Probing the string tension with external magnetic fields in Dyson-Schwinger equations

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Outline

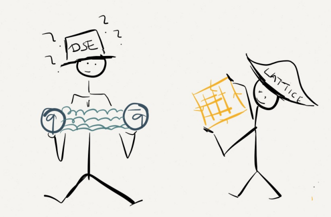
The String Tension..

..between two static color charges

- is an order parameter of confinement
- is accessible via the area-law fall off of the Wilson loop and therefore in the first place measurable on the lattice

However...

- Wilson loops not directly accessible via functional methods
- BUT: There is another way...

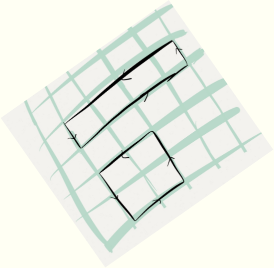


DUAL CONDENSATES...

- Fourier transforms of the chiral condensate wrt external parameters
- firstly introduced: Dressed Polyakov loop
  $\rightarrow$ loops of different length winding once around the time direction
  Gattringer,PRL 97(2006);
  Synatschke,Wipf,Wozar, PRD 75(2007);
  Bilgici, Bruckmann, Gattringer,Hagen, PRD 77(2008)

to probe the string tension σ:

- Dressed Wilson Loops
  Bruckmann and Endrödi,PRD 84, (2011)
- planar closed loops with the same area, but different geometries



The Dressed Wilson Loop: Main Idea

Bruckmann and Endrödi,PRD 84, (2011)

- Dual variables: magnetic field and the (by the loop) enclosed area

$$\langle\widetilde{\bar{\psi}\psi}\rangle(s) = \frac{1}{S_{\mu\nu}} \sum_{b=0}^{b_{max}} e^{-2i\pi bs/N_{\mu\nu}} \langle\bar{\psi}\psi\rangle(b)$$

- 'Dressed Wilson loops', since for $m \to \infty$: Contact to Wilson loops

$$\widetilde{\Sigma}(s)_{m\to\infty} \sim \exp(-\sigma S)$$

The dual condensate...

as response to an external magnetic field connects chiral symmetry breaking to confinement.

HOW TO...

probe for (spatial) string tension σ..

..in a Dyson–Schwinger framework ?

Need to...

- implement the external magnetic field

  Ritus (1972), Nikishov (1969), Fock (1937), Schwinger (1951)
- write down the Dyson–Schwinger equations

  Lee, Leung, Ng:PRD 55(1997), Miransky (1990s)
- find a tractable truncation/approximation scheme
- determine the condensate
- determine the dual condensate and extract the string tension

$\Rightarrow$ **Start to set the stage with the magnetic background.**

Magnetic Background

Implementation

- Abelian field in z-direction

$$A_\mu = (0, \mathcal{B}\, z, 0, 0)^\mathsf{T}$$

$$\Rightarrow \mathcal{F}_{\mu\nu,ab} = \mathcal{F}_{\mu\nu,ab} + F_{\mu\nu} \otimes \mathbb{1}_{ab}$$

- Principle of minimal coupling

$$\mathrm{D}_{\mu,\mathcal{B}} = \partial_\mu - ieA_\mu \qquad \mathcal{D}_{\mu,\mathcal{B}} = \mathrm{D}_{\mu,\mathcal{B}} + \mathrm{i}g\, t^a\, \mathcal{A}_{\mu,a}$$

- Leads to the underlying Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\not{\mathcal{D}}_{\mathcal{B}} - m)\psi + \frac{1}{4}\mathcal{F}_{\mu\nu}\,\mathcal{F}^{\mu\nu}$$

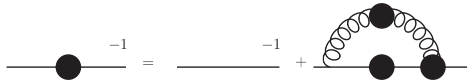
The Dyson-Schwinger Equations in sight

With the Lagrangian $\mathcal{L}$ at hand,

- derive the Green's functions' equation of motion

$$0 = \int \mathcal{D}\phi \frac{\delta}{\delta\phi} e^{-S[\phi]+\int J\cdot\phi},$$

- obtain an infinite tower of coupled integral equations for the propagators...



...and meet the challenges introduced by the magnetic field.

SETTING THE EQUATIONS

- propagator from Green's function identity:

$$(\mathrm{i}\not{D}_{\mathcal{B}} - m)\, S_0(x, x') = \delta(x - x')$$

  with $\mathrm{D}_{\mu,\mathcal{B}} = \partial_\mu - eA_\mu$
- 'standard' approach: expand in plane wave functions find diagonal propagator in momentum space
- challenge: $[\mathrm{D}_{\mu,\mathcal{B}}, p_\nu] \neq 0$
  $\rightarrow$'standard' not applicable

Following Ritus' method

...to obtain the (inverse) propagator in momentum space

V.I. Ritus: Annals of Phys. 69,555 (1972)

Ritus' Method

The Idea

- Observation I:
  S can only depend on scalar structures built from γ^μ contracted with $\mathrm{D}_{\mu,\mathcal{B}}, F_{\mu\nu}, ...$
- Observation II:
  $[(\not{D}_{\mathcal{B}})^2, S(x,x')] = 0$

