# The G2 QCD Phase Diagram

**Axel Maas** 

20<sup>th</sup> of March 2013 Quarks, Gluons, and Hadronic Matter under Extreme Conditions II St. Goar Germany





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Mini-review: Maas & Wellegehausen

Lattice'12 proceedings, 1211.5301



Conceptual – Quenched G2 QCD

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  - Full phase diagram accessible
    - Test of methods and models
    - Qualitative insights

• QCD is a gauge theory

QCD is a gauge theory

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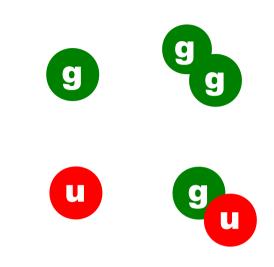


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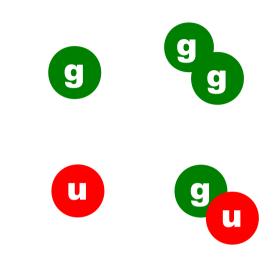
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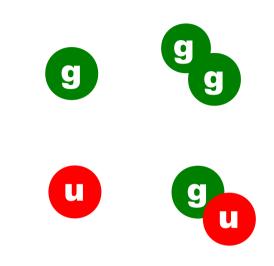
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- Here: G2

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    - (Unbroken) U(1) subgroup of chiral symmetry
- Non-anomalous chiral symmetry breaking for 1 flavor possible

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  - Fermionic 5 and 7 quark states

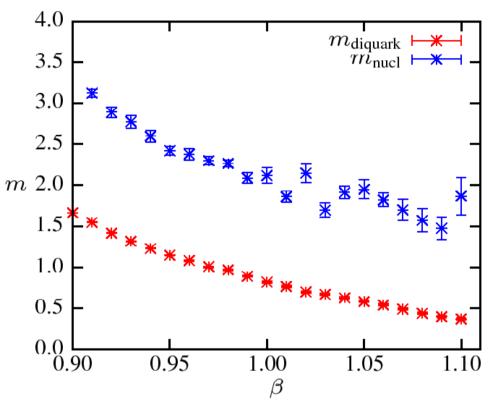
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- Glueballs

[Wellegehausen, Maas, Wipf, von Smekal unpublished]

#### Nucleons

 Fermionic baryon with three constituent quarks

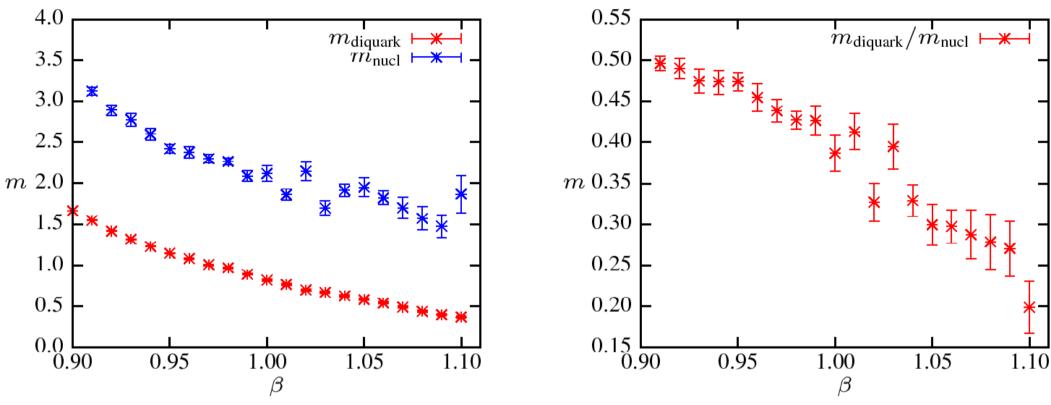
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#### Nucleons

- Fermionic baryon with three constituent quarks
- Chiral limit different to QCD

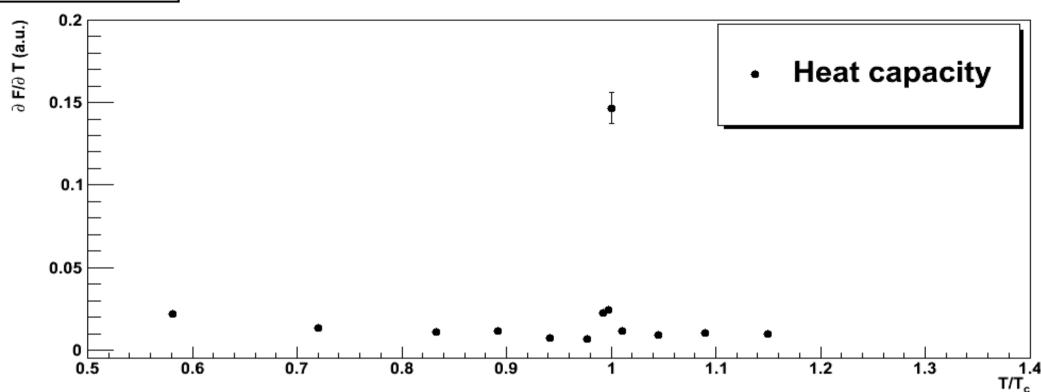
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[Danzer, Gattringer, Maas, JHEP09]

