
TECHNISCHE UNIVERSITÄT DARMSTADT

HIC for FAIR
Helmholtz International Center

# QCD-like theories at finite density

## HIC for FAIR Workshop

## Quarks, Gluons & Hadronic Matter under Extreme Conditions II

### St. Goar, 20 March 2013

**Lorenz von Smekal**

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# Contents

- Introduction
- QCD with Isospin Chemical Potential ↔ Two-Color QCD

  **N. Strodthoff, B.-J. Schaefer & L.v.S., Phys. Rev. D85 (2012) 074007**

  **K. Kamikado, N. Strodthoff, L.v.S. & J. Wambach, Phys. Lett. B 718 (2013) 1044**

- Isospin & Baryon Chemical Potential ↔ Polarised Fermi Gas
- Summary and outlook

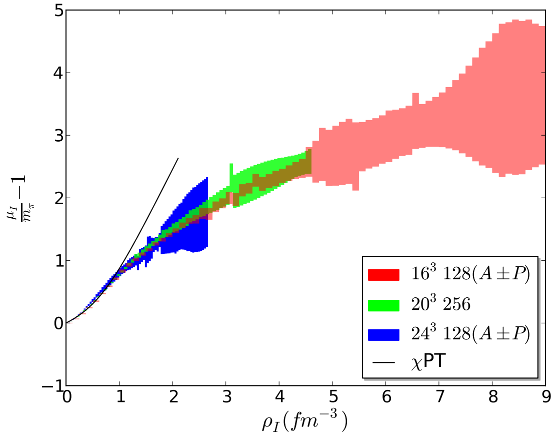
**See also: L.v.S. in "Physics at all scales: The Renormalization Group," the 49th Schladming Winter School on Theoretical Physics, Nucl. Phys. B (PS) 228 (2012) pp. 179 - 220 [arXiv:1205.4205]**

# QCD-like Theories

## Functional methods and effective models:

- compare with lattice simulations
  where there's no sign problem

**QCD at finite isospin density**



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- **apply to ultracold fermi gases**
  exploit analogies and
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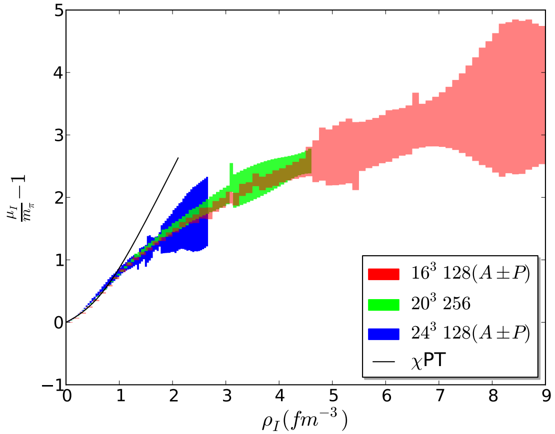
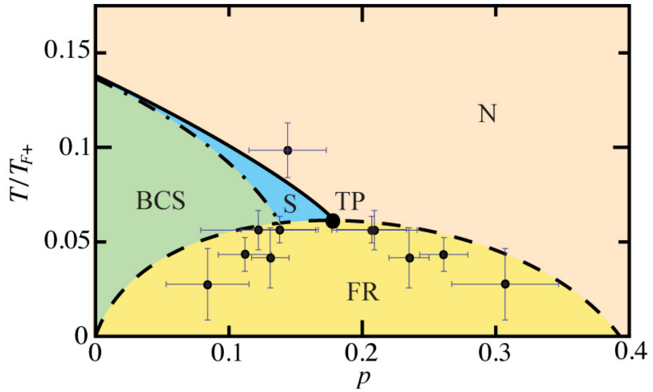



**polarised fermi gas at unitarity**



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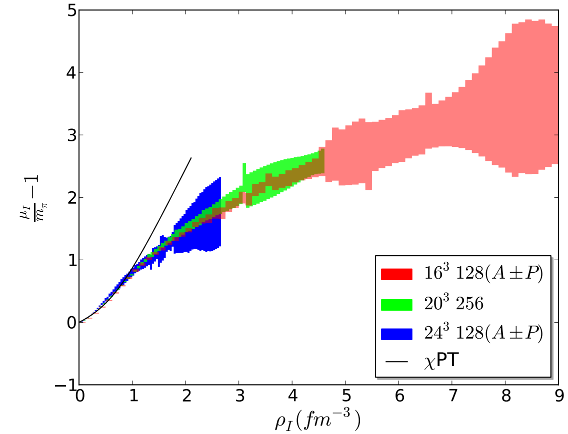


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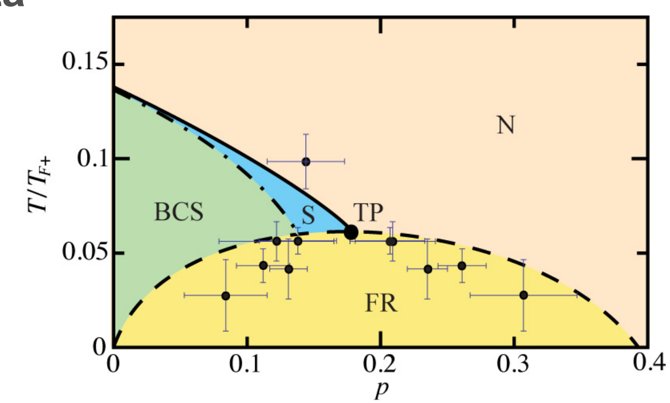
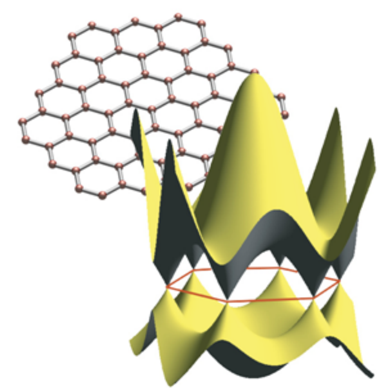



graphene



- **strongly correlated fermions in 2+1 dimensions**

  $\text{QED}_3$ (semimetal-insulator transition, $N_f < 4$),

  electronic properties of Graphene (half-filling, $N_f = 2$) − SFB 634

# Fermion-Sign Problem

- **Dirac operator:** $D(\mu) = \gamma_\mu(\partial^\mu + iA^\mu) - \gamma^0\mu$

anti-Hermitian Hermitian

$\Rightarrow \quad \gamma_5 D(\mu)^\dagger \gamma_5 = D(-\mu)$ **or** $\big(\mathrm{Det}\, D(\mu)\big)^* = \mathrm{Det}\, D(-\mu)$

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**(i) pseudo-real** $\quad T = \Sigma C \gamma_5, \; T^2 = 1$

color, $\Sigma^2 = -1$

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$\beta = 1$

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**(ii) real** $\quad T = C\gamma_5,\ T^2 = -1$ **adjoint QCD, or $G_2$-QCD** $\quad \beta = 4$

Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D86 (2012) 111901(R)

# Fermion-Sign Problem

...except if:

$$\big(\mathrm{Det}\, D(\mu)\big)^* = \mathrm{Det}\, D(-\mu)$$

Dyson index:

(b) two degenerate flavors with isospin chemical potential

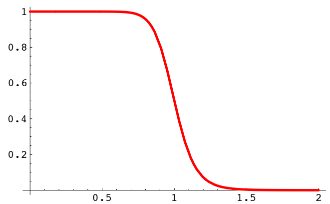
$$\beta = 2$$

fermion determinant $\rightsquigarrow \mathrm{Det}\big(D(\mu_I)D(-\mu_I)\big)$

**QCD at finite isospin density**

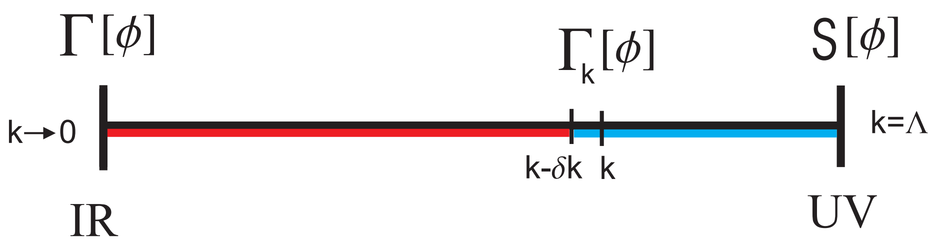
| | |
|---|---|
| $\chi$PT: | Son & Stephanov, Phys. Rev. Lett. 86 (2001) 592 |
| Silver Blaze: | Cohen, Phys. Rev. Lett. 91 (2003) 222001 |
| Lattice: | Kogut & Sinclair, Phys. Rev. D 70 (2004) 094501; PoS LAT2006 147 |
| | de Forcrand, Stephanov & Wenger, PoS LAT2007 237 |
| | Detmold, Orginos & Shi, Phys. Rev. D 86 (2012) 054507 |
| NJL: | He, Jin & Zhuang, Phys. Rev. D 71, (2005) 116001 |
| | Mu, He & Liu, Phys. Rev. D 82 (2010) 056006 |

# Functional RG (Flow) Equations



**Effective action:** $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j]$

**Legendre transform**



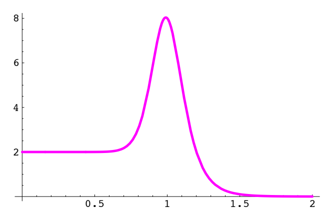

$\longrightarrow$ **1PI vertex functions**

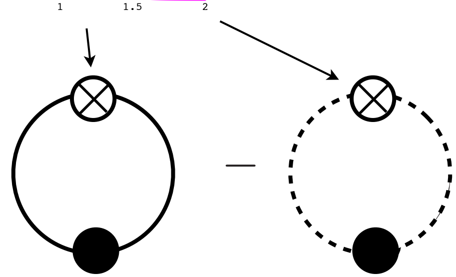
$\Gamma^{(n)}(x_1, \ldots x_n)$

$\longrightarrow$ **grand potential**

$\Omega(T, \mu)$, at

$$\phi_{\min} = \langle \phi \rangle_{T,\mu}$$



$k\partial_k \Gamma_k[\phi] = \frac{1}{2}$ 

**bosons** **fermions**

Wetterich, Phys. Lett. B 301 (1993) 90

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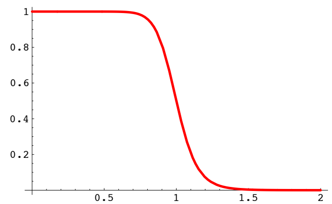


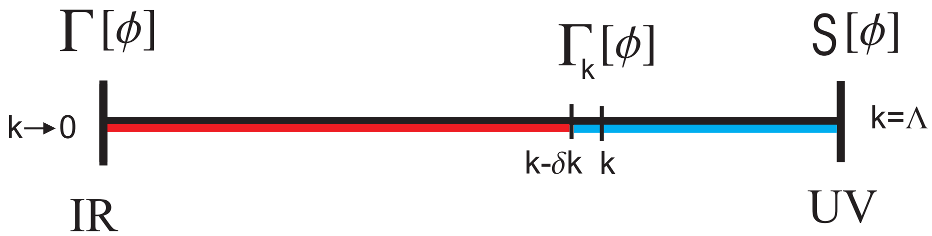


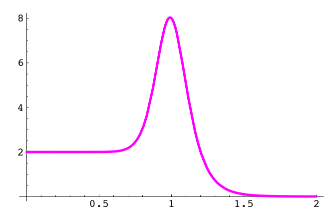


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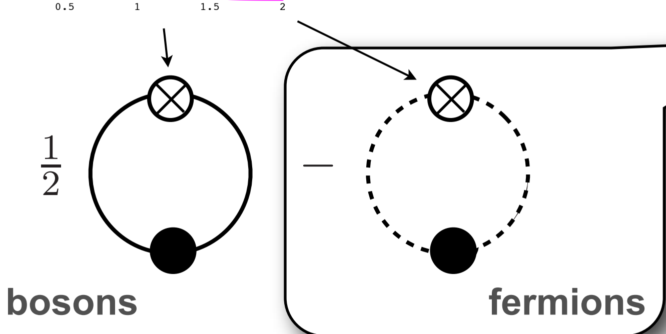


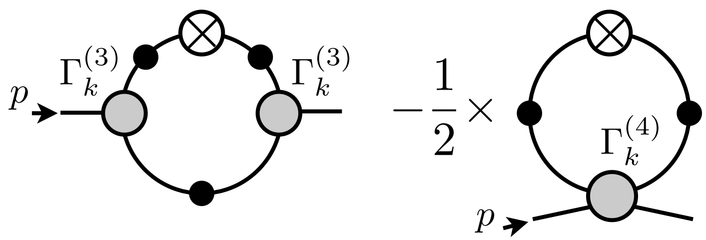

**extended mean field (eMF)**

Wetterich, Phys. Lett. B 301 (1993) 90

# Flow Equations for Correlation Functions

- **e.g. O(4) linear sigma model:**

$$k\partial_k\Gamma_k^{(2)}(p_0,\vec{p}) =$$




- **continue to real time:**

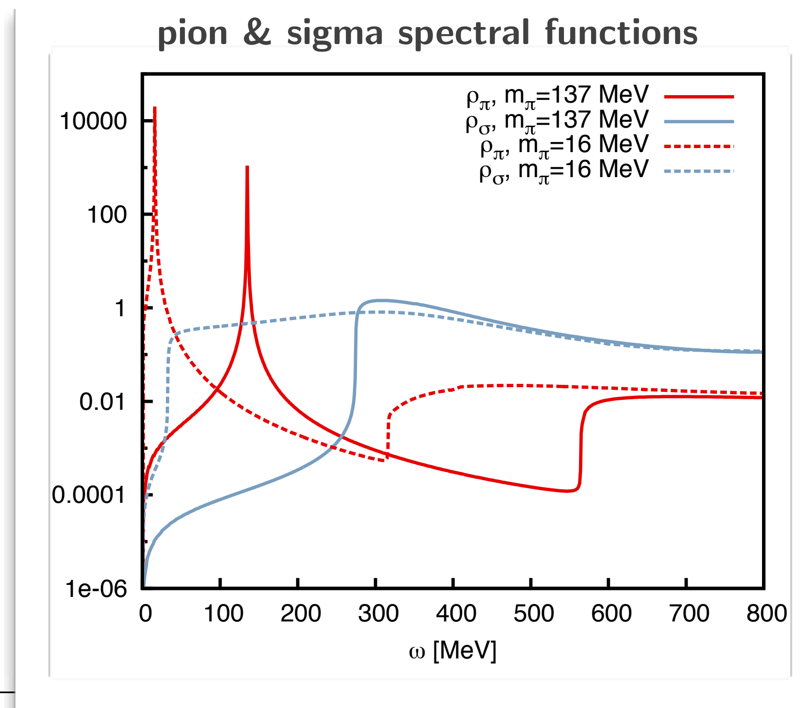
$$p_0 = -i(\omega + i\varepsilon) \quad \textbf{(retarded)}$$

$T = \mu = 0$:

**Kamikado, Strodthoff, LvS & Wambach, arXiv:1302.6199**

**finite $T$:**

**Tripolt, Strodthoff, LvS & Wambach, in prep.**



# QM Model with Isospin Chemical Potential

- ***N_f* = 2 quarks & mesons with Yukawa coupling:**

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}(\partial\!\!\!/ + g(\sigma + i\gamma^5 \vec{\pi}\vec{\tau}) - \mu\gamma^0 - \mu_I \tau_3 \gamma^0)\psi \\
&+ \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_0)^2 + U(\rho^2, d^2) - c\sigma \\
&+ \frac{1}{2}\big((\partial_\mu + 2\mu_I \delta^0_\mu)\pi_+ (\partial_\mu - 2\mu_I \delta^0_\mu)\pi_-\big)
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- **chemical potentials:**

$$\mu_u = \mu + \mu_I \quad \mu_d = \mu - \mu_I$$

$\mu \gg \mu_I$ : $\mu_I \rightsquigarrow$ imbalance between up and down

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- **$\mu = 0$, map to QMD model for $QC_2D$:**

$$N_c\colon\ 3 \to 2 \qquad (\psi_u, \psi_d) \to (\psi_r, \tau_2 C\bar{\psi}_g) \qquad \mu_I \to \mu$$

$$\pi_+, \pi_- \to \Delta, \Delta^* \qquad \pi_0 \to \vec{\pi}$$

# Two-Color QCD

- **extended flavor symmetry (Pauli-Gürsey), at $\mu = 0$**

$SU(N_f) \times SU(N_f) \times U(1)$ becomes $SU(2N_f)$

$N_f = 2$: connects pions and $\sigma$-meson with scalar (anti)diquarks.

