Landau-gauge propagators from lattice QCD at finite temperature

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- Why studying gauge-fixed n-point functions on the lattice?

Lattice results

- $N_f = 0,\, 2$: Landau gauge gluon & ghost propagators

Summary

Motivation

Lattice QCD

- successful in addressing problems of QCD at zero and finite temperature

Examples

- T=0: today's calculations quite close to physical point
- T>0: progress towards understanding of QCD phase diagram

Motivation

Lattice QCD

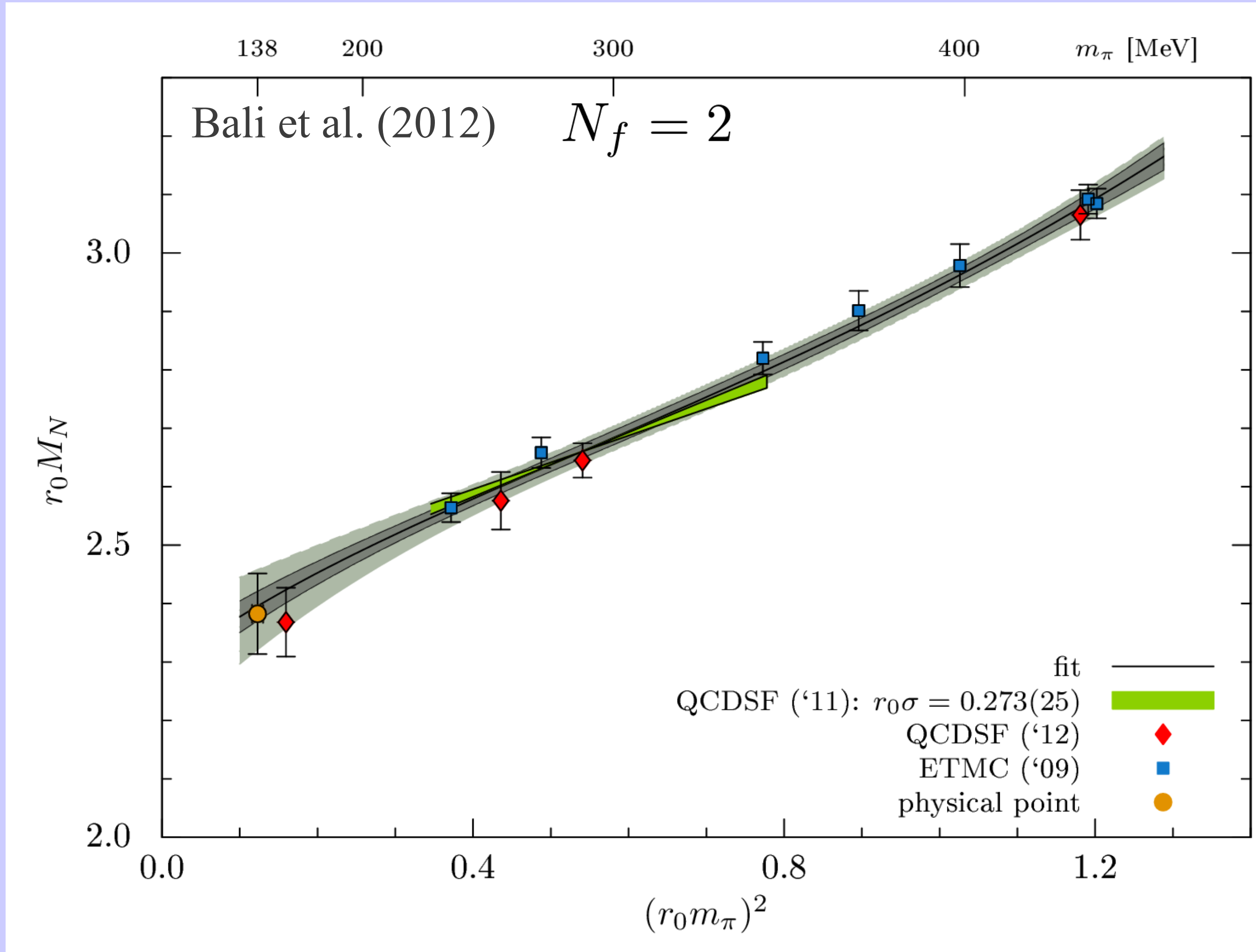
– successful ir

Examples

– T=0: today's

– T>0: progres

Benchmark I: Nucleon mass vs. pion mass squared



Motivation

Lattice QCD

– successful ir

Examples

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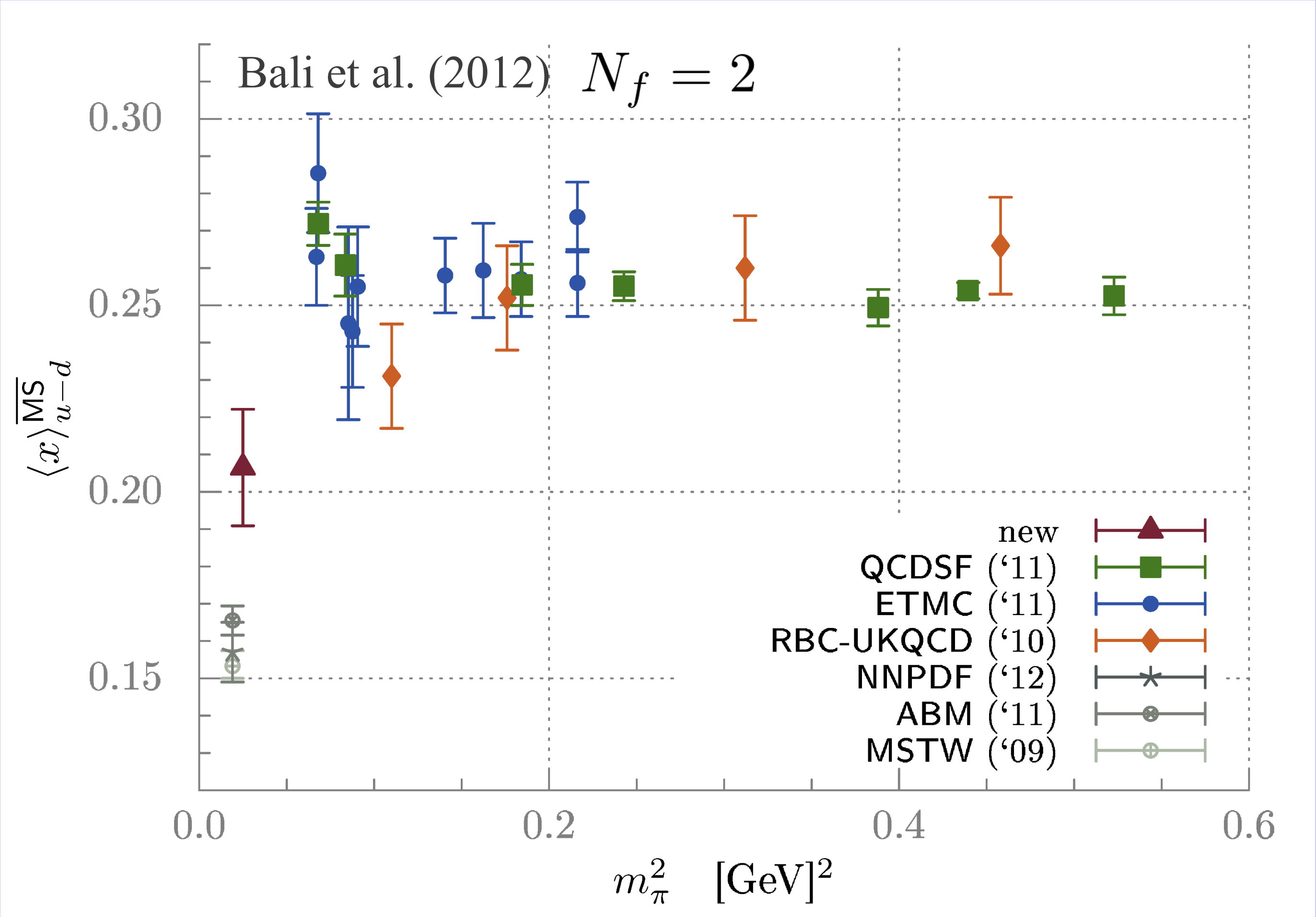
– T>0: progres

Benchmark II: 1.moment of Nucleon PDF vs. pion mass squared



Motivation

Lattice QCD

- successful in addressing problems of QCD at zero and finite temperature

Examples

- T=0: today's calculations quite close to physical point
- T>0: progress towards understanding of QCD phase diagram

Current problems

- T=0: continuum limit (freezing of top. charge)
- T=0: simulations numerically expensive the closer to physical point
- T>0: finite chemical potential ↔ sign-problem
- ….

Motivation

Functional methods

- Dyson-Schwinger equations (DSEs) + Bethe-Salpether/Faddeev equations
- Functional renormalization Group equations (FRGE)

Complementary approach to lattice QCD

- Hadron physics (form factors, masses, decay constants, g_μ-2,...)
- QCD phase diagram, ...

Motivation

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- QCD phase diagram, ...

Advantages and disadvantages

- no sign-problem, no continuum + infinite-volume limit
- all momenta accessible, easy change of quark mass
- less rigorous than lattice QCD due to truncations

→ conservation laws, symmetries and lattice results help to improve upon these truncations

DSE & FRGE ↔ Lattice

T=0: Strong coupling

Von Smekal, Hauck, Alkofer (1997)

$$\alpha_s(p^2) = \frac{g_0^2}{4\pi} Z(p^2) \cdot J^2(p^2)$$

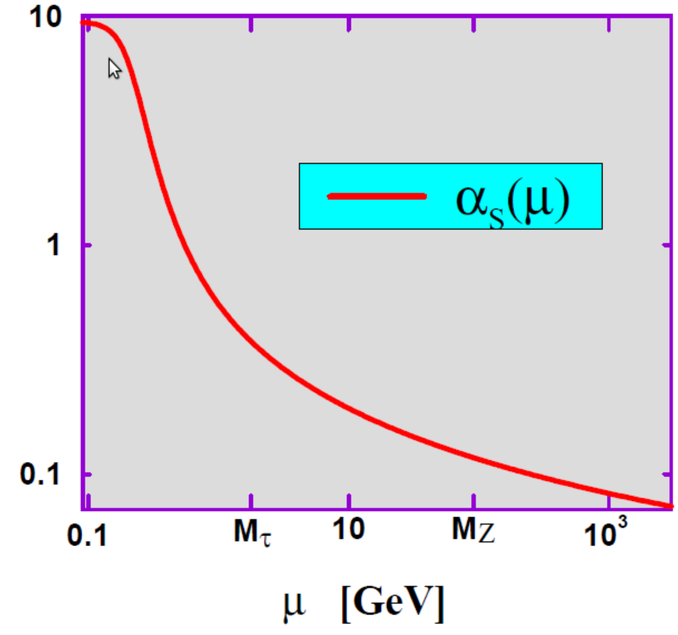
- Gluon propagator (Landau gauge)

$$D(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

- Ghost propagator

$$G(p^2) = -\frac{J(p^2)}{p^2}$$

DSE:
Von Smekal, Hauck, Alkofer (1997)]

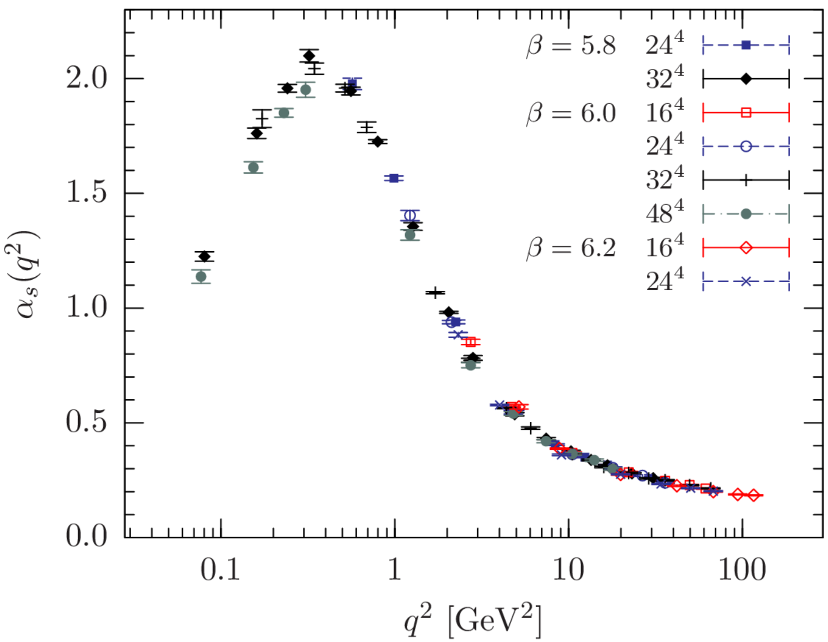


DSE & FRGE ↔ Lattice

T=0: Strong coupling

$$\alpha_s(p^2) = \frac{g_0^2}{4\pi} Z(p^2) \cdot J^2(p^2)$$

Lattice:
A.S., Ilgenfritz, Müller-Preussker, Schiller. (2005)



