Quark number susceptibilities in QCD via resummed perturbation theory

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Quarks, Gluons, and Hadronic Matter under Extreme Conditions

St. Goar, March 21, 2013.

Main references:
Jens Oluf Andersen, SM, Nan Su, and Aleksi Vuorinen, arXiv:1210.0912 (to be published in PRD)
JOA, SM, NS, AV, together with Michael Strickland, in preparation.

1. Introduction
   - Motivations, definitions
2. Hard-Thermal-Loop perturbation theory
   - Framework
   - Full 1-loop pressure
3. Dimensional Reduction within resummation
   - Formalism
   - 4-loop pressure, finite μ_f resummation
4. Results and comparison with Lattice
   - Choice of parameters
   - From 2nd to 6th order light QNS
   - Kurtosis: 4th/2nd order light QNS
5. Conclusion
   - Summary, perspectives

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Why hot and dense QCD via resummed perturbation theory?

- Connect lattice region to asymptotically high energies
- Qualitative understanding of plasma properties
- Show the usefulness of weak coupling methods for finite μ_f problems

Why diagonal quark number susceptibilities?

- Natural quantity for direct comparison with lattice
- Carry information about response of the system to nonzero μ_f

A few reminders:

- Implementing μ_f in (imaginary time formalism) thermal pQCD:
  $k_0^f = (2n+1)\pi T \to (2n+1)\pi T - i\,\mu_f$
- Quark number susceptibilities (QNS), as derivatives of the pressure:

$$\chi_{u_i\, d_j\, s_k \ldots}(T) \equiv \left.\frac{\partial^{i+j+k+\cdots}\, p(T,\{\mu_f\})}{\partial\mu_u^i\,\partial\mu_d^j\,\partial\mu_s^k \ldots}\right|_{\{\mu_f\}=0}$$

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- Reorganizes the perturbative series of thermal pQCD:

$$\mathcal{L}_{\rm HTLpt} \;=\; \Big(\mathcal{L}_{\rm QCD}+\mathcal{L}_{\rm HTL}\Big)\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}$$

- With the gauge invariant HTL improvement term:

$$\mathcal{L}_{\rm HTL} \;=\; -\frac{1}{2}(1-\delta)\, m_D^2\, {\rm Tr}\left(F_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y F^\mu_\beta\right)$$
$$+(1-\delta)\, i\sum_f^{N_f} m_{q_f}^2\, \bar\psi_f\, \gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi_f$$

- δ: Formal expansion parameter, $\delta = 1$ in the end $\to$ QCD!
  Count the # of HTL dressed loops
  $\Delta\mathcal{L}_{\rm HTL}$: HTL counterterm(s); y^μ: Lightlike 4-vector
  m_D/m_{q_f}: Debye/quark thermal mass parameters
- Adding $\mathcal{L}_{\rm HTL}$ "SHIFTS" the ground state around which one expands, to an ideal gas of thermal (massive!) quasiparticles

- At 1-loop, Debye m_D and quark thermal mass parameters m_{q_f}: identification with their weak coupling values $\Rightarrow m_D \sim m_{q_f} \sim gT$
- 1-loop truncated pressure through $\mathcal{O}\left((m_D/T)^5, (m_{q_f}/T)^5\right)$:
  JOA, SM, NS, and AV, [1210.0912]
  2-loop truncated pressure through $\mathcal{O}\left((m_D/T)^5, (m_{q_f}/T)^5\right)$, $\mathcal{O}\left((\mu_f/T)^4\right)$:
  Haque, Mustafa, and Strickland, [1212.1797]
- Here:
  Full 1-loop results, accurate to all orders in m_D/T and m_{q_f}/T
  Hence:
  1. No approximations $\rightarrow$ Exact (numerical) result!
  2. Refines the comparison between the truncated 1-loop HTLpt results [1210.0912], and lattice data, plus...
  3. ...gives access to higher order QNS!
  4. Probes quantitatively the convergence of the truncation, respect to the full results (pressure and QNS)

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- Separation of the pressure into different contributions:

$$p_{\rm QCD} \;=\; p_{\rm HARD} + T\, p_{\rm EQCD}(m_{\rm E}, \lambda, g_{\rm E})$$