- **Dirac mass (quark condensate)**

$SU(4) \to Sp(2)$

or $SO(6) \to SO(5)$

Coset: $S^5$ 5 Goldstone bosons: pions and scalar (anti)diquarks

- **color-singlet diquarks (bosonic baryons)**

**$\chi$PT:** Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl. Phys. B 582 (2000) 477

**Lattice:** Hands, Montvay, Scorzato & Skullerud, Eur. Phys. J. C 22 (2001) 451

Hands, Kenny, Kim & Skullerud, Eur. Phys. J. A 47 (2011) 60

Cotter, Giudice, Hands & Skullerud, arXiv:1210.4496

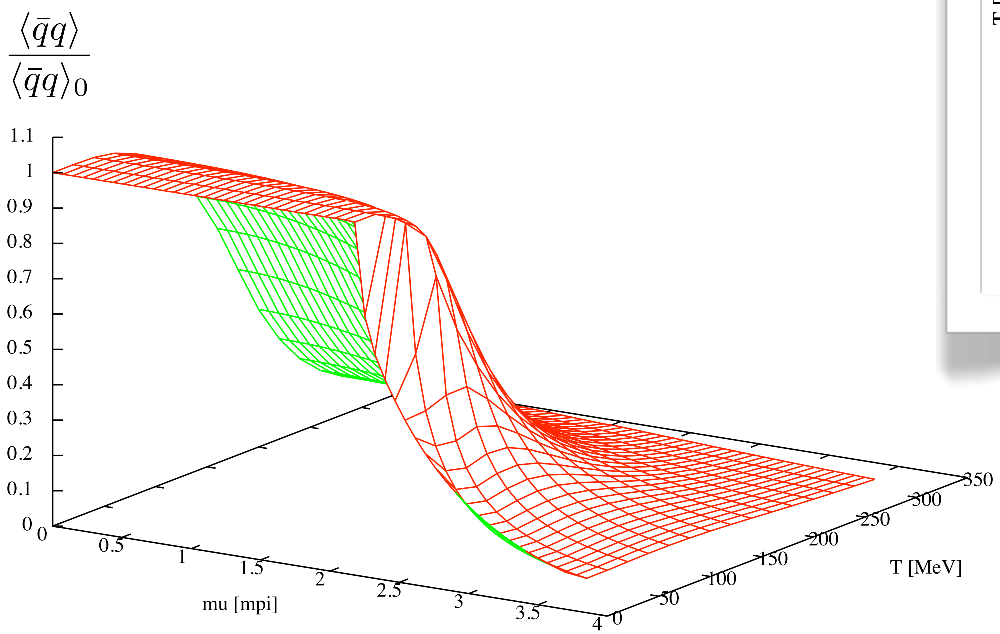
**NJL:** Ratti & Weise, Phys. Rev. D 70 (2004) 054013

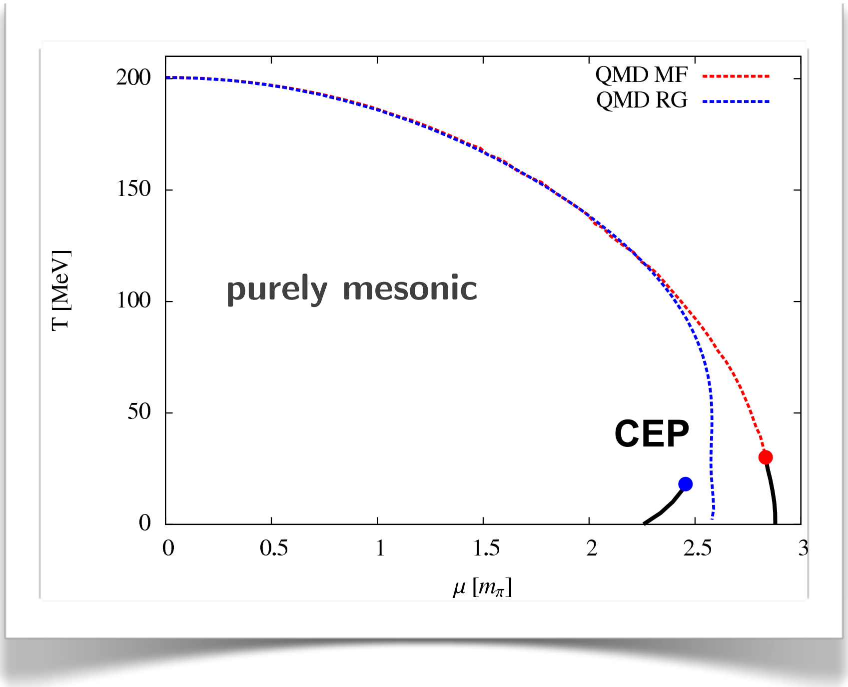
He, Phys. Rev. D 82 (2010) 096003.

**PNJL:** Brauner, Fukushima & Hidaka, Phys. Rev. D 80 (2009) 074035

# Two-Color QCD

- **QMD model phase diagram**




- $1^{st}$ order chiral transition and CEP at $\mu \approx 2.5\, m_\pi$

Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007

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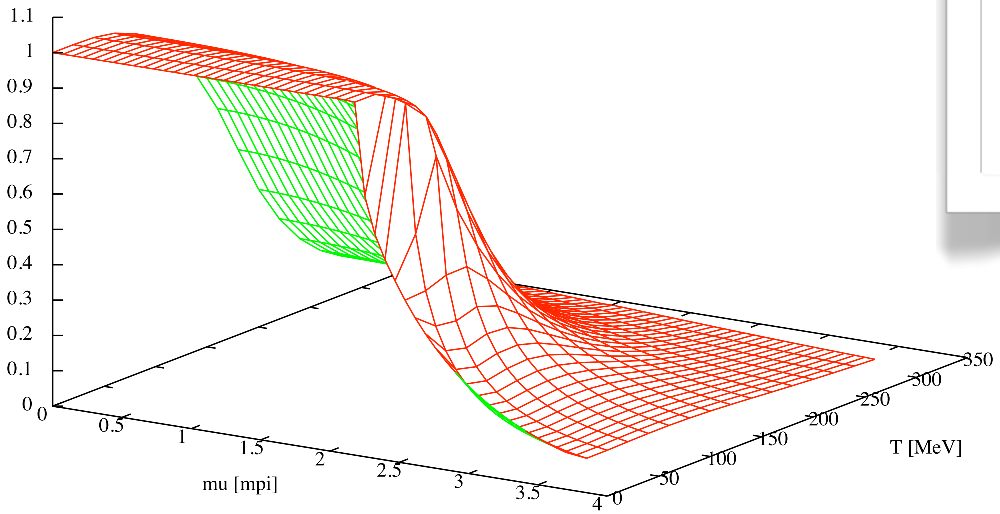
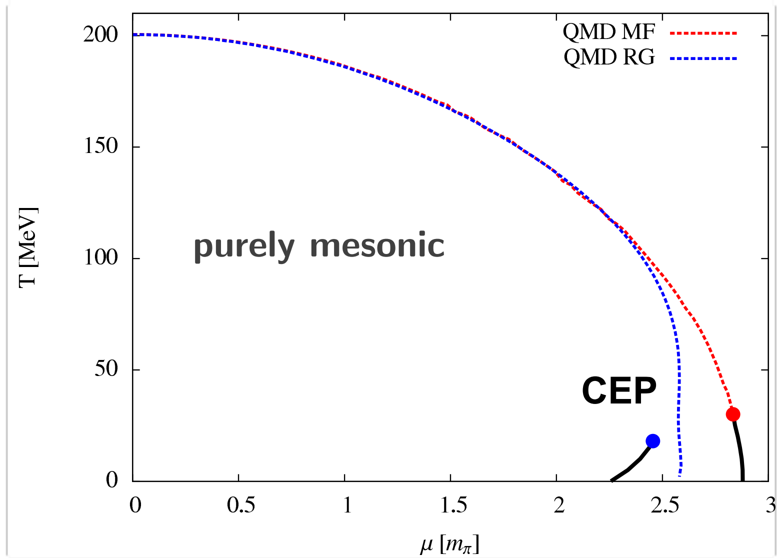
- **QMD model phase diagram**

- **$\mu$ = 0 axis:**

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0}$$

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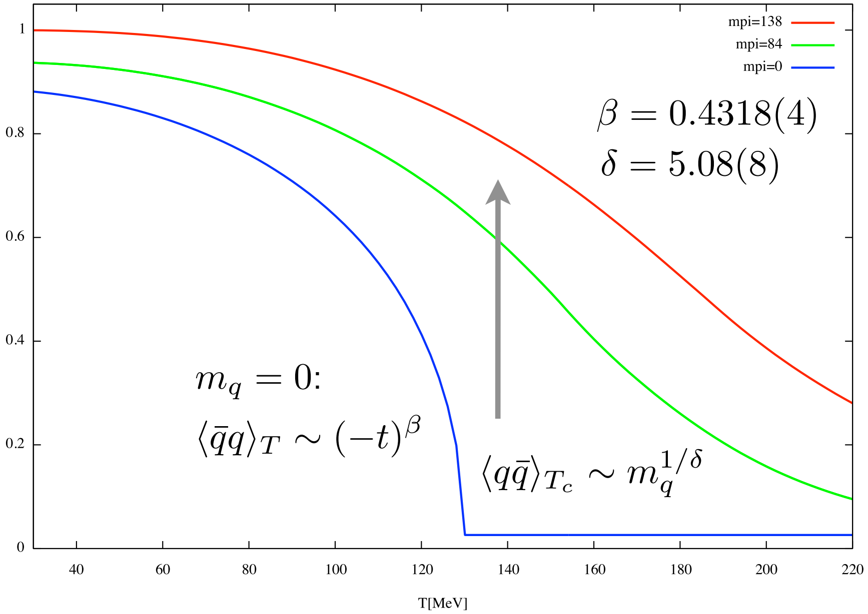
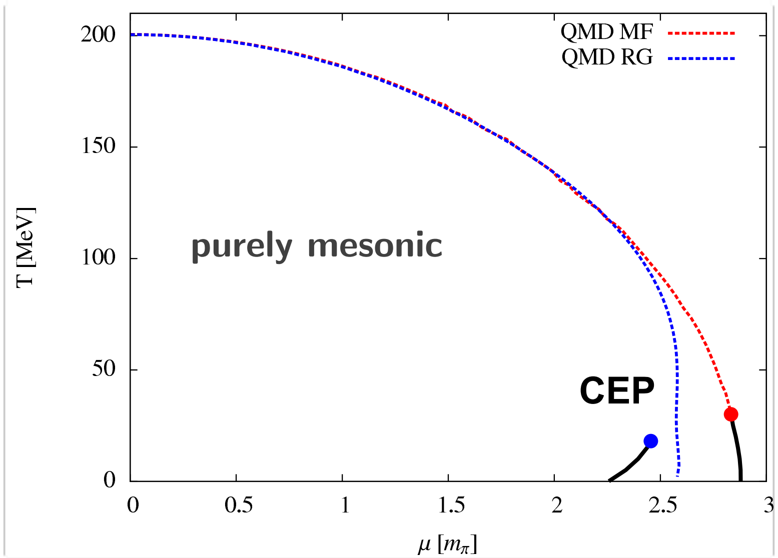
- **$\mu$ = 0 axis: O(6) scaling**

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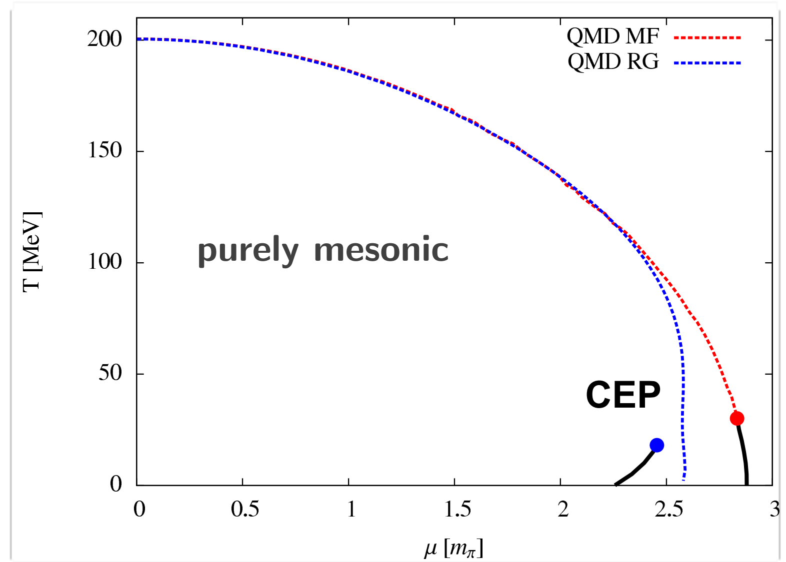
- **Clausius-Clapeyron:**

  along $1^{\rm st}$ order line

$$\frac{dT_c}{d\mu_c} = -\frac{\Delta n}{\Delta s}$$

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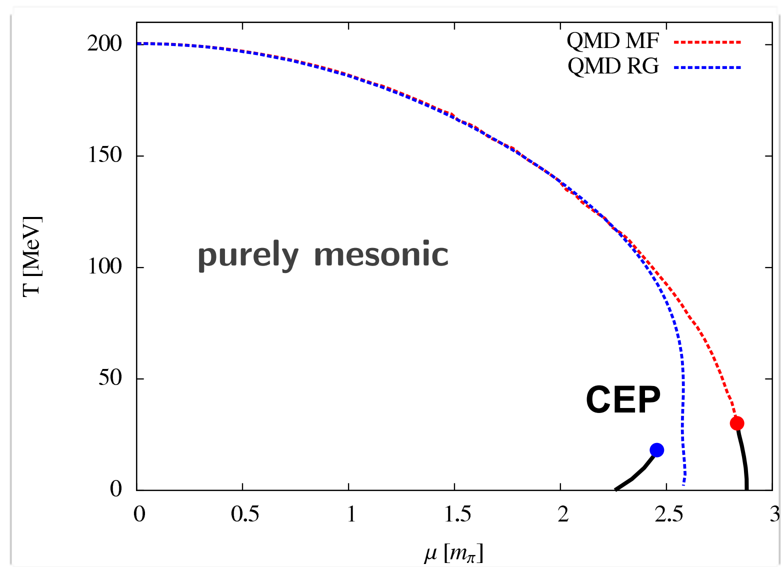
  - **mean field:**

    Lee-Wick, chiral transition

$$\Delta n > 0,\ \Delta s > 0$$

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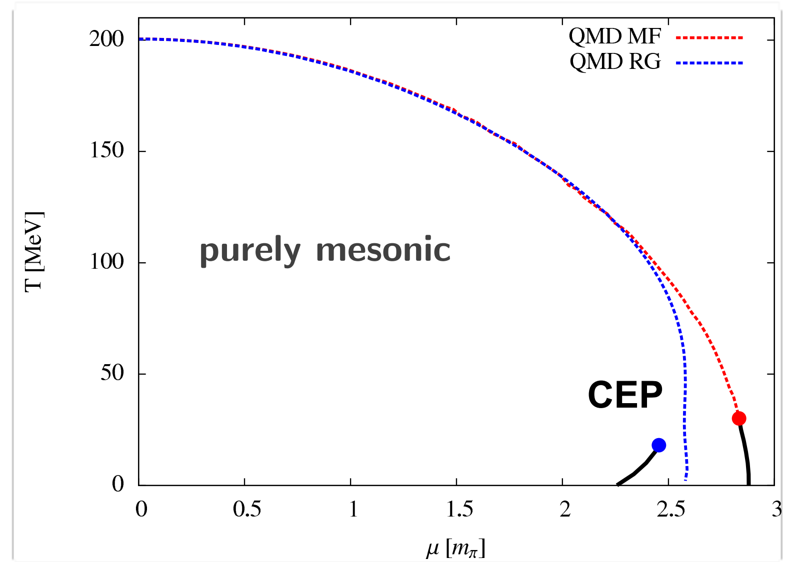
$$\Delta n > 0, \ \Delta s > 0$$

