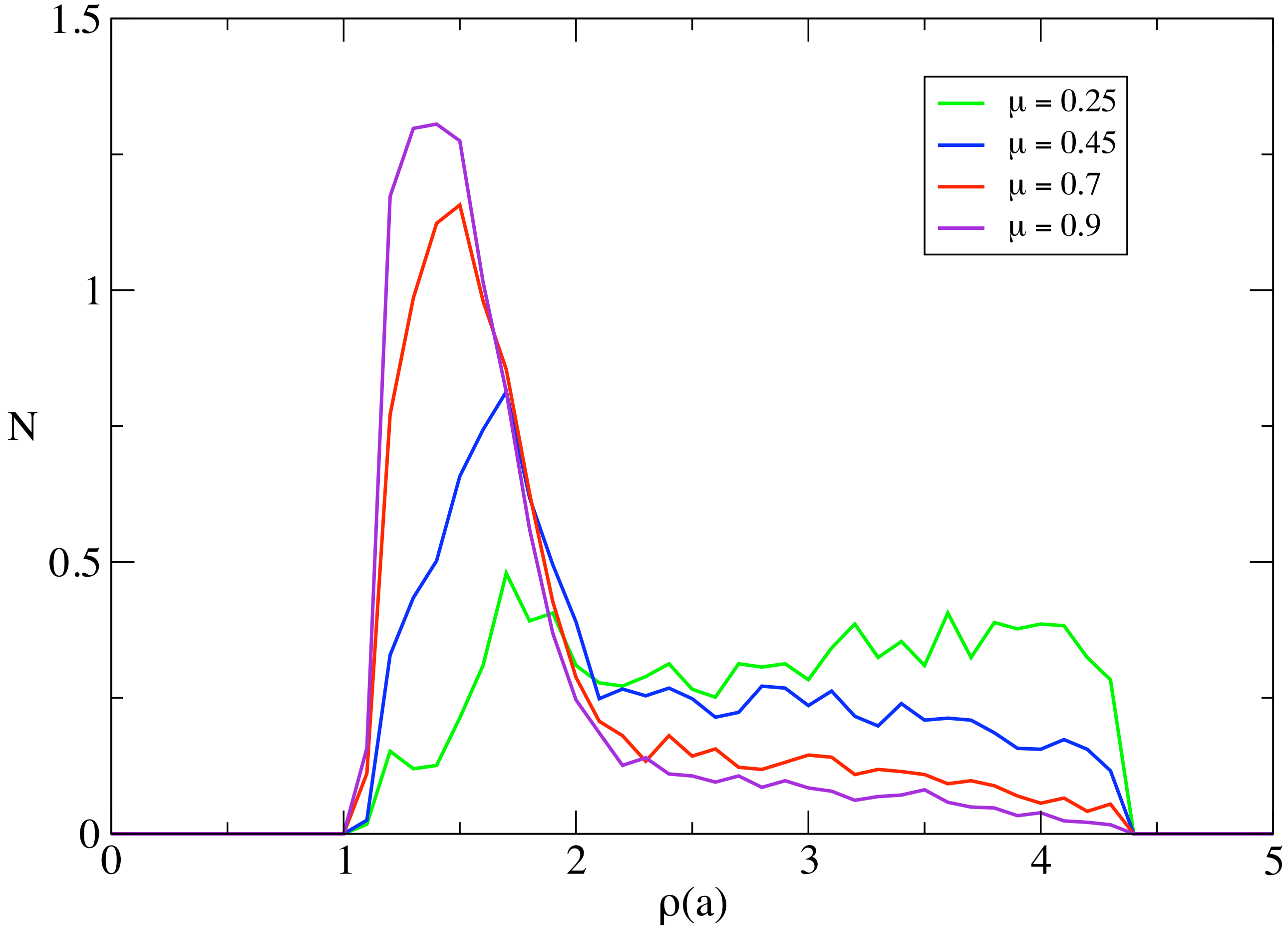
One-loop Debye screening:

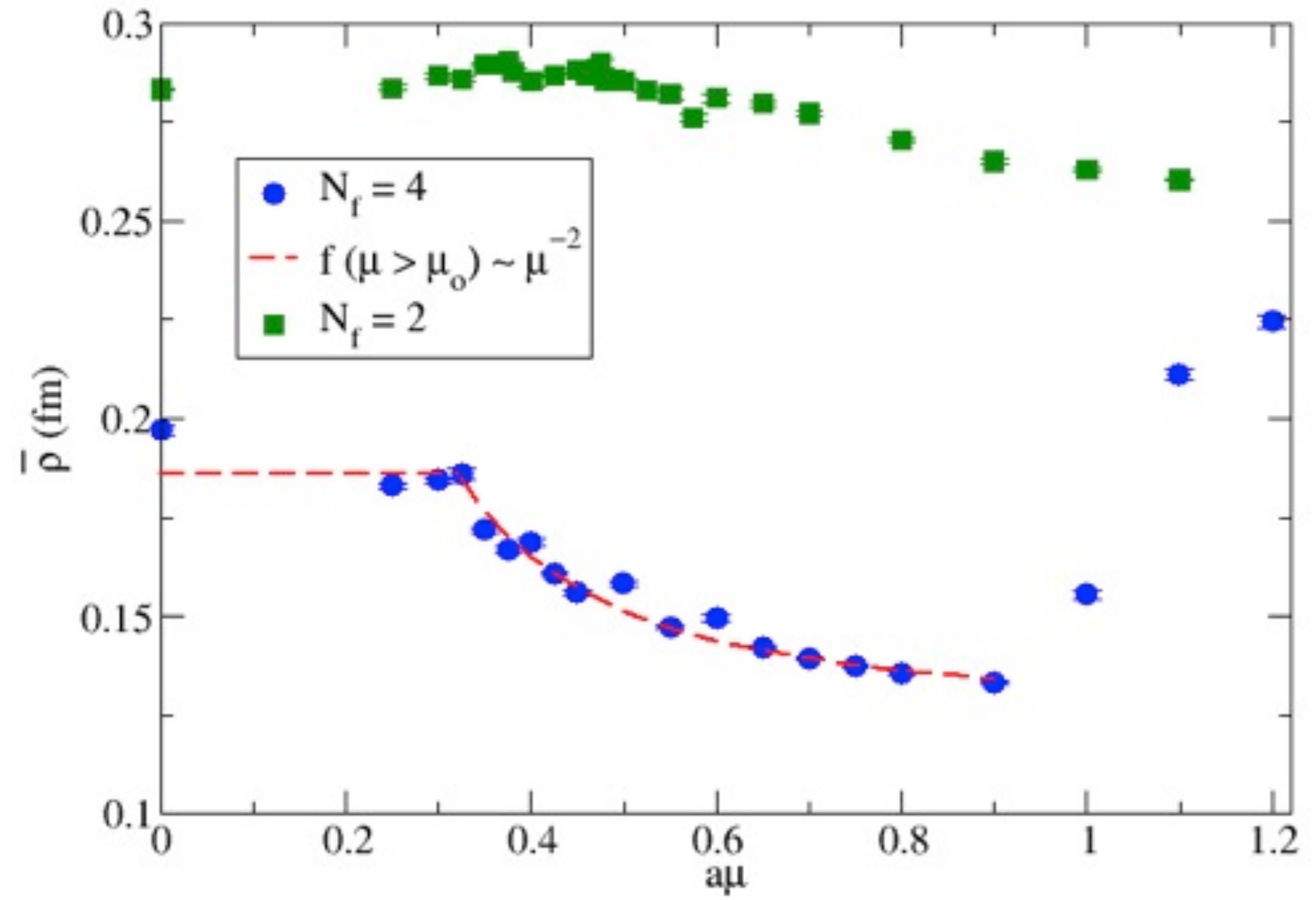
Schäfer & Shuryak RMP 70(1998)323

$$n_I(\mu) \;\propto\; \exp\left[-N_f\rho_I^2\mu^2\right]$$

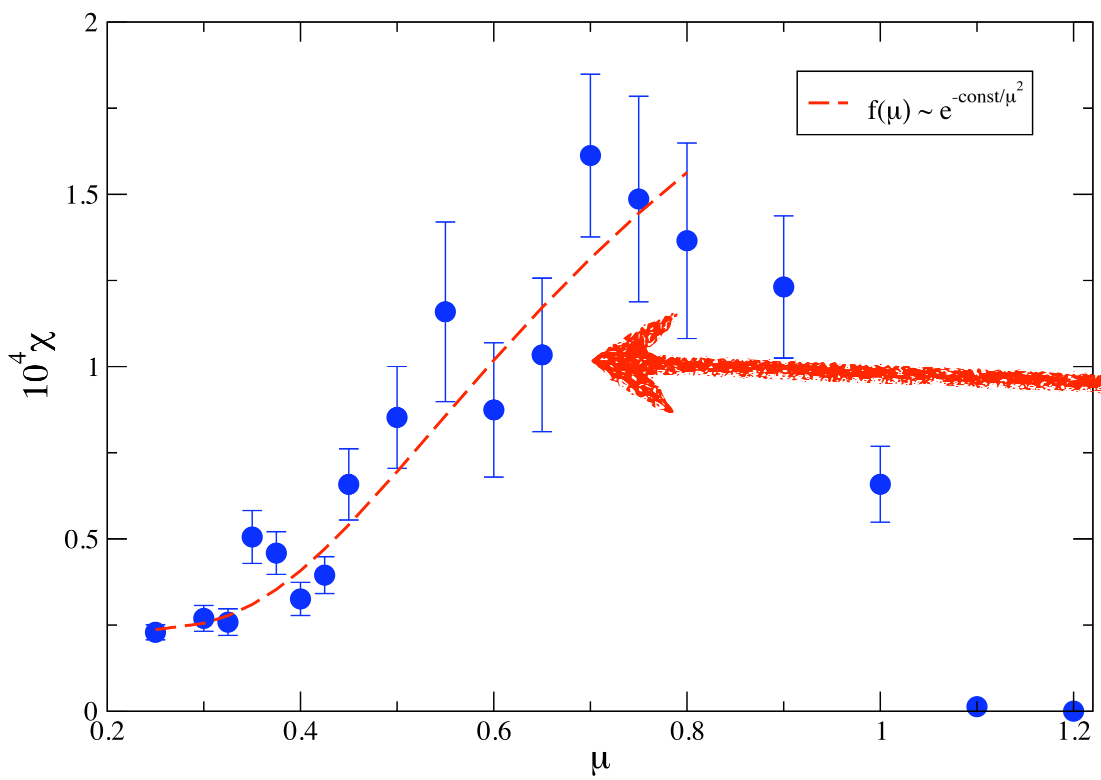
$$\propto\; \exp\left[-\frac{\text{const}}{\mu^2}\right]$$

In QCD$_{2+1}$: Enhancement of $\mathrm{U}(1)_A$ breaking ⇒ first-order region of Columbia plot grows with μ?





For $\mu_o<\mu<\mu_d$ the mean instanton size ρ_I decreases



One-loop Debye screening:

Schäfer & Shuryak RMP 70(1998)323

$$n_I(\mu) \;\propto\; \exp\left[-N_f\rho_I^2\mu^2\right]$$

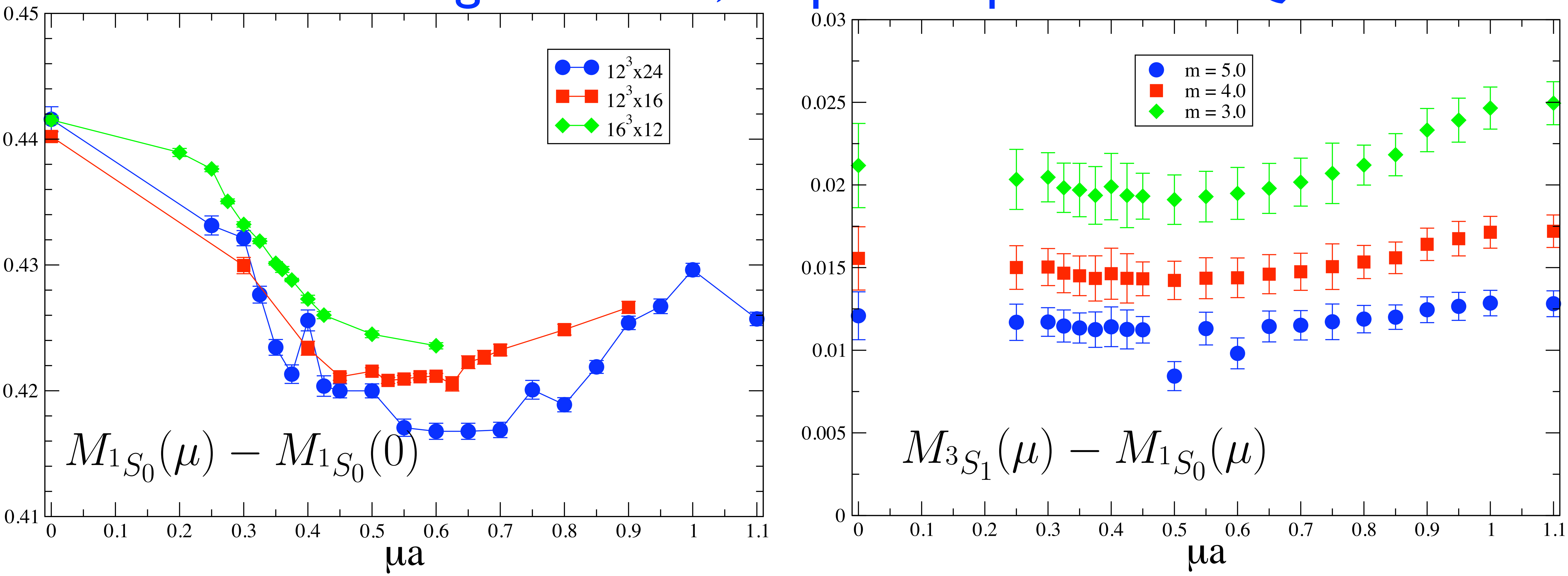
$$\propto\; \exp\left[-\frac{\mathrm{const}}{\mu^2}\right]$$

In QCD$_{2+1}$: Enhancement of $U(1)_A$ breaking $\Rightarrow$ first-order region of Columbia plot grows with μ?

