Universality of Phase Diagrams in QCD and QCD-like Theories

Naoki Yamamoto

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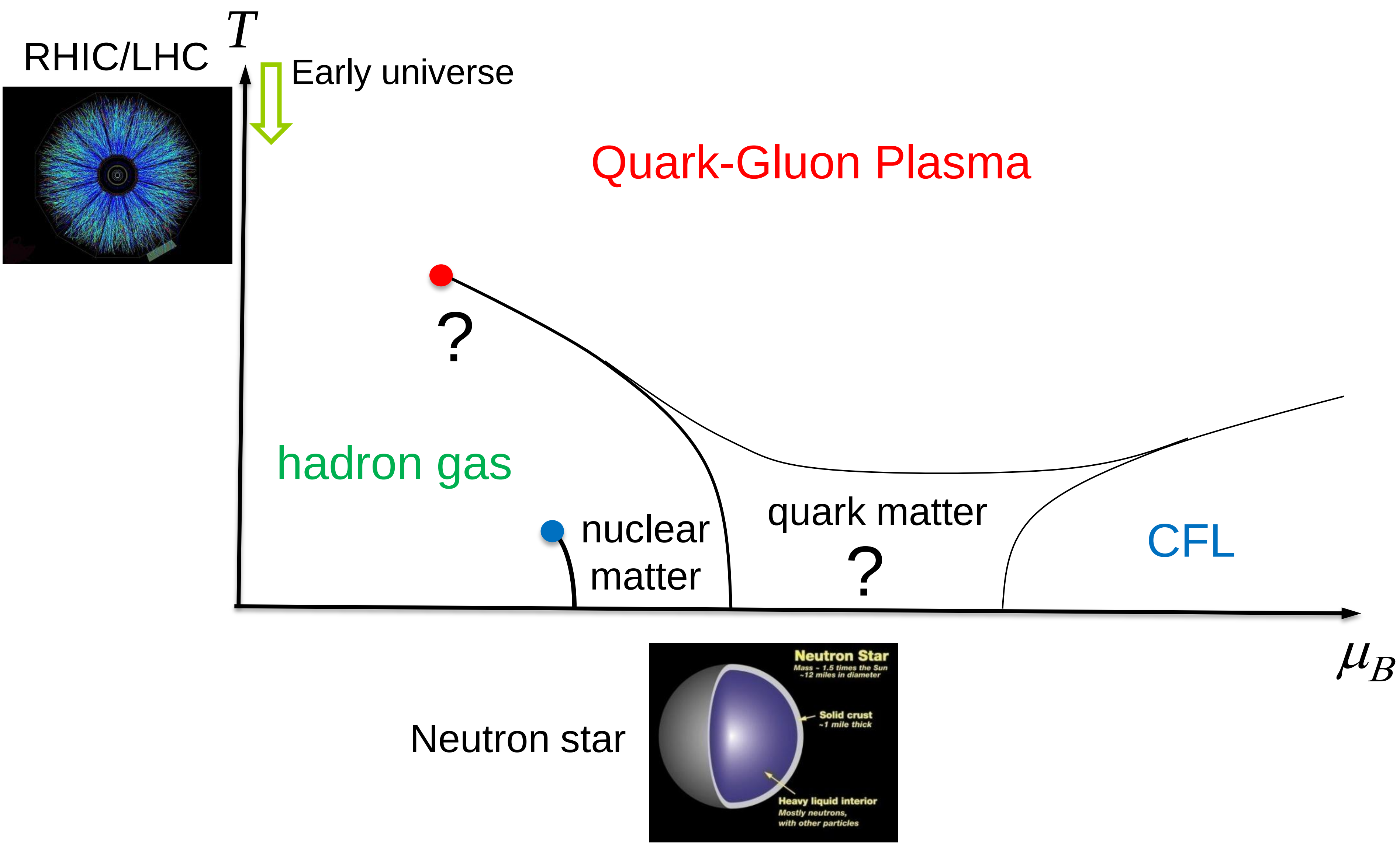
"Quarks, Gluons, and Hadronic Matter under Extreme Condition," March 22, 2013

Outline

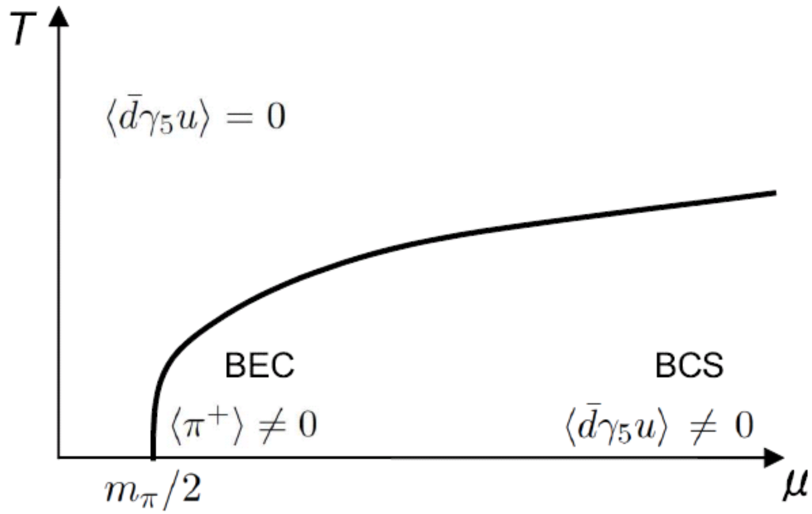
1. Universality of phases in QCD and QCD-like theories
2. A theorem on the QCD critical point
3. Summary and discussion

Refs.: **M. Hanada, NY, (JHEP 2012)**
Y. Hidaka, NY, (PRL 2012)
M. Hanada, Y. Matsuo, NY, (PRD 2012)