  - **with fluctuations:**

    liquid-gas, bound quark matter,
    $n > 0$ & $\langle \bar{q}q \rangle > 0$, partially 'polarised'

$$\Delta n > 0, \ \Delta s < 0$$

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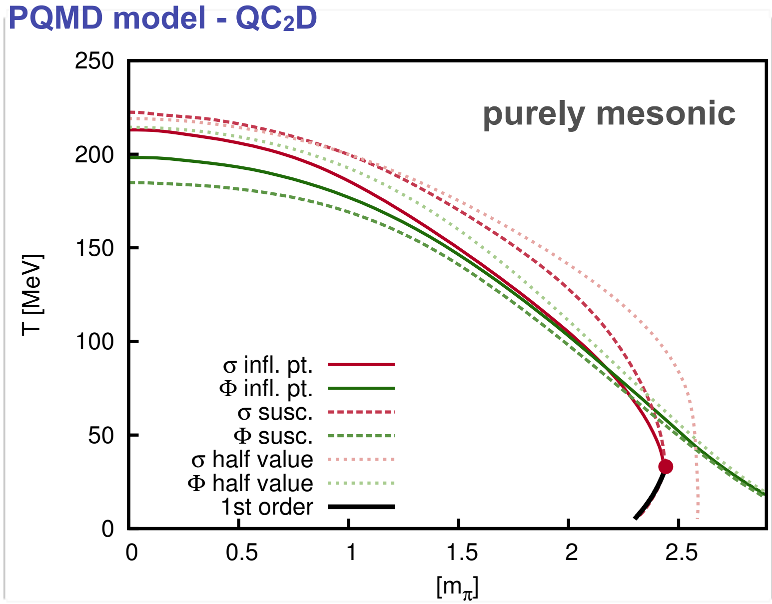
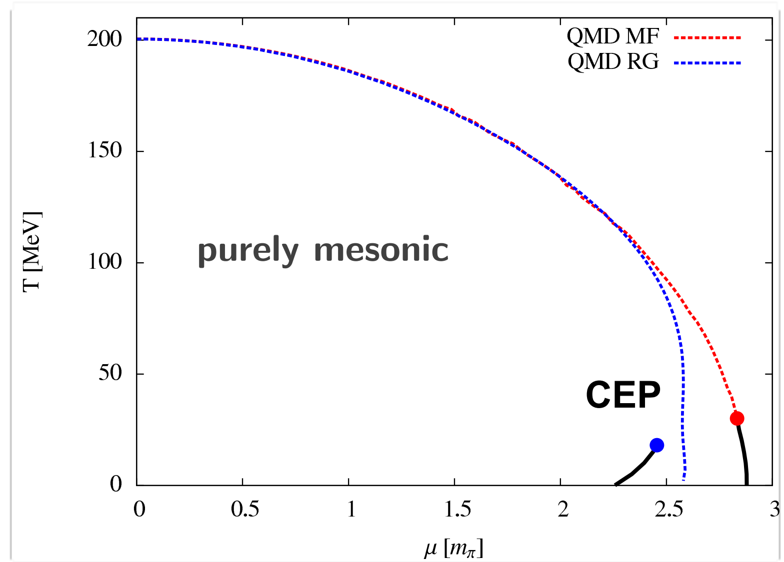



Strodthoff & L.v.S., in preparation

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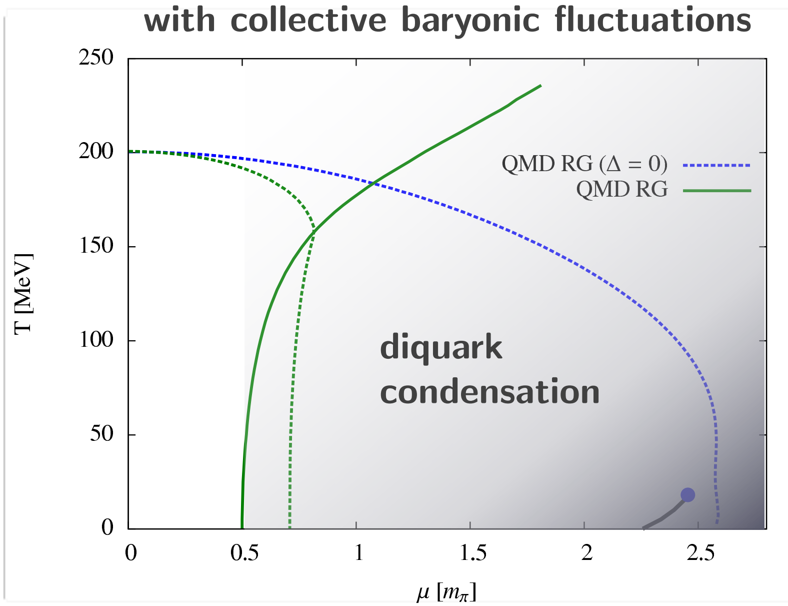


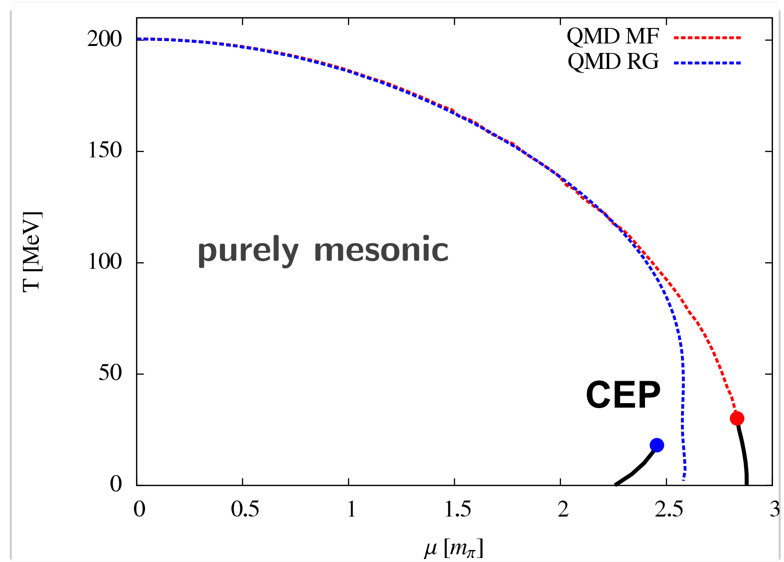


- no low-$T$ $1^{\mathrm{st}}$ order transition, no CEP at $\mu \sim 2.5\, m_\pi$ !

- $1^{\mathrm{st}}$ order chiral transition and CEP at $\mu \approx 2.5\, m_\pi$

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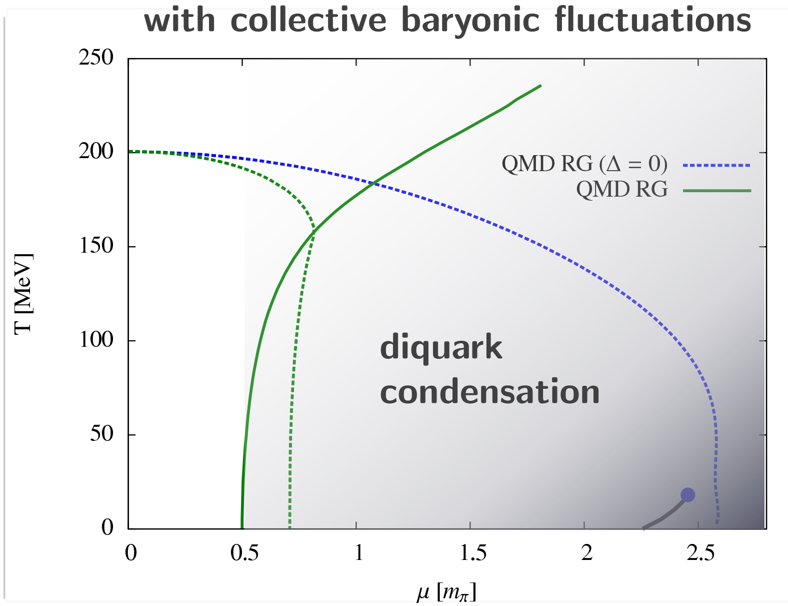
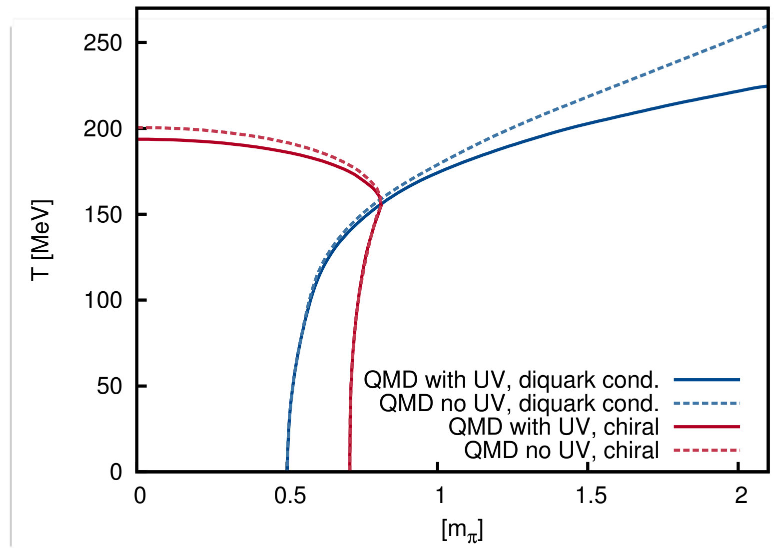


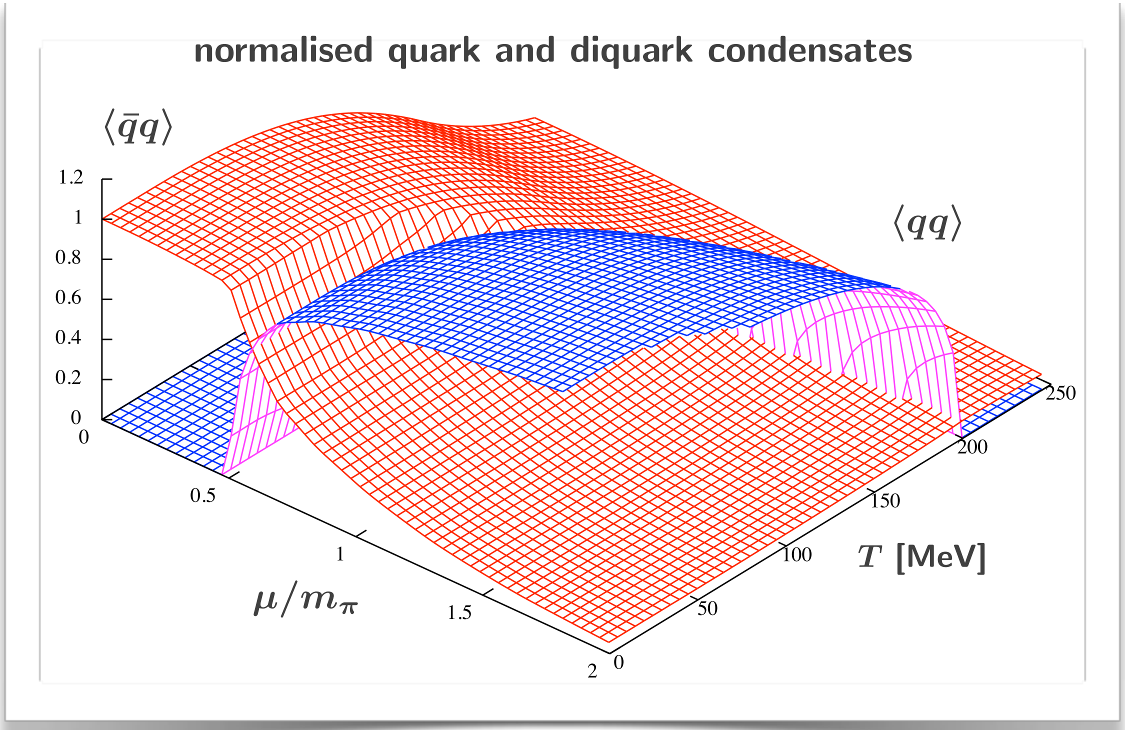

Strodthoff & L.v.S., in preparation

- no low-$T$ $1^{\mathrm{st}}$ order transition, no CEP at $\mu \sim 2.5\ m_\pi$ !

Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007

# Two-Color QCD

- **QMD model phase diagram**




Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007

# Two-Color QCD

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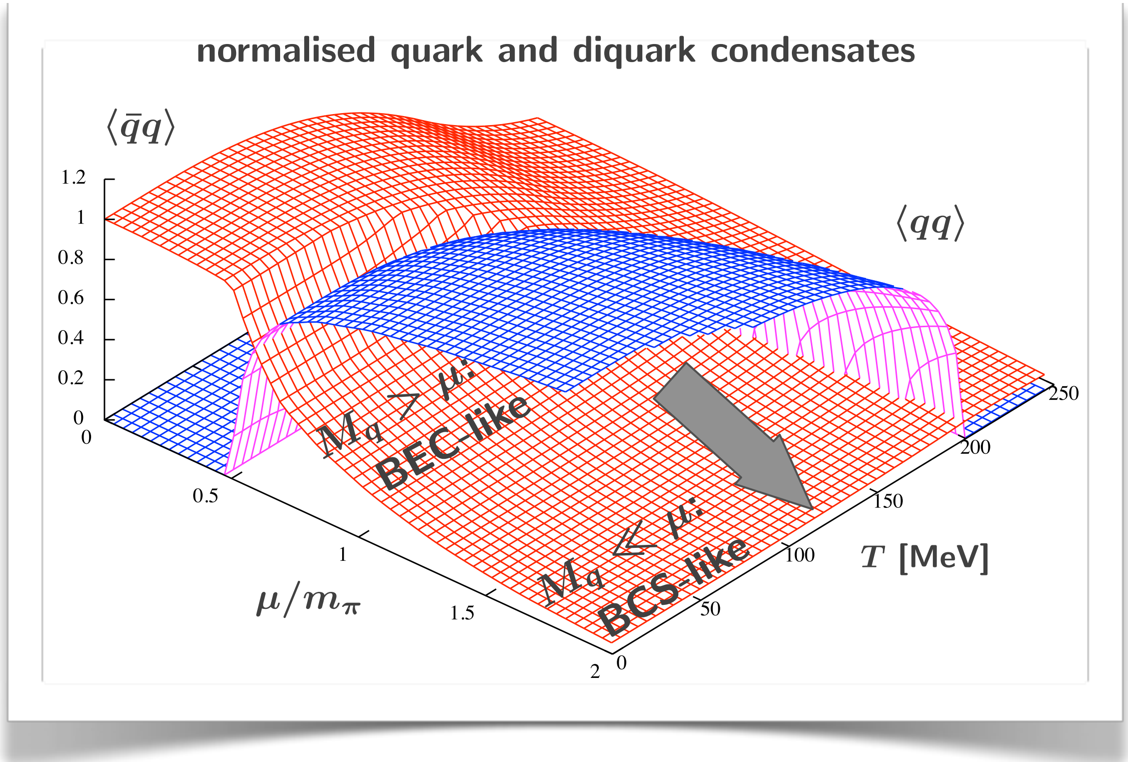


Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007

# Two-Color QCD

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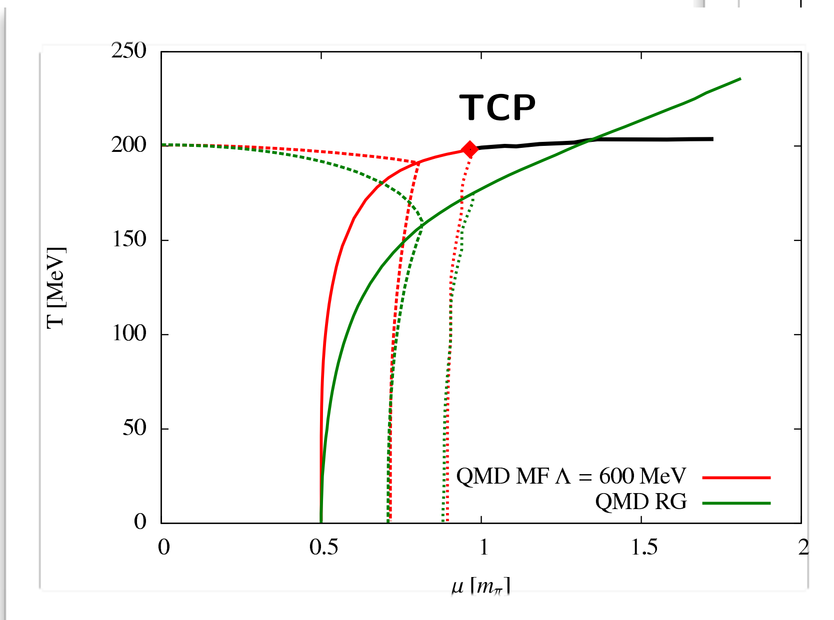