Lagrangian density of the effective 3d Yang-Mills plus adjoint Higgs theory aka Electrostatic QCD (EQCD):

$$\mathcal{L}_{\rm EQCD} \;=\; g_3^{-2}\Big\{\frac{1}{2}\,{\rm Tr}\,[F_{ij}]^2 + {\rm Tr}\big[(D_i A_0)^2\big] + m_{\rm E}^2\,{\rm Tr}\,\big[A_0^2\big] + i\zeta\,{\rm Tr}\,\big[A_0^3\big] + \lambda_{\rm E}\,{\rm Tr}\,\big[A_0^4\big]\Big\} + \delta\mathcal{L}_{\rm E}$$

where $\delta\mathcal{L}_{\rm E}$ includes higher order operators.

- Note that:
  1. $p_{\rm hard}$: From hard modes ($\propto 2\pi T$), via strict loop-expansion in the 4d theory
  2. $p_{\rm soft}$: From soft modes ($\propto gT$), via effective EQCD

$\Rightarrow$ Gives the correct momentum scale contributions

- Known 4-loop pressure from Vuorinen, PRD **68**, 054017
- Natural resummation scheme (Laine and Schröder, PRD **73**, 085009)

  $\Rightarrow$ Keep the EQCD parameters unexpanded

  Straightforward application to finite μ_f case
  1. Substantially reduces the renormalization scale dependence!
     (by a factor larger than 5 for the 2nd order QNS)
  2. Resums a certain class of contributions to all orders
- No fitting: Unknown $\mathcal{O}\left(g^6\right)$ term in the pressure set to 0!

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- $N_c = 3$ and $N_f = 3$
- Strange mass: $m_s = 0$
- g perturbative running: HTLpt/DR $\rightarrow$ 1/2-loop
- QCD scale $\Lambda_{\overline{\text{MS}}}$:
  Matching 1/2-loop running to lattice value at reference scale 1.5 GeV
  (Bazavov et al., PRD **86** 114031)
  Gives $\Lambda_{\overline{\text{MS}}}^{\text{HTLpt/DR}} \rightarrow 176/283$ MeV, plus variation ± 30MeV
- Renormalization scale $\bar{\Lambda}$: Central value $\bar{\Lambda}_{\text{opt}} \approx 1.44 \times 2\pi T$
  Obtained by optimizing the EQCD gauge coupling g_3
  (Kajantie et al., Nucl. Phys. B **503** 357).
  $\bar{\Lambda}$ is then varied by a factor of 2 around the central value.