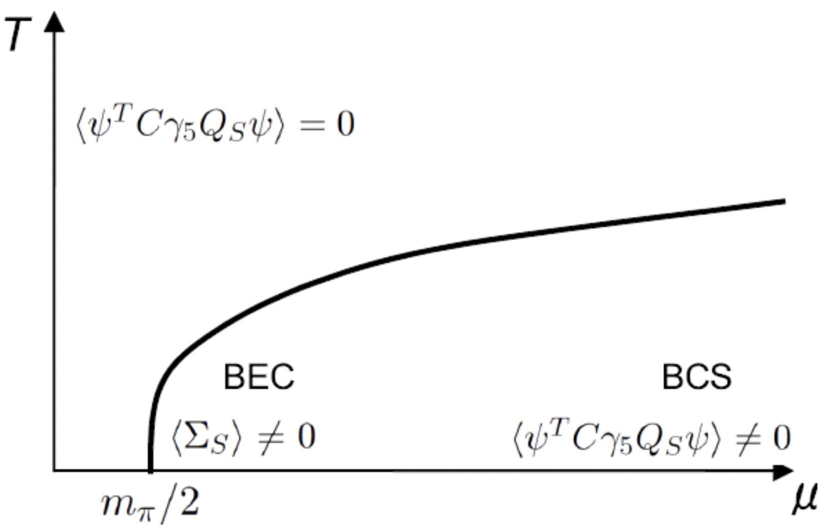
A conjectured QCD phase diagram



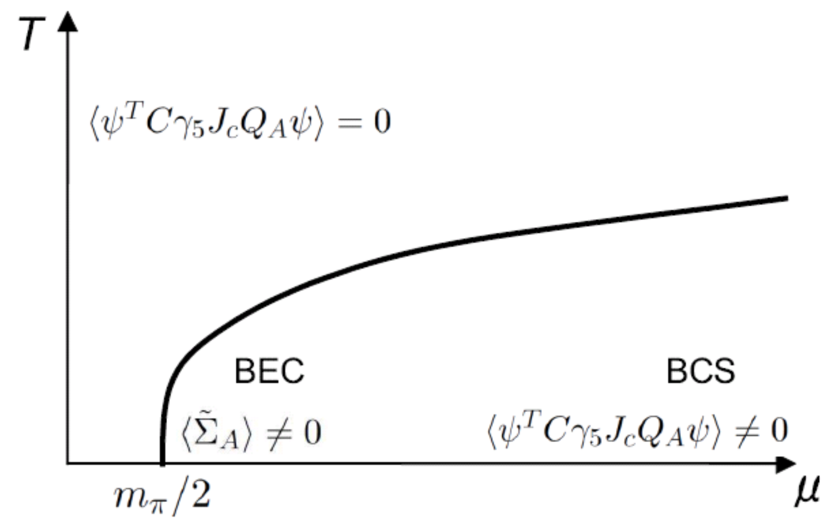
Phase diagrams of QCD-like theories



QCD at $\mu_I \neq 0$
Son-Stephanov (2001)



SO(N_c) gauge theory at $\mu_B \neq 0$

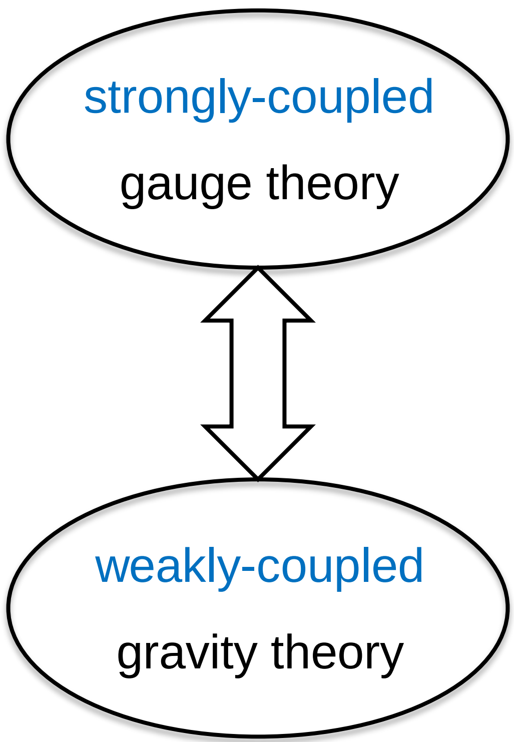


Sp(N_c) gauge theory at $\mu_B \neq 0$

Holography vs. Orbifolding

Gauge/gravity duality

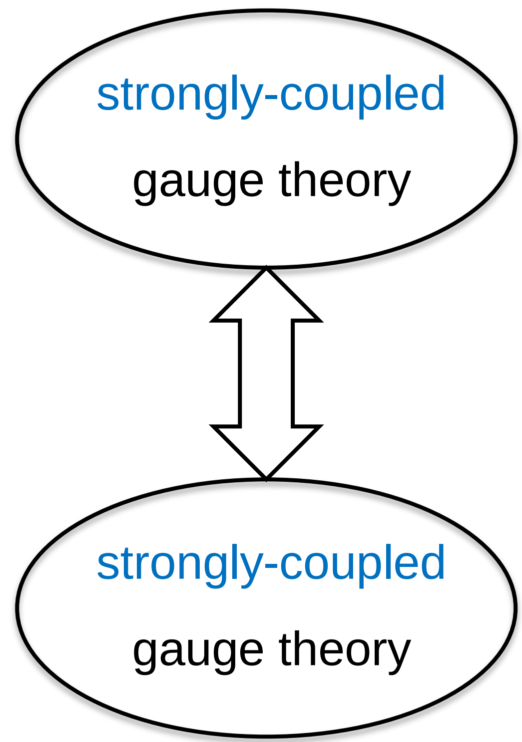
Maldacena (1998), ...



AdS/QGP,
AdS/QCD,
AdS/CMT, ...

Orbifold equivalence

Kachru-Silverstein (1998), ...



Cherman-Hanada-Robles-Llana;
Cherman-Tiburzi;
Hanada-NY; Hidaka-NY (2012)

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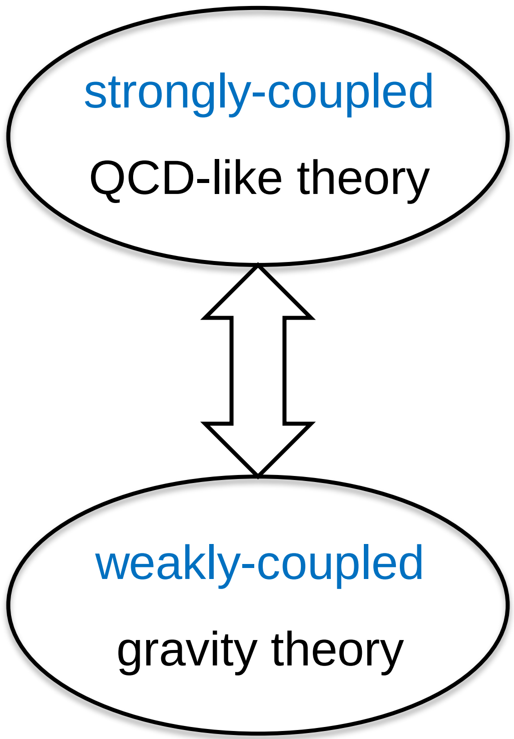

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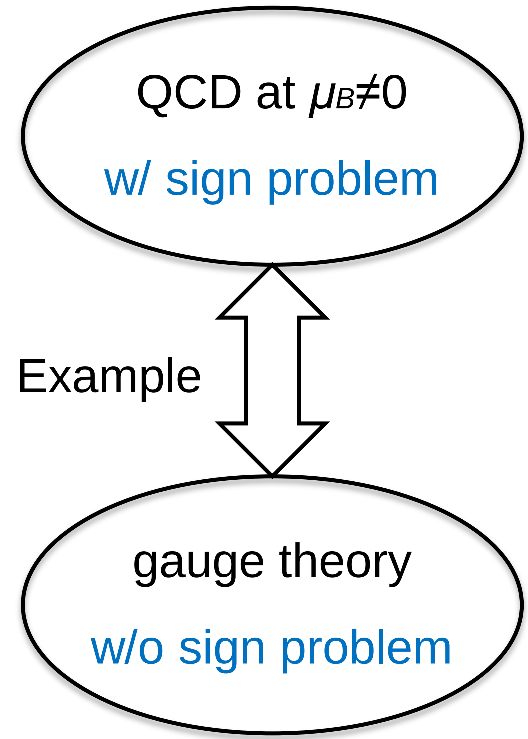

- Monte Carlo
- Exact results

Cherman-Hanada-Robles-Llana;
Cherman-Tiburzi;
Hanada-NY; Hidaka-NY (2012)

Orbifold equivalence

[How to use]

1. Identify a discrete global symmetry *S* of a theory *P* (parent).

2. Eliminate all the d.o.f.s of parent not invariant under *S* (projection).

3. This gives a new theory *D* (daughter).

4. A class of observables are identical between *P* and *D* at large N_c or MFA.

   Valid as long as *S* is not broken spontaneously.

Refs: Bershadsky-Johansen (1998); Kovtun-Ünsal-Yaffe (2003, 2005, 2006);

Generalization w/ fermions at finite μ/T, Hanada-NY (JHEP, 2012).