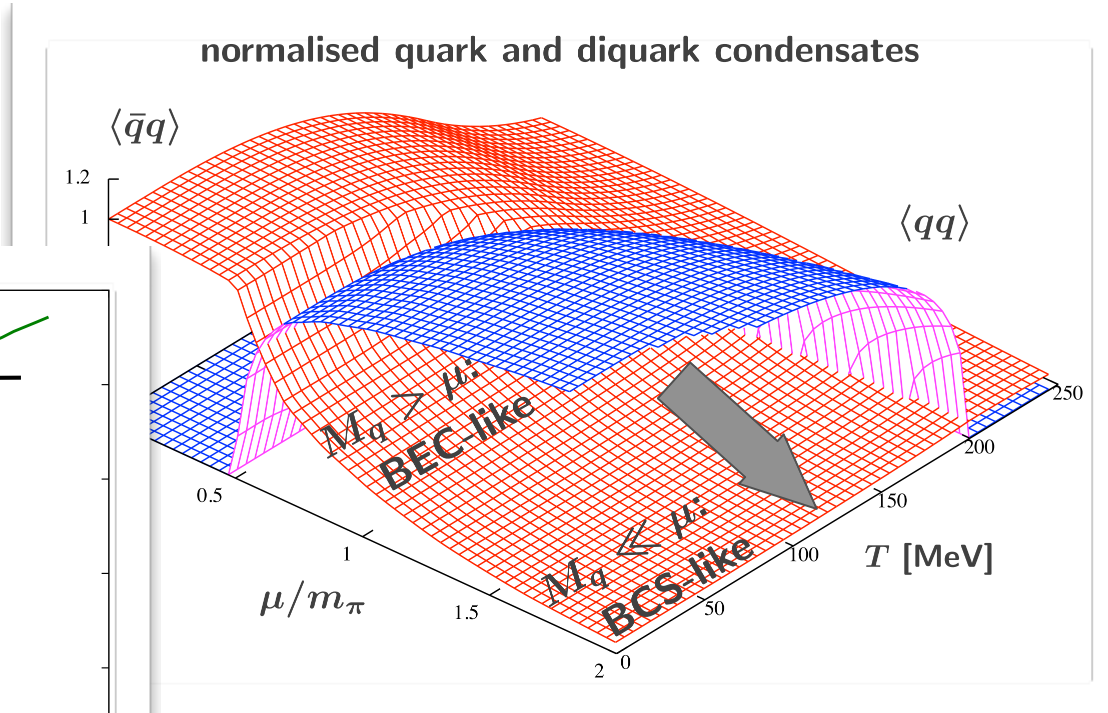


- Tricritical point predicted in:

  Splittorff, Toublan & Verbaarschot, Nucl. Phys. B 620 (2002) 290

Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007



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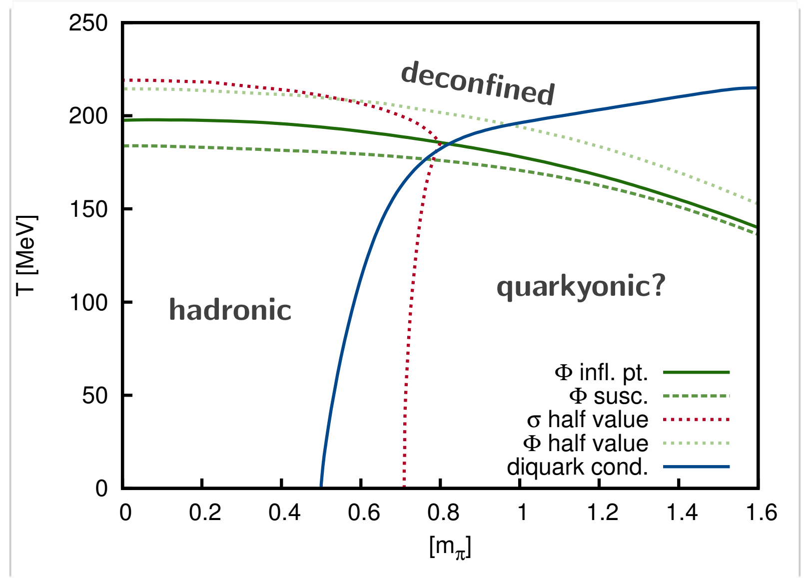
# Two-Color QCD

- **Polyakov-Quark-Meson-Diquark model phase diagram:**




Strodthoff & L.v.S., in preparation

# Two-Color QCD

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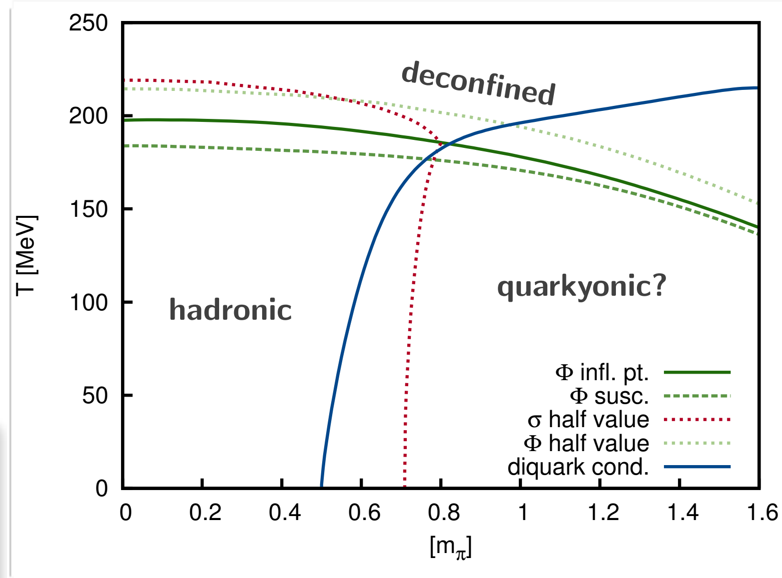
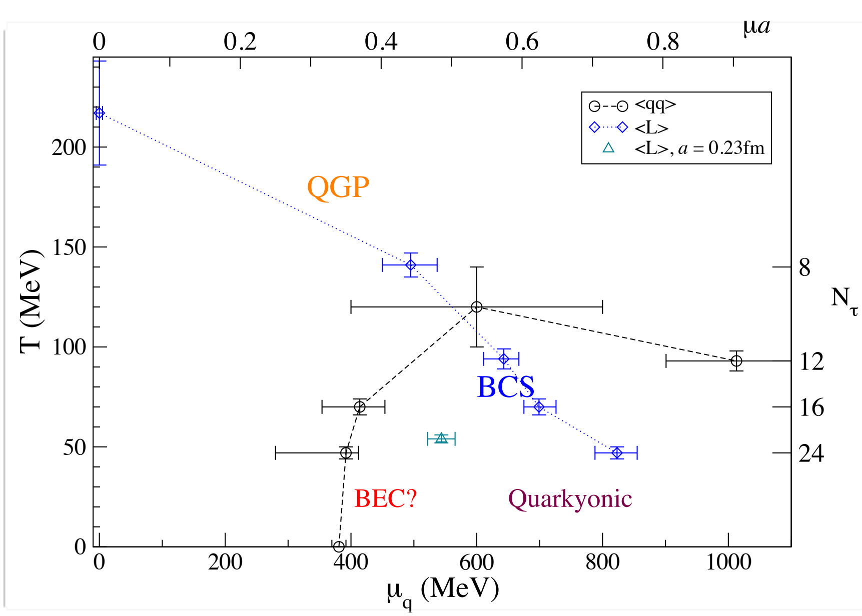



Strodthoff & L.v.S., in preparation

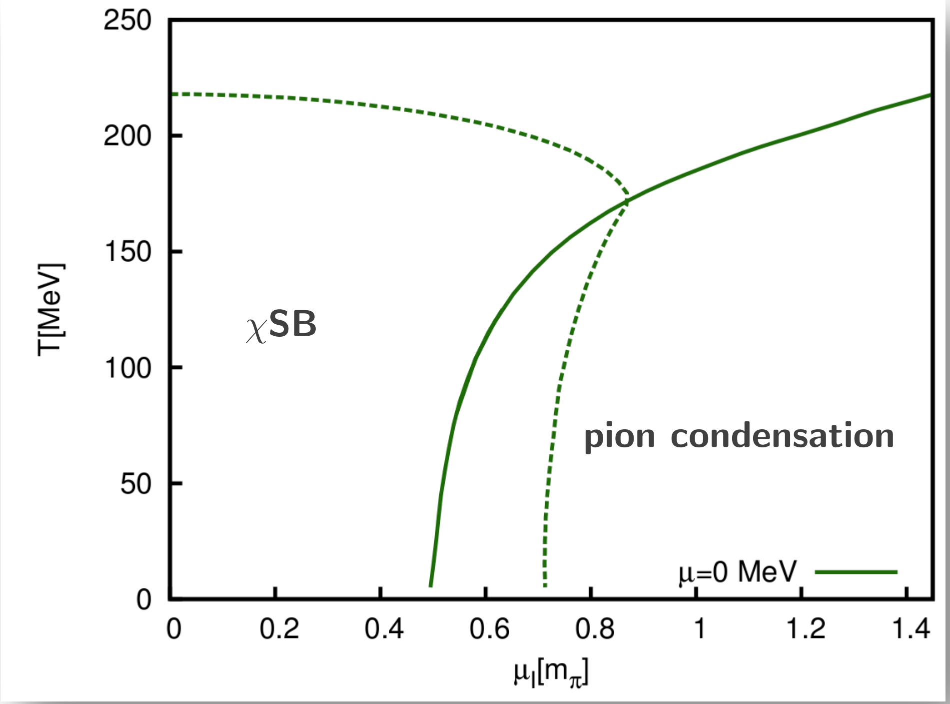
- **Lattice simulations:**




Cotter, Giudice, Hands & Skullerud, arXiv:1210.4496

# QCD with Isospin Chemical Potential

- **QM Model with fluctuating chiral & pion condensates**



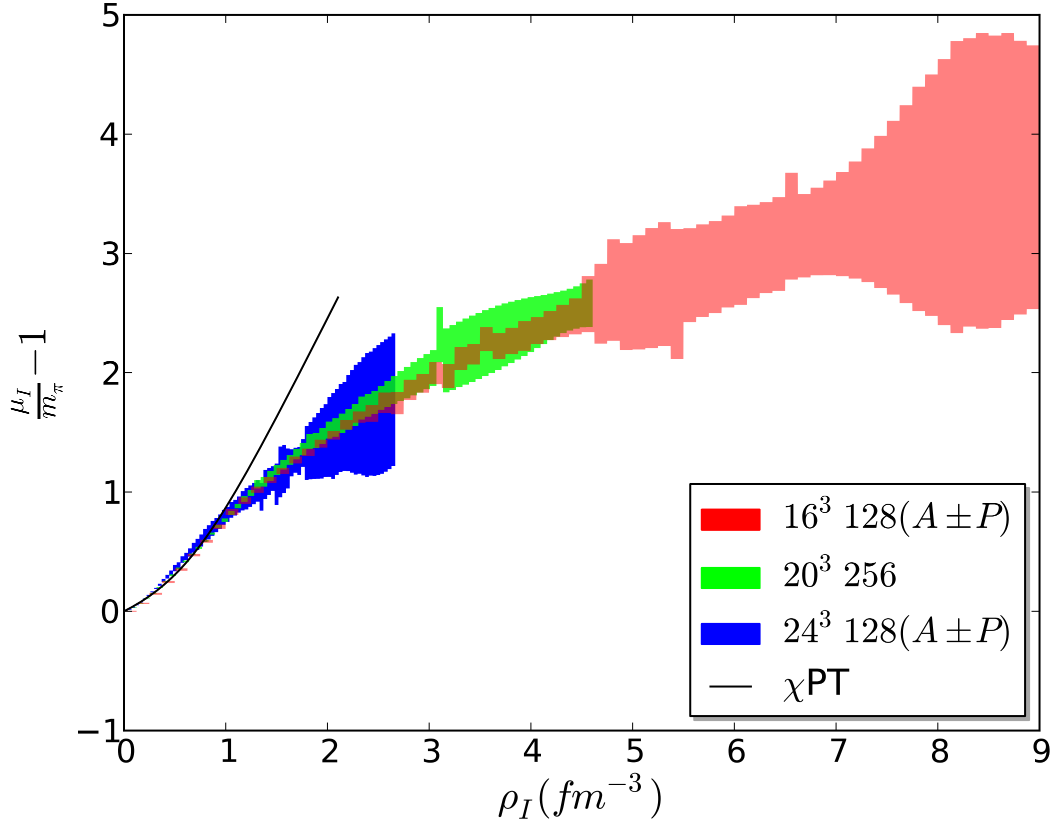

- **need 2 fields in effective potential**

$U = U(\rho^2, d^2)$, but replace $\rho^2 = \sigma^2 + \vec{\pi}^2$ and $d^2 = |\Delta|^2$
by $\rho^2 = \sigma^2 + \pi_0^2$ and $d^2 = \pi_1^2 + \pi_2^2 = \pi_+ \pi_-$

Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

# QCD with Isospin Chemical Potential

- ***T* = 0 isospin density - lattice QCD:**




Detmold, Orginos & Shi, Phys. Rev. D86 (2012) 054507

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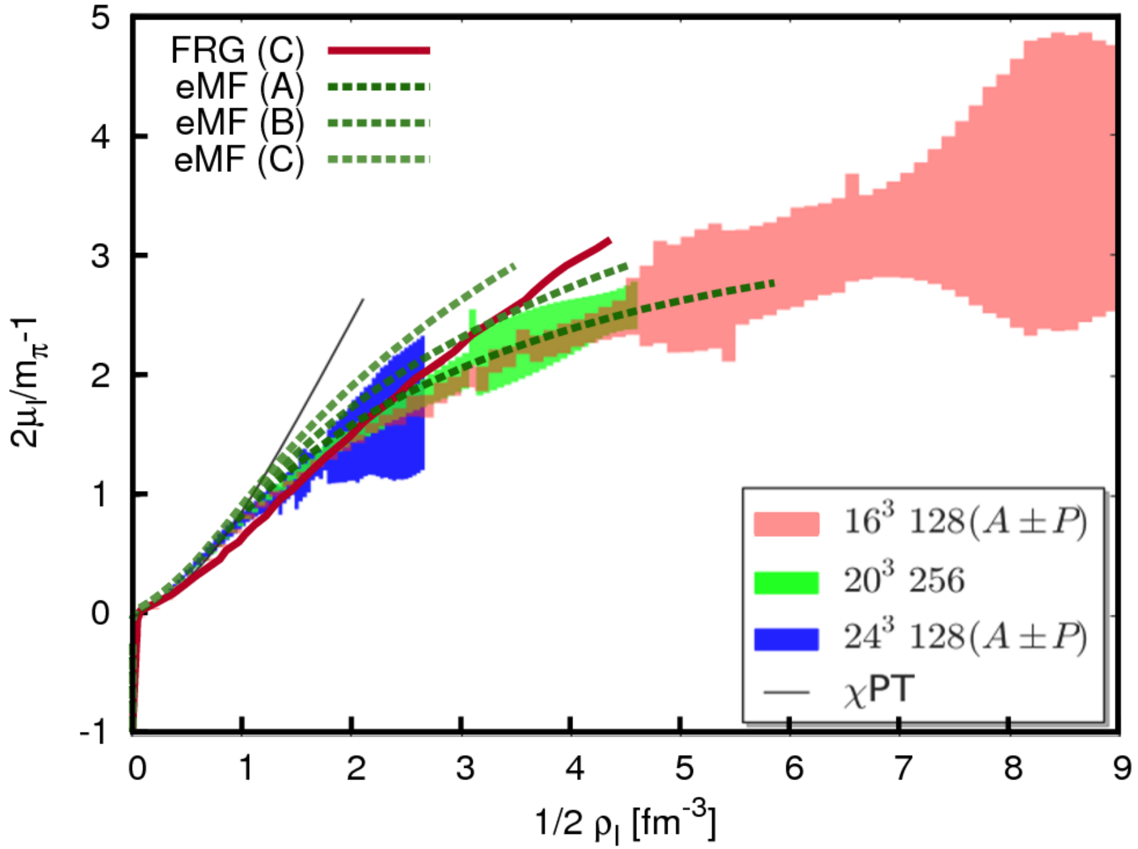
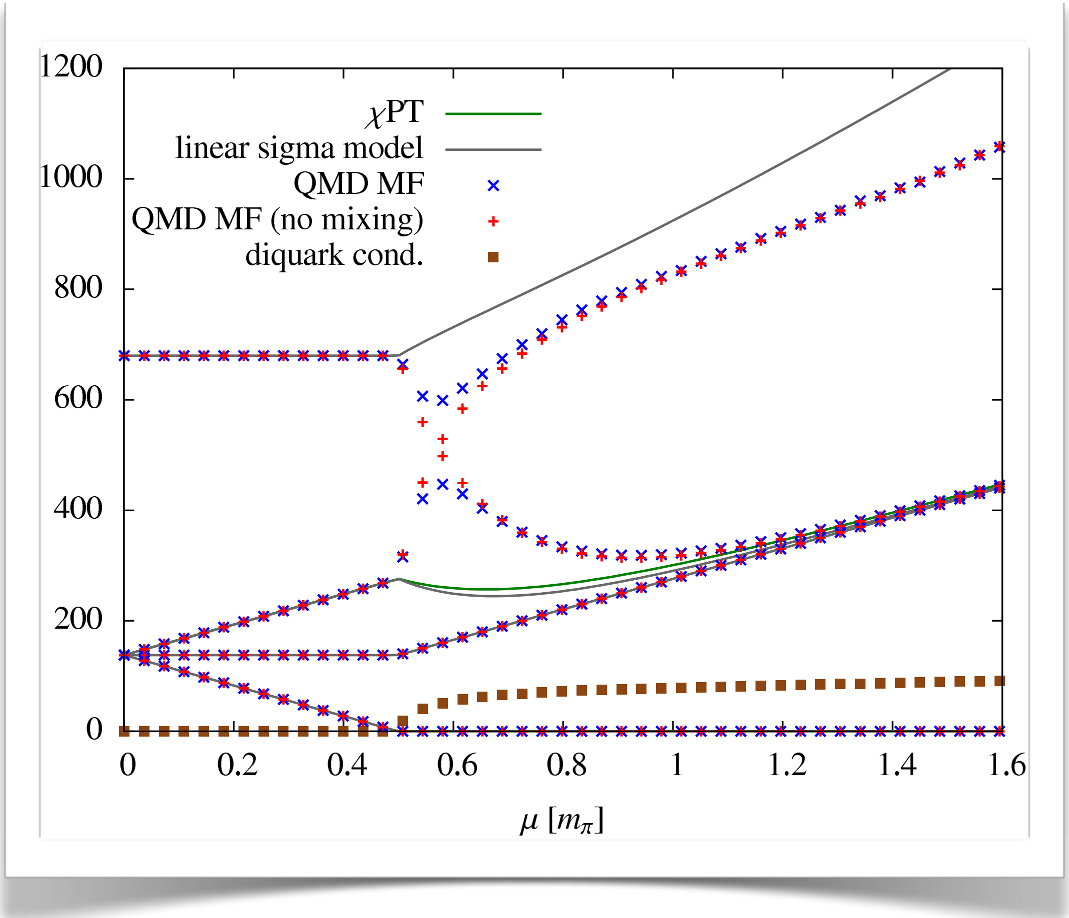