DSE:
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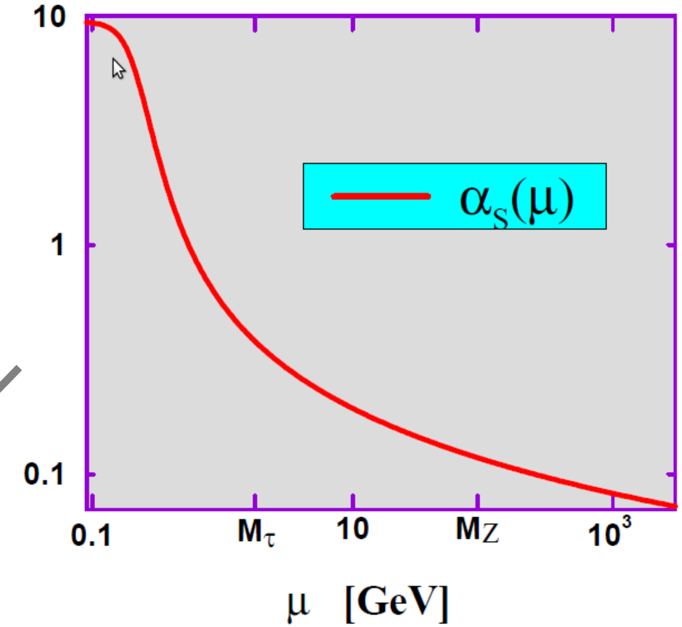

DSE & FRGE ↔ Lattice

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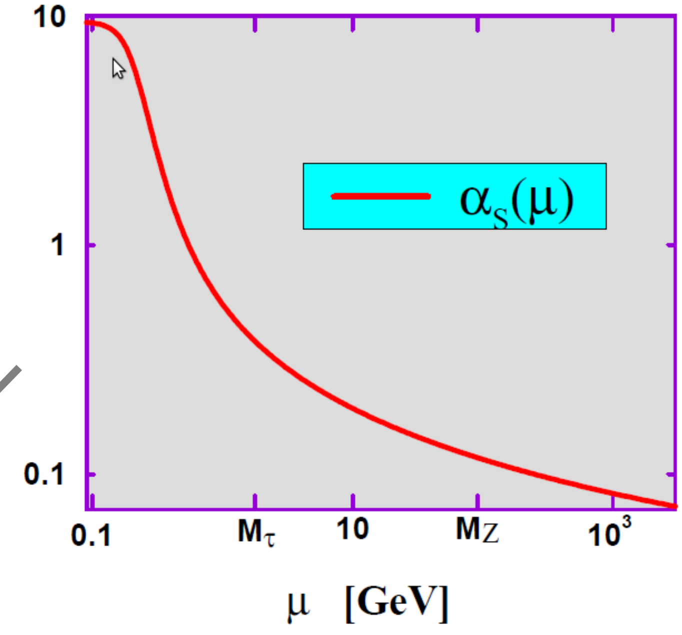
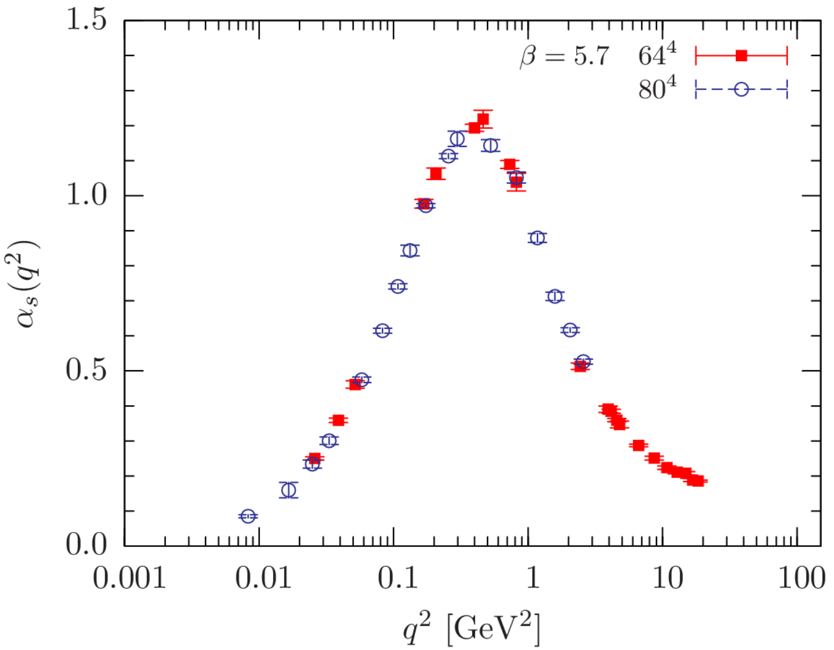


Lattice:
Bogolubsky, Ilgenfritz, Müller-Preussker, *A.S.* (2009)



DSE & FRGE ↔ Lattice

T=0: Strong coupling

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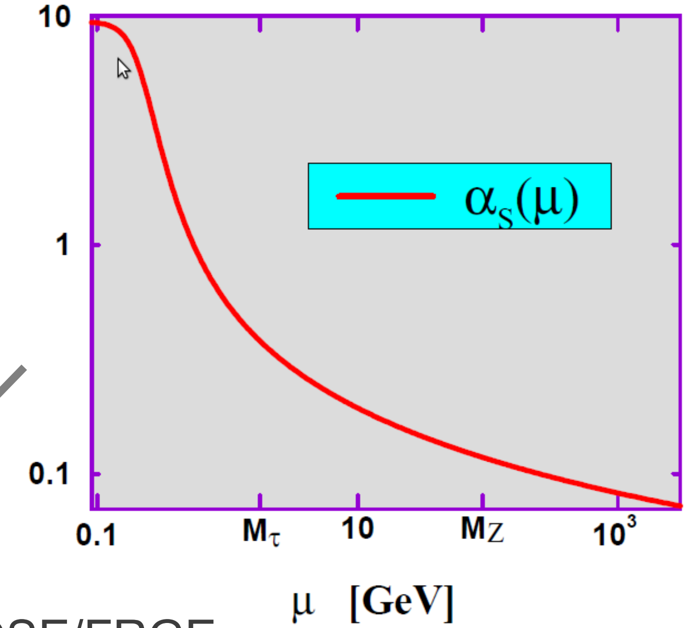

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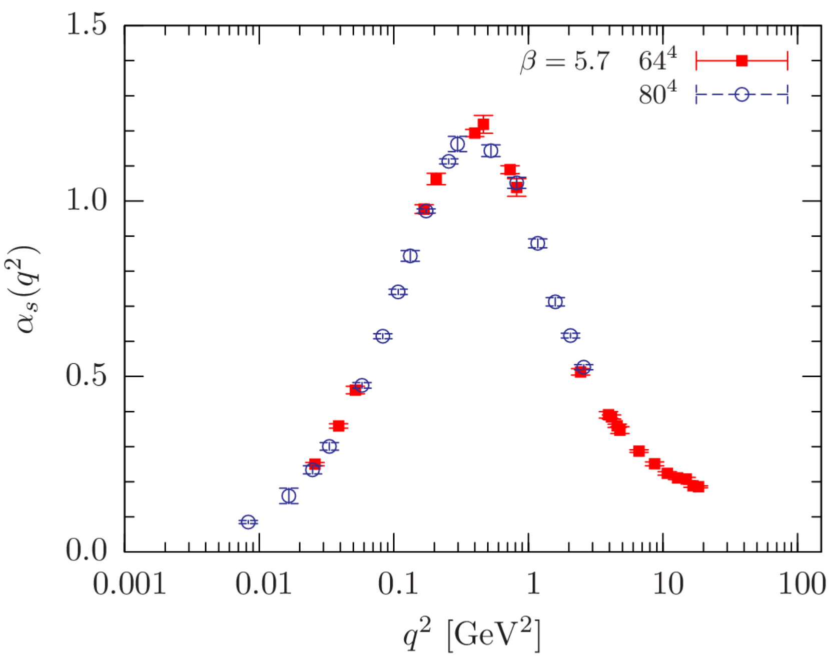
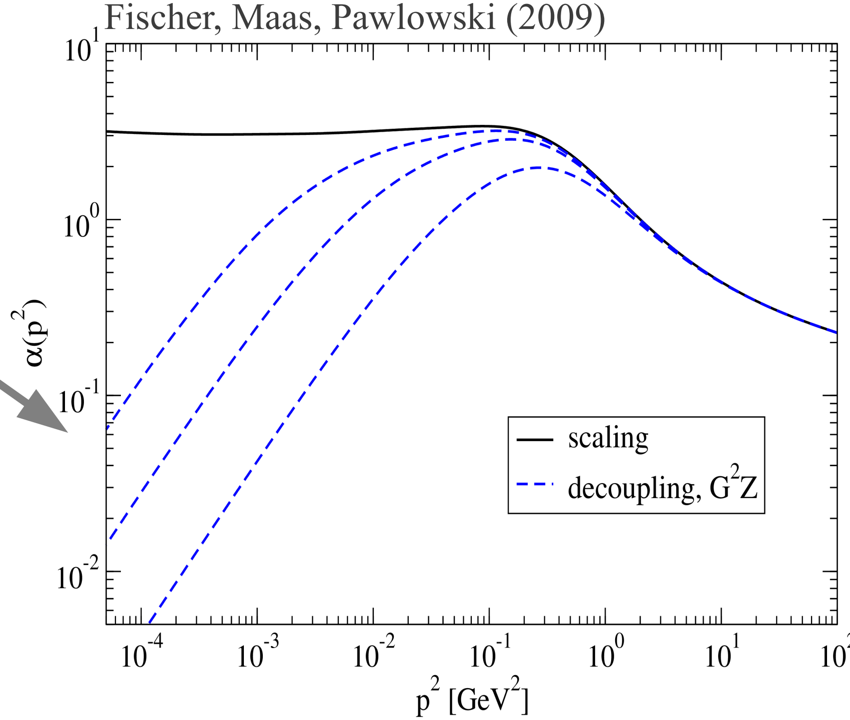


DSE/FRGE:
Fischer, Maas, Pawlowski (2009)



DSE & FRGE ↔ Lattice

T=0: Today

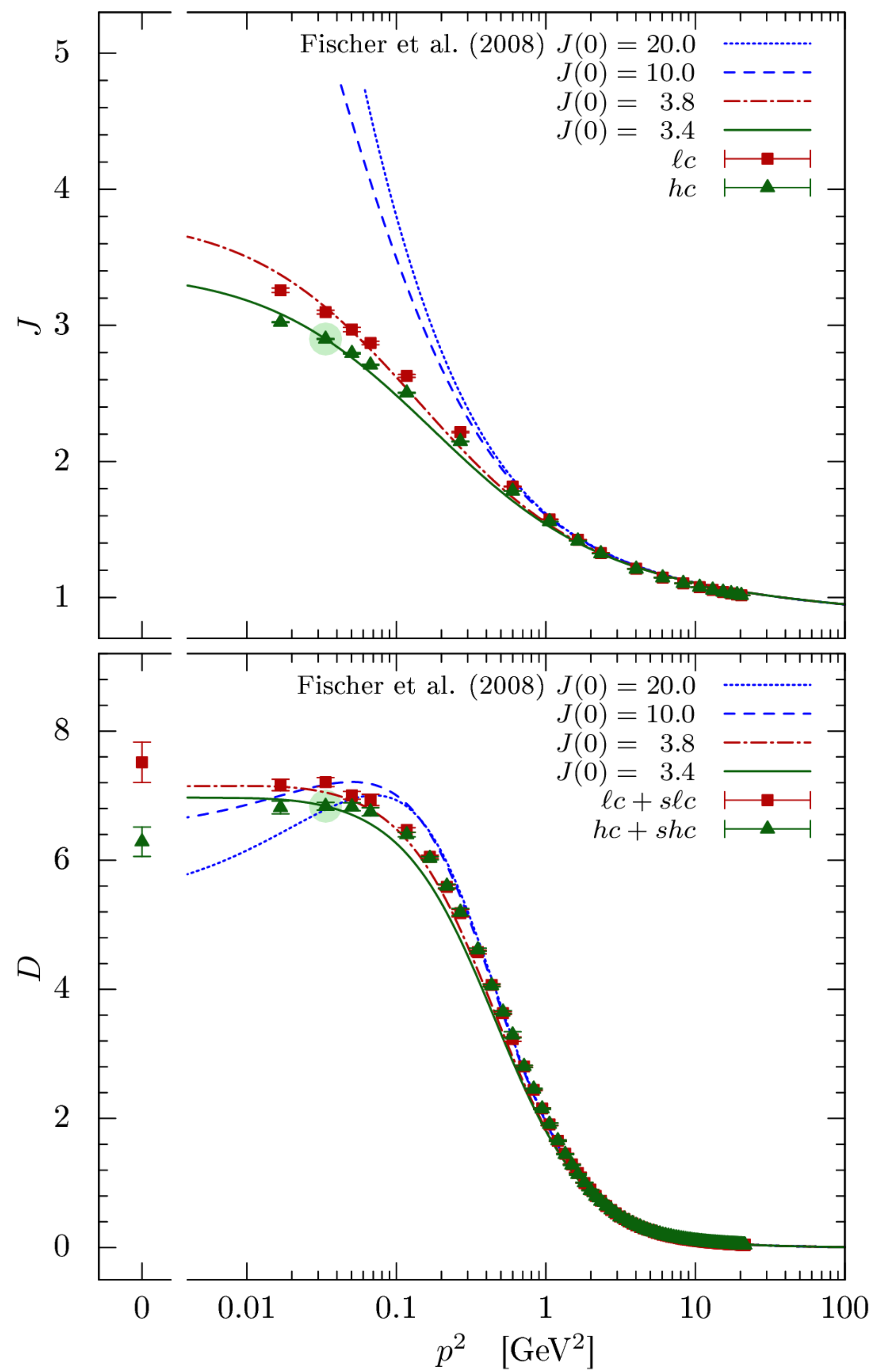
1) a few members of family of decoupling solutions can be reproduced qualitatively on the lattice