FULL 2nd order light QNS, PRELIMINARY

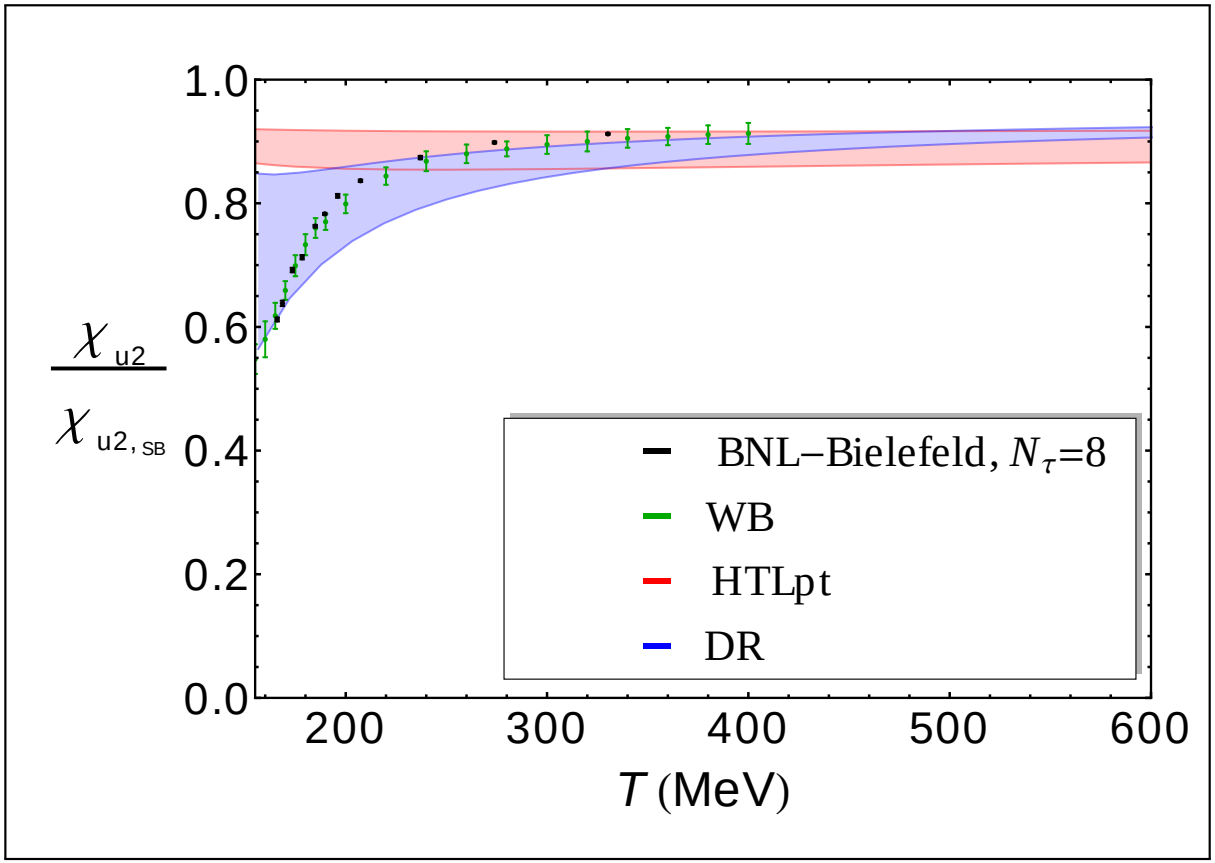

Figure : BNL-B data from Schmidt [1212.4278], [1212.4283]. WB data from Bórsanyi et al., JHEP **1201** 138.

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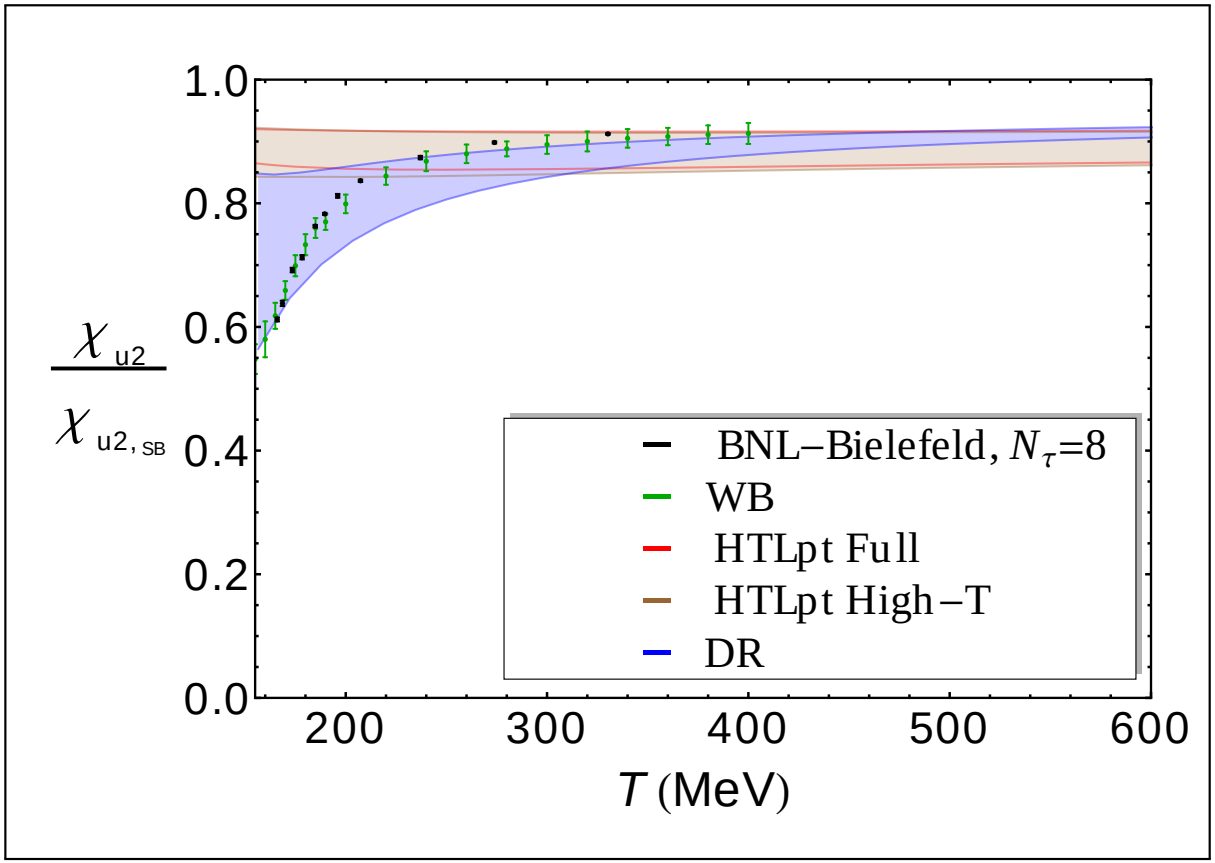