Detmold, Orginos & Shi, Phys. Rev. D86 (2012) 054507

# Vacuum Alignment, $T$=0

- **RPA pole masses, QMD model:**




- **PNJL model:**

  Brauner, Fukushima & Hidaka, Phys. Rev. D 80 (2009) 074035

- **NJL with isospin chemical potential:**

  He, Jin & Zhuang, Phys. Rev. D 71 (2005) 116001

  Xiong, Jin & Li, J. Phys. G 36 (2009) 125005

- **Functional RG:**

  Strodthoff, Schaefer & LvS, Phys. Rev. D 85 (2012) 074007

  Kamikado, Strodthoff, LvS & Wambach, Phys. Lett. B 718 (2013) 1044



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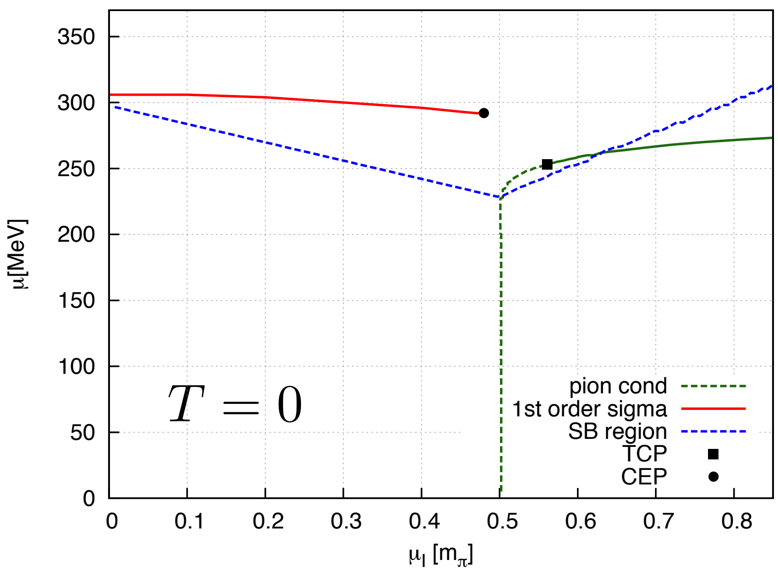
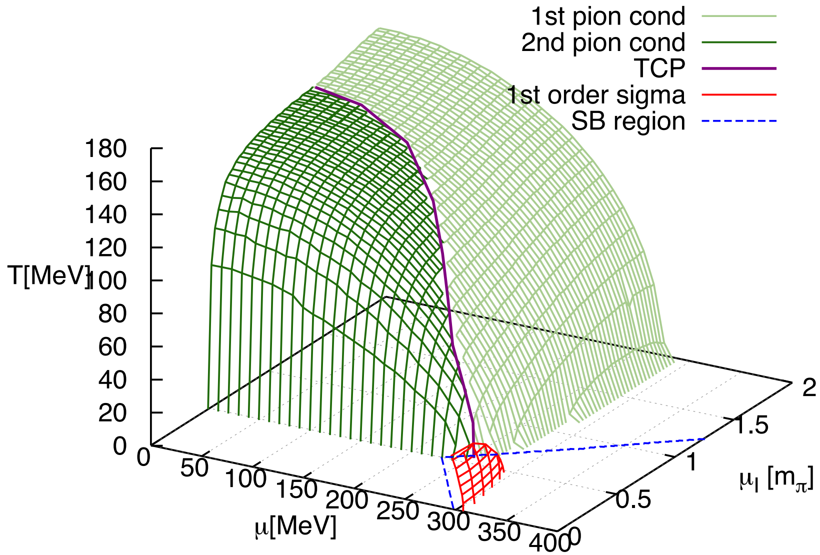
# Baryon & Isospin Chemical Potential

- **Fermionic flow (extended mean-field):**

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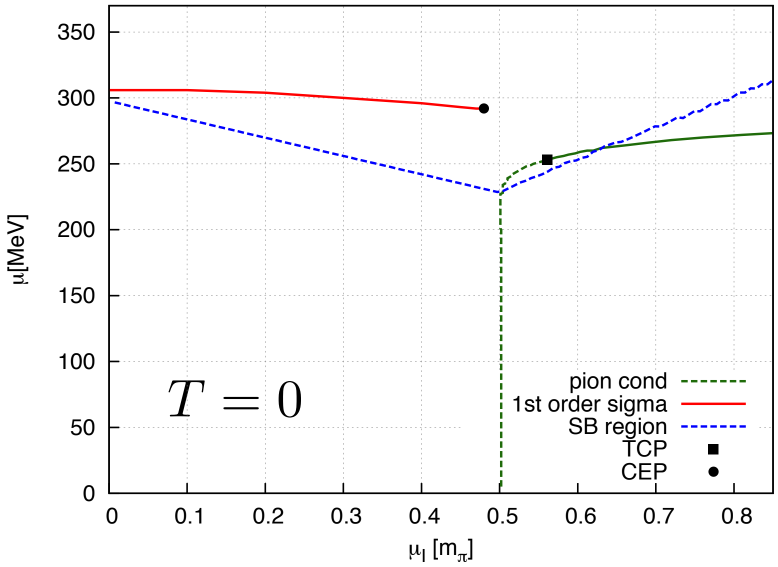
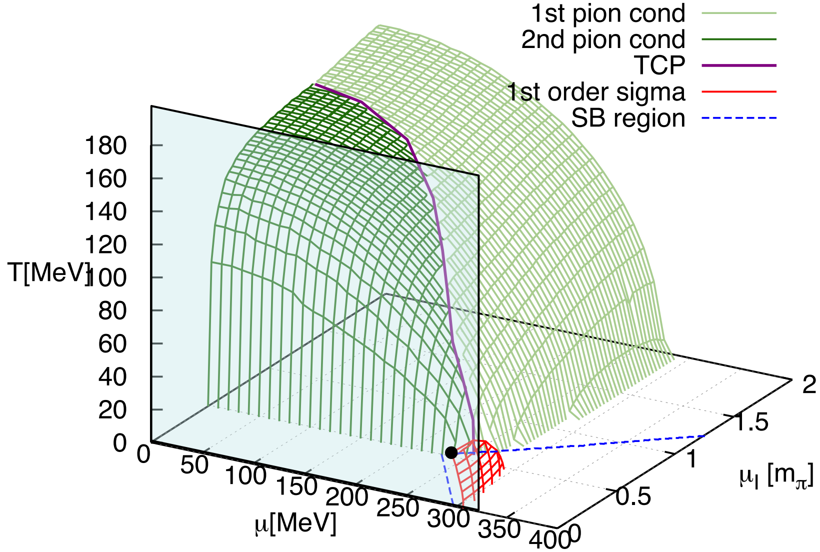


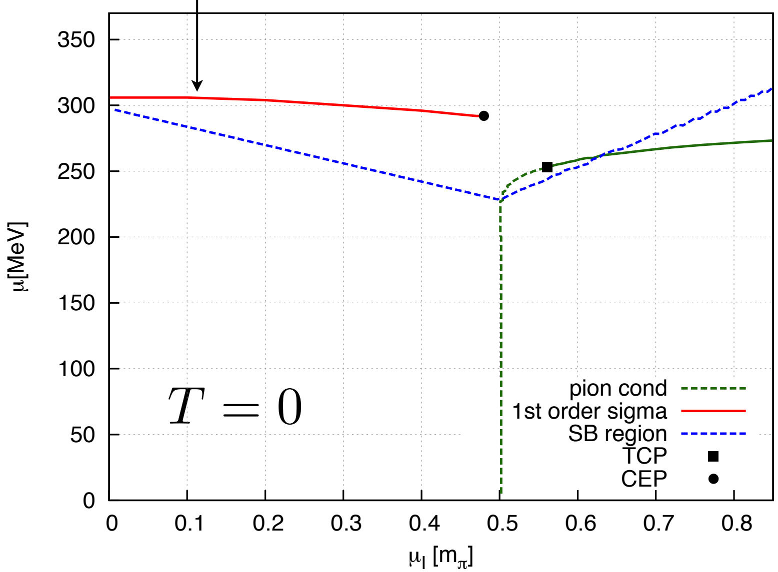



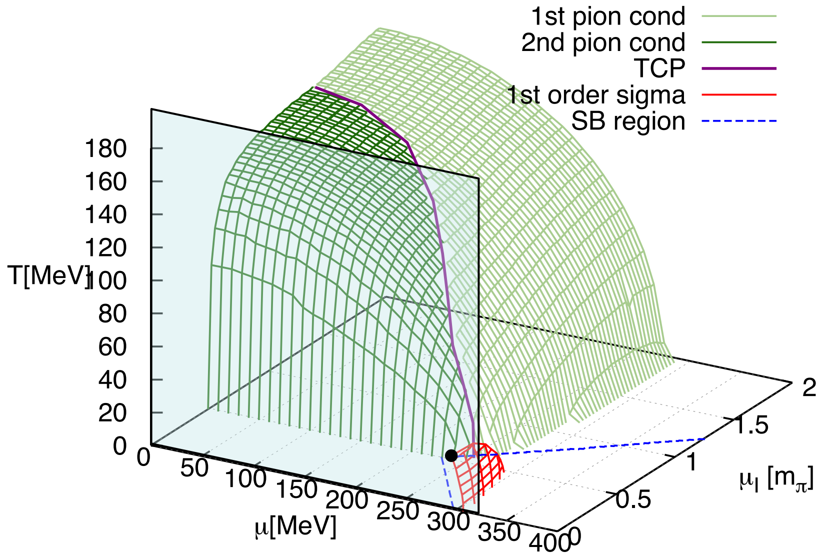

Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

# Baryon & Isospin Chemical Potential

- **Fermionic flow (extended mean-field):**

Lee-Wick




Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

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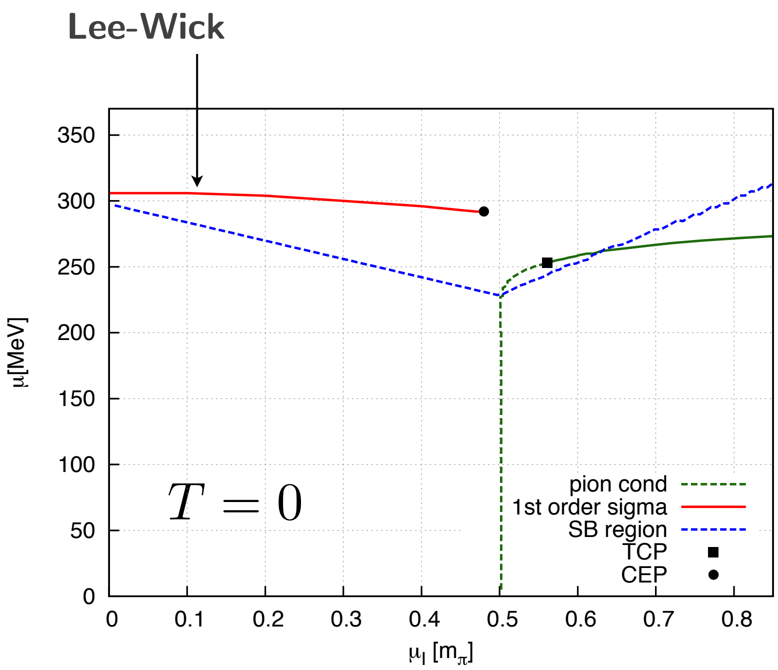


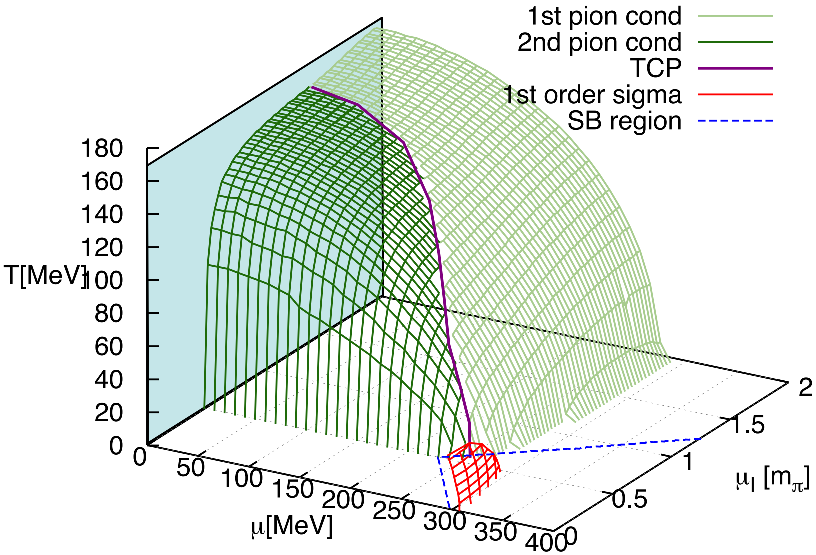


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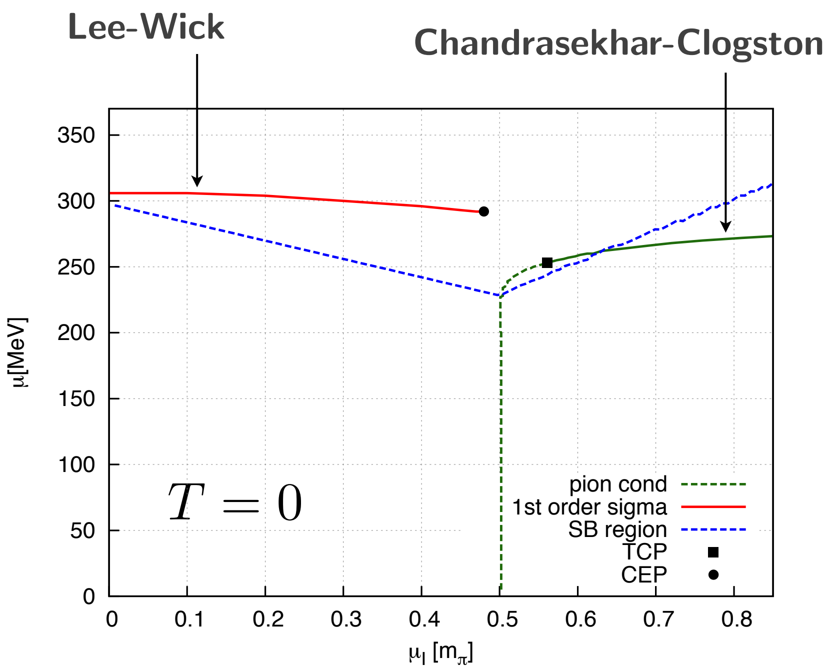
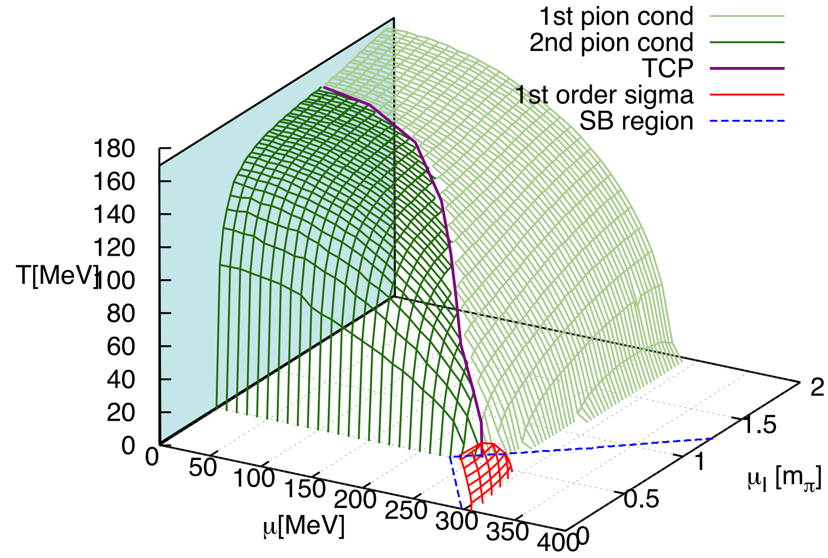