Phase transition

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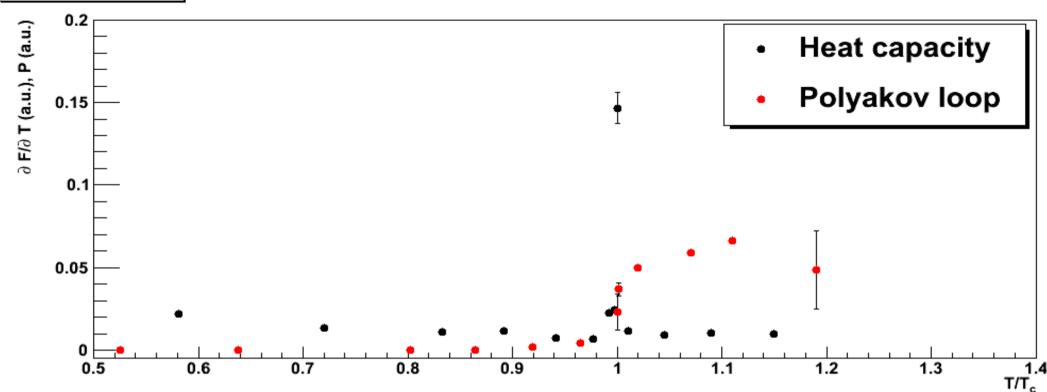
• First order transition [Pepe et al. NPA07, Greensite et la. PRD07, Cossu et al. JHEP07]

• Observed in free energy

- Complicated by a bulk transition
  - Requires fine lattice [Cossu et al. JHEP 07]

Phase transition

[Danzer, Gattringer, Maas, JHEP09]



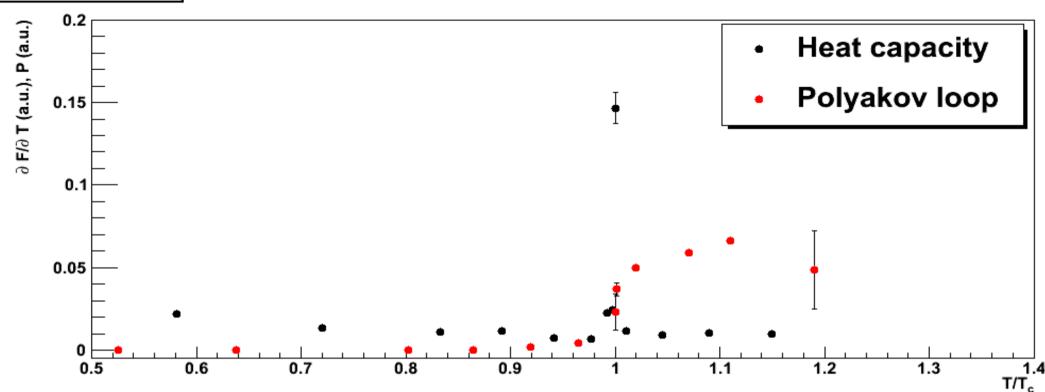
#### Polyakov loop transition

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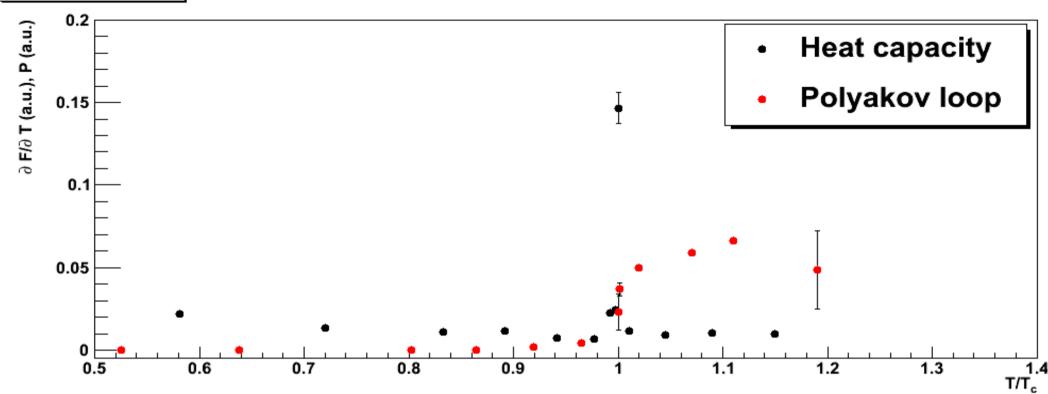
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#### • Coincident with free energy