The Procedure

- Use *eigen*functions of $\not{D}^2_{\mathcal{B}}$ to diagonalize propagator
- End up with 'modified' propagator
  diagonal in momentum space
  depending on special subset of momenta

$$
\begin{aligned}
(\mathrm{i}\not{D}_{\mathcal{B}} - \mathrm{m})\, S_0(x,x') &= \int \mathrm{dp}\, \mathbb{E}_p\, (\bar{\not{p}} - \mathrm{m}) S_0(p)\, \bar{\mathbb{E}}_p \\
&\overset{!}{=} \delta(x - x')
\end{aligned}
$$

THE PROPAGATOR

- Diagonalization procedure shows

$$S_0(\bar{p}) = (\bar{\not{p}} - \mathrm{m})^{-1}$$

- With momenta given by

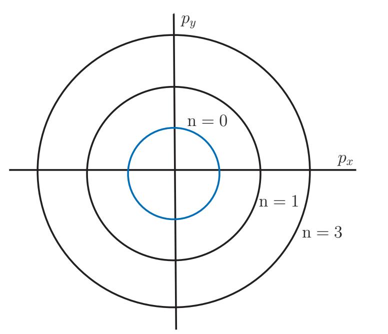
$$\bar{p} = (p_0, 0, (p_2)_{n,\sigma}, p_3)^{\mathrm{T}}$$

- And $(p_2)_{n,\sigma}$ encoding the particles' Landau Levels

$$(p_2)_{n,\sigma} = \sqrt{|e\mathcal{B}|(2\mathrm{n}+1+\sigma)} \leftrightarrow (p_2)_\kappa = \sqrt{2\kappa|e\mathcal{B}|}$$

- States per unit area:
  $\frac{|e\mathcal{B}|}{2\pi}$ for $\kappa = 0$
  $\frac{|e\mathcal{B}|}{\pi}$ for $\kappa \geqslant 1$

THE LANDAU LEVELS

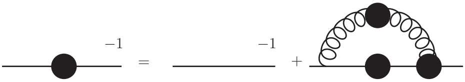


- Lowest Landau Level approach: $\kappa = 0$
- Beyond LLLA: include $\kappa = 1, 2, \ldots$
  $\rightarrow$ fixed cutoff Λ_{UV} for all $\mathcal{B}$
  $\rightarrow$ sets limit $\kappa_{max}(\mathcal{B})$
- 'Dimensional reduction'
  $\rightarrow$ will set $p_0 = p_3 = 0$

$$\bar{p} = (0, 0, (p_2)_\kappa, 0)^T$$

- NO application of the Mermin–Wagner theorem
  $\rightarrow$ gluons are 4-dimensional
- we do expect to see chiral symmetry breaking...

THE DYSON-SCHWINGER EQUATIONS



with the dressed propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\,(A_\mu(\bar{p})\bar{p}_\mu\gamma^\mu)$$

- start with the 'simplest' tensor structure for first probe
- include higher terms $(\sigma \cdot F,\ (F \cdot \mathrm{D})^2,\ \gamma_5(F \cdot \widetilde{F}))$ ...in a next step.
- 'modified' bare vertex approximation: with the phenomenological *ansatz* function

$$\Gamma(q) = \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2 + 1]}{4\pi}\right)^{2\delta}$$

THE DYSON-SCHWINGER EQUATIONS

- Landau gauge
- zero temperature for now
- Gluonic input from lattice calculations
  $\rightarrow$ expression for dressing function $Z(k)$

$$Z(k) = \frac{k^2\Lambda^2}{(k^2+\Lambda^2)^2}\left\{\left(\frac{c}{k^2+\Lambda^2 a}\right)^b + \frac{k^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[k^2/\Lambda^2+1]}{4\pi}\right)^\gamma\right\}$$

Fischer, Maas, Mueller: Eur. Phys. J. C 68(2010)

- probe plane perpendicular to magnetic field
- solution in a finite volume
  $\rightarrow$ compactify xy-direction (periodic bc)

Magnetic Fields In A Finite Volume

Magnetic Flux

$$\int dx_\mu A_\mu = \mathcal{B} \cdot \mathrm{F}$$

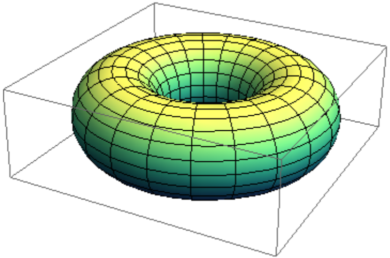
$$\int dx_\mu A_\mu = \mathcal{B} \cdot (\mathrm{F} - L_x L_y)$$

Charged Particles

$$\exp(\mathrm{i} q \mathcal{B} \mathrm{F}) \overset{!}{=} \exp(\mathrm{i} q \mathcal{B} (\mathrm{F} - L_x L_y))$$

$$\Rightarrow q\mathcal{B} = \frac{2\pi}{L_x L_y} b$$

with b= 0,1,2...