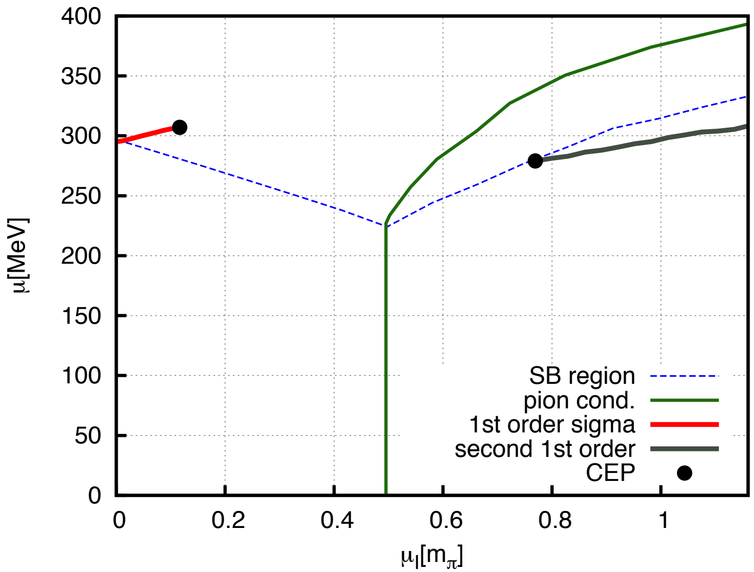



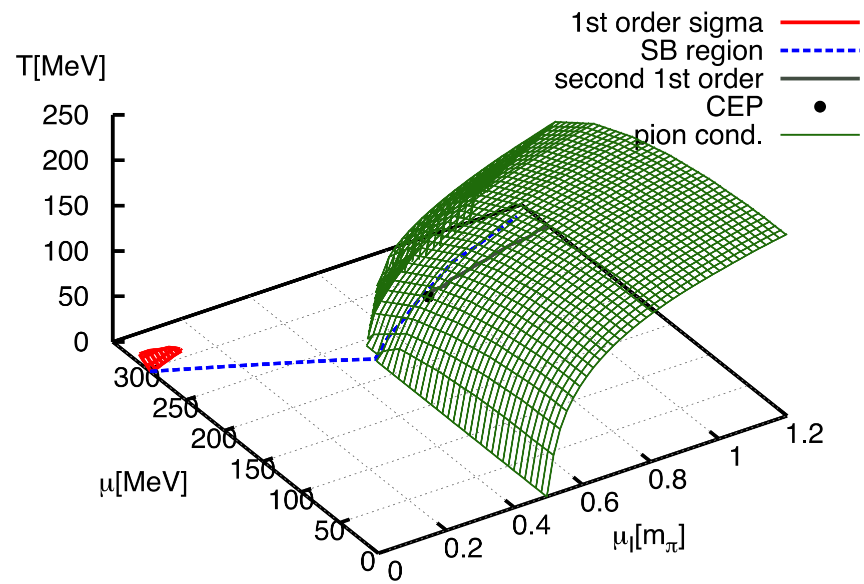

Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

# Baryon & Isospin Chemical Potential

- **Full mesonic flow (2 dimensional):**

$T = 0$:




Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

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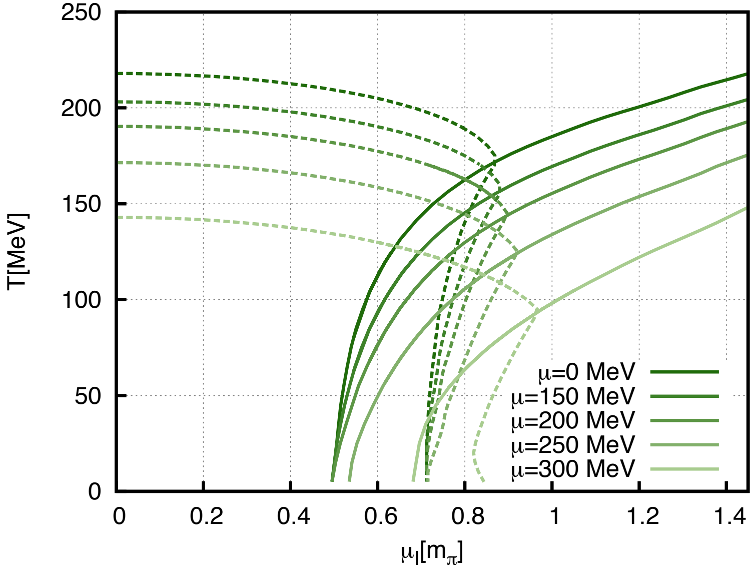


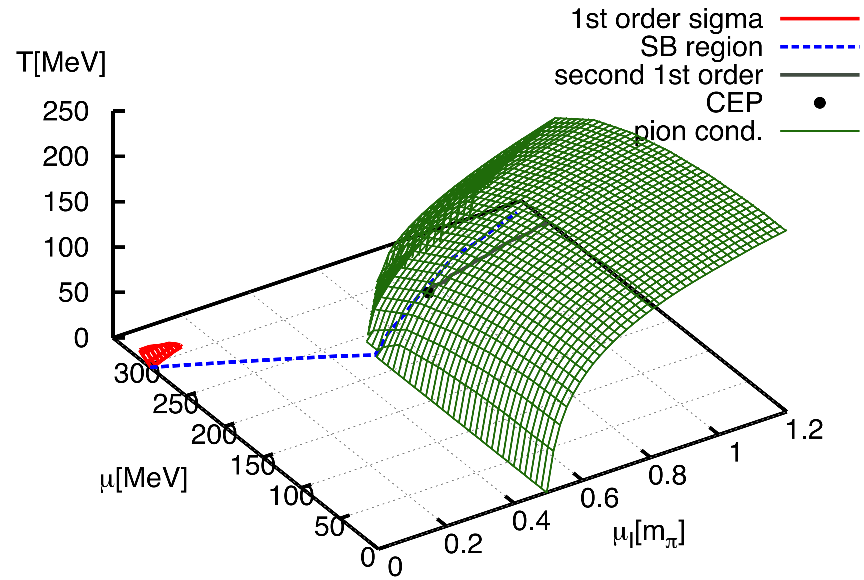




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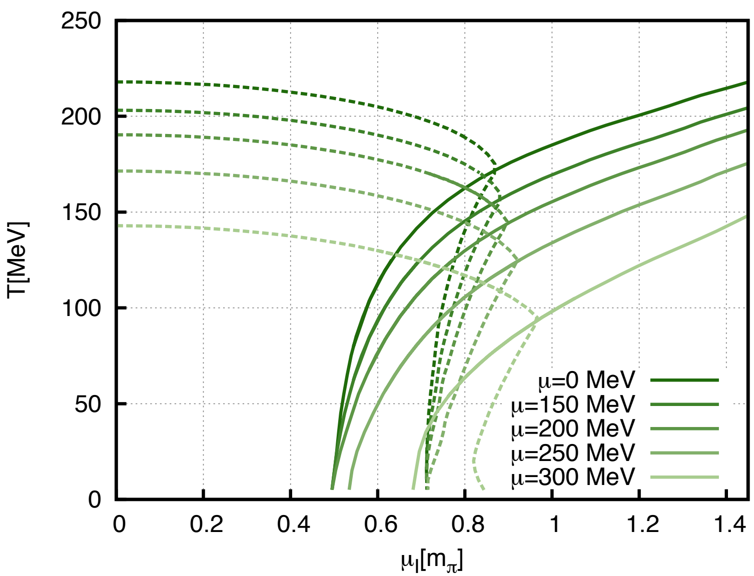
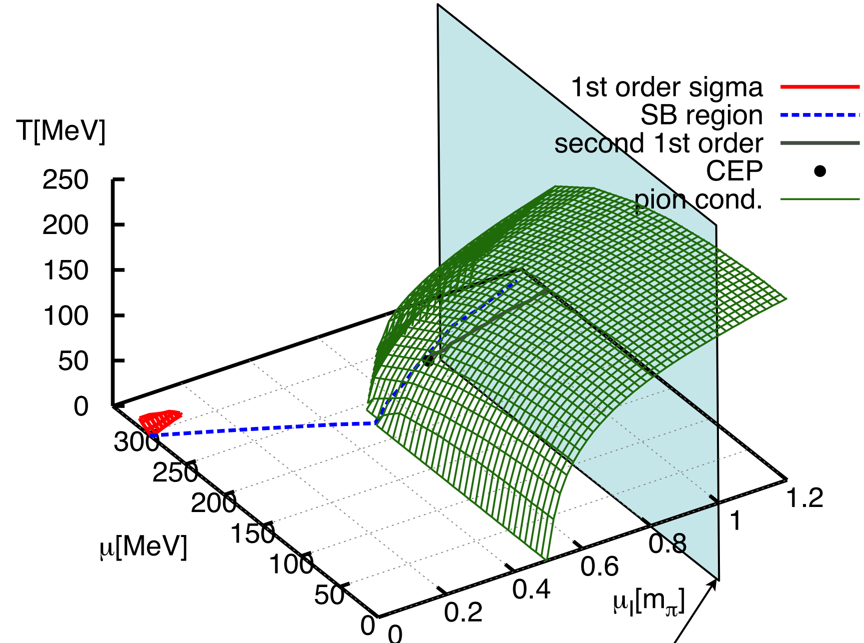


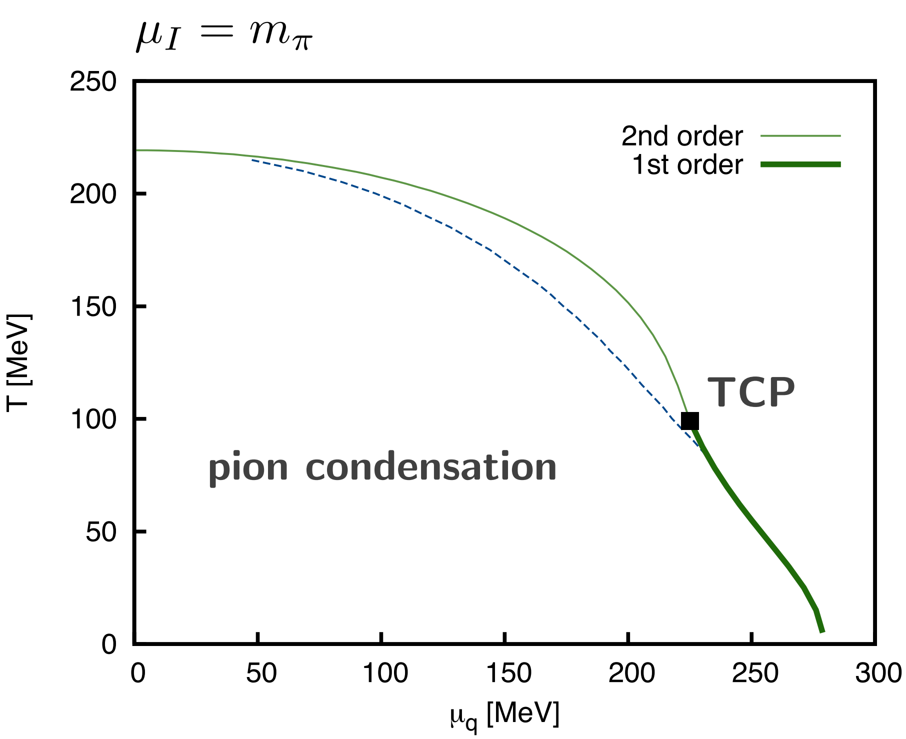




fixed $\mu_I = m_\pi$

$\mu$ for (up/anti-down) imbalance

# Up-Antidown Population Imbalance

- **Fermionic flow (extended mean-field):**



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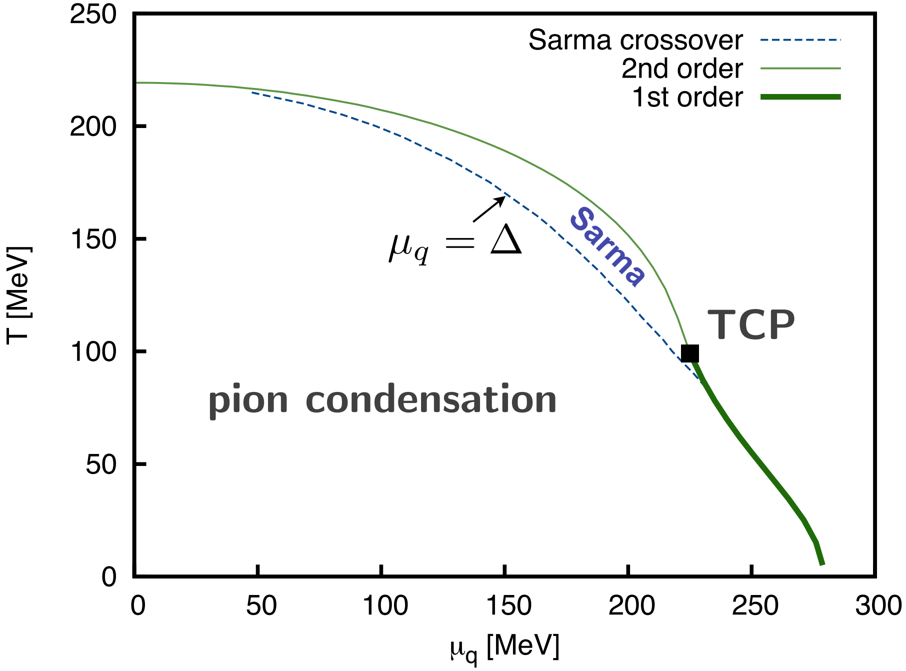