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   $J_c = -i\sigma_2 \times \mathbf{1}_{N_c} \in SO(2N_c)$, $\omega = e^{i\pi} \in U(1)_B$

2. Eliminate all the d.o.f.s of parent not invariant under *S* (projection).

   $A_\mu^{SO} = J_c A_\mu^{SO} J_c^{-1}$, $\psi^{SO} = \omega J_c \psi^{SO}$

3. This gives a new theory *D* (daughter).

   D = SU(N_c) QCD.

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Universality of phase diagrams

sign-free

$\boxed{\mathrm{SO}(N_c) \text{ or } \mathrm{Sp}(N_c) \text{ YM} + \mu_B}$

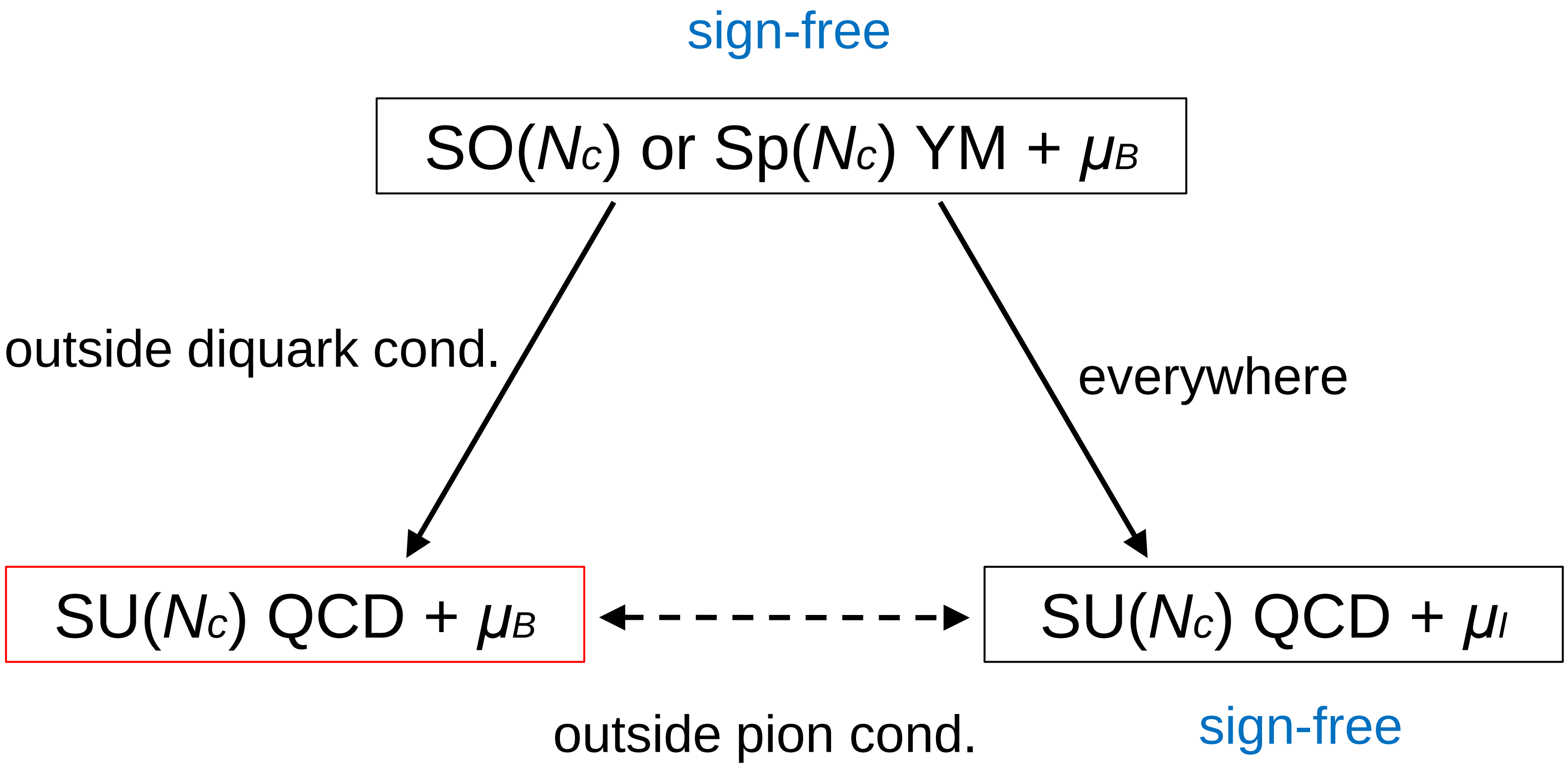
$\boxed{\mathrm{SU}(N_c) \text{ QCD} + \mu_B}$

$\boxed{\mathrm{SU}(N_c) \text{ QCD} + \mu_I}$

sign-free

Hanada-NY (JHEP, 2012)

Universality of phase diagrams



Hanada-NY (JHEP, 2012)

Sign problem and phase quenching in finite-density QCD: models, holography, and lattice

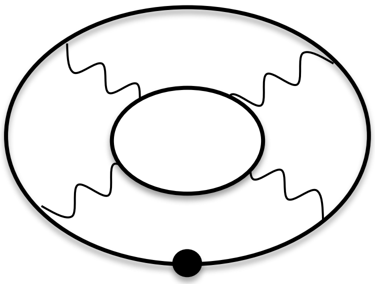
Masanori Hanada[1,2], Yoshinori Matsuo[1], and Naoki Yamamoto[3,4]

arXiv:1205:1030 (PRD, 2012)

CONTENTS

Intuitive derivation

- Consider $\langle \bar{q}q \rangle$ in QCD at finite μ_I or μ_B with $N_f = 2$ (u & d).

$\langle \bar{q}q \rangle(\mu_B)$ = 

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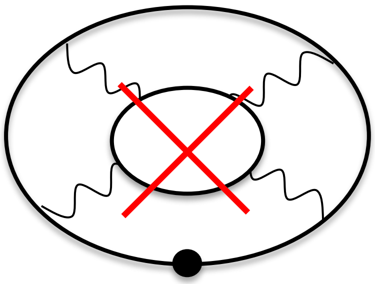

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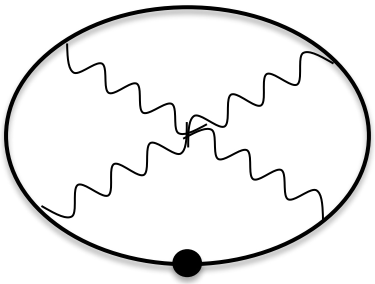

$= f(\mu_u) + f(\mu_d)$

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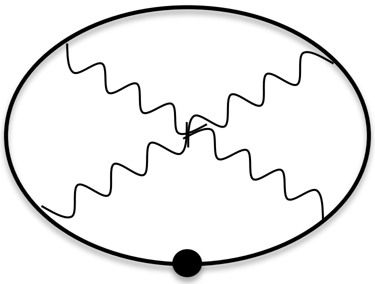
$\langle \bar{q}q \rangle(\mu_B)$ =  $= f(\mu_u) + f(\mu_d) = f(\mu_u) + f(-\mu_d) = \langle \bar{q}q \rangle(\mu_I)$

see also T. D. Cohen (2004)

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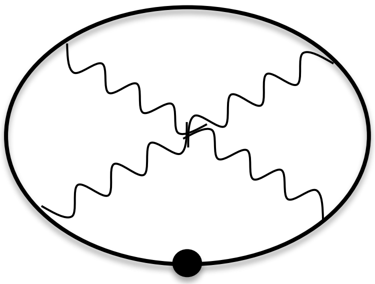