No deconfinement – like in QCD

Phase transition

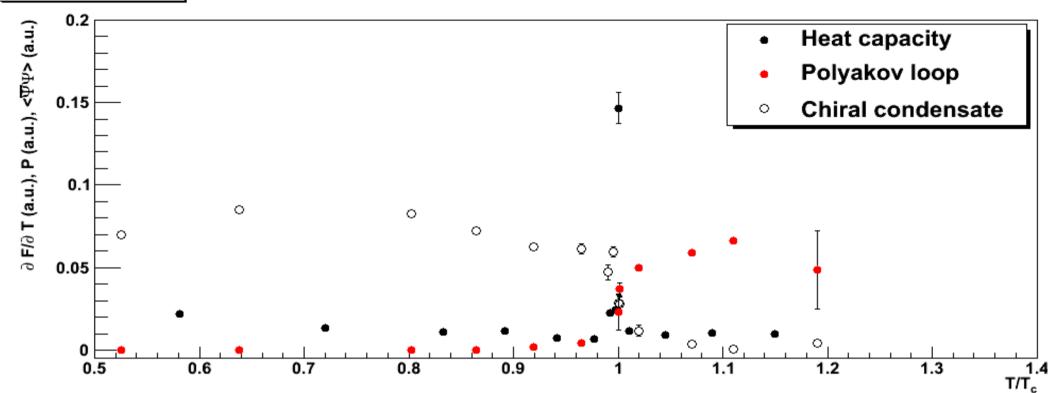
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Phase transition

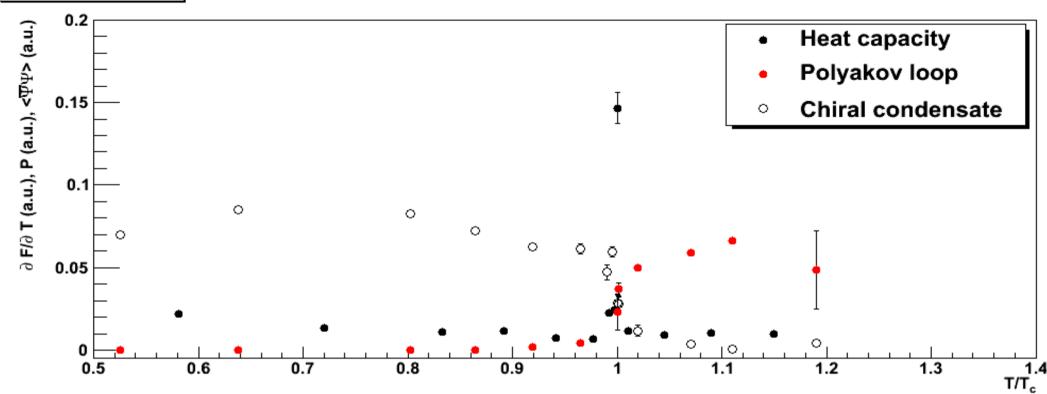
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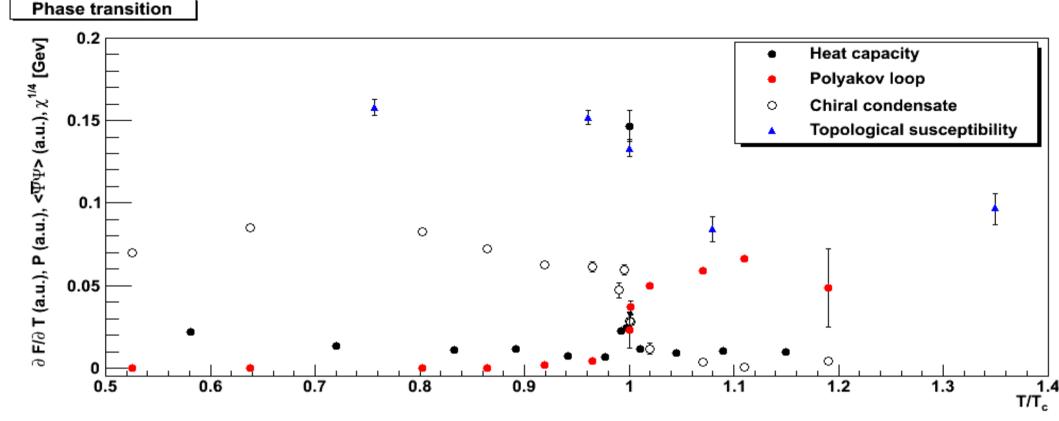
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- Chiral symmetry 'broken' in quenched G2 QCD
- 'Restoration' at the phase transition
  - Like in QCD
  - Unlike adjoint QCD [Bilgici, Ilgenfritz, Gattringer, Maas JHEP 09]

#### **Topological susceptibility**

[Ilgenfritz & Maas PRD'12]



