Confinement/deconfinement and χsb

in gauge theory/string theory correspondence

and some applications

Alex Buchel

(Perimeter Institute & University of Western Ontario)

$\Longrightarrow$ Confinement and χsb in gauge theories are strongly coupled phenomena which are difficult to study from first principles[a]

$\Longrightarrow$ I will use gauge theory/string theory correspondence of Maldacena, where the strongly coupled dynamics of certain gauge theories is mapped to essentially classical dynamics of higher dimensional gravitational theories

$\Longrightarrow$ I consider a specific string theory example of gauge gravity correspondence, rather than a phenomenological model of thereof.

---

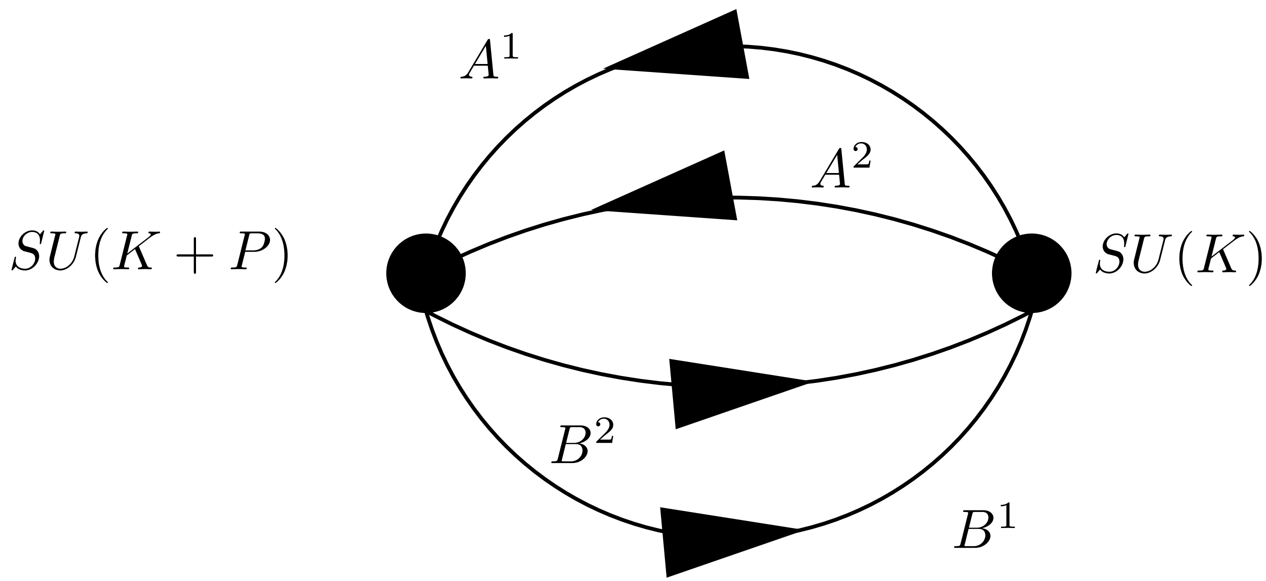
[a]For dynamical questions.

Outline of the talk:

- The (cascading) gauge theory: $\mathcal{N}=1$ supersymmetric $SU(K+P)\times SU(K)$ +bi-fundamental matter
- Cascading gauge theory plasma:
  - first-order confinement/deconfinement transition
  - chiral symmetry breaking instabilities
  - critical point
- Transport coefficients: shear and bulk viscosities
- Application: cavitation in the vicinity of the phase transition

Klebanov-Strassler model (a cascading gauge theory)

- Consider a following quiver gauge theory:



The gauge group and the matter content:

$$
\begin{array}{cccc}
\{g_1, g_2\}: & SU(K+P) & \times & SU(K) \\
A_i: & (K+P) & \times & \overline{K} \\
B_i: & \overline{(K+P)} & \times & K
\end{array}
$$

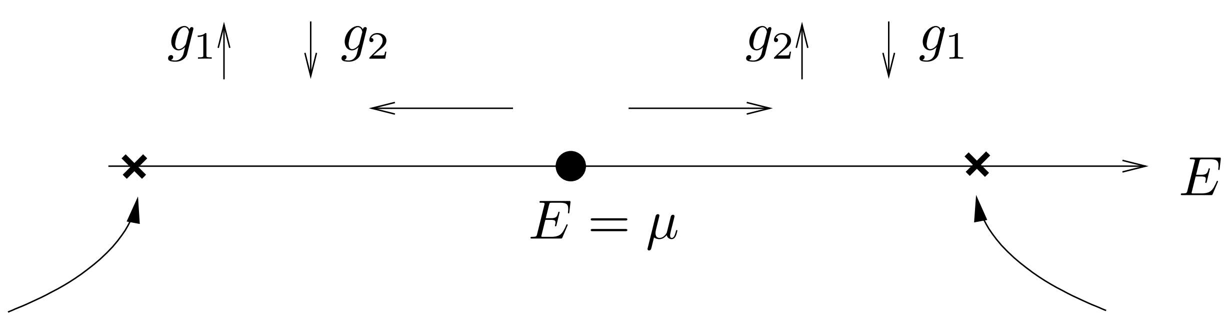
Compute β-functions corresponding to RG running of $\{g_1, g_2\}$ gauge couplings:

$$\beta_1 \sim 3(K+P) - 2K(1-\gamma_{A^i} - \gamma_{B^j}) = 3P + \mathcal{O}(P^3/K^2)$$
$$\beta_2 \sim 3K - 2(K+P)(1-\gamma_{A^i} - \gamma_{B^j}) = -3P + \mathcal{O}(P^3/K^2)$$

From the β-functions:

$$\frac{4\pi}{g_1^2(\mu)} + \frac{4\pi}{g_2^2(\mu)} = \text{const}\,, \qquad \frac{4\pi}{g_1^2(\mu)} - \frac{4\pi}{g_2^2(\mu)} \sim P\ \ln\frac{\mu}{\Lambda}$$

where Λ is the strong coupling scale of the theory



$\frac{1}{g_1^2} = 0\,,\ SU(K+P)$ strongly coupled

$\frac{1}{g_2^2} = 0\,,\ SU(K)$ strongly coupled

What is the effective description of the theory past the Landau poles?

What is the effective description of the theory past the Landau poles?

$\Longrightarrow$ Using Seiberg duality for $\mathcal{N}=1$ SUSY gauge theory, the extension of the model past the Landau poles results in self-similarity cascade (Klebanov and Strassler):

$$K \to K(\mu) \sim 2P^2 \ \ln \frac{\mu}{\Lambda}$$

$$\text{UV}: \qquad K \to K+P\,, \qquad\qquad \text{IR}: \qquad K \to K-P$$

$\Longrightarrow$ If K is a multiple of P, the theory in the deep infrared is $\mathcal{N}=1$ $SU(P)$ SYM[a]; this theory:

- confines
- spontaneously breaks $U(1)_R$ chiral symmetry

$\Longrightarrow$ Can compute: gap in spectrum, gaugino condensate, tension of domain walls between P vacua...

[a] YM + massless Weyl fermions in adjoint representation

Klebanov-Strassler model (a supergravity story)

It is possible to derive an effective 5d action from string theory dual to KS model:

$$S_5 = \frac{108}{16\pi G_5}\int_{\mathcal{M}_5} d^5\xi\sqrt{-g}\;\Omega_1\Omega_2^2\Omega_3^2\;\Bigg\{R_{10} - \frac{1}{2}\left(\nabla\Phi\right)^2 - \frac{1}{2}e^{-\Phi}\left(\frac{(h_1-h_3)^2}{2\Omega_1^2\Omega_2^2\Omega_3^2} + \frac{1}{\Omega_3^4}\left(\nabla h_1\right)^2 + \frac{1}{\Omega_2^4}\left(\nabla h_3\right)^2\right) - \frac{1}{2}e^{\Phi}\left(\frac{2}{\Omega_2^2\Omega_3^2}\left(\nabla h_2\right)^2 + \frac{1}{\Omega_1^2\Omega_2^4}\left(h_2 - \frac{P}{9}\right)^2 + \frac{1}{\Omega_1^2\Omega_3^4}h_2^2\right) - \frac{1}{2\Omega_1^2\Omega_2^4\Omega_3^4}\left(4\Omega_0 + h_2\left(h_3-h_1\right) + \frac{1}{9}Ph_1\right)^2\Bigg\},$$

where:

$$R_{10} = R_5 + \left(\frac{1}{2\Omega_1^2} + \frac{2}{\Omega_2^2} + \frac{2}{\Omega_3^2} - \frac{\Omega_2^2}{4\Omega_1^2\Omega_3^2} - \frac{\Omega_3^2}{4\Omega_1^2\Omega_2^2} - \frac{\Omega_1^2}{\Omega_2^2\Omega_3^2}\right) - 2\Box\ln\left(\Omega_1\Omega_2^2\Omega_3^2\right) - \Big\{\left(\nabla\ln\Omega_1\right)^2 + 2\left(\nabla\ln\Omega_2\right)^2 + 2\left(\nabla\ln\Omega_3\right)^2 + \left(\nabla\ln\left(\Omega_1\Omega_2^2\Omega_3^2\right)\right)^2\Big\},$$