2) strong coupling constant

$$\alpha_s^{\mathrm{MM}}(p^2) = \frac{g_0^2}{4\pi} Z(p^2) \cdot J^2(p^2)$$

is used to calculated Λ_{QCD} on the lattice [*A.S.* et al. 1212.2039]

A.S., Müller-Preussker (2013)



Functional methods for QCD at finite T

DSEs (biased selection)

- Grüter, Alkofer, Maas, Wambach (2005)
  - Temperature-dependence of gluon+ghost propagator

- Fischer, Luecker, Mueller (2009-2012):
  - Phase diagram of quenched QCD and unquenched QCD with finite real μ
  - (dual) quark condensate and dressed Polyakov loop
  - input: Landau gauge quark (DSE) + lattice gluon propagator (nf=0),
+ quark-gluon vertex model
+ quark pol.tensor from gluon DSE (~ unquenching)

- Müller, Buballa, Wambach (2013)
  - Color superconductivity at finite T and μ
  - Input: similar as above

Functional methods for QCD at finite T

DSEs (biased selection)

– Grüter, Alk
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– Fischer, Lu
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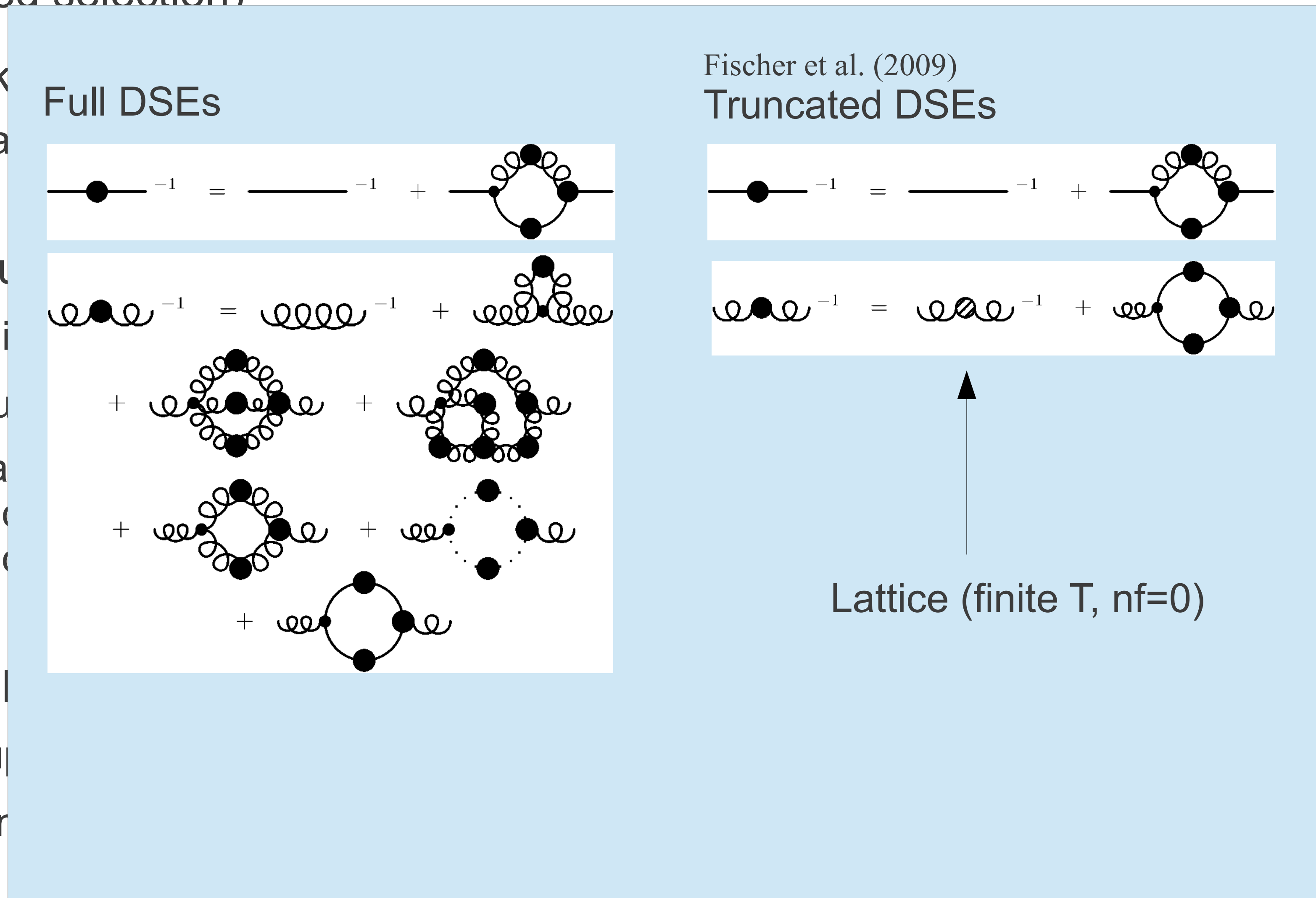


Functional methods for QCD at finite T

FRGEs (biased selection)

- Braun, Haas, Marhauser, Pawlowski (2009):
  - Phase diagram of Nf=2 QCD at imag. chem. potential in chiral limit
  - Input gluon and ghost propagators Landau gauge YM

- Fister, Pawlowski (2011):
  - Landau-gauge gluon, ghost propagators and the ghost-gluon vertex Nf=0
  - agree with lattice findings, as far as available

Lattice studies

$$D^{ab}_{\mu\nu}(k) = \langle A^a_\mu(k) A^b_\mu(-k)\rangle$$

Can provide (untruncated) input to DSEs

- Lattice Landau gauge gluon propagator most reliable source
- High-precision data for ($\underline{N_f = 0,\ 2,\ 2{+}1}$, $2{+}1{+}1$) for T=0, T>0 available
- also: ghost/quark propagator + vertex functions partly available

In Landau gauge, for $T > 0$

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}\left(P^T_{\mu\nu} D_T(\vec{k}, k_4) + P^L_{\mu\nu} D_L(\vec{k}, k_4)\right)$$

D_T: transverse ("chromomagnetic")

D_L: longitudinal ("chromoelectric")

- projectors:

$$P^T_{\mu\nu} = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}\right)$$

$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}\right) - P^T_{\mu\nu}$$

momenta:

$$k_\mu \in (-L_\mu/2, L_\mu/2]$$

$$q_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{L_\mu}\right)$$

Gluon propagator on the lattice

Gluon propagator

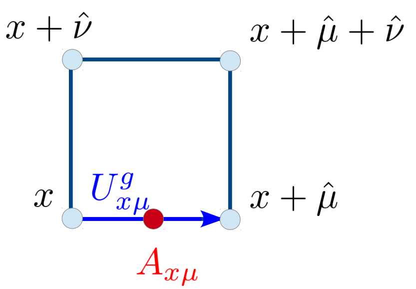
$$\langle A^a_\mu(k) A^b_\nu(-k) \rangle_{U^g} = \frac{\sum_{U^g} e^{-S[U]} A^a_\mu(k) A^b_\nu(-k)}{\sum_U e^{-S[U]}}$$

Gauge-fix configurations $\boldsymbol{U_{x\mu} \to U^g_{x\mu}}$

- Iteratively maximize Landau-gauge functional $F_U[g] = \frac{1}{3} \sum_{x,\mu} \underbrace{g_x U_{x\mu} g^\dagger_{x+\hat{\mu}}}_{U^g_{x\mu}}$
- Stop if: $\max_x [\nabla_\mu A_{x\mu} \nabla_\nu A^\dagger_{x\nu}] < 10^{-13}$

Gluon fields $(A_\mu = A^a_\mu T^a)$

$$A_{x\mu} \equiv A_\mu(x + \hat{\mu}/2) = \frac{1}{2iag_0}(U^g_{x\mu} - U^{g\dagger}_{x\mu}) \,|_{tr.less}$$



Gluon propagator on the lattice

Gluon propagator

$$D_L = \frac{1}{N_g}\left(1+\frac{q_4^2}{\vec{q}^{\,2}}\right)\left\langle \widetilde{A}_4^a(k)\widetilde{A}_4^a(-k)\right\rangle \qquad D_T = \frac{1}{2N_g}\left\langle \sum_{i=1}^{3}\widetilde{A}_i^a(k)\widetilde{A}_i^a(-k) - \frac{q_4^2}{\vec{q}^{\,2}}\widetilde{A}_4^a(k)\widetilde{A}_4^a(-k)\right\rangle$$

Gauge-fix configurations $\boldsymbol{U_{x\mu} \to U_{x\mu}^g}$

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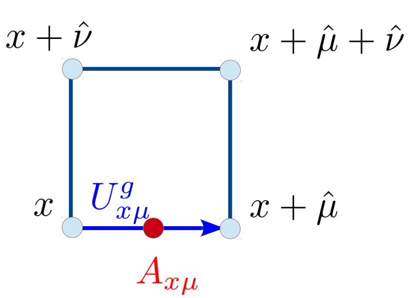

Lattice studies

Gluon propagator at finite T $(N_f = 0)$

- SU(3): Mandula, Ogilvie (1988)
- SU(2): Heller, Karsch , Rank (1995-1998)

electric and magnetic screening masses in deconfinement phase

- SU(2): Cucchieri, Karsch, Petreczky (2000-'01)

screening masses + momentum dependence, 3d+4d

- SU(2): Cucchieri, Maas, Mendes (2007)

momentum dependence + comparison DSE vs. lattice, 3d+4d

- SU(2) + SU(3): Fischer, Maas, Müller (2010)

Comparison SU(2) + SU(3)

- SU(2): Cucchieri, Mendes (2011-'12)

low-momentum dependence around T_c

- SU(2): Bornyakov, Mitrjushkin (2011)

momentum dependence $T>T_c>T$, Gribov-copy effects,screening masses

- SU(2) + SU(3): Maas, Pawlowski, von Smekal, Spielmann (2012)

order of phase transition is reflected in momentum dependence, screening masses, 3d+4d

- SU(3): Aouane et al. (2011):

phase transition reflected in momentum dependence, continuum limit → input to DSE studies, also ghost propagator

Gluon dressing functions $N_f = 0$

Setup

- Wilson gauge action
  $\beta = 6.337 \quad (fix) \quad \Rightarrow \quad N_T = 12 \Leftrightarrow T \approx T_c \qquad T^{-1} = N_T a(\beta)$
- Vary temporal extension
  $\frac{T}{T_c} \equiv \frac{12}{N_T} \in \left[\frac{12}{18}, \frac{12}{4}\right]$
- Spatial volume
  $N_s = 48 \quad (2.6\,\mathrm{fm})$