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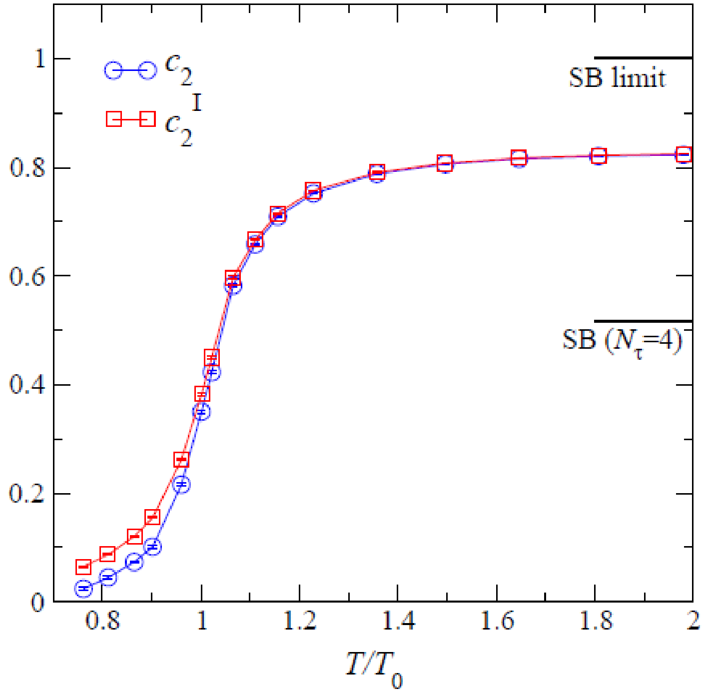
see also T. D. Cohen (2004)

- For gluonic observables, equivalence holds up to $O(N_c^{-2})$. D. Toublan (2005)
- Not applicable when u & d are mixed → MFA outside π condensation
- Orbifold equivalence: more systematic for a larger class of theories.

Lattice results

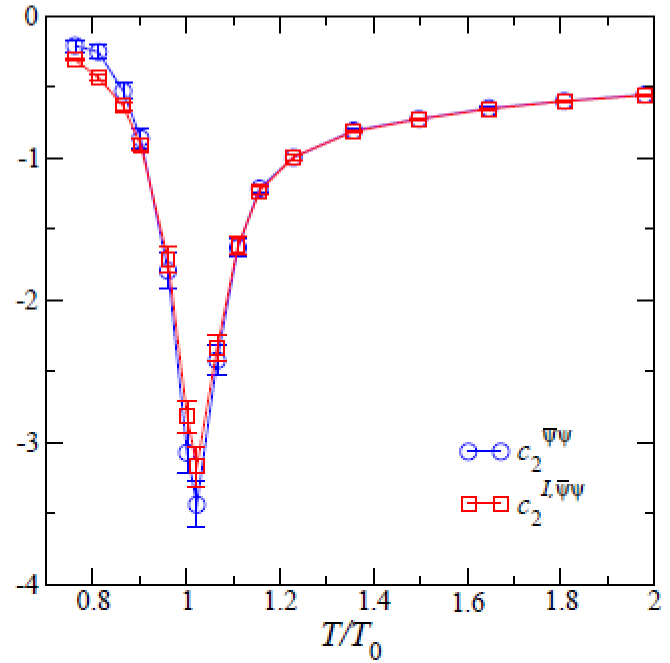
Pressure

$$\frac{p}{T^4} \equiv \Omega(T, \mu_q, \mu_q) = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n$$



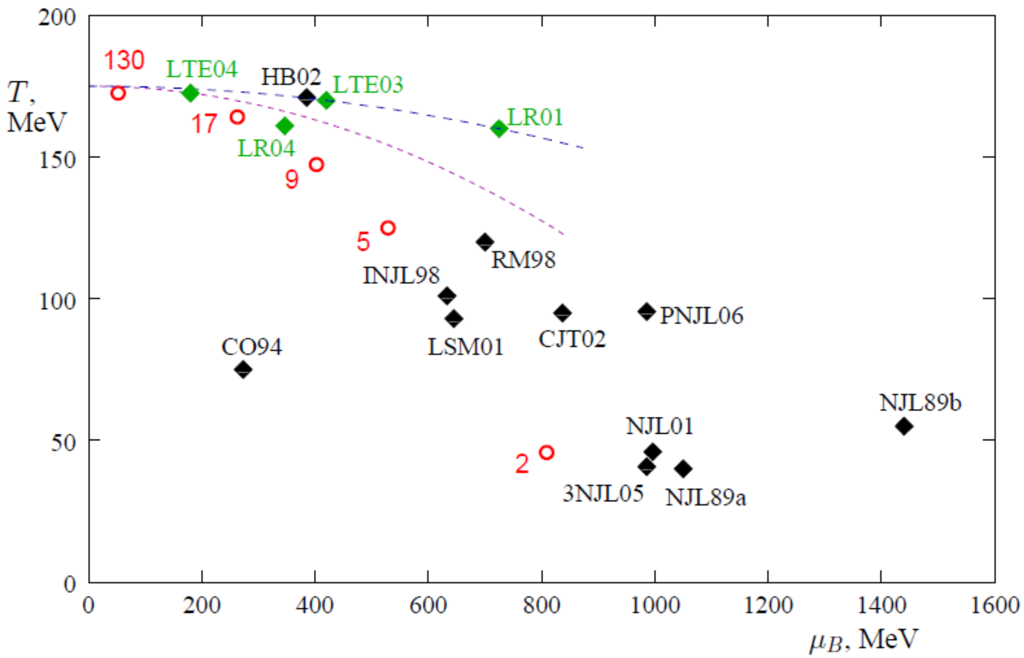
Chiral condensate

$$\frac{\langle\bar{\psi}\psi\rangle}{T^3} = \left(\frac{N_\tau}{N_\sigma}\right)^3 \frac{\partial \ln \mathcal{Z}}{\partial m/T} = \sum_{n=0}^{\infty} c_n^{\bar{\psi}\psi}(T) \left(\frac{\mu_q}{T}\right)^n$$

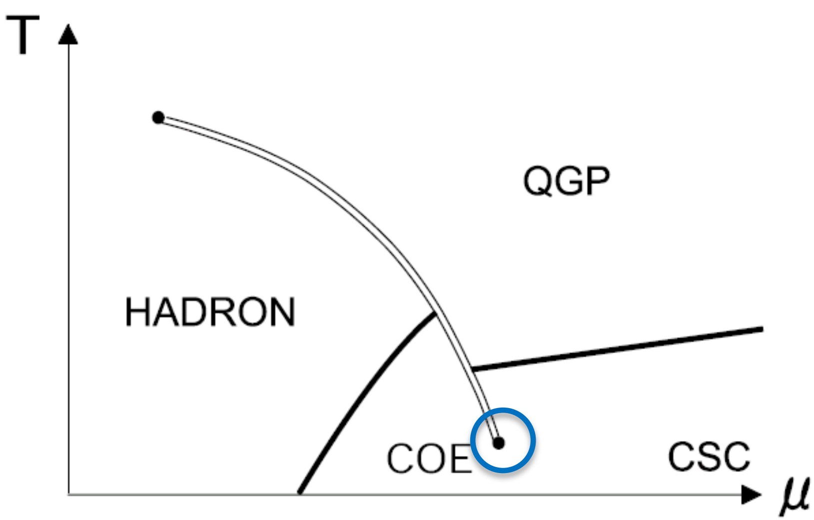


Allton *et al.* (Bielefeld-Swansea collaboration), 2005, PRD

Where are QCD critical point(s)?