$\Longrightarrow$ Euclidean gravitational solutions in this 5-dim theory of gravity coupled to various scalars fields with compactified time-direction describe **confined equilibrium states** of the cascading plasma. As usual,

$$t_E \sim t_E + \frac{1}{T_{plasma}}$$

$\Longrightarrow$ Black holes with translationary invariant horizon describes **deconfined equilibrium states** of the cascading gauge theory plasma, with:

$$T_{plasma} \qquad \Longleftrightarrow \qquad T_{Hawking}$$

$$s_{plasma} \qquad \Longleftrightarrow \qquad s_{Bekenstein-Hawking}$$

$$\mathcal{E}_{plasma} \qquad \Longleftrightarrow \qquad \text{Black hole mass density}$$

$$\mathcal{F}_{plasma} \qquad \Longleftrightarrow \qquad \text{Black hole gravitational action}$$

$\Longrightarrow$ Spectrum of physical excitation in deconfined gauge theory plasma corresponds to spectrum of black-hole quasinormal modes

$\Longrightarrow$ Comments on confinement/deconfinement transition in $N \to \infty$ gauge theories:

- In the deconfined phase the free energy density and the entropy density

$$\mathcal{F}_{deconfined} \propto \mathcal{O}\left(N^2\right) , \qquad s_{deconfined} \propto \mathcal{O}\left(N^2\right)$$

- In the confined phase the free energy density

$$\mathcal{F}_{confined} \propto \mathcal{O}\left(N^0\right) , \qquad s_{confined} \propto \mathcal{O}\left(N^0\right)$$

- Since

$$\lim_{N\to\infty} \left.\frac{\mathcal{F}}{N^2}\right|_{deconfined} \neq 0 \,, \qquad \text{or} \qquad \lim_{N\to\infty} \left.\frac{\mathcal{F}}{sT}\right|_{deconfined} \neq 0$$

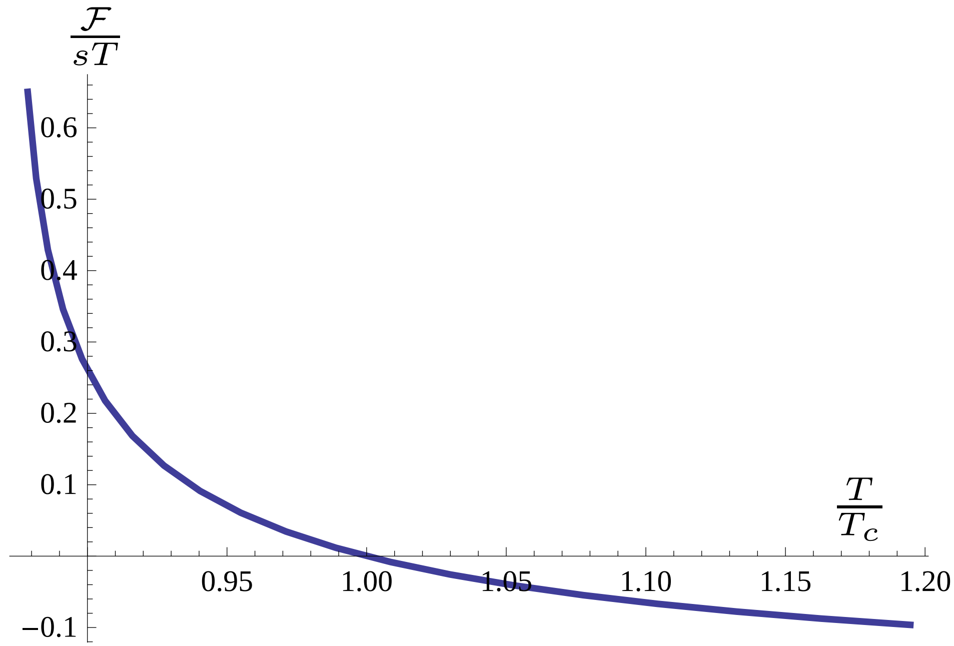
  and

$$\lim_{N\to\infty} \left.\frac{\mathcal{F}}{N^2}\right|_{confined} = 0 \,,$$

  the confined phase of plasma is thermodynamically favourable once

$$\frac{\mathcal{F}}{sT} > 0 \,, \qquad \text{provided} \qquad s \sim \mathcal{O}\left(N^2\right)$$

$\Longrightarrow$ Confinement/deconfinement phase transition in cascading plasma



- T_C is the critical temperature

$$T_c = 0.6141111(3)\Lambda$$

- The phase transition is of the first-order, between the **deconfined chirally symmetric** phase and the **confined phase with broken chiral symmetry**

$\Longrightarrow$ Is the deconfined chirally symmetric phase of the cascading plasma perturbatively stable?