Study systematic effects

- Finite volume
- Gribov copies
- Discretization
- **Temperature dependence**

Gluon dressing functions

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Study systematic effects

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- Gribov copies
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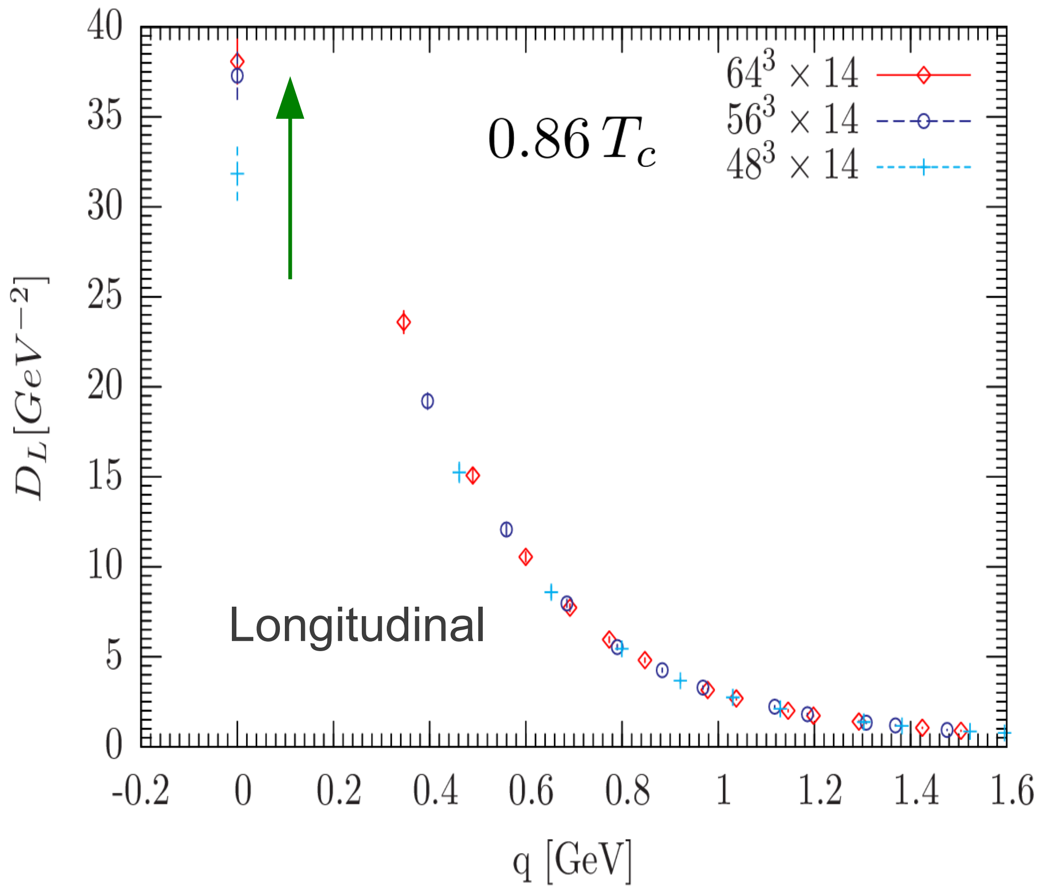
- Temperature dependence



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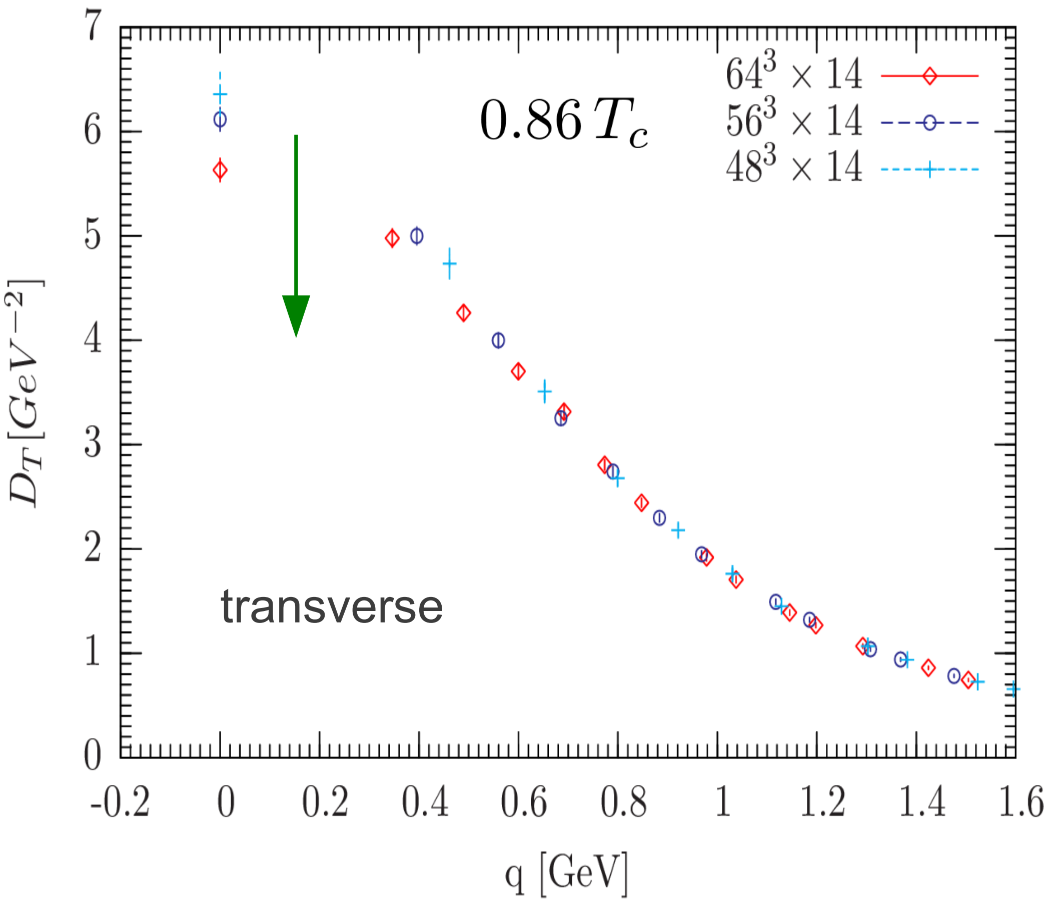

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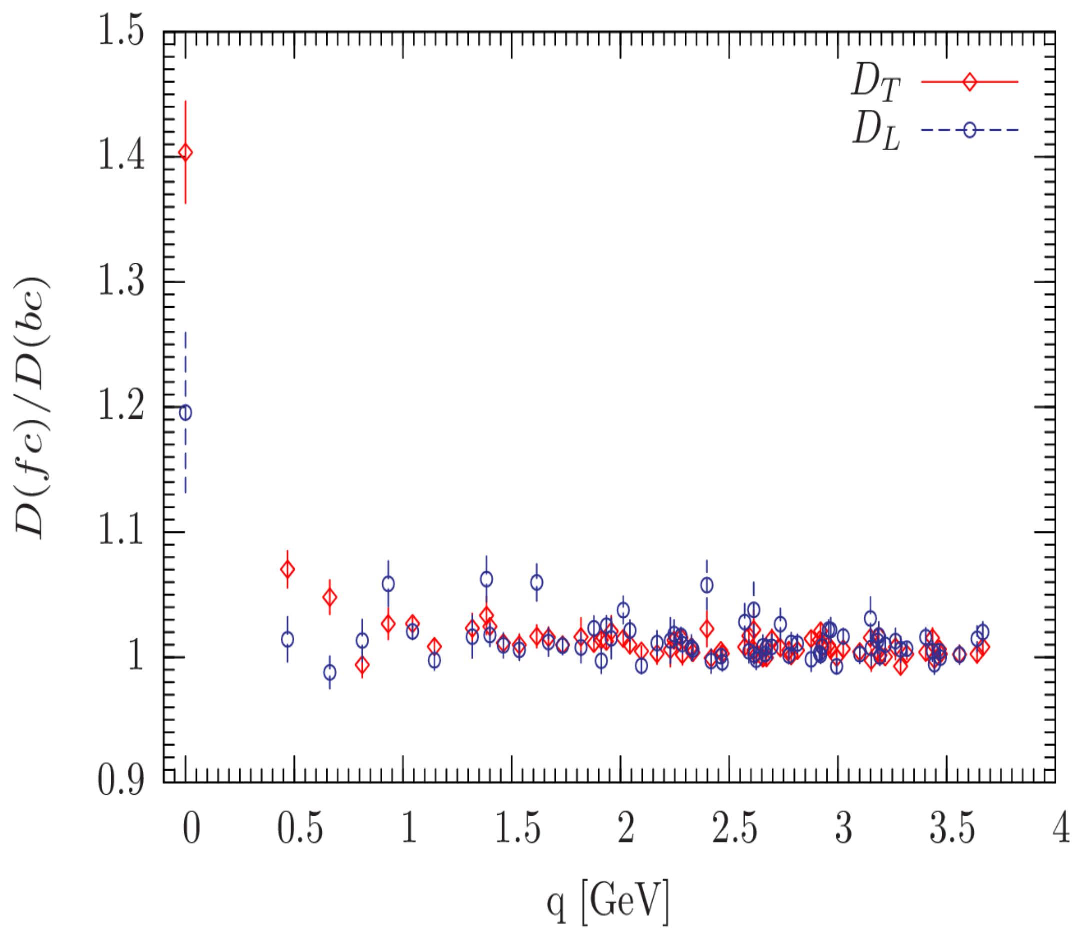
Study systematic effects

- Finite volume (→ for q<0.6GeV)
- Gribov copies (→ visible)
- Discretization
- Temperature dependence



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Study systematic effects

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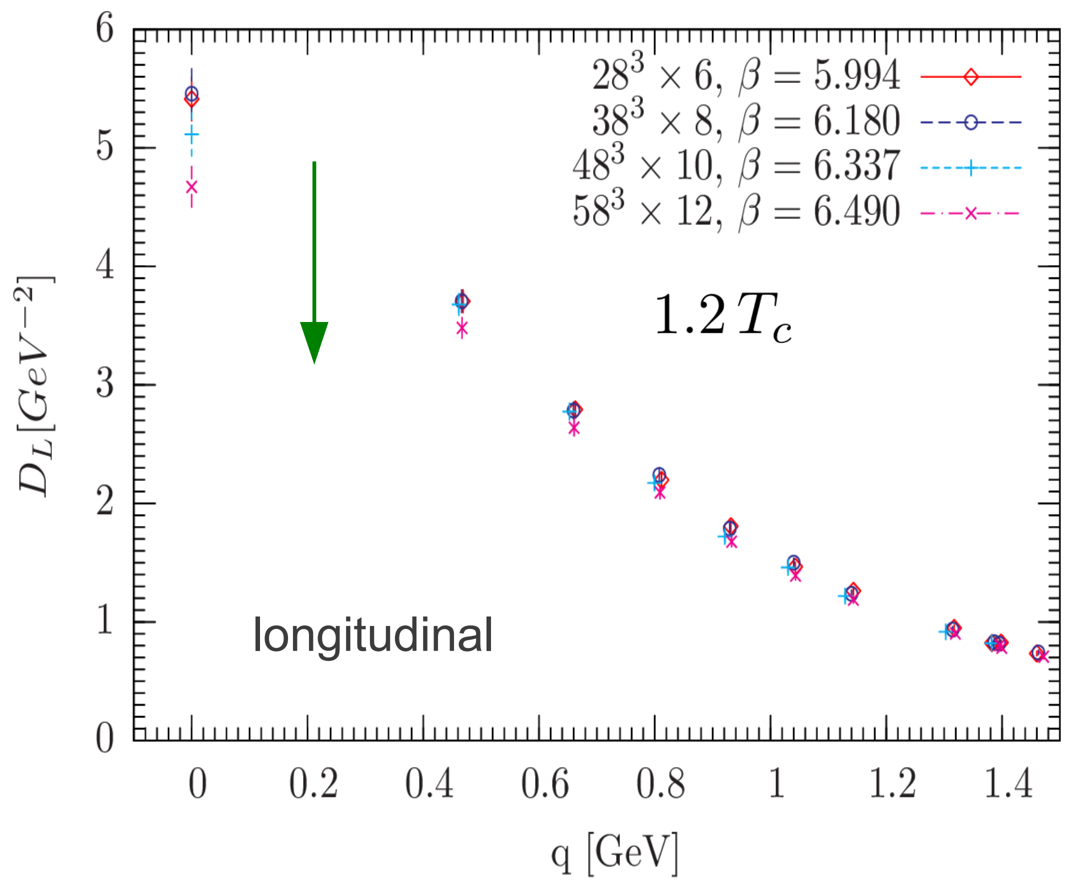

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Study systematic effects

- Finite volume (→ for q<0.6GeV)
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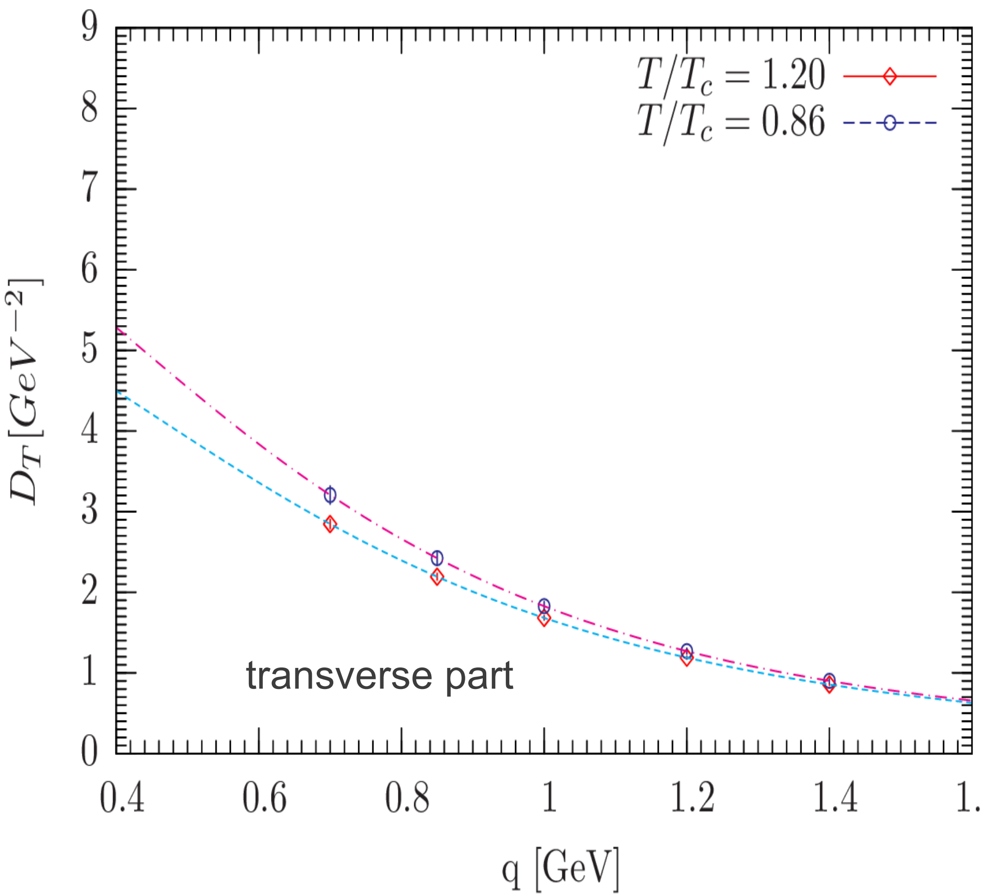
- Discretization (→ systematic)
- Temperature dependence