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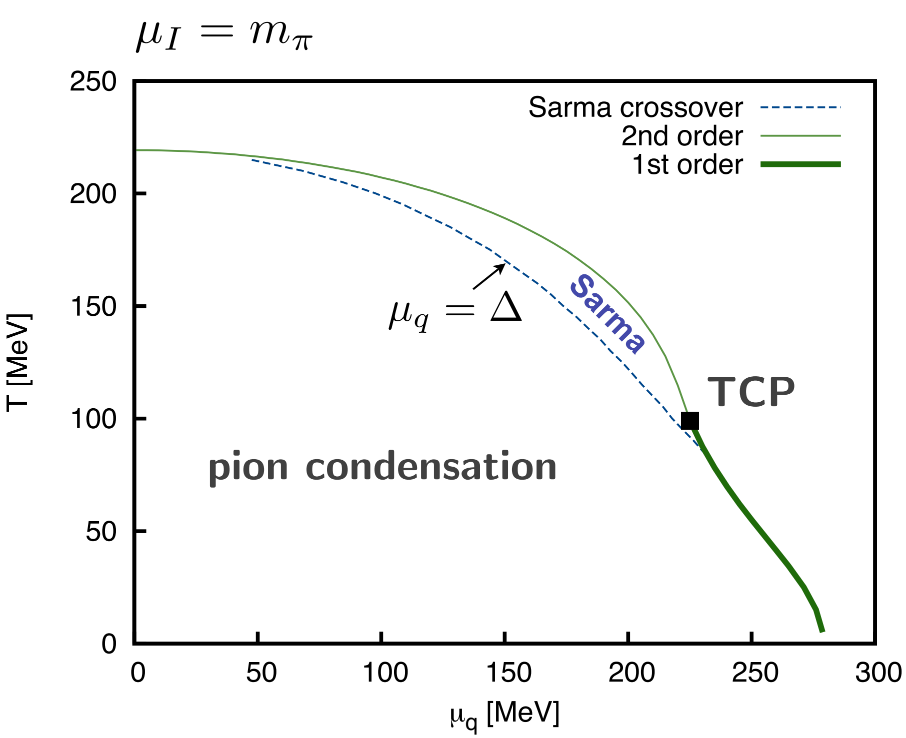


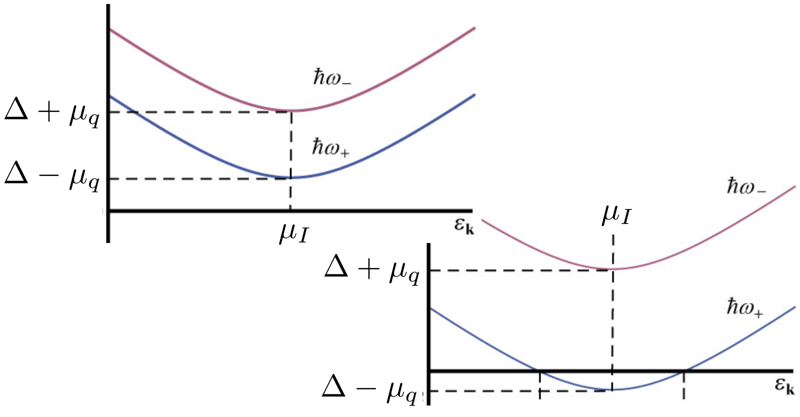

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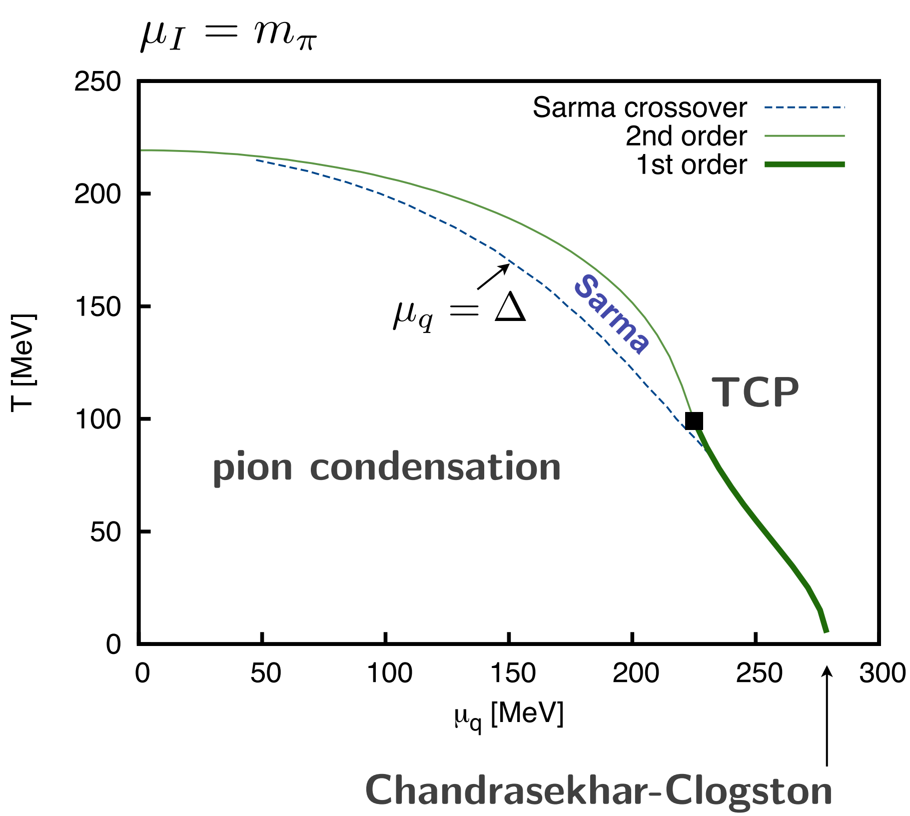


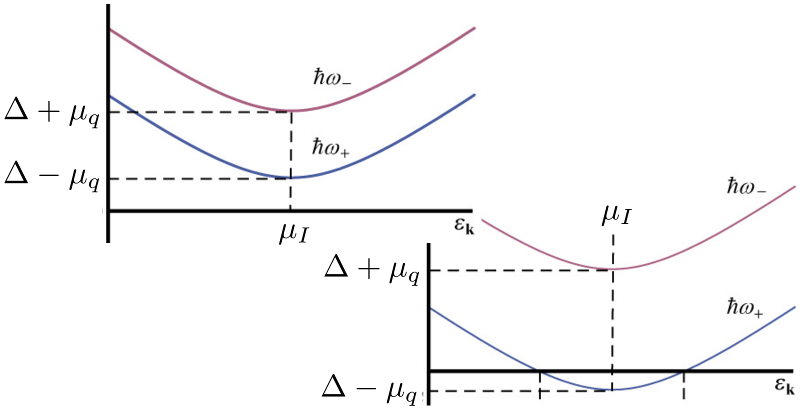

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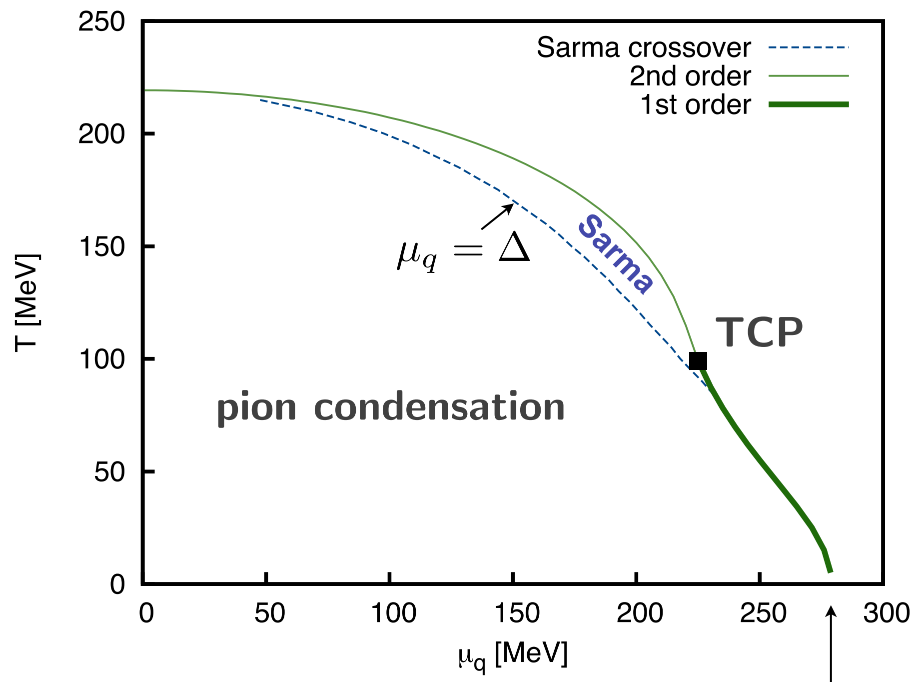
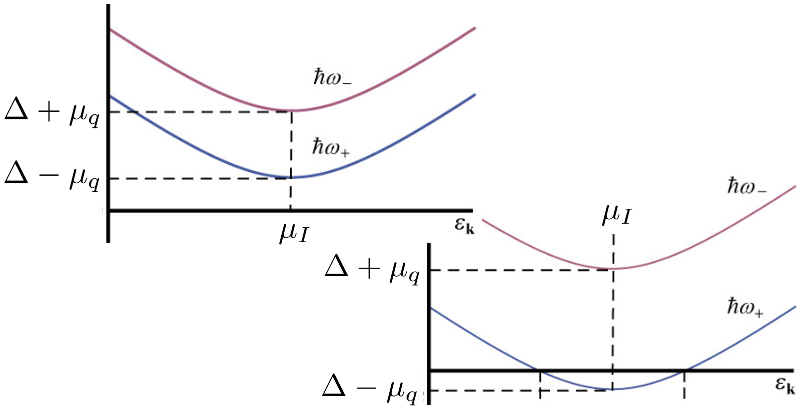
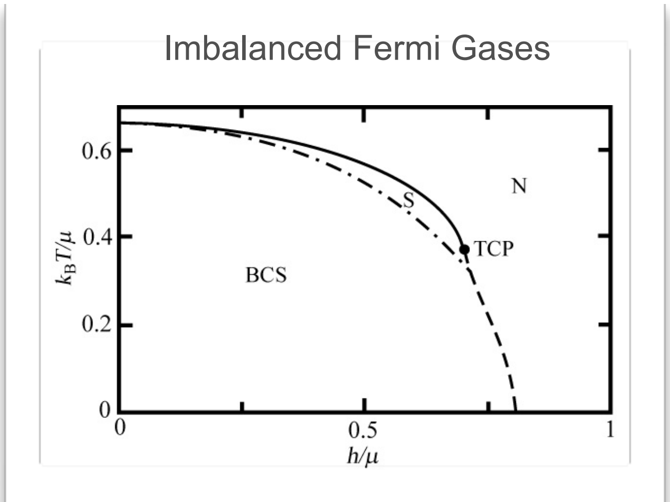
$$\mu_I = m_\pi$$




- compare:




Gubbels, Stoof, arXiv:1205.0568



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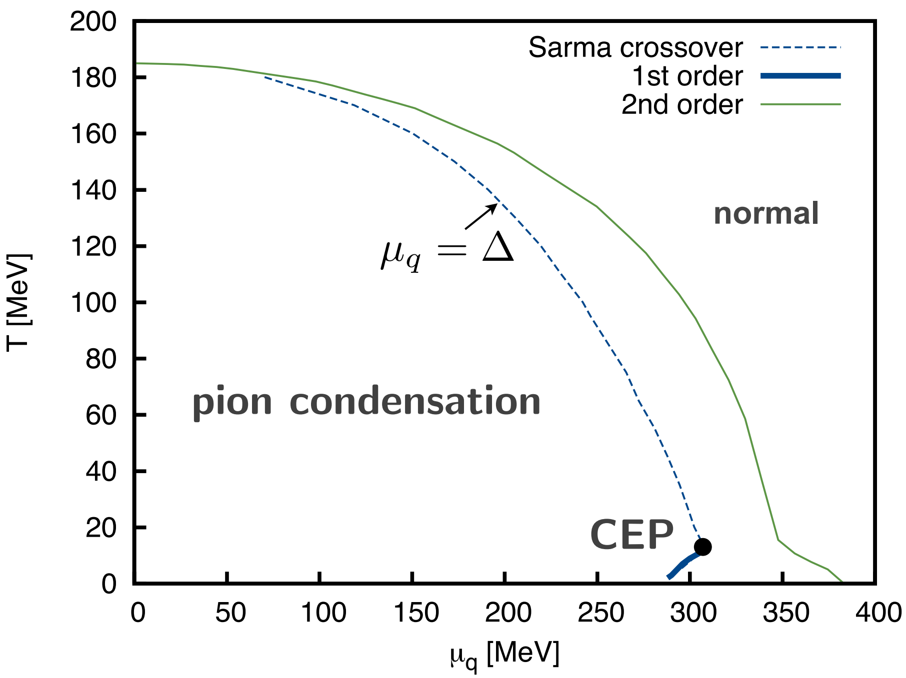
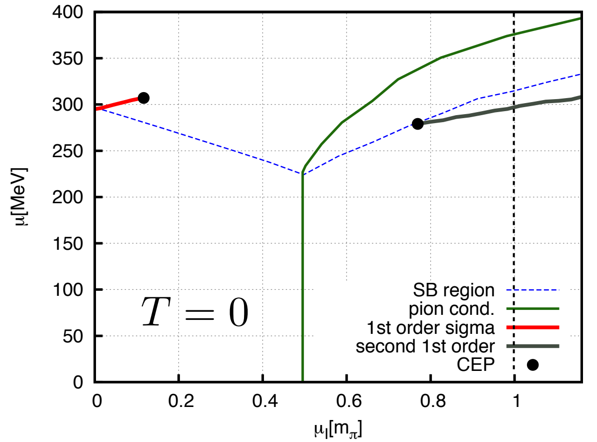
# Up-Antidown Population Imbalance

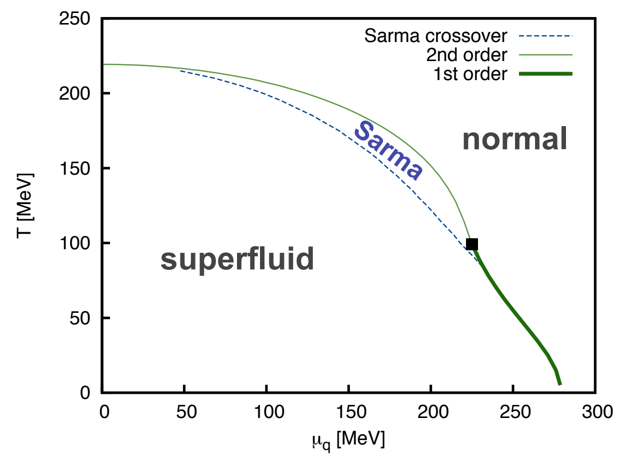
- **Full flow with mesonic fluctuations:**

$\mu_I = m_\pi$




extended mean field



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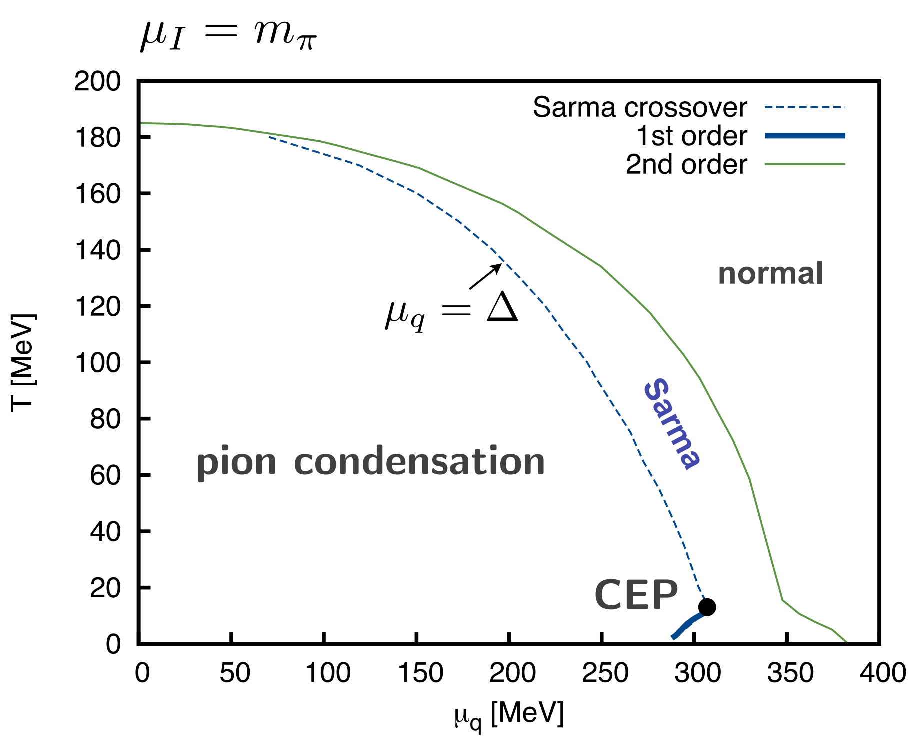
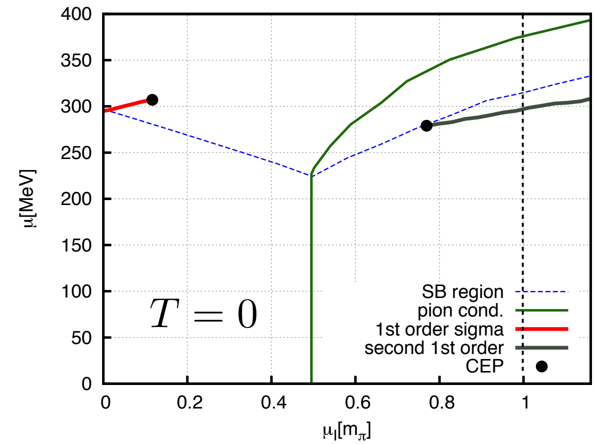