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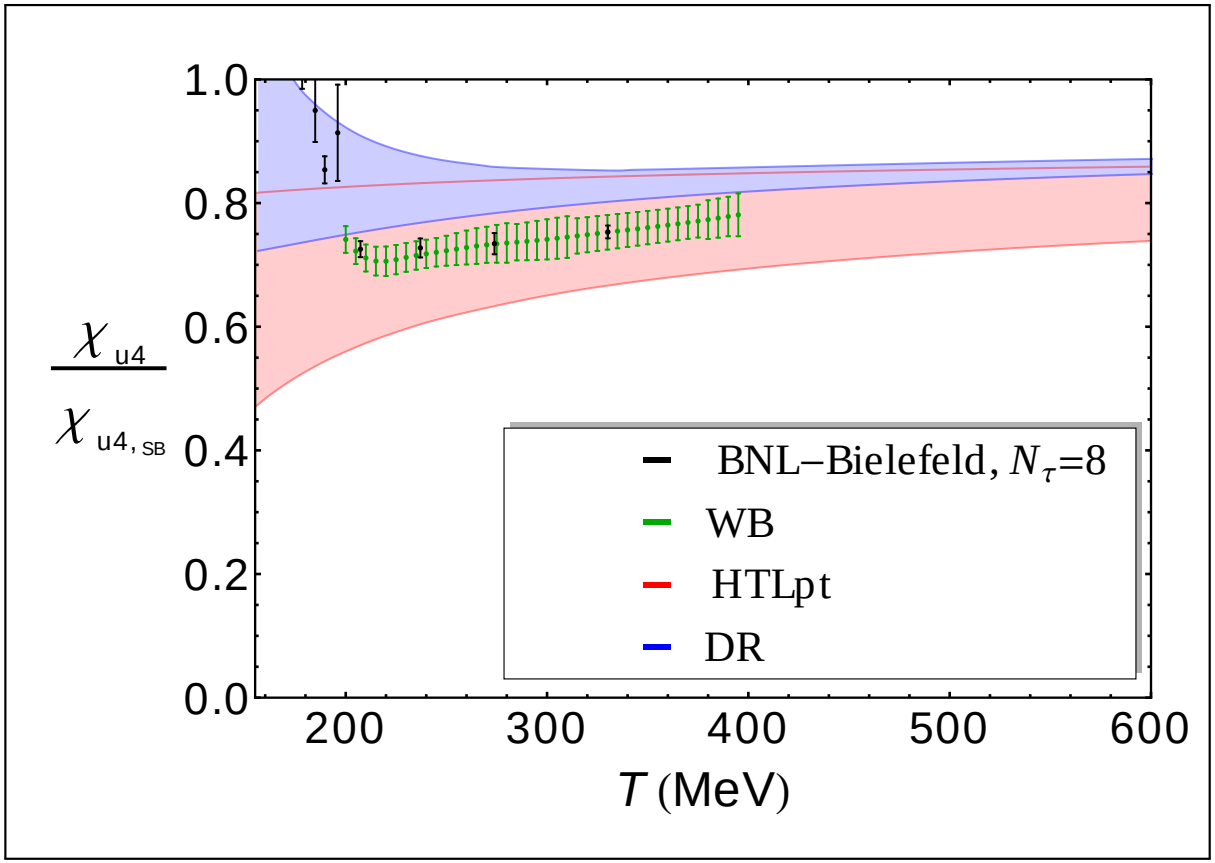