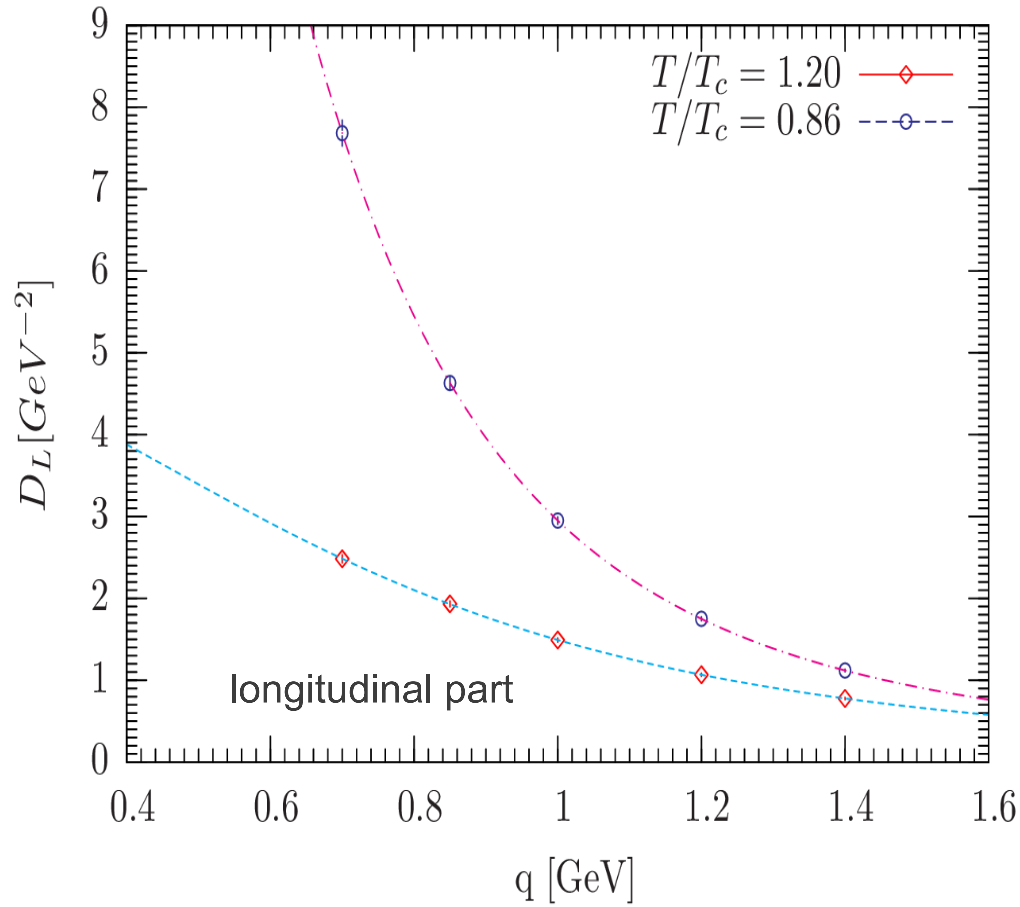
Take continuum limit

- For fixed volume
- For two temperatures

Gluon dressing functions $N_f = 0$

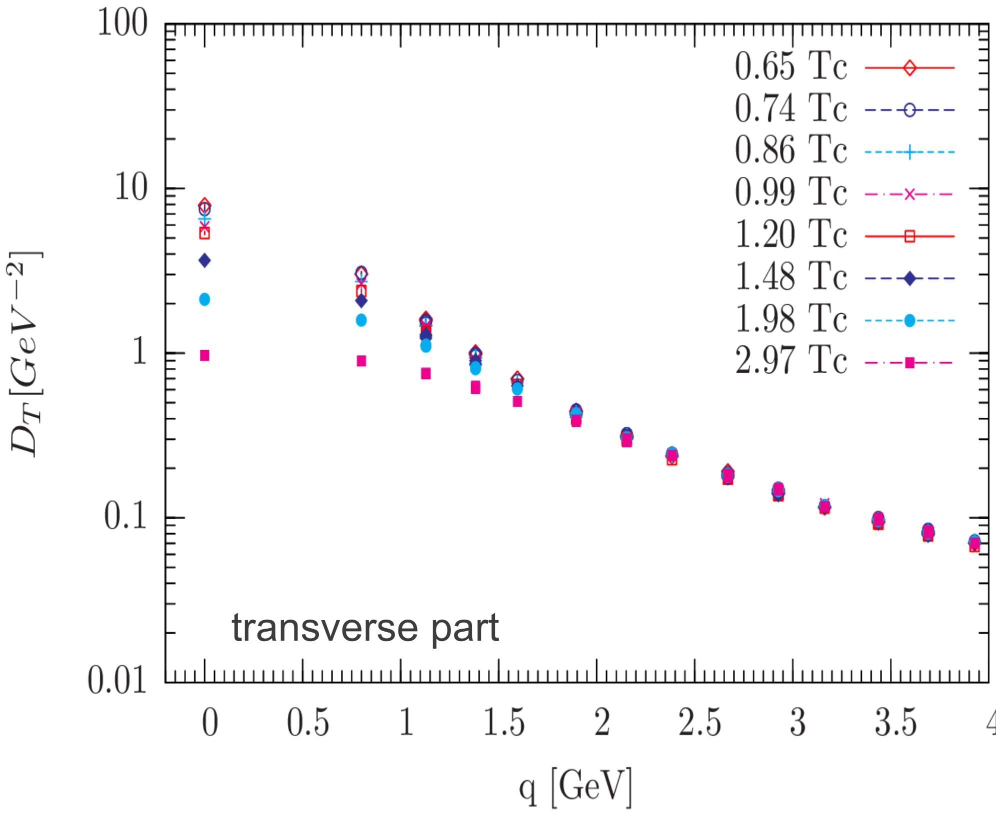
Continuum-limit extrapolated data + interpolation lines