Preliminary: The Planar Chiral Condensate

Full Quark Propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\,\gamma^2 A_2(\bar{p})\,(p_2)_\kappa$$

Renormalized condensate

$$\langle\bar{\psi}\psi\rangle_{ren} = \langle\bar{\psi}\psi\rangle(m_l) - \frac{m_l}{m_h}\langle\bar{\psi}\psi\rangle(m_h)$$

Chiral Condensate

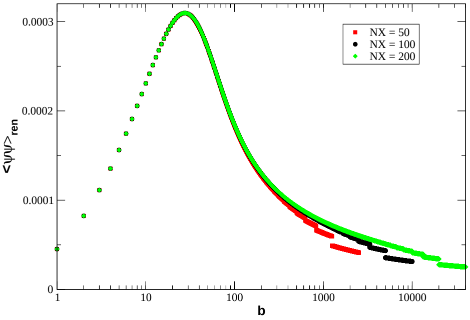
$$\langle\bar{\psi}\psi\rangle(b) \sim b \sum_\kappa \frac{B_b(\bar{p})}{B_b(\bar{p})^2 + ((A_{2b}(\bar{p})\,p_2)^2}$$

Quantized B-Field

$$|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$$
$$b \in [0, L_x \cdot L_y]$$

Typical Tori

- Box Length: 6 fm



⇒ **Calculate dual condensate in** b, s

Preliminary: The dressed Wilson Loop

Dual Condensate

$$\langle\widetilde{\bar\psi\psi}\rangle_{ren}(s) = \frac{1}{S_{\mu\nu}} \sum_{b=0}^{b_{max}} e^{-2i\pi bs/N_{\mu\nu}} \langle\bar\psi\psi\rangle_{ren}(b)$$

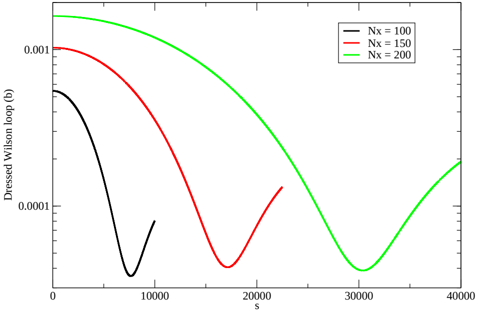
B-field

$$|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$$
$$b = 1, 2, \ldots$$

Area

pierced by magn. flux

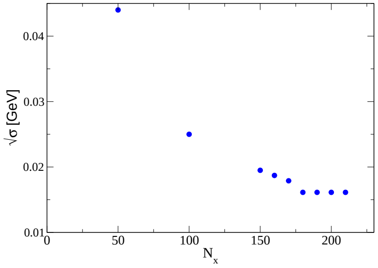
$$S_{(xy)} = s\frac{L_x L_y}{N_{(xy)}}$$
$$s = 1, 2, \ldots$$



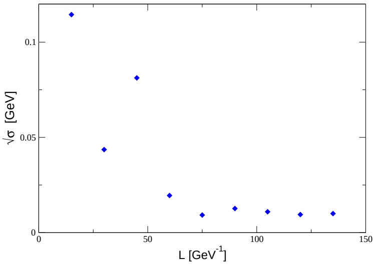
$\Rightarrow$ **Extract string tension**

Preliminary: String tension

Fixed volume calculations



Fixed cutoff calculations



In the limit $m \to \infty$

$$\langle \widetilde{\bar{\psi}\psi} \rangle_{ren}(s) \sim \exp(-\sigma_s A)$$

Findings:

$\rightarrow$ find area-law

$\rightarrow$ find limit for string tension for large enough cutoff and volume

SUMMARY

- Concept of constant external magnetic fields
- Effects on particles' propagators and momenta
- Chiral condensate and dressed Wilson loop

Still many 'technical' open questions...

- understand sensitivity of condensate to dealing with Landau Levels in different volumina
- compare finite volume to continuum studies

  → Niklas Müller
- include higher tensor structures in propagator
- overcome bare vertex approximation

OUTLOOK

What is left to do ...

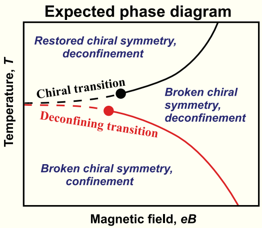
- Charge dependence studies

  $\longrightarrow$ D'Elia, Negro, PRD 83 (2011)

- Finite temperatures

  $\longrightarrow$ Bali et al. JHEP 1202 (2012), Bali et al. arXiv:1206.4205, Fukushima, Pawlowski arXiv:1203.4330

- Finite chemical potential



Mizher, Chernodub, Fraga: PRD 82,105016 (2010)

Thank you for your attention!

Questions??

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NPQCD

Supplement

- periodic boundary conditions for bosons
- antiperiodic boundary conditions for fermions

$\Rightarrow$ discretized momentum space

$$\int \frac{d^2q}{(2\pi)^2} \cdots \to \frac{1}{L^2} \sum_{\text{all momenta}} \cdots$$

- the temperature T
- the number of lattice points in momentum space in time direction: NT
- the number of lattice points in momentum space in space direction: NX