Quarkonia

SJH, S. Kim, J.I. Skullerud PLB711 (2012) 199

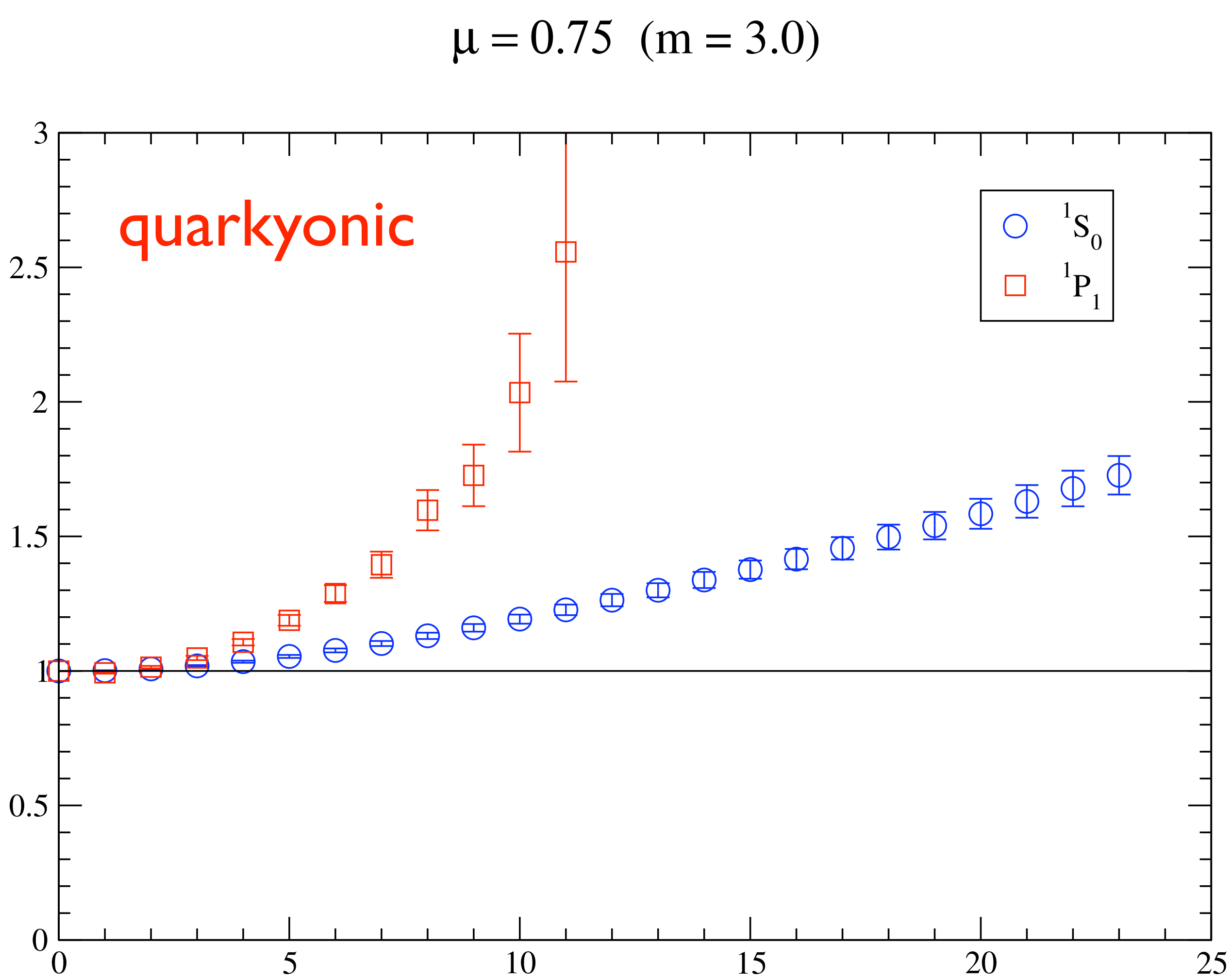
Study propagation of heavy $Q\bar{Q}$ (QQ) states through baryonic medium using tree-level, tadpole-improved NRQCD