Summarized in
M. Stephanov, hep-lat/0701002



Baym-Hatsuda-Tachibana-NY
(PRL, 2006)

A theorem

- $\pi^{\pm}$ are the lightest in QCD at finite μ_I Son-Stephanov (2001)

  ∵ QCD inequalities: $\pi^{\pm}$ propagator ≧ any meson propagator,

  which follows from $\tau_1\gamma_5 D\gamma_5\tau_1 = D^{\dagger}$ and Cauchy-Schwarz inequality.

- $m_\sigma \geqq m_\pi > 0$ for $m_q>0$ outside π condensation phase.

- QCD critical point ($\xi=\infty$ or $m_\sigma=0$) is prohibited there.

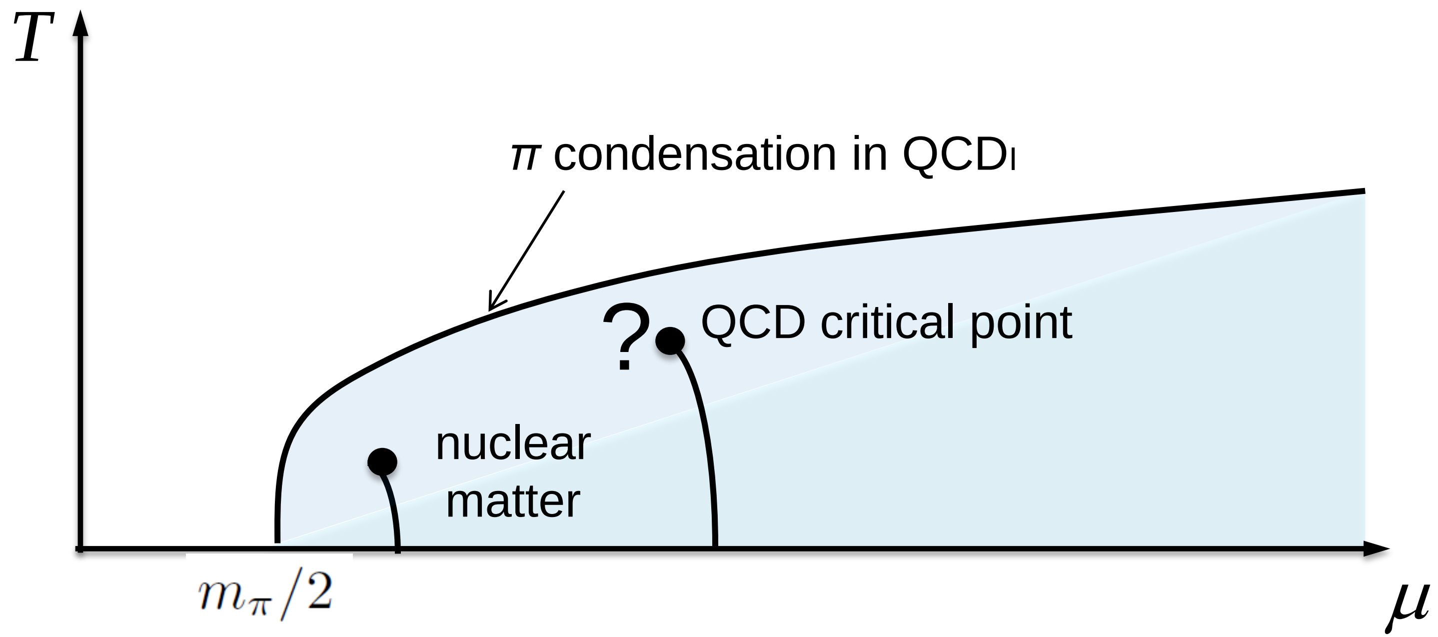
  → Via the orbifold equivalence, it also holds in QCD at finite μ_B.

  Hidaka-NY (PRL, 2012)

- The only assumption: suppression of disconn. diagrams (OZI rule)

A theorem

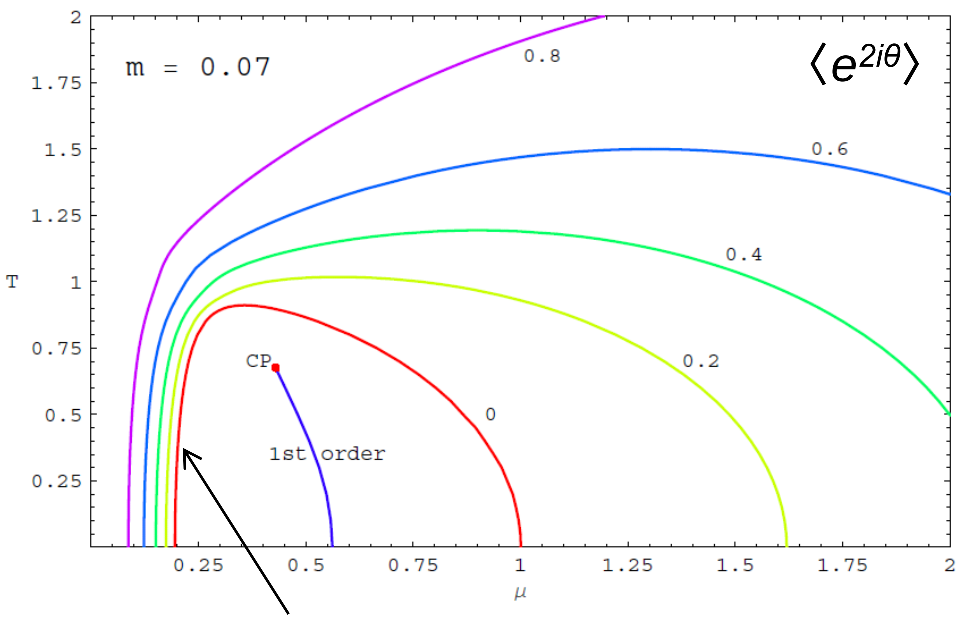
QCD critical point in QCD$_B$ can only be inside π condensation of QCD$_I$ where reweighting method breaks down.



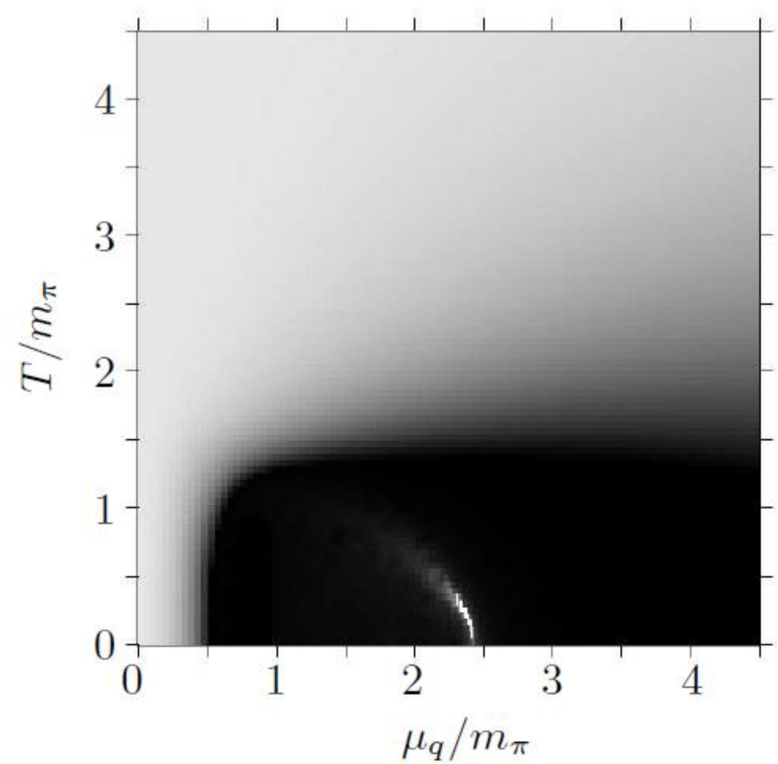
Model results (N_f=2)