$\Longrightarrow$ To answer this question:

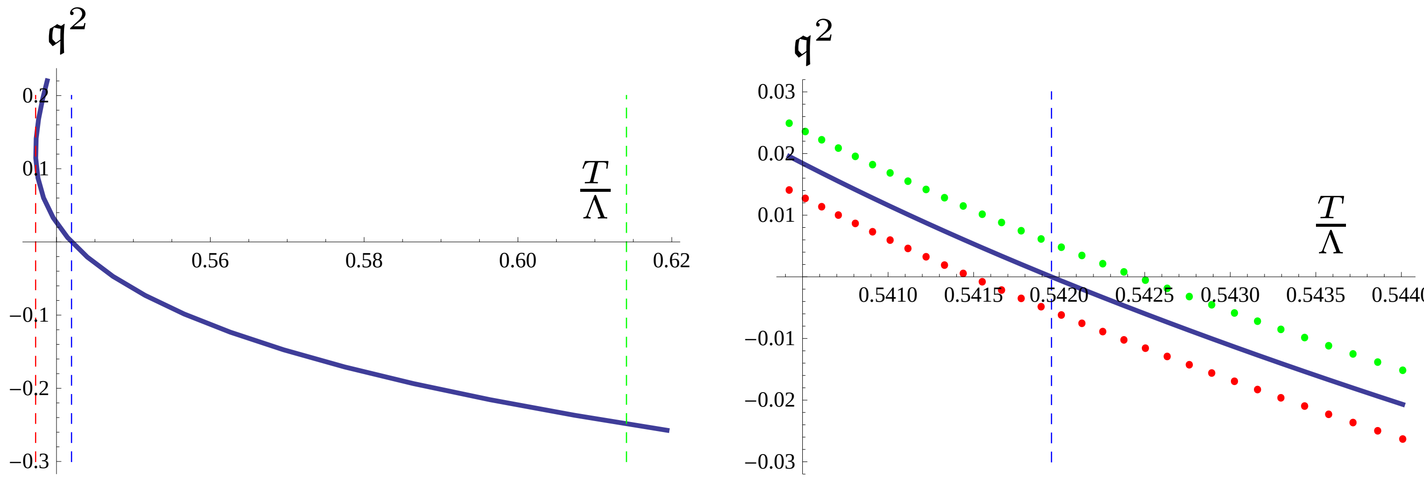
- we look at linearized χsb fluctuations $\propto e^{-i\omega t+i\vec{k}\cdot\vec{x}}$ about chirally symmetric thermal state. Suppose that these fluctuations have a dispersion relation

$$\mathfrak{w} = \mathfrak{w}(\mathfrak{q}^2)\,, \qquad \mathfrak{w} \equiv \frac{\omega}{2\pi T}\,, \qquad \mathfrak{q} = \frac{|\vec{k}|}{2\pi T}$$

- These χsb fluctuations are unstable, provided

$$\mathrm{Im}\,(\mathfrak{w}) > 0 \qquad \text{for} \qquad \mathrm{Im}\,(\mathfrak{q}) = 0$$

- Using the holographic duality, one can precisely map these fluctuations into quasinormal modes of the 5d black hole solution, describing the deconfined chirally symmetric equilibrium phase of the cascading plasma



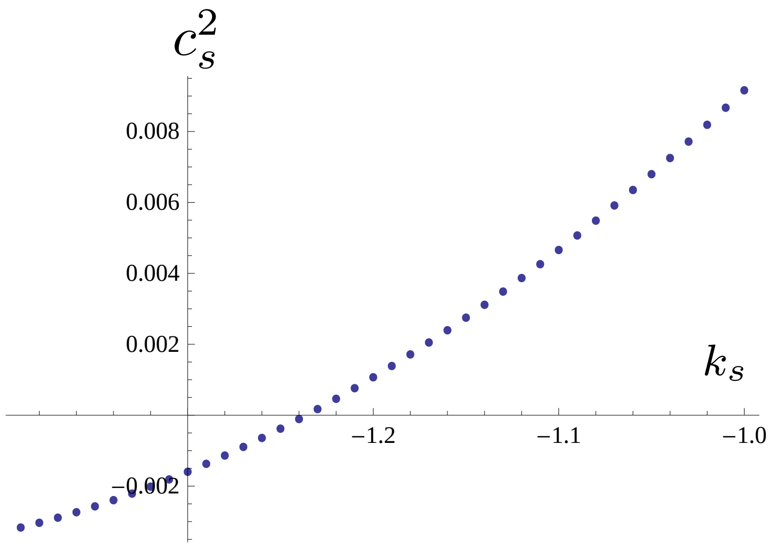
$\Longrightarrow$ The left plot represents the dispersion relation of the chiral fluctuations at the threshold of instability, *i.e.*, , with $\mathfrak{w}(\mathfrak{q}^2) = 0$. The blue dashed vertical lines represent the onset of instability: $T = T_{\chi \rm SB}$, such that $(i\mathfrak{w} = 0, \mathfrak{q}^2 = 0)$. The vertical green dashed line represents the confinement/deconfinement critical temperature T_c,