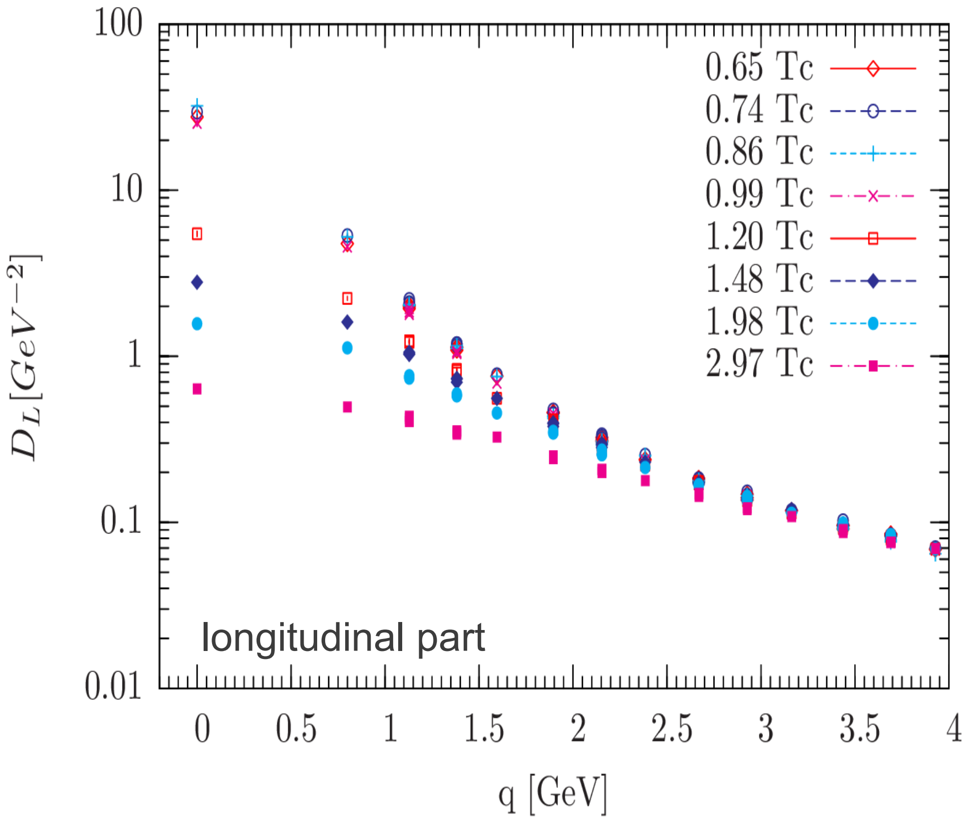




Gluon dressing functions $N_f = 0$

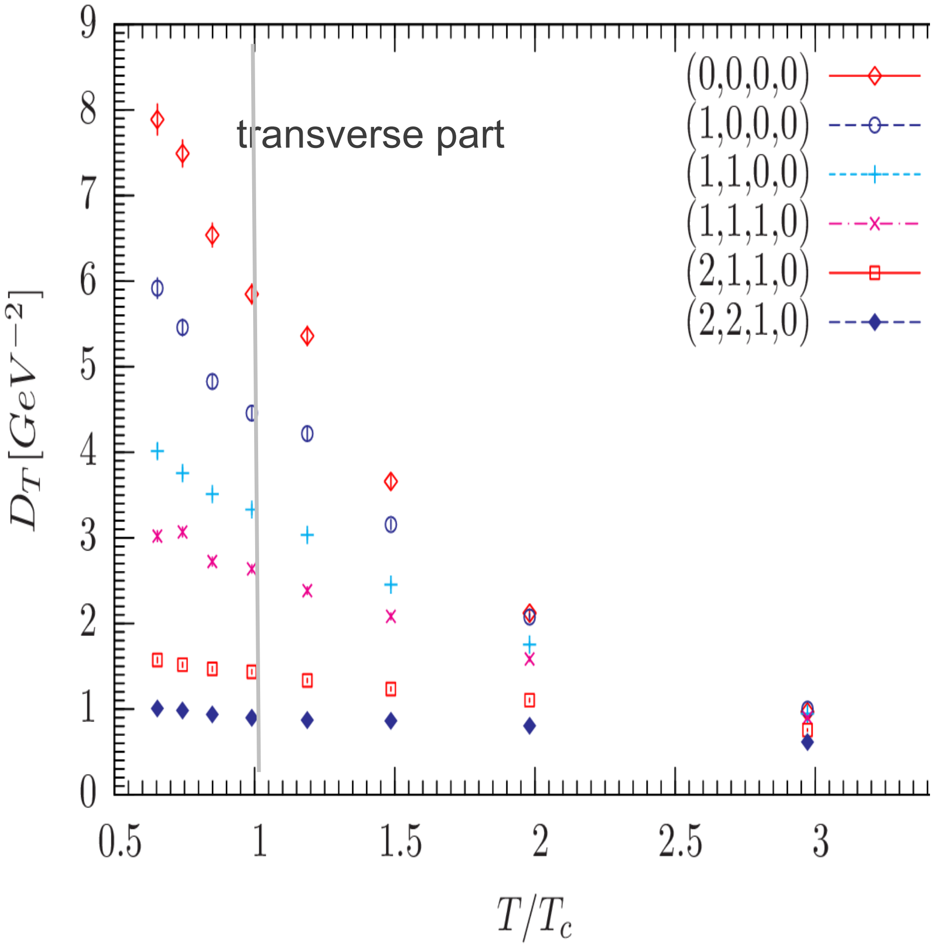
Temperature dependence

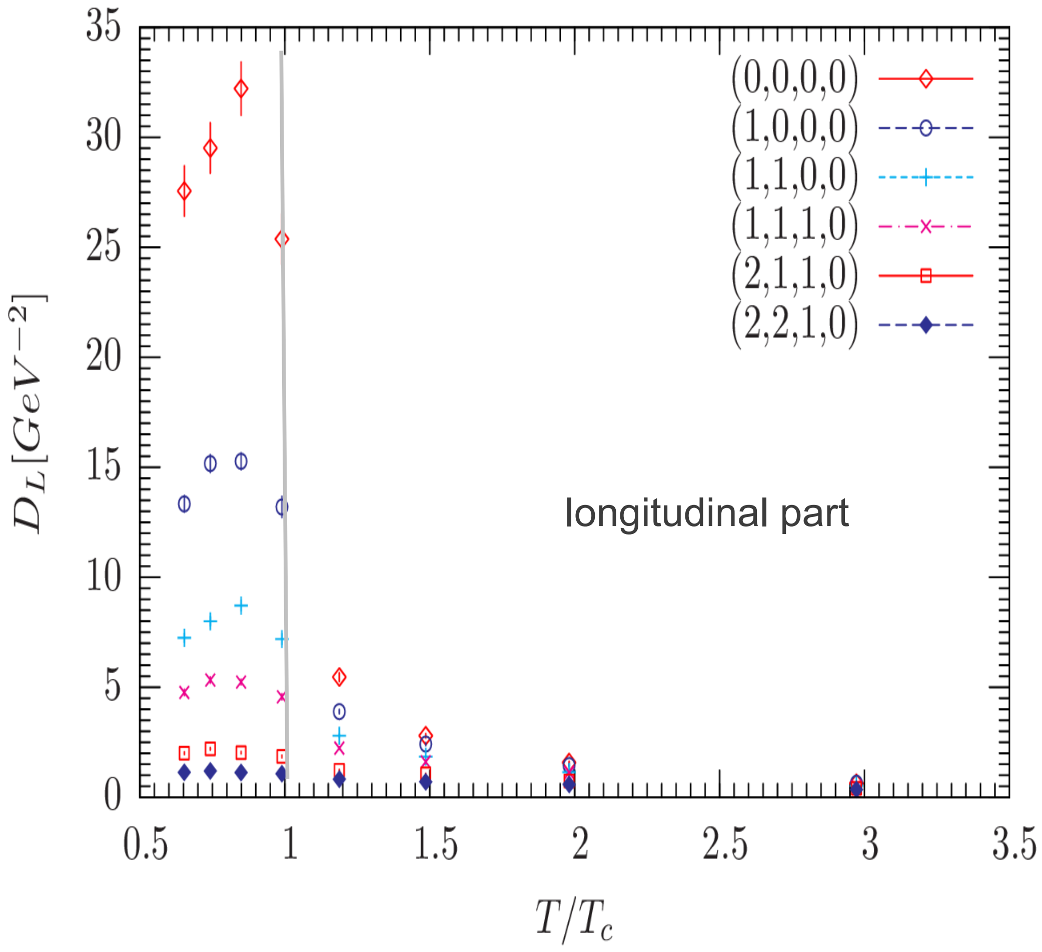




Gluon dressing functions $N_f = 0$

Temperature dependence





Summary $N_f = 0$

Gluon

- Z_T changes smoothly from below to above T_c
- Z_L changes significantly from below to above T_c
- temperature dependence as found, e.g., in [Fischer, Maas, Müller (2010)]
- Find clear Gribov-copy and finite-volume effects at low momenta
- For $0.6 \leq q \leq 8.0\,\mathrm{MeV}$ we give interpolation formula

$$Z^{\mathrm{fit}}_{Z,L} = q^2 \frac{c \cdot (1 + d q^2)}{(q^2 + r^2)^2}$$

$\rightarrow$ input to FRGE / DSE studies

Ghost

- Shows a weak temperature dependence at low momenta

Functional methods for QCD at finite T

DSEs (biased selection)

- Grüter, Alk
  - Tempera
- Fischer, Lu
  - Phase di
  - (dual) qu
  - input: La
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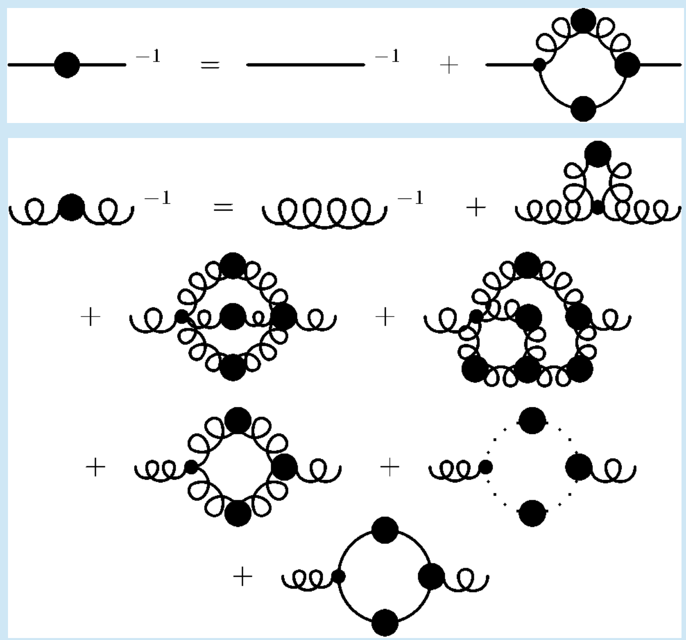
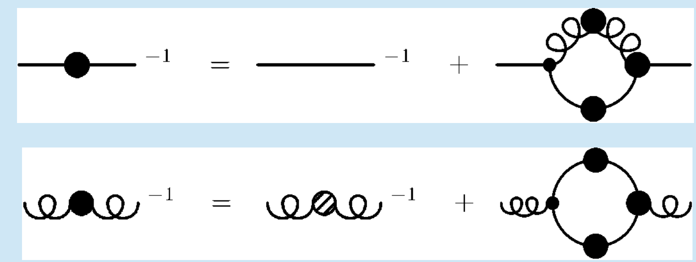
- Müller, Bul
  - Color su
  - Input: sir

Full DSEs



Fischer et al. (2009)

Truncated DSEs



Lattice (fin.T, nf=0)

Lattice data for finite T, nf>0 needed

How good is this truncation?

Lattice studies $(N_f = 2)$

Gluon + ghost propagators at finite T

- Furui and Nakajima (2007): momentum dependence $T \rightarrow T_c \rightarrow T$, screening masses, ghost condensate
- Bornyakov, Mitrjushkin (2011): Only gluon propagator, momentum dependence $T > T_c > T$, Gribov-copy effects, screening masses
- Aouane et al. (2012): momentum dependence changes smoothly for $T \rightarrow T_c \rightarrow T$, m_π=320..470MeV, interpolation of data $\rightarrow$ input to DSE studies

Gluon dressing functions $N_f = 2$

Setup

- Tree-level Symanzik-improved Wilson gauge action

$$3.84 \leq \beta \leq 4.07 \qquad T^{-1} = N_T a(\beta)$$

- Two flavours of twisted-mass fermions (tmfT collaboration)

$$N_s = 32, \quad N_T = 12$$

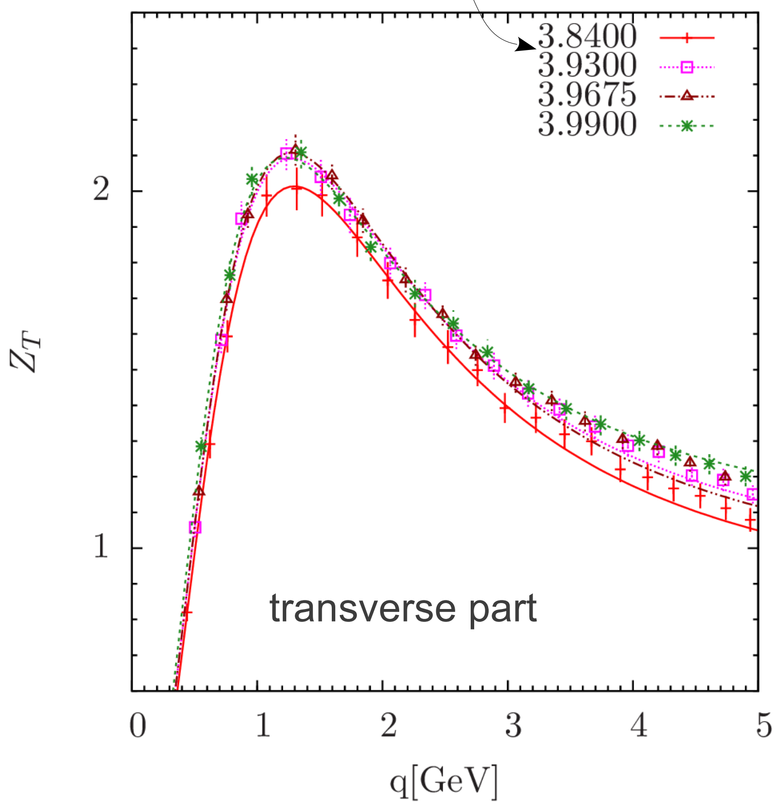
- Pion masses: $m_\pi = 316, 398, 469\,\mathrm{MeV}$ (smooth crossover expected)

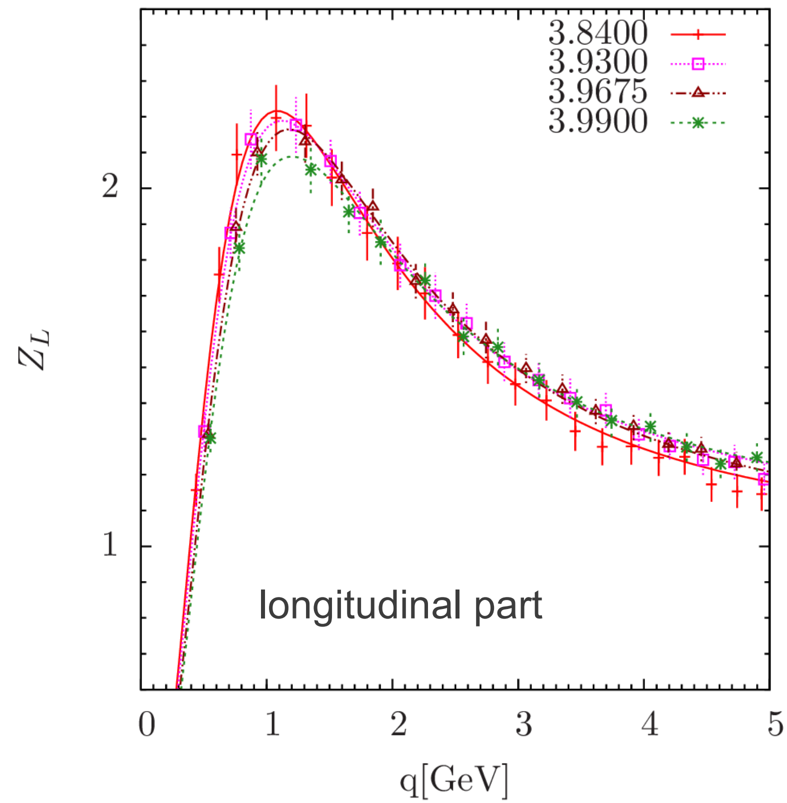
Gluon dressing functions

$N_f = 2$

$m_\pi \approx 316\,\mathrm{MeV}$

$\beta_c^\chi = 3.89(3)$





(unrenormalized)

Interpolation function:
(Gribov-Stingl)

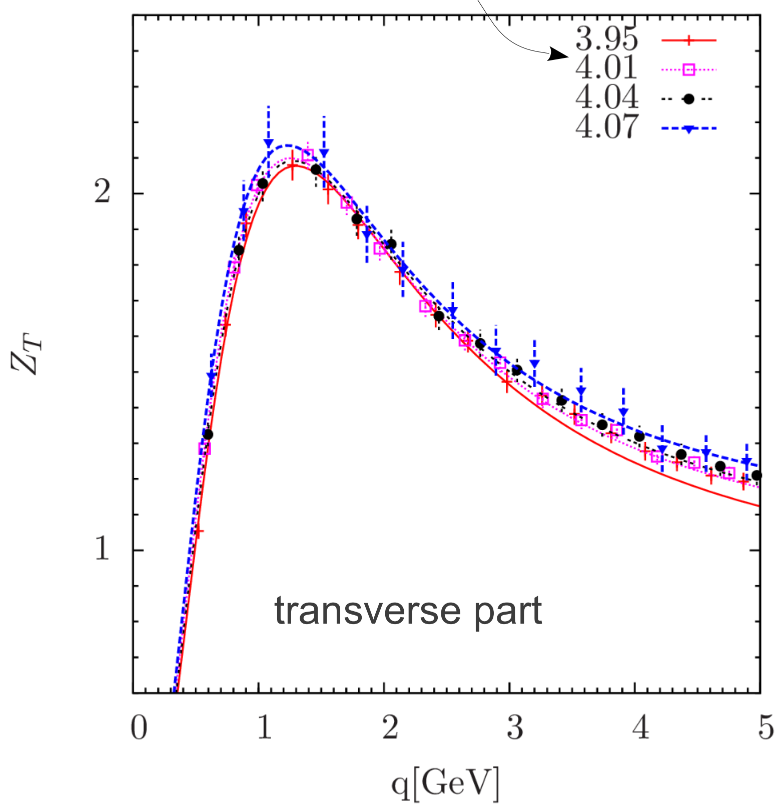
$$Z^{\mathrm{fit}}_{Z,L} = q^2 \frac{c(1+dq^2)}{(q^2+r^2)^2}$$