Random matrix model: Han-Stephanov (2008)

NJL model: Andersen-Kyllingstad-Splittorff (2009)



onset of pion condensation phase

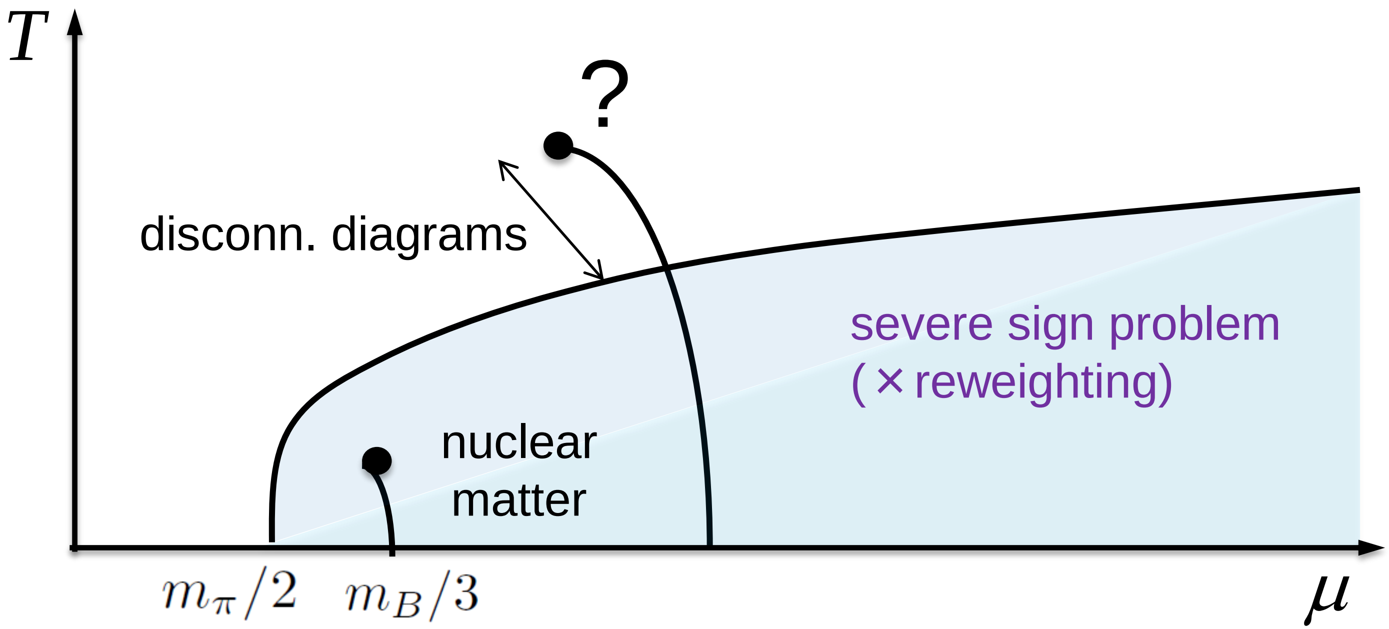


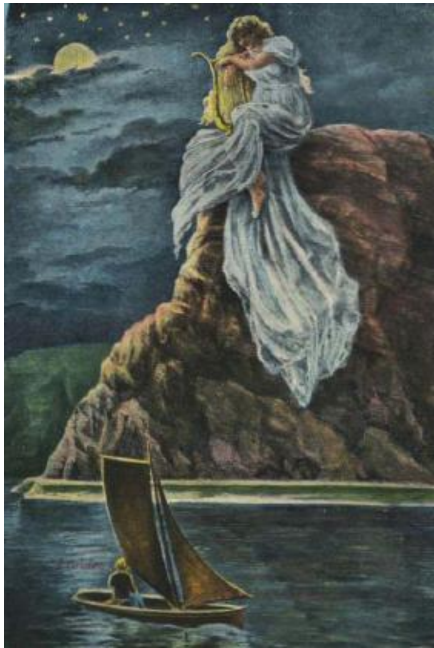
$$e^{2i\theta} = \frac{\det(D + \mu_q \gamma_0 + m)}{\det(D + \mu_q \gamma_0 + m)^*}$$

➢ Similar result in PNJL model: Sakai-Sasaki-Kouno-Yahiro (2010)

Summary & Outlook

- Universality of phase diagrams in QCD and QCD-like theories
- Importance of disconn. diagrams beyond MFA → DS & Functional RG?
- Many more applications of the orbifold equivalence.
- Challenge for theory: physics inside π condensation.





Back up slides

Discussion

- At MFA, chiral transition cannot be 1st outside π condensation.

  $\rightarrow$ only beyond-MFA effects can make it 1st.

Discussion

- At MFA, chiral transition cannot be 1st outside π condensation.

  → only beyond-MFA effects can make it 1st.

  ⇕ competition

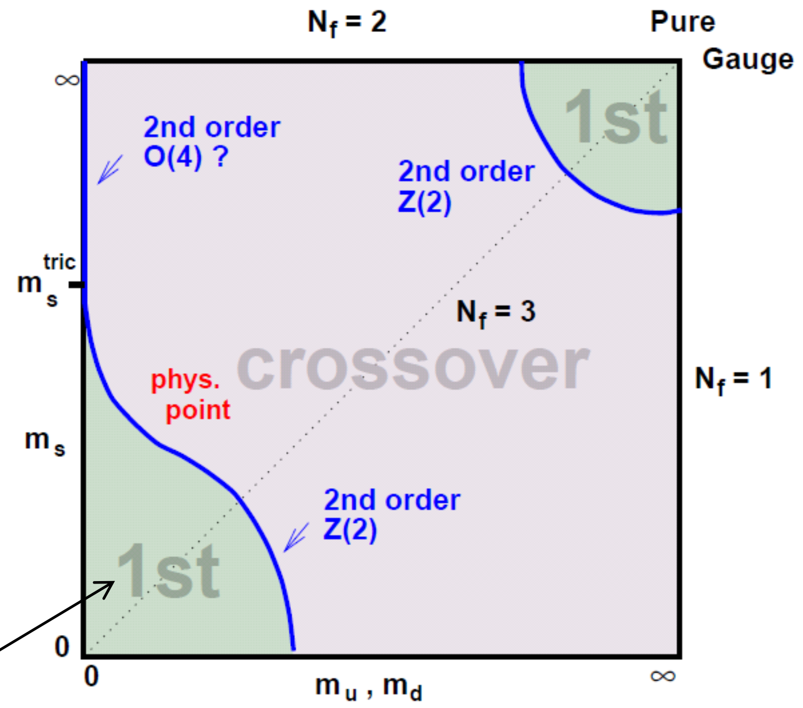
  quark mass weakens the chiral transition (1st → 2nd → crossover)

- At $\mu_B = 0$, chiral transition is crossover; beyond-MFA effects < quark mass.

- QCD critical point is accessible on the lattice (outside π condensation) if beyond-MFA effects are enhanced at $\mu_B \neq 0$ for some reason.

A counter example?