- **stable Sarma phase down to *T* = 0**

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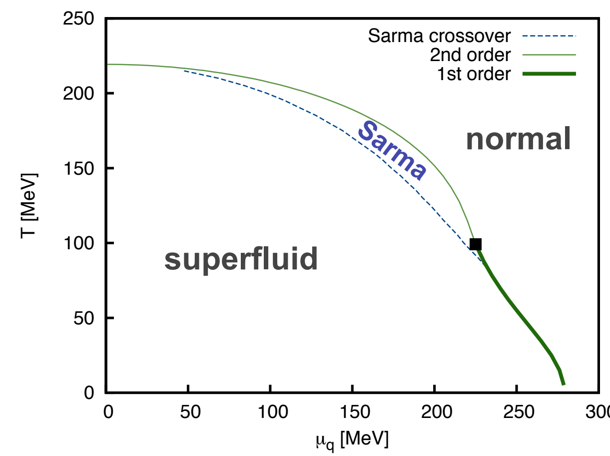




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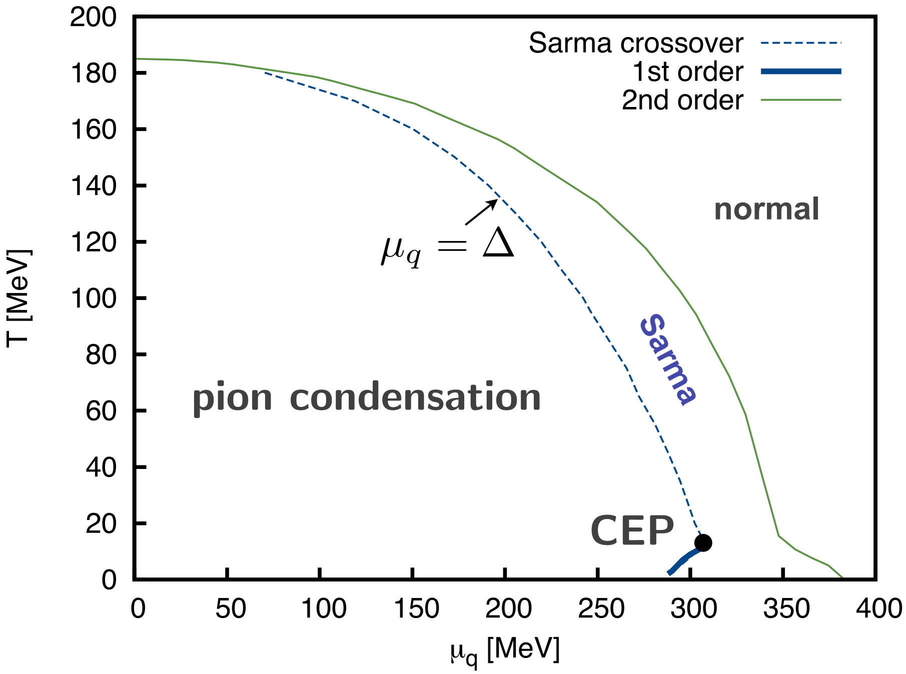
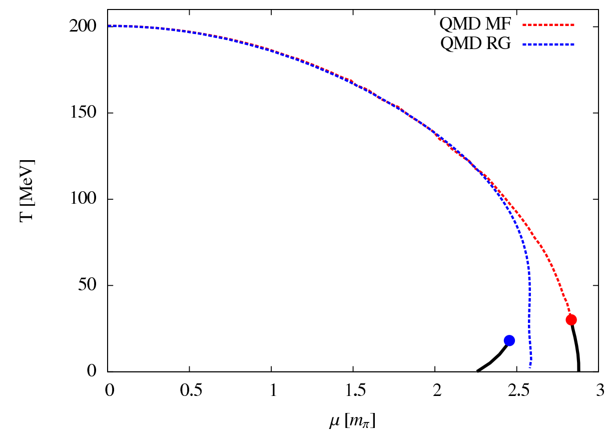



- **stable Sarma phase down to *T* = 0**

- **compare:**

Chiral Transition

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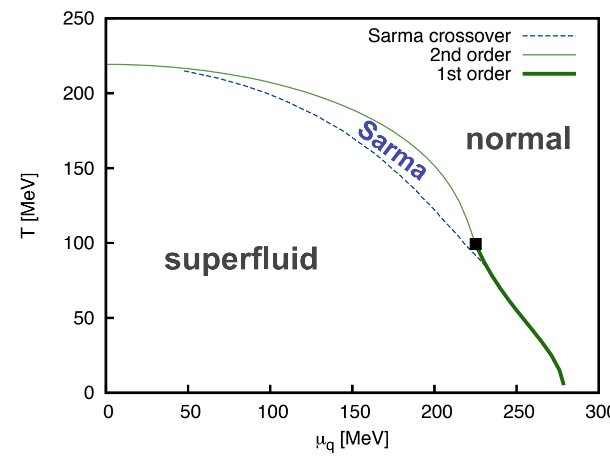

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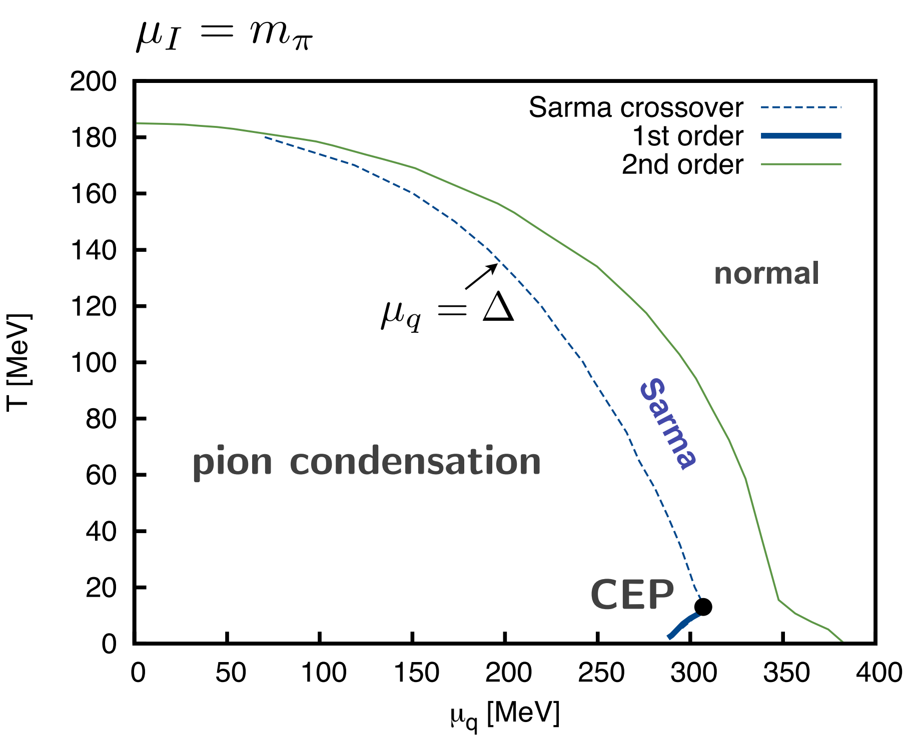
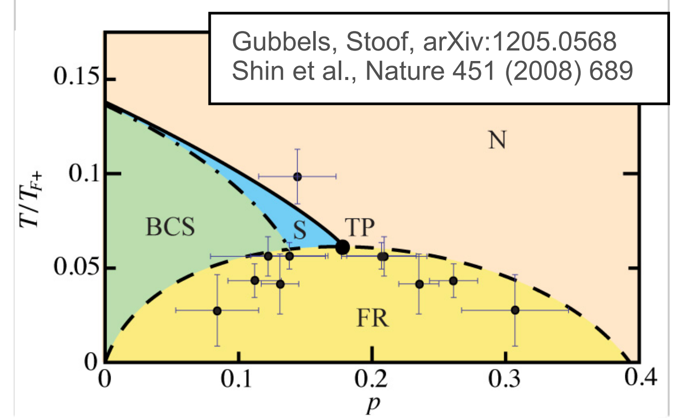
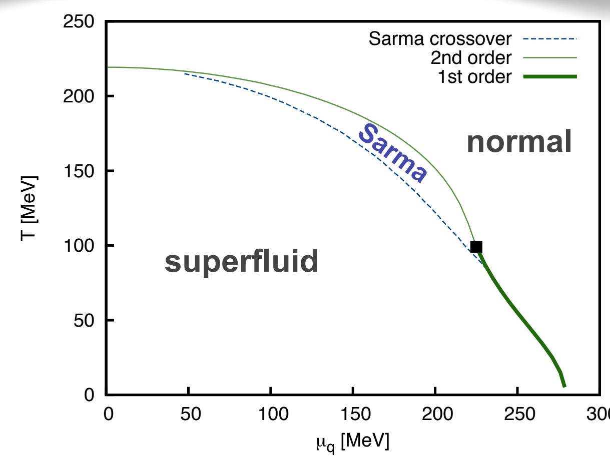



- **compare:**

polarised fermi gas at unitarity




extended mean field

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**part. pol. superfluid near interface in fermi gases?**

Strodthoff & LvS, in preparation

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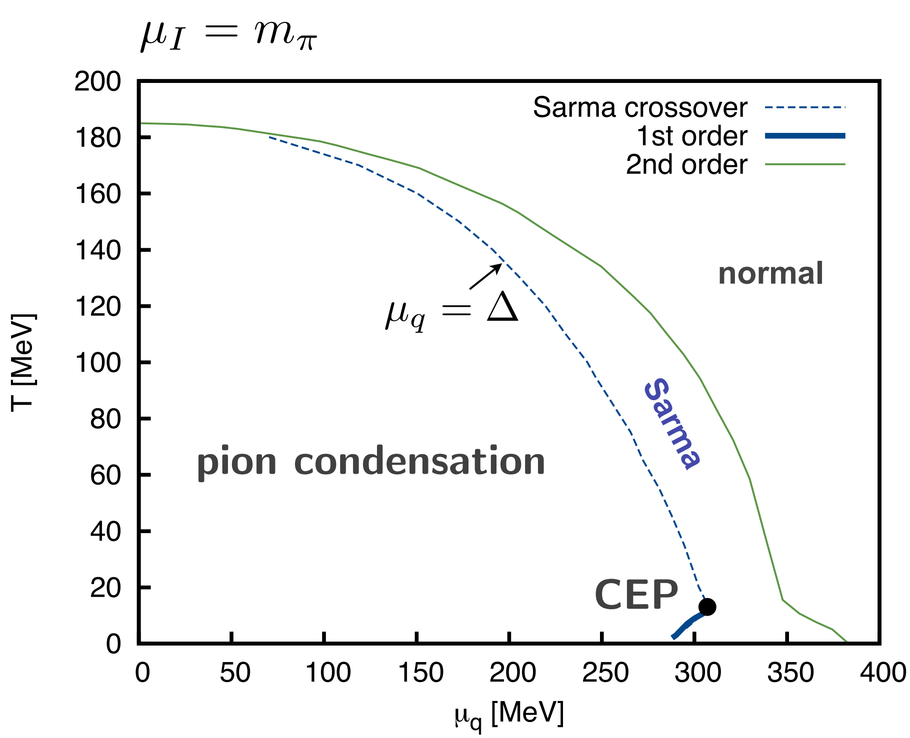


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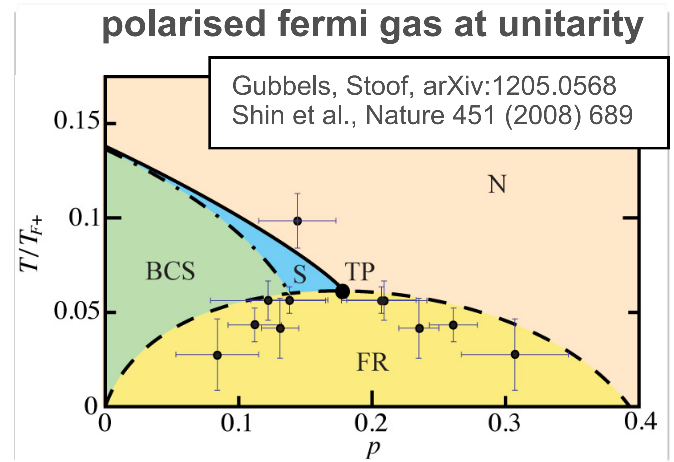
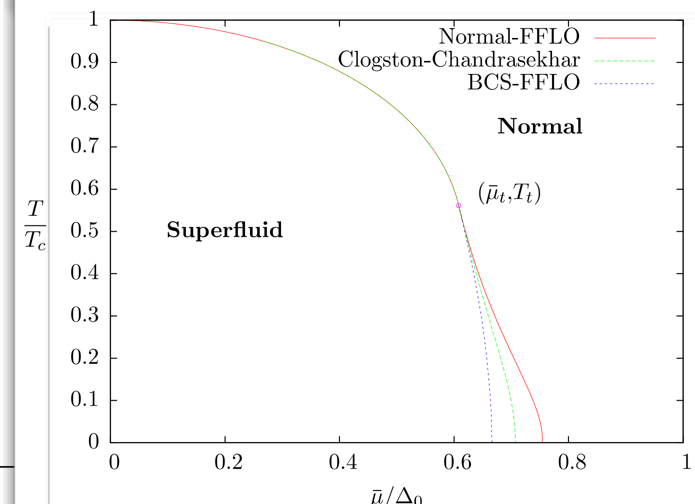
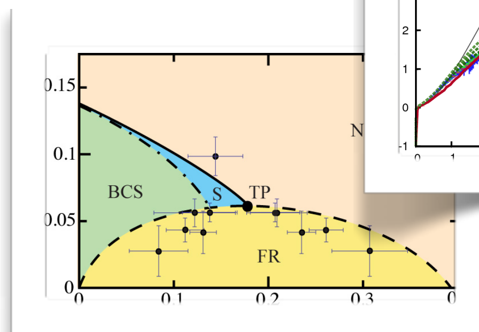


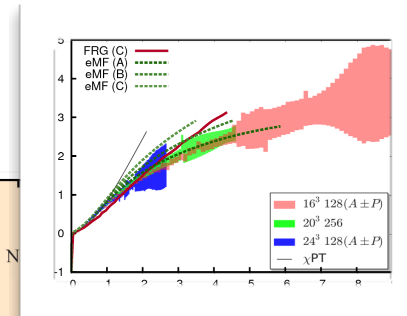




Chevy & Mora, Rep. Prog. Phys 73 (2010) 112401

# Summary & Outlook

- **Finite Isospin Density in QCD and Baryon Density in Two-Color QCD**
  - detailed understanding of phase diagram
  - functional methods and models vs. lattice MC
  - analogies with ultracold fermi gases
    BEC-BCS crossover, population imbalance
    with universal phase diagram...




- **Phase Diagram of $G_2$ Gauge Theory**
  - no sign problem − fermionic baryons
- **Fermions in 2+1 Dimensions**
  - quantum phase transitions, transport properties, topological aspects...
- **QCD Phase Diagram**
  - refined functional methods & models, baryonic dofs, finite volume...

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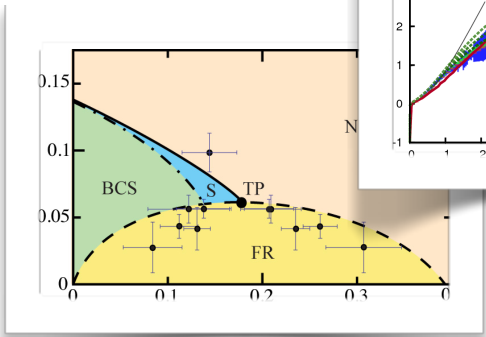


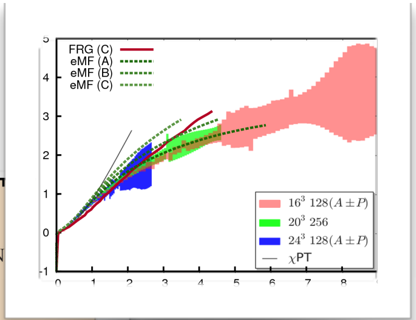


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**Thank You for Your Attention!**

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