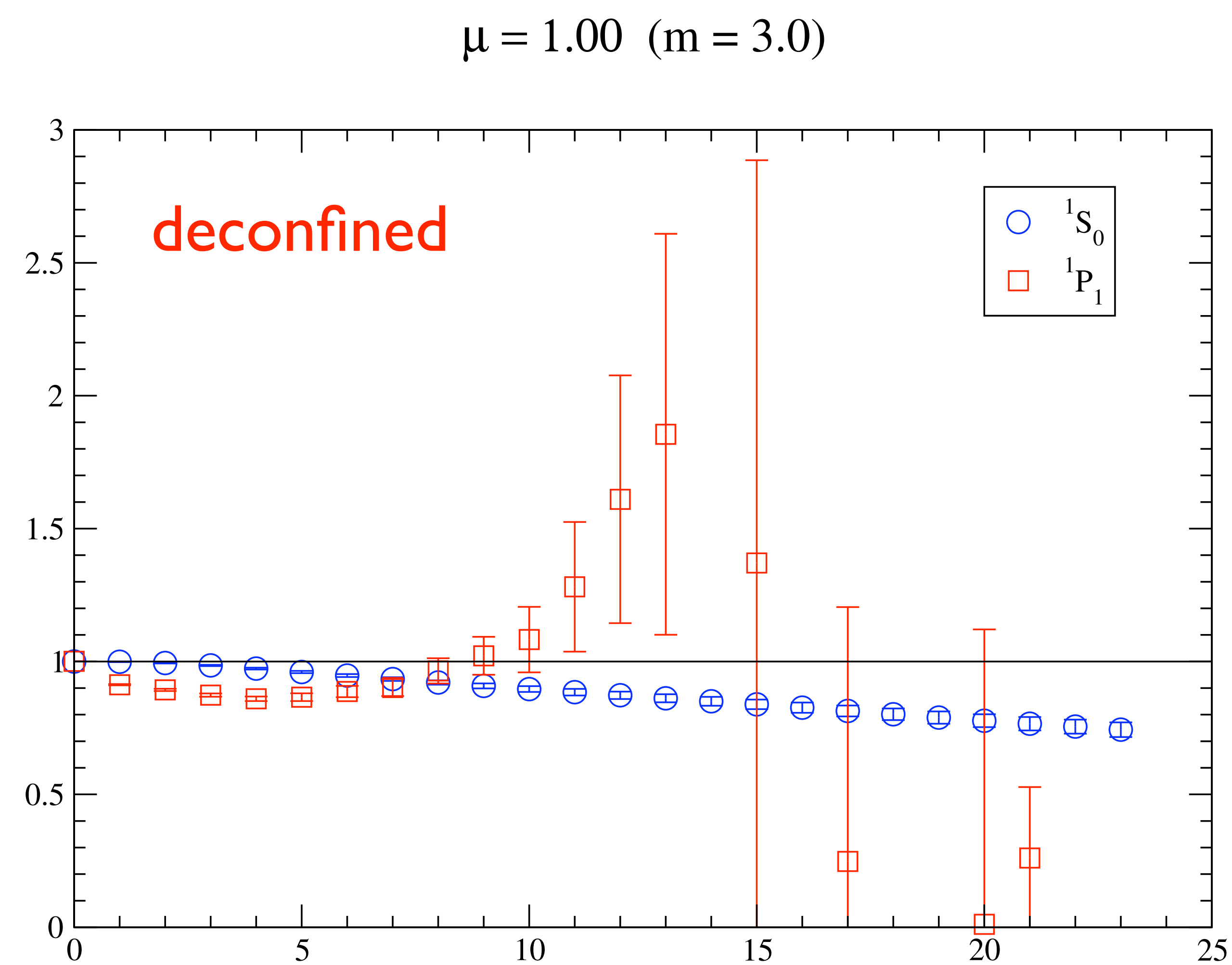


Mass of singlet s-wave state shows interesting μ-dependence. Also see weak effect in hyperfine splitting

Interpret as QQ → Qq + qQ (tetraquark?) as $n_q \nearrow$?

p-wave states not well-fitted by a simple pole





Here we plot the propagator ratio $C_{QQ}(t;\mu)/C_{QQ}(t;0)$ for spin-singlet *s*-and *p*-wave states

Qualitative difference between quarkyonic and deconfined regimes

Summary

★ QC$_2$D offers an accessible theoretical laboratory for dense baryonic matter

(Cf. recent studies of G_2 Maas, v. Smekal, Wellegehausen & Wipf PRD86 (2012) 111901 & QCD with isospin chemical potential in canonical approach Detmold, Orginos & Shi PRD86 (2012) 054507)

★ Despite artifacts a robust picture is emerging.

For low T (at least) 3 distinct regions:

Vacuum for $\mu < \mu_o$

Confined "Quarkyonic" superfluid for $\mu_Q < \mu < \mu_d$

Deconfined and strongly-bound phase for $\mu > \mu_d$

★ For larger T "deconfinement" may mean different things for bulk and Fermi surface phenomena

★ Not discussed today:

hadron spectrum (w/ Peter Sitch, JIS)

QC$_2$D in the attoworld (w/ Joyce Myers, Tim Hollowood)

static quark potential, gluon propagator (Boz *et al*)