- Columbia plot:

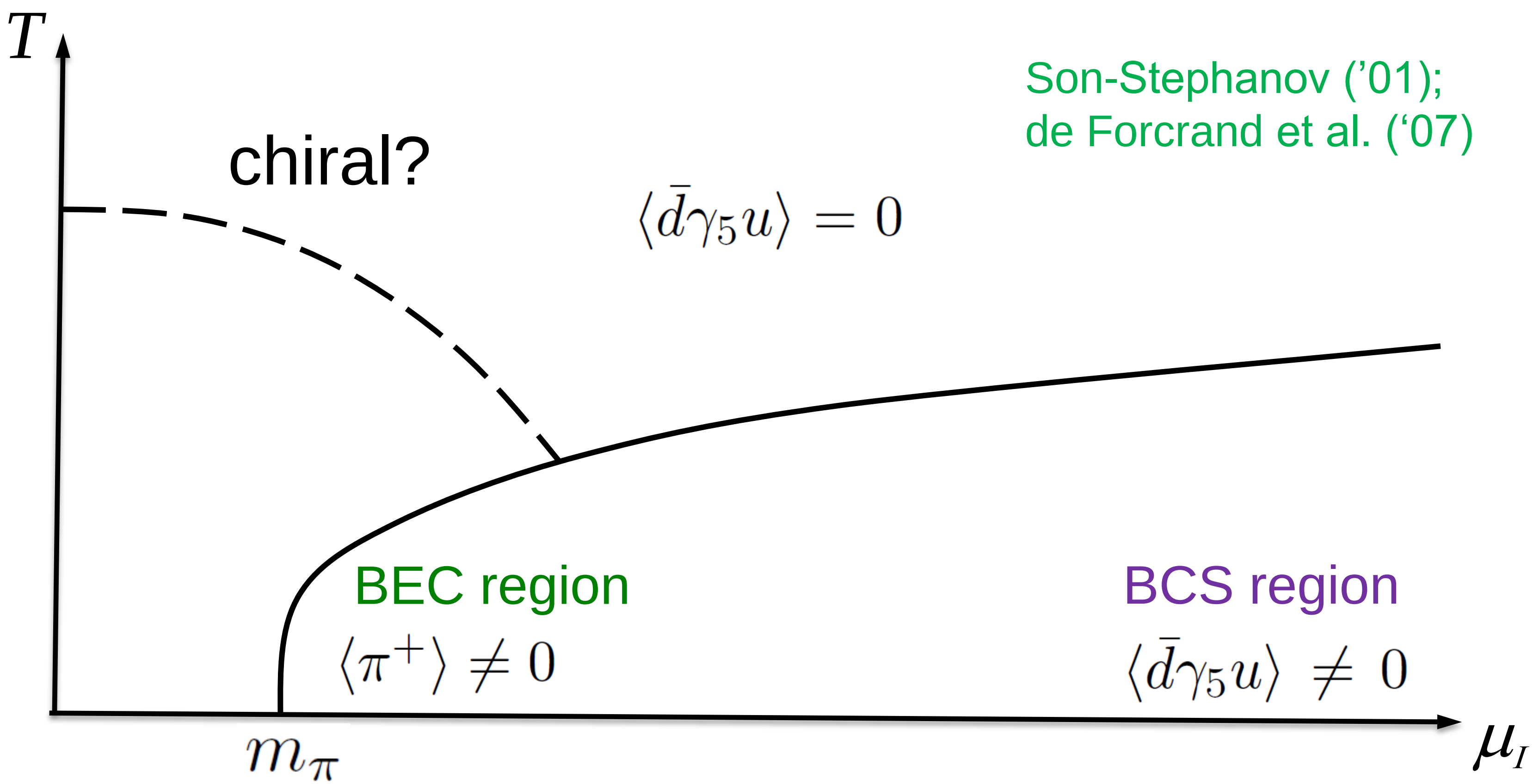


axial anomaly (disconnected diagrams)

- One can make the critical point on the T-axis (outside the pion condensation) by fine tuning m_q ∼ anomaly.

- Ignoring disconn. diagrams: anomaly < m_q

QCD at finite isospin density

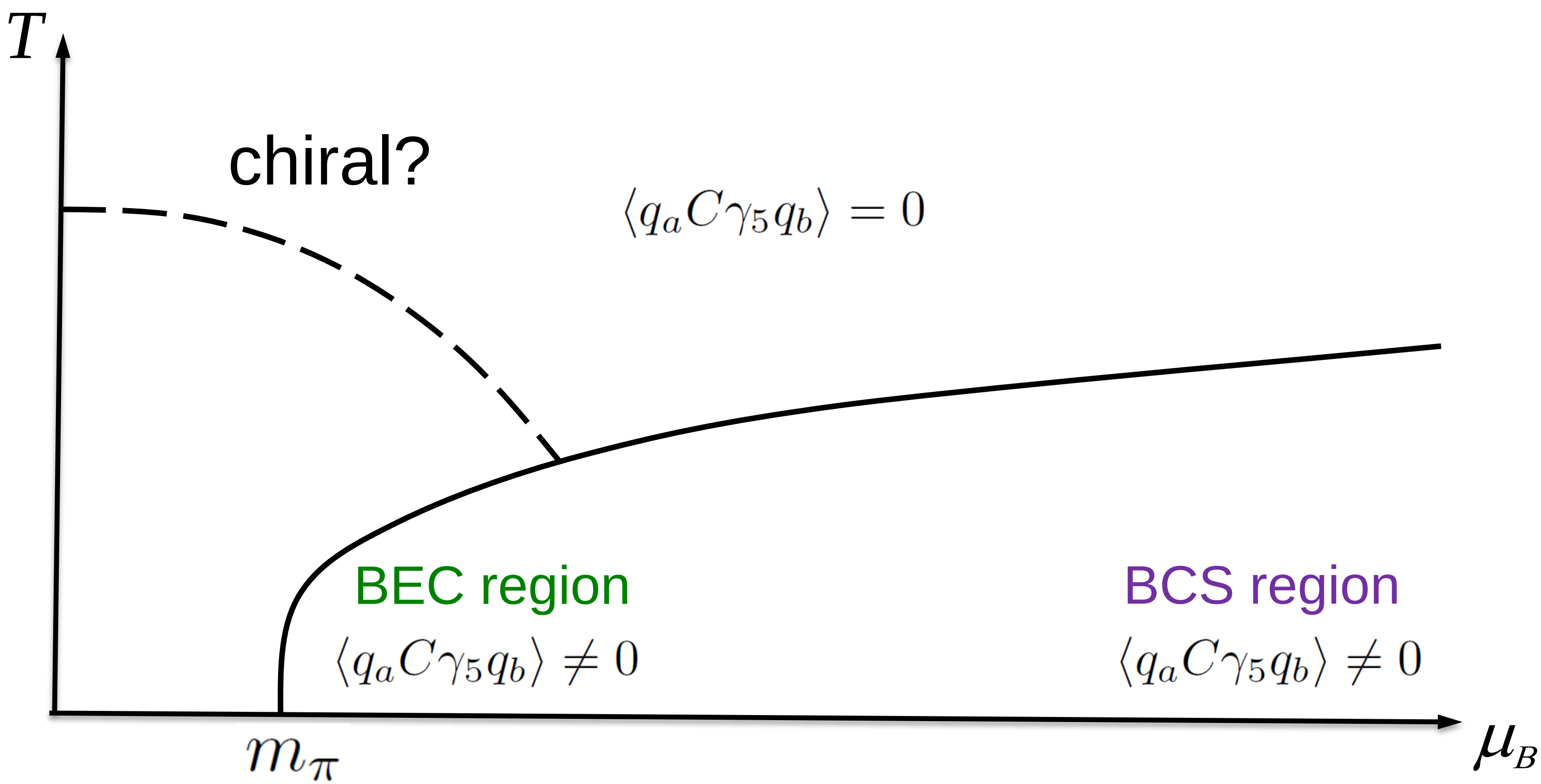


Finite μ_I=2μ: chemical potentials μ & -μ for *u* & *d* quarks

→ Dirac eigenvalues for *u* & *d* are complex conjugate.