- Topological structure similar
  - Topology reflects phase transition
  - Fewer topological lumps the higher the temperature

[Maas, von Smekal, Wellegehausen, Wipf '12]

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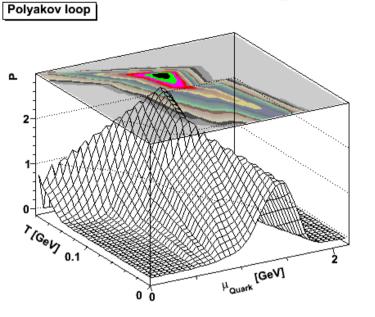
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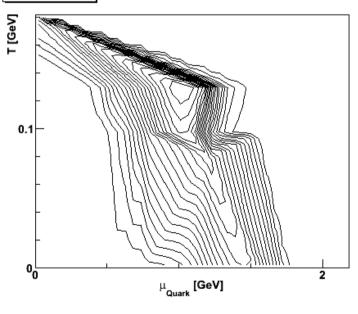
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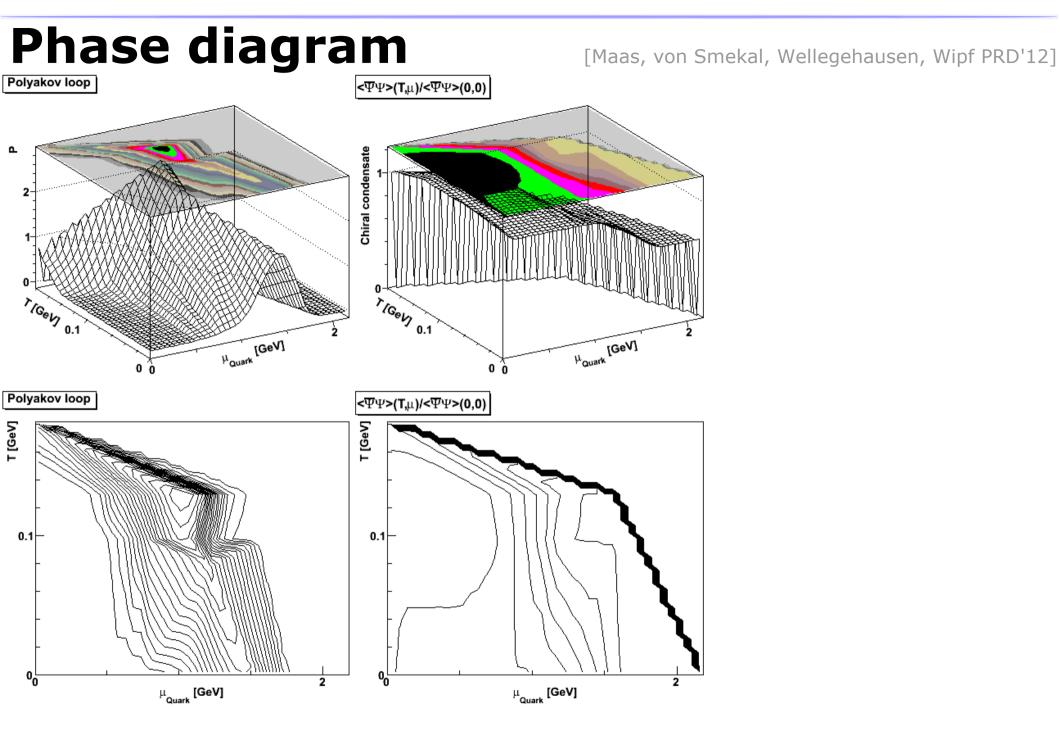
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  - Preliminary: Not very different for 2 flavors