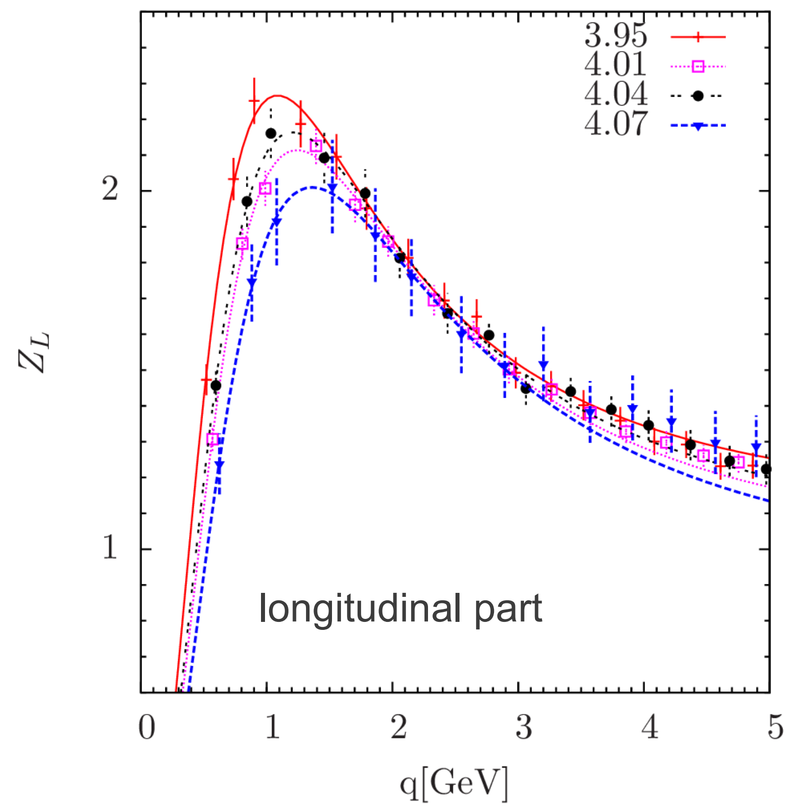
Gluon dressing functions

$N_f = 2$

$m_\pi \approx 469\,\mathrm{MeV}$

$\beta_c^\chi = 3.97(3)$





(unrenormalized)

Interpolation function:
(Gribov-Stingl)

$$Z^{\mathrm{fit}}_{Z,L} = q^2 \frac{c(1+dq^2)}{(q^2+r^2)^2}$$

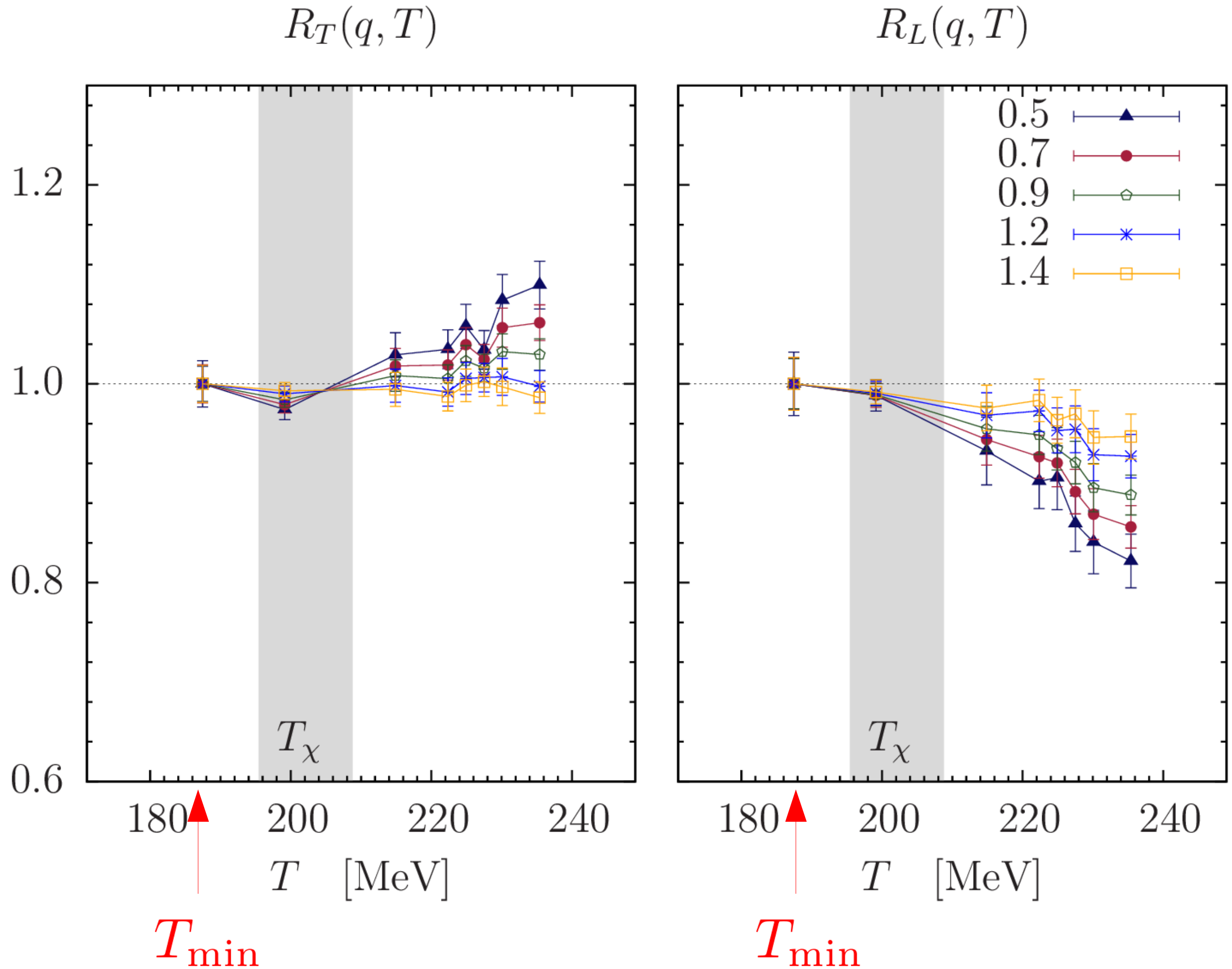
Gluon dressing function

$N_f = 2$

$m_\pi \approx 316\,\mathrm{MeV}$

Ratios for selected momenta

$$R_{T,L}(q,T) = \frac{D^{\mathrm{ren}}_{T,L}(q,T)}{D^{\mathrm{ren}}_{T,L}(q,T_{\mathrm{min}})}$$



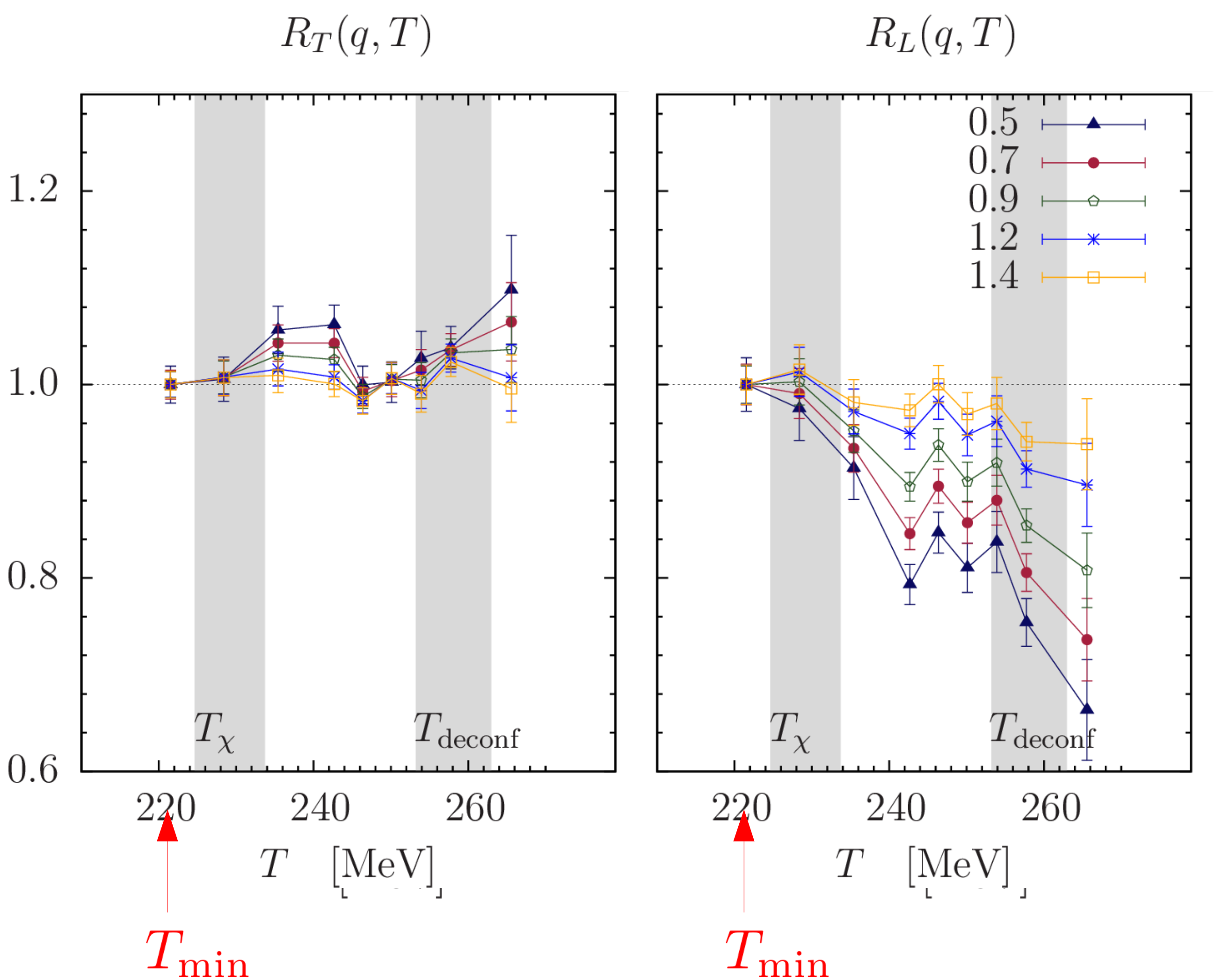
Gluon dressing function

$N_f = 2$

$m_\pi \approx 469\,\mathrm{MeV}$

Ratios for selected momenta

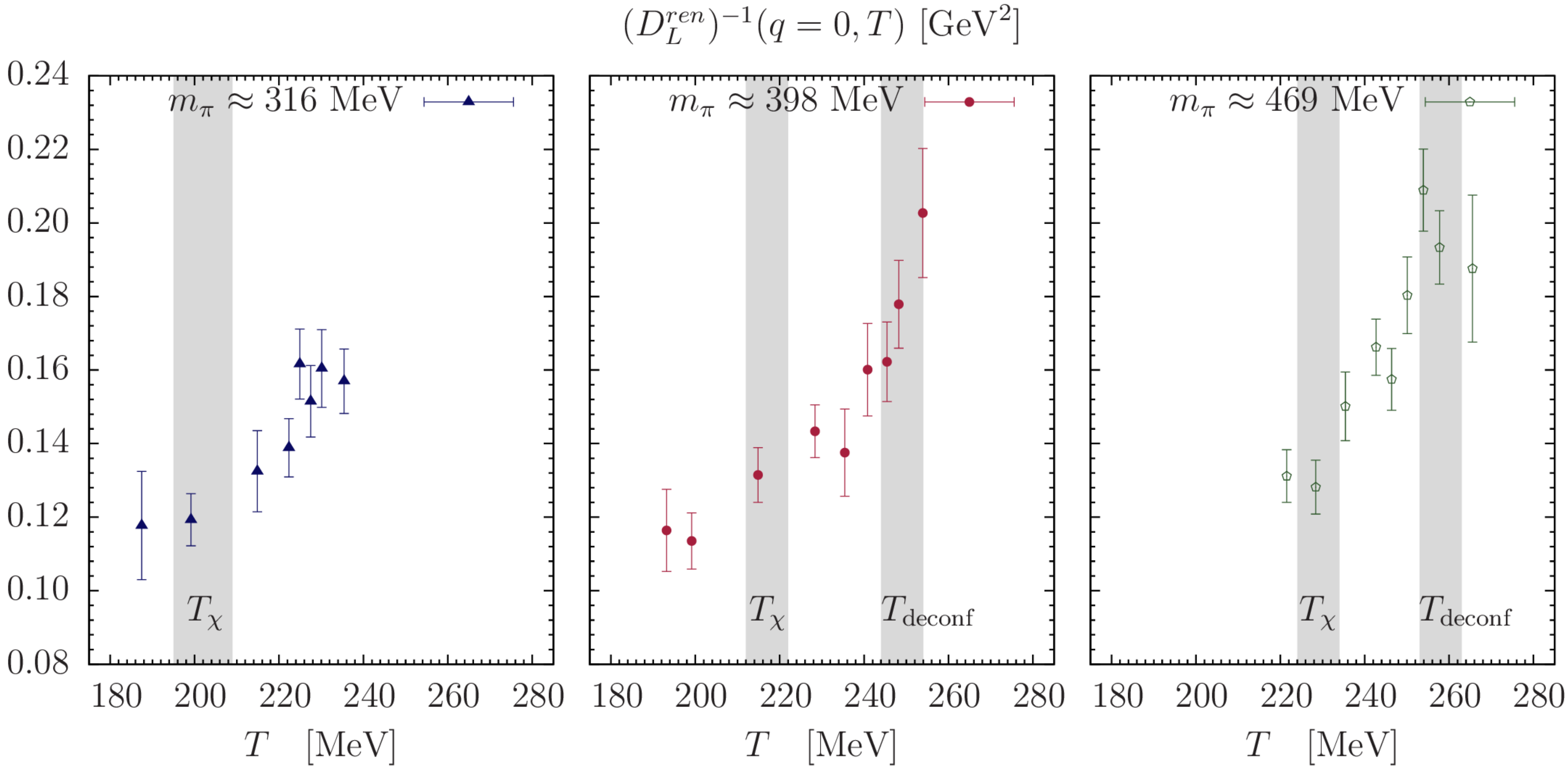
$$R_{T,L}(q,T) = \frac{D^{\mathrm{ren}}_{T,L}(q,T)}{D^{\mathrm{ren}}_{T,L}(q,T_{\mathrm{min}})}$$