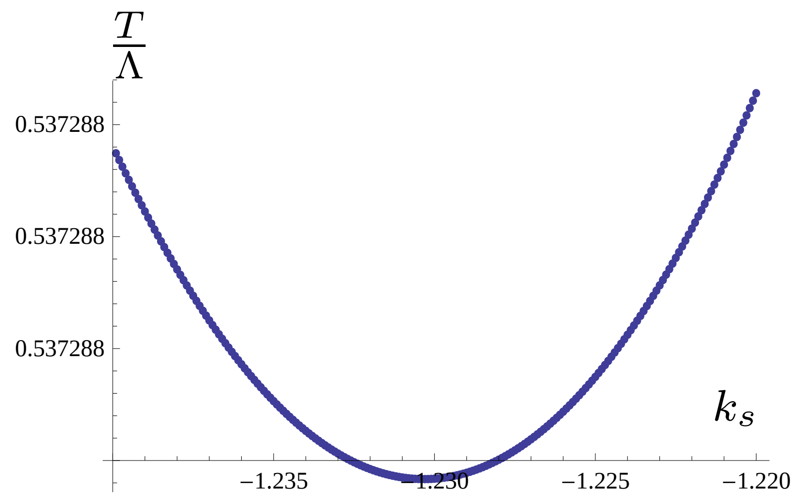
$$T_{\chi \rm SB} = 0.882503(0) T_c$$

$\Longrightarrow$ On the right plot: the green dots indicate quasinormal modes with $(\mathfrak{w} = -i0.01, \mathfrak{q}^2)$ as a function of $\frac{T}{\Lambda}$ — these fluctuations are stable. The red dots indicate quasinormal modes with $(\mathfrak{w} = i0.01, \mathfrak{q}^2)$ as a function of $\frac{T}{\Lambda}$ — these fluctuations are genuine tachyons whenever $\mathfrak{q}^2 > 0$.

Critical end point of the chirally symmetric deconfined phase

$\Longrightarrow$ Consider sound waves in deconfined phase:





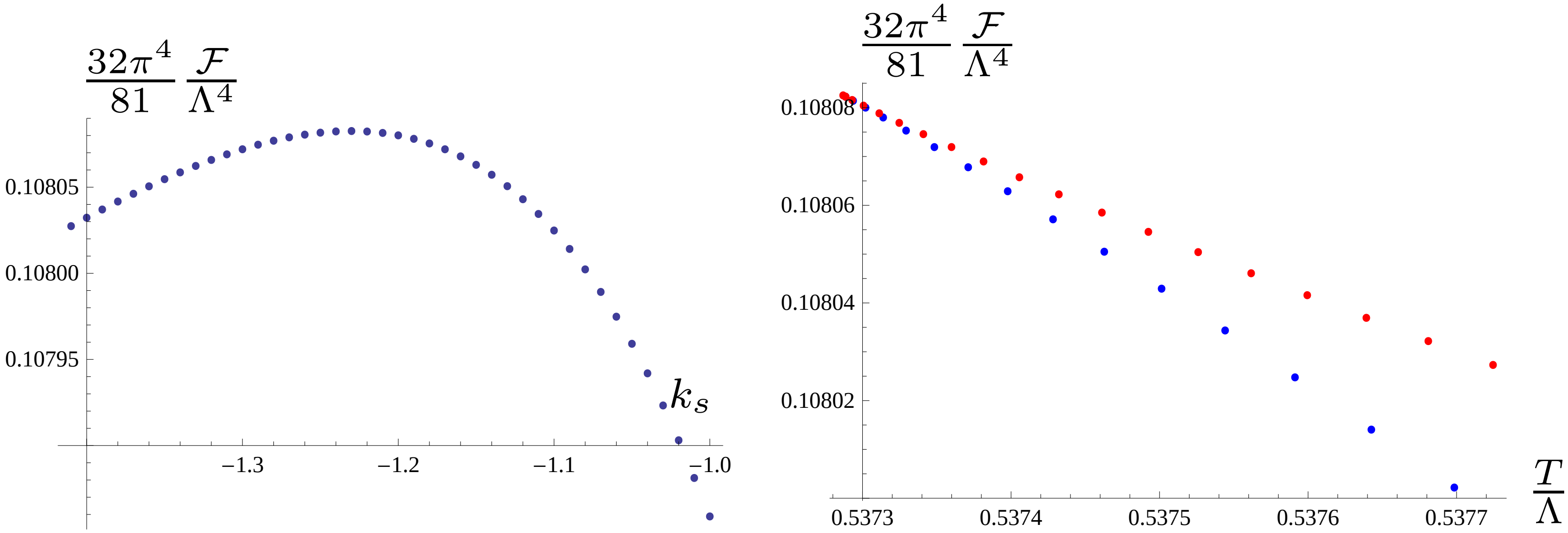
$\Longrightarrow$ There is a minimal temperature of the deconfined phase:

$$T_{end} = 0.8749(0)T_c$$

Vanishing of the speed of sound as above implies that the specific heat c_V of the cascading plasma diverges near the end point with the critical exponent $\alpha = \frac{1}{2}$:

$$c_V = s/c_s^2 \propto |1 - T_{end}/T|^{-1/2}$$

$\Longrightarrow$ At the critical end point the free energy develops a cusp:



$\Longrightarrow$ We can also compute remaining critical exponents:

$$(\alpha, \beta, \gamma, \delta, \nu, \eta) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{4}, 0\right)$$

$\Longrightarrow$ Why do we care about critical point?