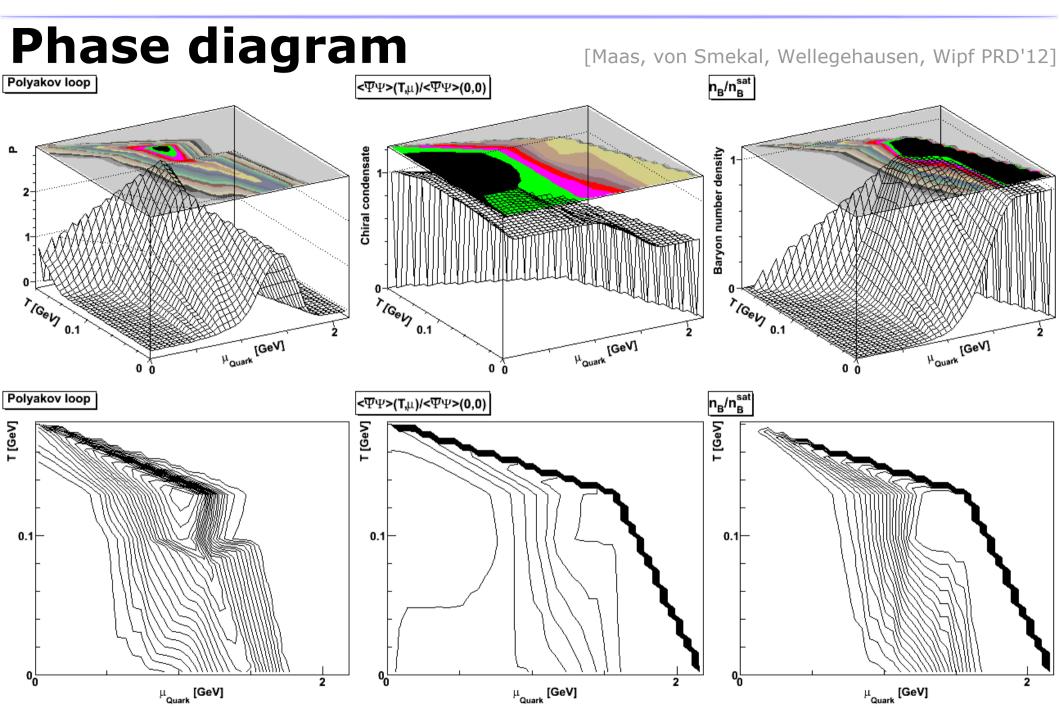
### Phase diagram

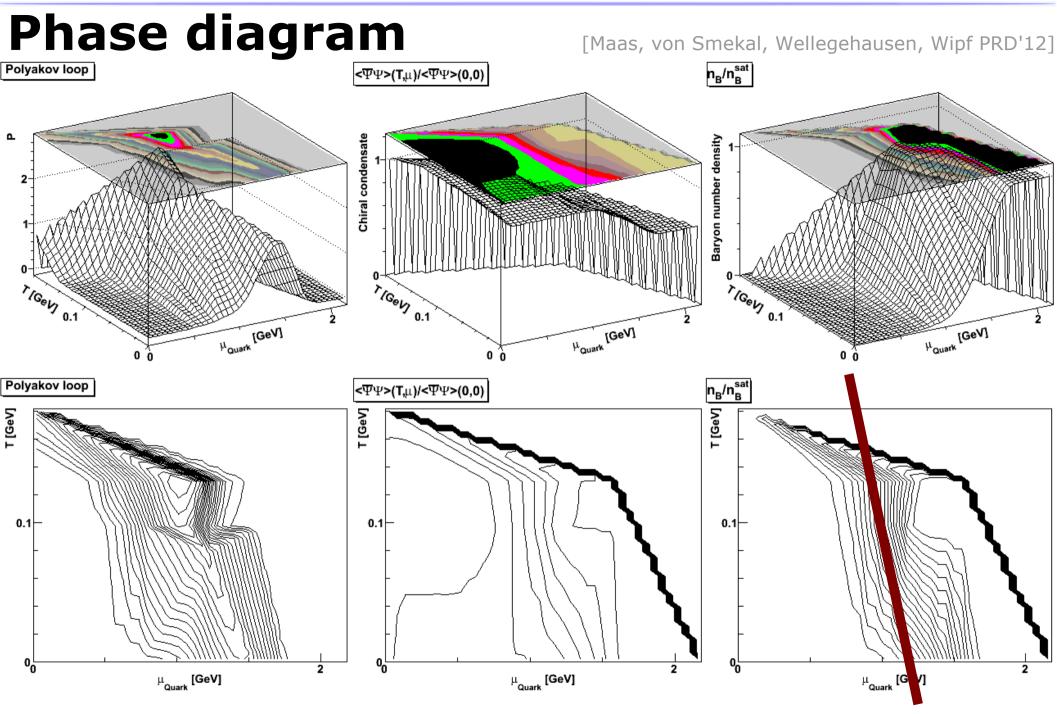












Start of lattice artifacts

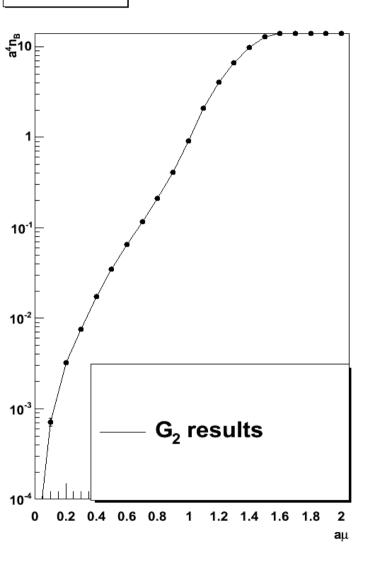