→ Positive fermion determinant (no sign problem)

SO & Sp gauge theories at μ_B>0



SO(2N_c): real → positive fermion determinant (no sign problem)

(similarly for pseudo-real Sp(2N_c) with even N_f)

From SO(2N_c) to SU(N_c) at finite μ_I

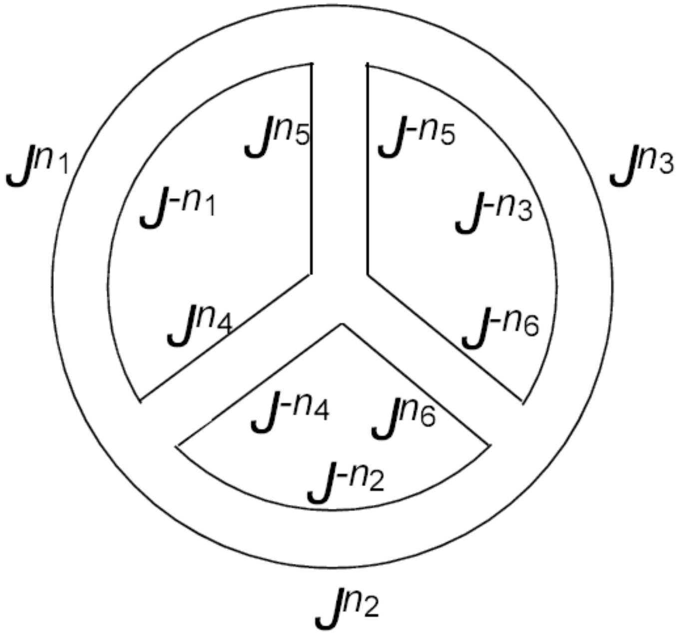
Start with SO(2N_c) gauge theory at μ_B>0.

1. Discrete symmetry: $J_c = -i\sigma_2 \otimes \mathbf{1}_{N_c} \in \mathrm{SO}(2N_c)$
$$J_i = -i\sigma_2 \otimes \mathbf{1}_{N_f/2} \in \mathrm{SU}(2)_{\mathrm{iso}}$$
2. Projection: $A_\mu^{\mathrm{SO}} = J_c A_\mu^{\mathrm{SO}} J_c^{-1}, \quad \psi^{\mathrm{SO}} = J_c \psi^{\mathrm{SO}} J_i^{-1}$
3. Daughter theory: U(N_c) ≈ SU(N_c) gauge theory at μ_I>0.
$$A_\mu^{\mathrm{proj}} = \begin{pmatrix} (A_\mu^{\mathrm{U}})^C & 0 \\ 0 & A_\mu^{\mathrm{U}} \end{pmatrix}, \quad \psi^{\mathrm{proj}} = \begin{pmatrix} \psi_+^{\mathrm{U}} \\ \psi_-^{\mathrm{U}} \end{pmatrix}$$
4. Orbifold equivalence: $\langle \bar\psi\psi \rangle^{\mathrm{SO}} = \langle \bar\psi\psi \rangle^{\mathrm{SU}}$ etc.

[Caution]

- $Z_4 \in \mathrm{SU}(2)_{iso}$ unbroken from SO(2N_c) to QCD at μ_I>0 everywhere.

Perturbative proof



Bershadsky-Johansen ('98)

- Insert $\mathcal{P}(A_\mu^{\mathrm{SO}}) = \frac{1}{2}\left(A_\mu^{\mathrm{SO}} + J_c A_\mu^{\mathrm{SO}} J_c^{-1}\right)$ for each propagator.
- Take the same 't Hooft coupling.
- Difference comes from color factors.
- Condition: $\mathrm{tr}(J_c^n) = 0$, when $J_c^n \neq \pm \mathbf{1}_{2N_c}$

$$\sum_{n_i=0,1} \left(\frac{1}{2}\right)^{N_P} \cdot \mathrm{tr}(J^{-n_1} J^{n_4} J^{n_5}) \cdot \mathrm{tr}(J^{-n_2} J^{-n_4} J^{n_6}) \cdot \mathrm{tr}(J^{-n_3} J^{-n_5} J^{-n_6}) \cdot \mathrm{tr}(J^{n_1} J^{n_2} J^{n_3})$$

$$= 2^{-6} \cdot 2^{6-3} \cdot 2^4 = 2$$

Generally, $2^{-N_P} \cdot 2^{N_P-(N_L-1)} \cdot 2^{N_L} = 2$ for any planar diagrams.

What are (aren't) equivalent?

Not all the quantities are equivalent in the orbifold equivalence.

- Projection symmetry must be unbroken.
- Observables must keep the projection symmetry (neutral).
- Symmetry breaking patterns, quantum numbers of the condensates can be different, but their magnitudes are the same.
- Example: BCS gap (inside the BEC-BCS crossover)

$$\Delta_{\mu_B}^{\mathrm{SU}} \sim \mu \exp\left(-\frac{\pi^2}{g}\sqrt{\frac{6N_c}{N_c+1}}\right) \longrightarrow 0$$

'*t* Hooft limit (large N_c, $g^2 N_c$ fixed)

$$\Delta_{\mu_I}^{\mathrm{SU}} \sim \mu \exp\left(-\frac{\pi^2}{g}\sqrt{\frac{6N_c}{N_c^2-1}}\right)$$

$$\Delta_{\mu_B}^{\mathrm{SO}} \sim \mu \exp\left(-\frac{\pi^2}{g}\sqrt{\frac{12}{2N_c-1}}\right) \longrightarrow \sim \mu \exp\left(-\pi^2\sqrt{\frac{6}{g^2 N_c}}\right)$$

$$\Delta_{\mu_B}^{\mathrm{Sp}} \sim \mu \exp\left(-\frac{\pi^2}{g}\sqrt{\frac{12}{2N_c+1}}\right)$$

QCD inequality

- Correlation function:

$$\begin{aligned} C_\Gamma(x,y) &\equiv \langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} \\ &= -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A \\ &= \langle \mathrm{tr}[S_A(x,y)\Gamma i\gamma_5 S_A^\dagger(x,y) i\gamma_5 \bar{\Gamma}]\rangle_A \\ &\leq \sqrt{\langle \mathrm{tr}[S_A S_A^\dagger]\rangle_A}\sqrt{\langle \mathrm{tr}[\Gamma\gamma_5 S_A^\dagger \gamma_5 \bar{\Gamma}(\Gamma\gamma_5 S_A^\dagger \gamma_5 \bar{\Gamma})^\dagger]\rangle_A} \\ &= \langle \mathrm{tr}[S_A(x,y) S_A^\dagger(x,y)]\rangle_A, \end{aligned}$$

$$\bar{\Gamma} \equiv \gamma_0 \Gamma^\dagger \gamma_0$$

$$S_A(x,y) \equiv \langle x|\mathcal{D}^{-1}|y\rangle$$

Inequality saturated when $\Gamma = i\gamma_5$.

- Cauchy-Schwarz inequality: $\mathrm{tr}(AB^\dagger) \leq \sqrt{\mathrm{tr}\, AA^\dagger}\sqrt{\mathrm{tr}\, BB^\dagger}$