$\Longrightarrow$ Why do we care about critical point?

$\Longrightarrow$ Because we can also compute transport coefficients —
the shear and bulk viscosity; and thus can answer the question whether bulk viscosity diverges at criticality
(at least for the mean-field critical point under consideration)

Hydrodynamic fluctuations in cascading plasma

Hydrodynamics is a universal framework to describe strongly coupled systems at energy scales much lower than their characteristic microscopic scales (masses, temperature, etc). The basic hydrodynamic equation is that of the conservation of the stress-energy tensor

$$\nabla_\mu T^{\mu\nu} = 0\,, \qquad T^{\mu\nu} = T^{\mu\nu}_{ideal} + \Pi^{\mu\nu}$$

$$T^{\mu\nu}_{ideal} = \mathcal{E}\ u^\mu u^\nu + \mathcal{P}\ \Delta^{\mu\nu}\,, \qquad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$\Pi^{\mu\nu} = -\eta\ \sigma^{\mu\nu} - \zeta\ (\nabla_\alpha u^\alpha)\Delta^{\mu\nu}$$

where the shear tensor is

$$\sigma^{\mu\nu} = \left(\Delta^{\mu\lambda}\nabla_\lambda u^\nu + \Delta^{\nu\lambda}\nabla_\lambda u^\mu\right) - \frac{2}{3}(\nabla_\alpha u^\alpha)\Delta^{\mu\nu}$$

and η and ζ are the shear and bulk viscosities

$\Longrightarrow$ There are 2 type of on-shell fluctuations of the stress-energy tensor, shear modes and the sound modes. The have the following dispersion relation:

- shear fluctuations

$$\mathfrak{w} = -i\ 2\pi\frac{\eta}{s}\ \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^4)$$

- sound fluctuations

$$\mathfrak{w} = \pm\ c_s\ \mathfrak{q} - i\ \frac{4\pi}{3}\ \frac{\eta}{s}\ \left(1 + \frac{3}{4}\ \frac{\zeta}{\eta}\right) + \mathcal{O}(\mathfrak{q}^3)$$

where as before

$$\mathfrak{w} \equiv \frac{\omega}{2\pi T}\,, \qquad \mathfrak{q} = \frac{|\vec{k}|}{2\pi T}$$

$\Longrightarrow$ In holography is it straightforward to directly compute spectrum of hydrodynamic fluctuations

Cascading plasma transport

$\Longrightarrow$ From the dispersion relation of the hydro fluctuations in deconfined phase we can read off:

- shear viscosity ratio

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

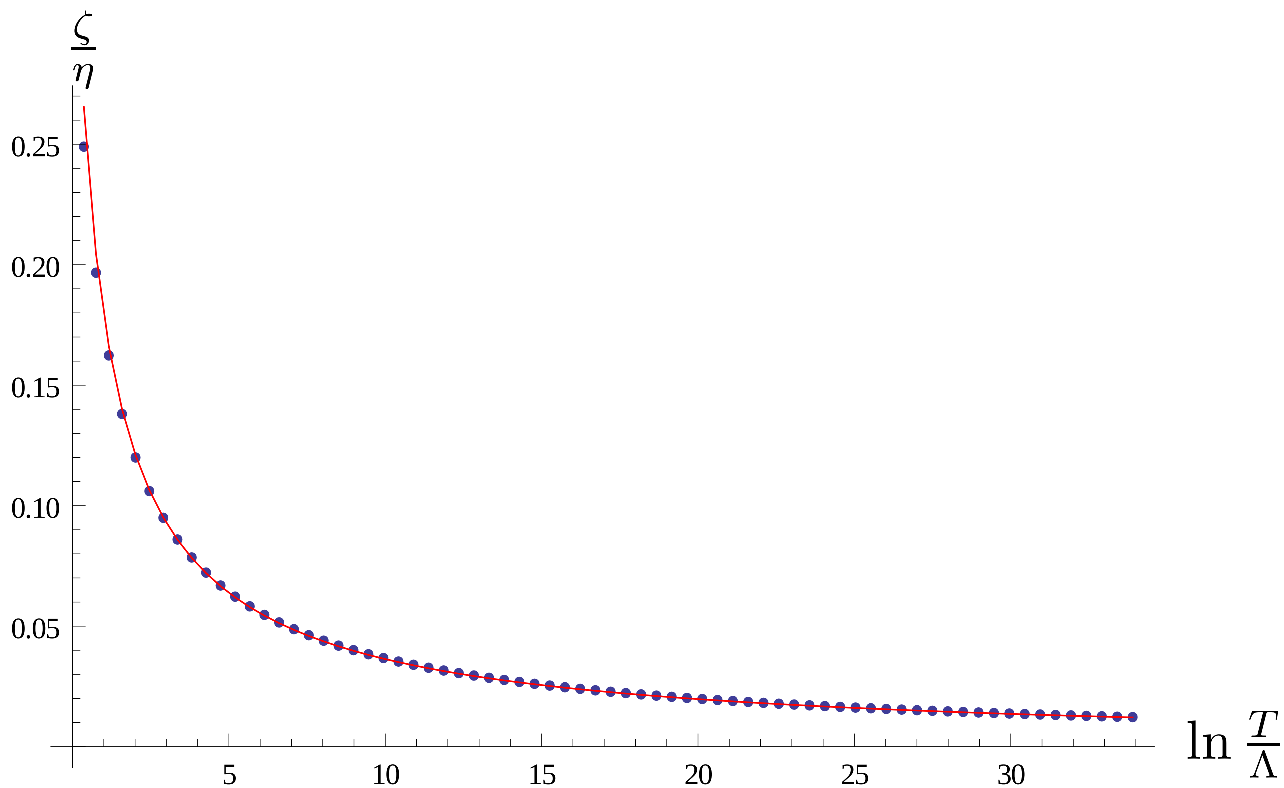
independent of the temperature

- bulk viscosity ratio

$$\frac{\zeta}{\eta}$$

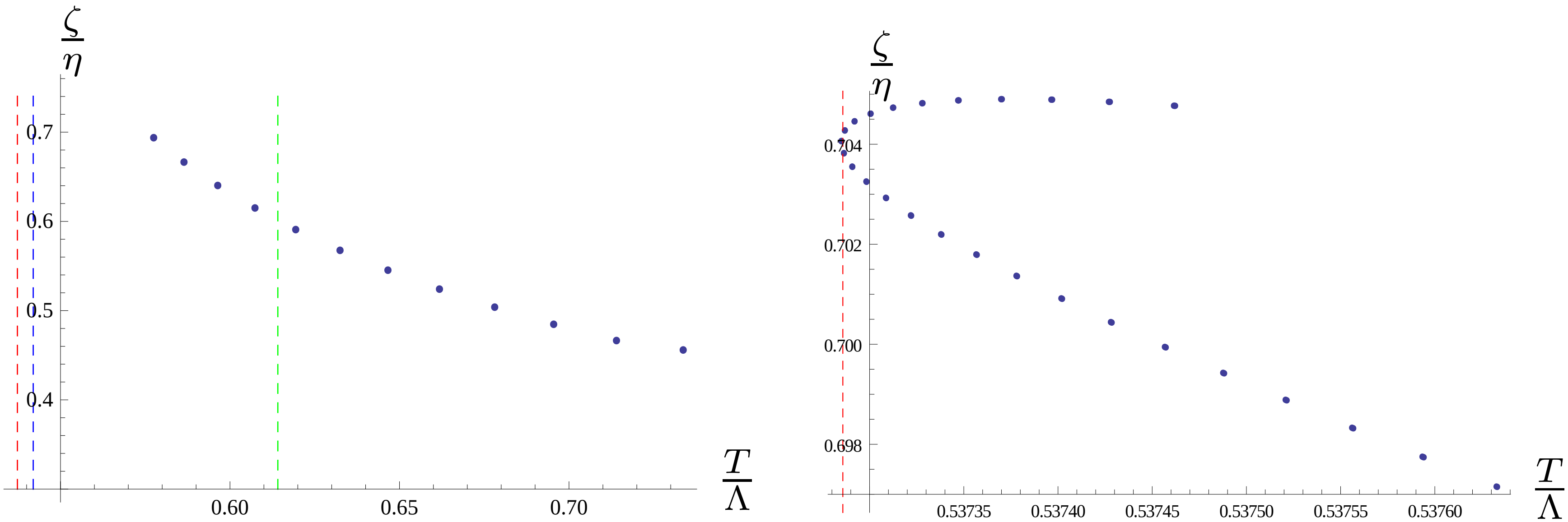
is temperature dependent.

$\Longrightarrow$ I will present now results for high temperature $\ln \frac{T}{\Lambda} \gg 1$, and in the IR (close to various transitions)



Ratio of bulk ζ to shear η viscosities in cascading plasma at high temperature. The solid red line is an analytic high-T approximation

$$\frac{\zeta}{\eta} = \frac{4}{9 \ln \frac{T}{\Lambda}} + \mathcal{O}\left(\ln^{-2} \frac{T}{\Lambda}\right)$$



- Green: confinement/deconfinement, $\frac{\zeta}{\eta} = 0.5(9)$
- Blue: chiral symmetry breaking instability
- Tricritical end point (maximum ratio): $\frac{\zeta}{\eta} = 0.7(0)$

Can cavitation affect the confinement/deconfinement phase transition?