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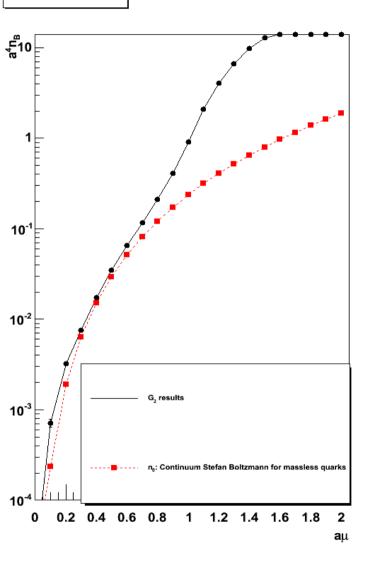
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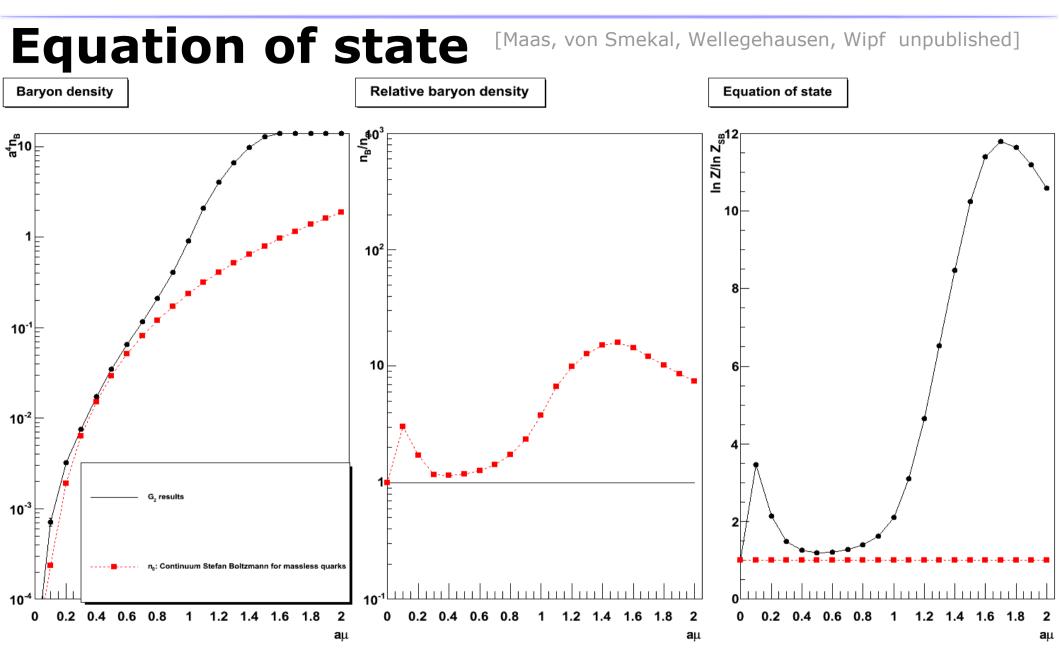