Gluon dressing function

$N_f = 2$

At **zero** momentum



Ghost propagator

Yet another Landau-gauge two-point function

- Relevant for example for the gluon DSE
- Can be estimated on the lattice for finite momentum
- Numerically much more expensive than ghost

$$q_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{L_\mu}\right)$$

"Inverse Faddeev-Popov matrix"

$$G^{ab}(q) = \delta^{ab} \frac{J(q)}{q^2} = a^2 \sum_{x,y} \left\langle e^{-2\pi i (k/N)\cdot(x-y)} [M^{-1}]^{ab}_{xy} \right\rangle$$

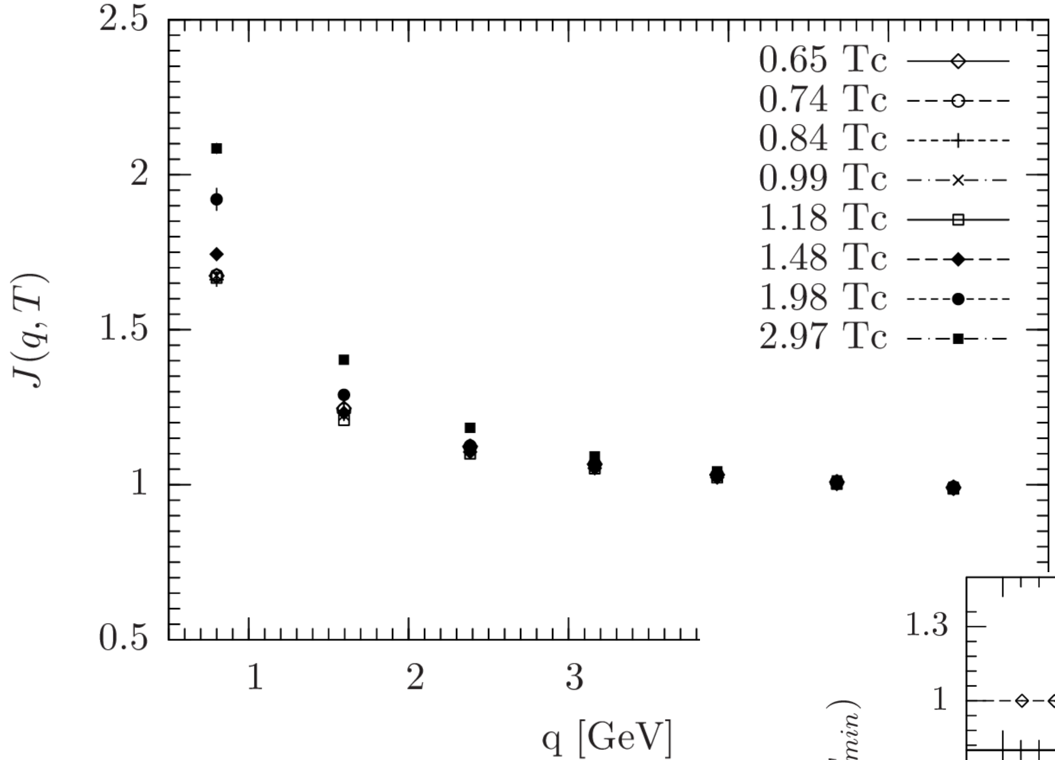
- separate inversion for each momentum
- restrict to diagonal momenta ((1,1,1), (2,2,2),...)
- use accelerated inverter [A.S., PRD75(2005)014507]
  - preconditioned CG
  - improves performance much

$$M[U=\mathbb{I}] = \Delta^{-1}$$

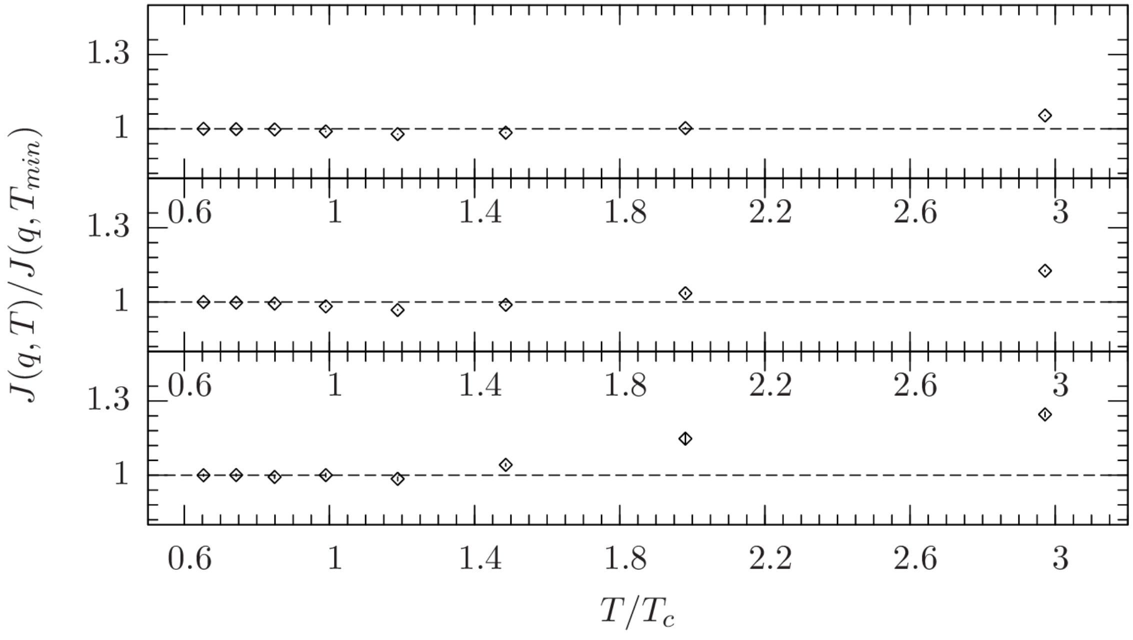
Ghost dressing function

$$N_f = 0$$

$$\beta = 6.337, N_s = 48$$

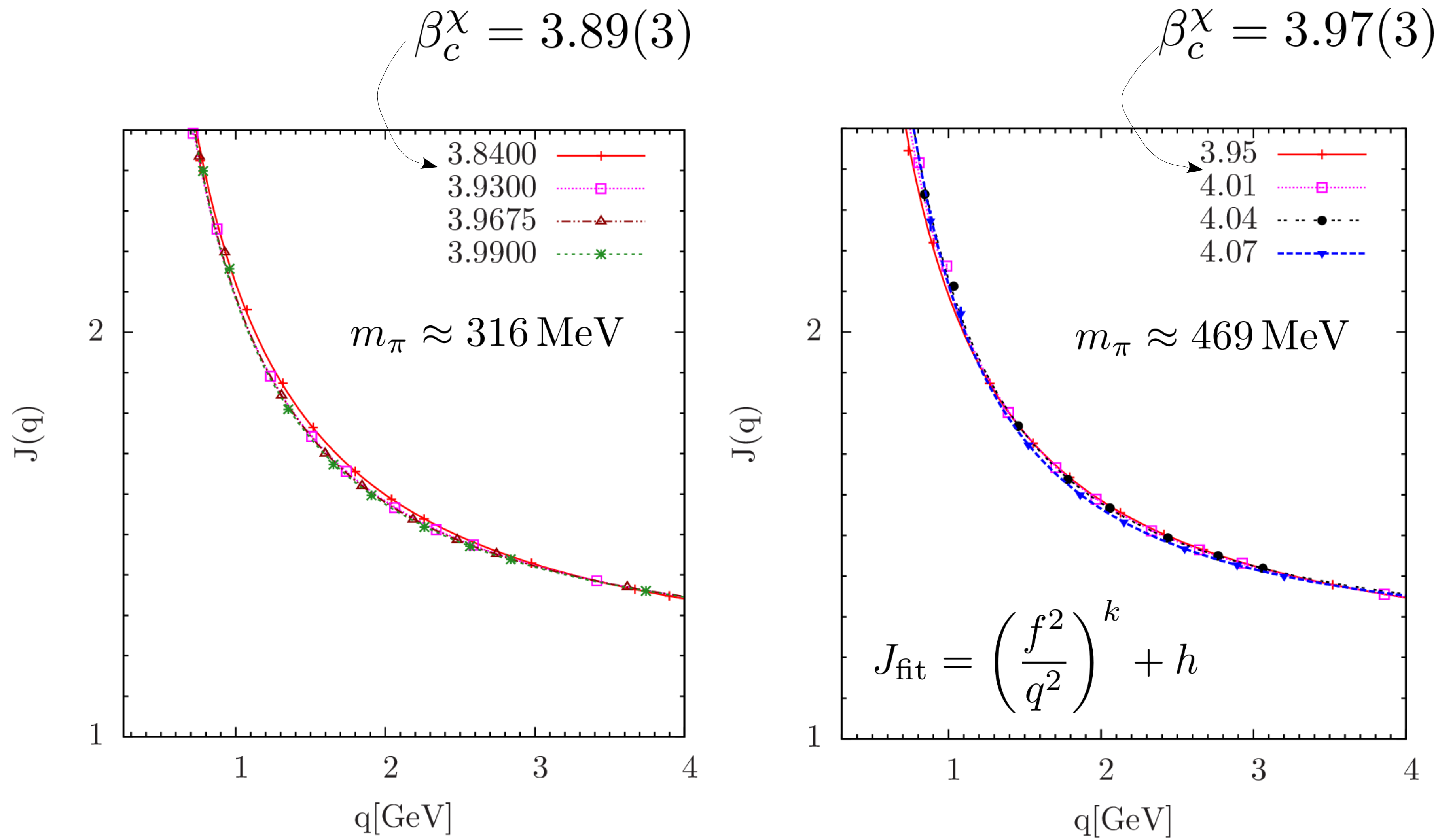


$$R(q,T) = \frac{J^{\rm ren}(q,T)}{J^{\rm ren}(q,T_{\rm min})}$$



Ghost dressing function

$N_f = 2$



Summary $N_f = 2$

Gluon

- Z_L and Z_T change smoothly in the crossover region
- Changes for Z_L are larger than for Z_T
- For $0.4 \leq q \leq 3.0\,\mathrm{MeV}$ we give interpolation formula

$$Z^{\mathrm{fit}}_{Z,L} = q^2 \frac{c \cdot (1 + dq^2)}{(q^2 + r^2)^2}$$

$\rightarrow$ input to FRGE / DSE studies

Ghost

- Almost no temperature dependence
- For $0.4 \leq q \leq 3.0\,\mathrm{MeV}$ we give interpolation formula

$$J_{\mathrm{fit}} = \left(\frac{f^2}{q^2}\right)^k + h$$

$\rightarrow$ input to FRGE / DSE studies

Conclusion

Lattice studies of gauge-fixed n-point functions

- can be performed to high-precision
- limit of infinite-volume / continuum has to be taken carefully
- provides important input to Functional methods (DSE,FRGE)
  (helps to improve truncations)

Mutual checks (DSE/FRGE $\leftrightarrow$ Lattice QCD)

- successfully performed in the past for T=0
- now also for T>0

Functional methods

- can complement lattice results
- may provide insight in a regime not accessible on a lattice

Thank you for your attention!