(motivated by K.Rajagopal and N.Tripuraneni, JHEP 1003:018,2010)

$\Longrightarrow$ Notice that viscous effects in fluid tend to reduce the pressure:

- isotropic expansion:

$$\mathcal{P}^{eff}|_{isotropic} = \mathcal{P} - \zeta \ (\nabla_\alpha u^\alpha) = \mathcal{P} - \zeta \ 3\frac{\dot{a}}{a} < \mathcal{P}$$

where $a = a(\tau)$ is the spatial expansion factor;

- boost invariant expansion:

$$\mathcal{P}_\perp^{eff} = \mathcal{P} + \frac{2\eta - 3\zeta}{3\tau}\,, \qquad \mathcal{P}_\xi^{eff} = \mathcal{P} - \frac{4\eta + 3\zeta}{3\tau}\,,$$

$$\langle \mathcal{P}^{eff} \rangle = \frac{2}{3}\ \mathcal{P}_\perp^{eff} + \frac{1}{3}\mathcal{P}_\xi^{eff} = \mathcal{P} - \zeta \ (\nabla_\alpha u^\alpha) = \mathcal{P} - \frac{\zeta}{\tau}$$

where τ is the proper time in boost invariant expansion

Consider now a system which in thermal equilibrium can exist in one of the two phases $_A$ or $_B$. A first-order phase transition between these two phases implies the existence of a critical temperature T_c, such that $\mathcal{P}_A > \mathcal{P}_B$ for $T > T_c$ and $\mathcal{P}_A < \mathcal{P}_B$ otherwise.

$\Longrightarrow$ If the system flows, the relevant pressure determining the stability of a phase is the effective one:

$$\mathcal{P}^{eff}_{A/B} = \mathcal{P}_{A/B} - \zeta_{A/B}\,(\nabla_\alpha u^\alpha)\,.$$

Close to T_c,

$$\mathcal{P}_{A/B} = \mathcal{P}_c + s_{A/B}\,(T - T_c) + \mathcal{O}\left((T - T_c)^2\right)\,,$$

where $s_{A/B}$ are entropy densities of the corresponding phases.

$\Longrightarrow$ the critical temperature get shifted:

$$\frac{|\delta T_c|}{T_c} \sim \frac{|\zeta_A - \zeta_B|}{|s_A - s_B|}\,\frac{|\nabla_\alpha u^\alpha|}{T_c} \lesssim \frac{|\zeta_A - \zeta_B|}{|s_A - s_B|}\,,$$

where the upper bound comes from applicability of first-order hydro: $|\nabla_\mu u^\nu| < T$

In cascading plasma:

- $_A$ — deconfined phase; $_B$ confined phase
- $\zeta_A \gg \zeta_B$, $s_A \gg s_B$ (large N_c suppression)

$$\frac{|\delta T_c|}{T_c} \lesssim \frac{\zeta_A}{s_A} < 0.04(8)$$

$\Longrightarrow$ It is reasonable to expect that the results is universal as it reflects the fact that large-N_c phase transitions are typically strong (as opposite to weak) first-order, and that the bulk viscosity at the critical point remains finite.

$\Longrightarrow$ Some phenomenological models suggest that QCD bulk viscosity might diverge at the critical point of the $T - \mu_B$ phase diagram. Since QCD critical point separates the line of the first-order transitions (at large chemical potential) from the crossovers (at low chemical potential) both of these effects tend to increase $|\delta T_c|/T_c$.

Summary

- I considered a cascading gauge theory, which is in the same universality class in the IR as $\mathcal{N}=1$ $SU(M)$ SYM, in the planar limit, and for (infinitely) large 't Hooft coupling.
- I argued that this theory undergoes a first order confinement phase transition (with spontaneous broken chiral symmetry) at T_c
- Below T_c, the metastable chirally symmetric deconfined phase in this theory becomes perturbatively unstable at

$$T_{\chi\text{SB}} = 0.882503(0) T_c$$

- Deconfined phase 'ends' at the mean-field critical point

$$(\alpha, \beta, \gamma, \delta, \nu, \eta) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{4}, 0\right) \text{ at } T_{end} = 0.8749(0)$$

- Computed transport of the theory in deconfined phase:

$$\frac{\eta}{s} = \frac{1}{4\pi}, \qquad \max\left(\frac{\zeta}{\eta}\right) = 0.704(1)$$

- Cavitation is not important

Future directions:

- study cascading gauge theory at finite chemical potential
- compute nonlocal observables (Wilson, t' Hooft loop tension)
- study dynamical thermalization