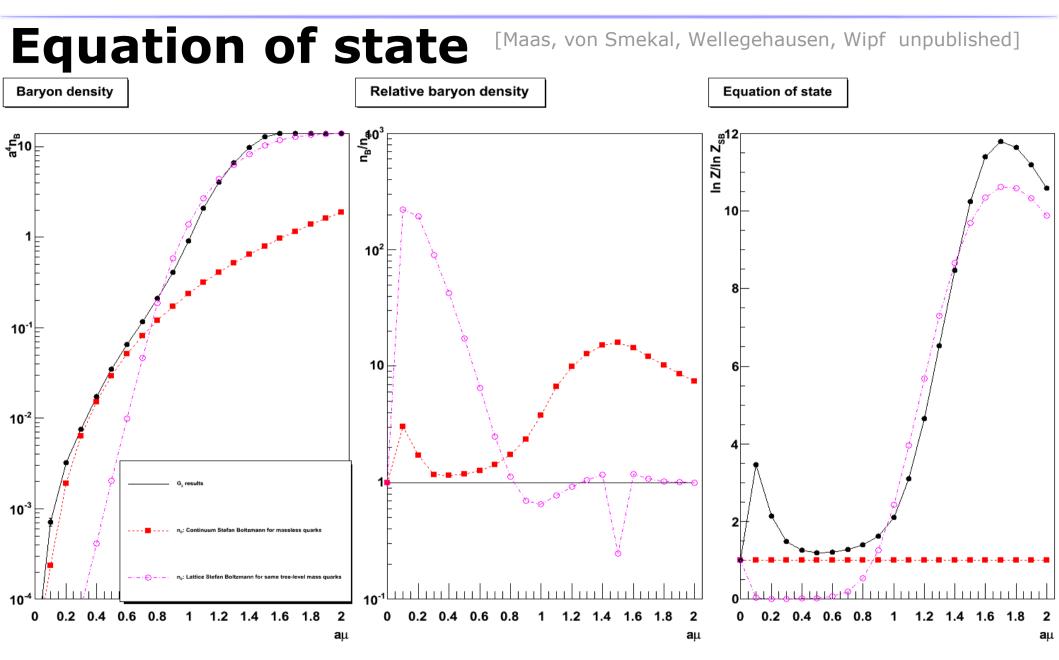
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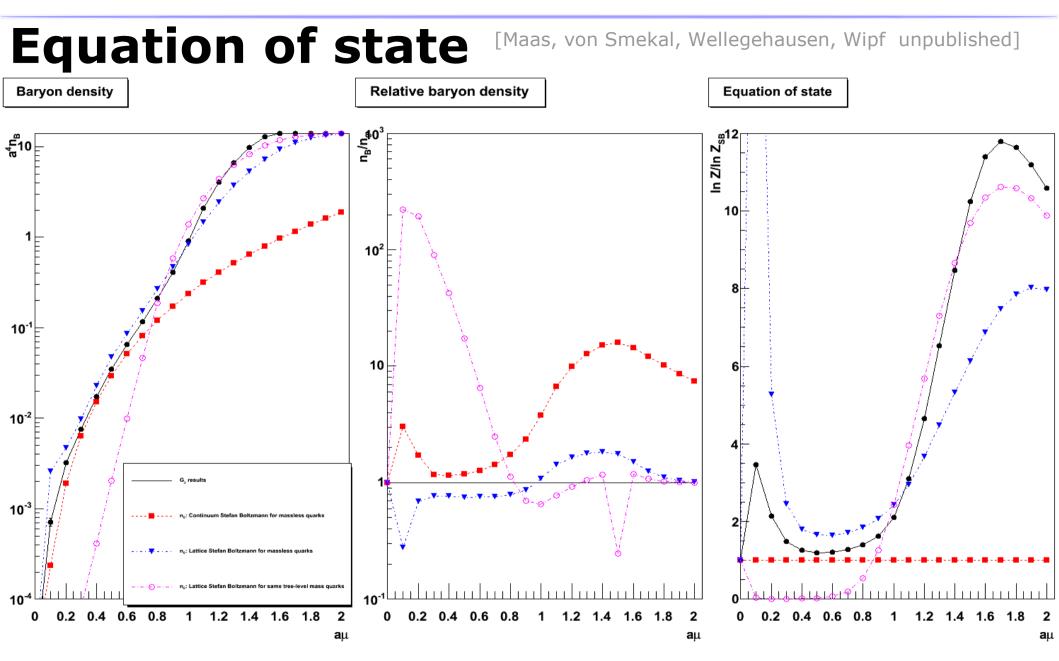
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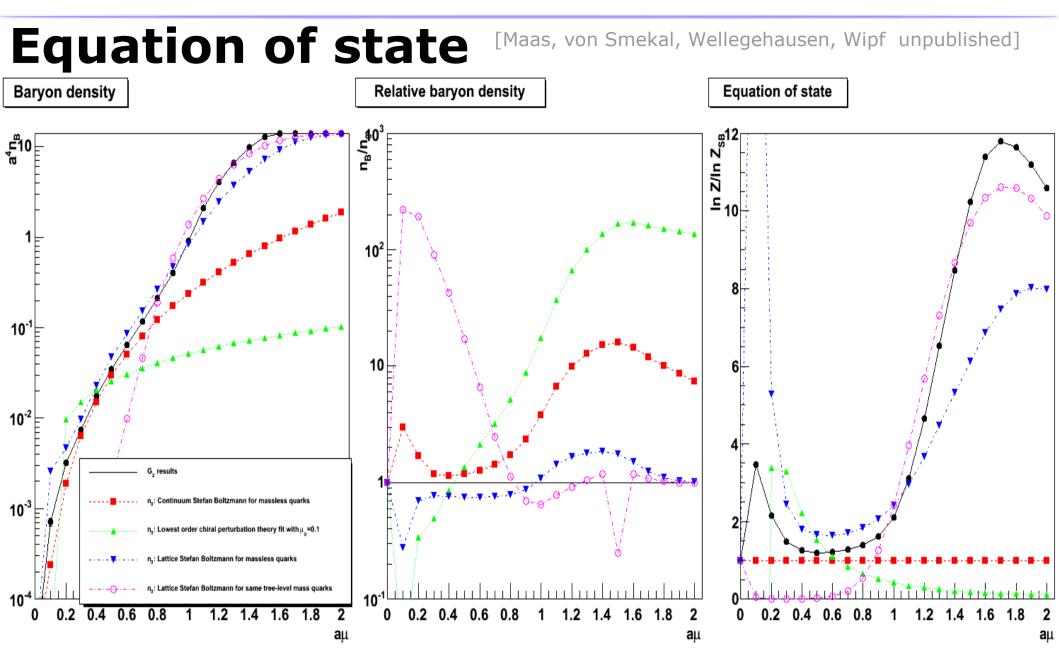
Can be calculated from baryon density

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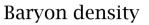


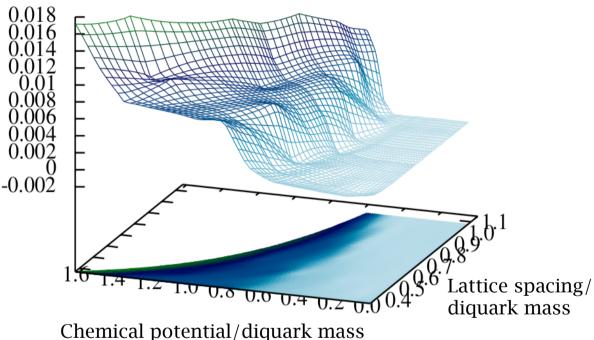




• Lattice artifacts still hard to control

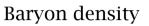
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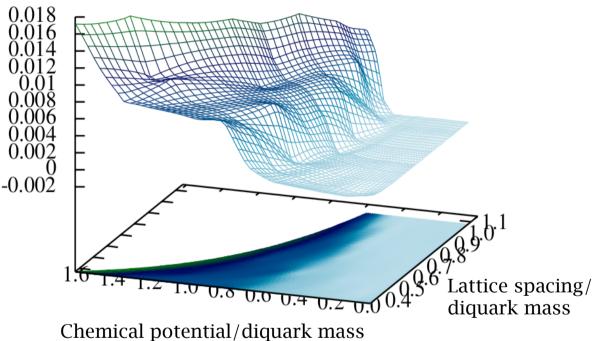




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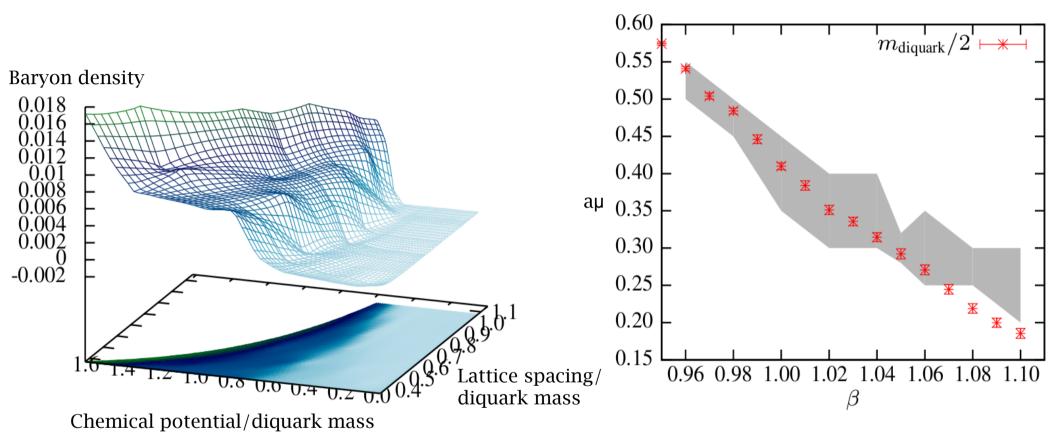
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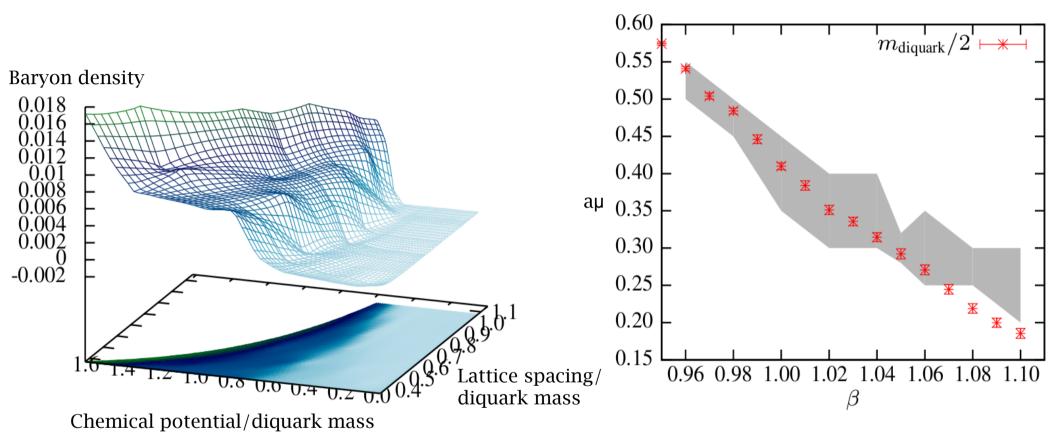
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- Plateau at small density: Intermediate phase?

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- Go off the lattice and combine
  - Derive effective theories
  - Tests for truncation/approximations in continuum methods
    - E.g. functional methods

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  - Rough shape of the phase diagram of a gauge theory is similar to the expected one