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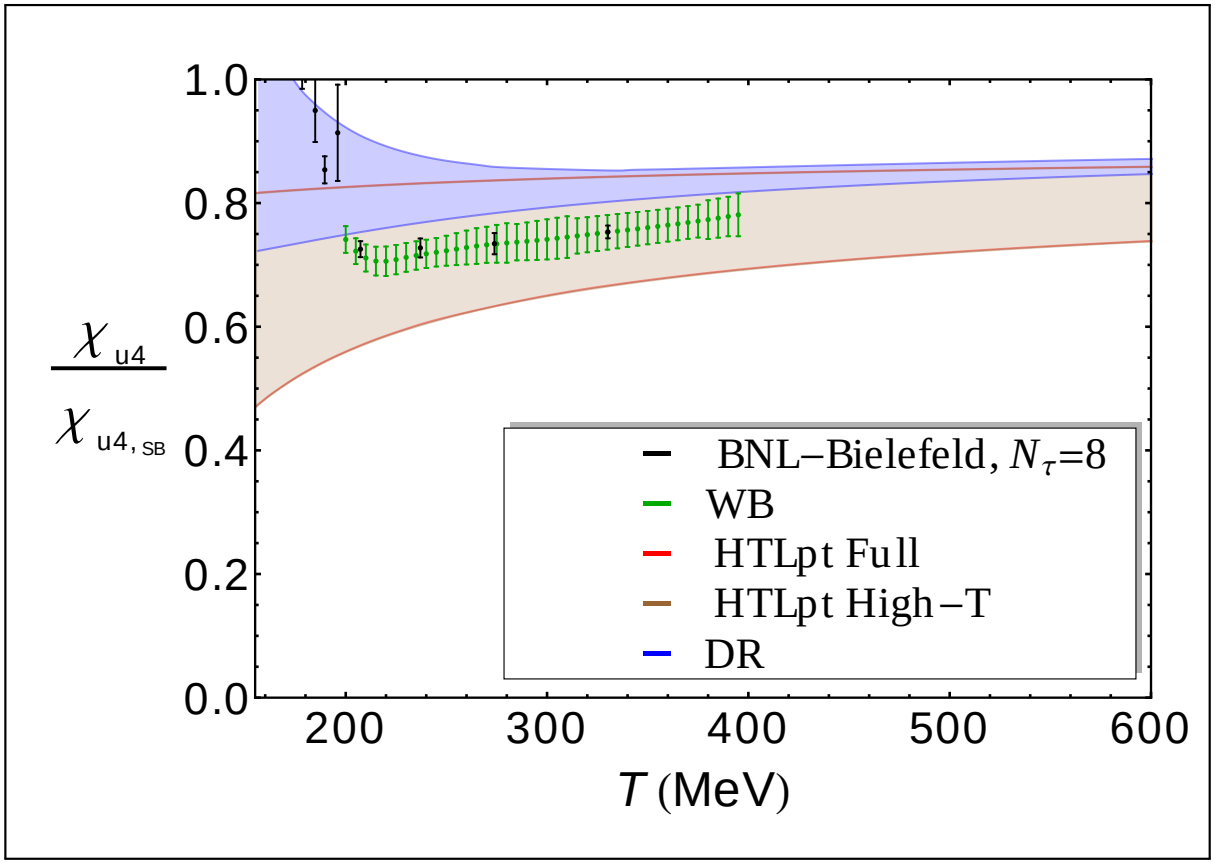

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FULL 6th order light QNS, (very!) PRELIMINARY

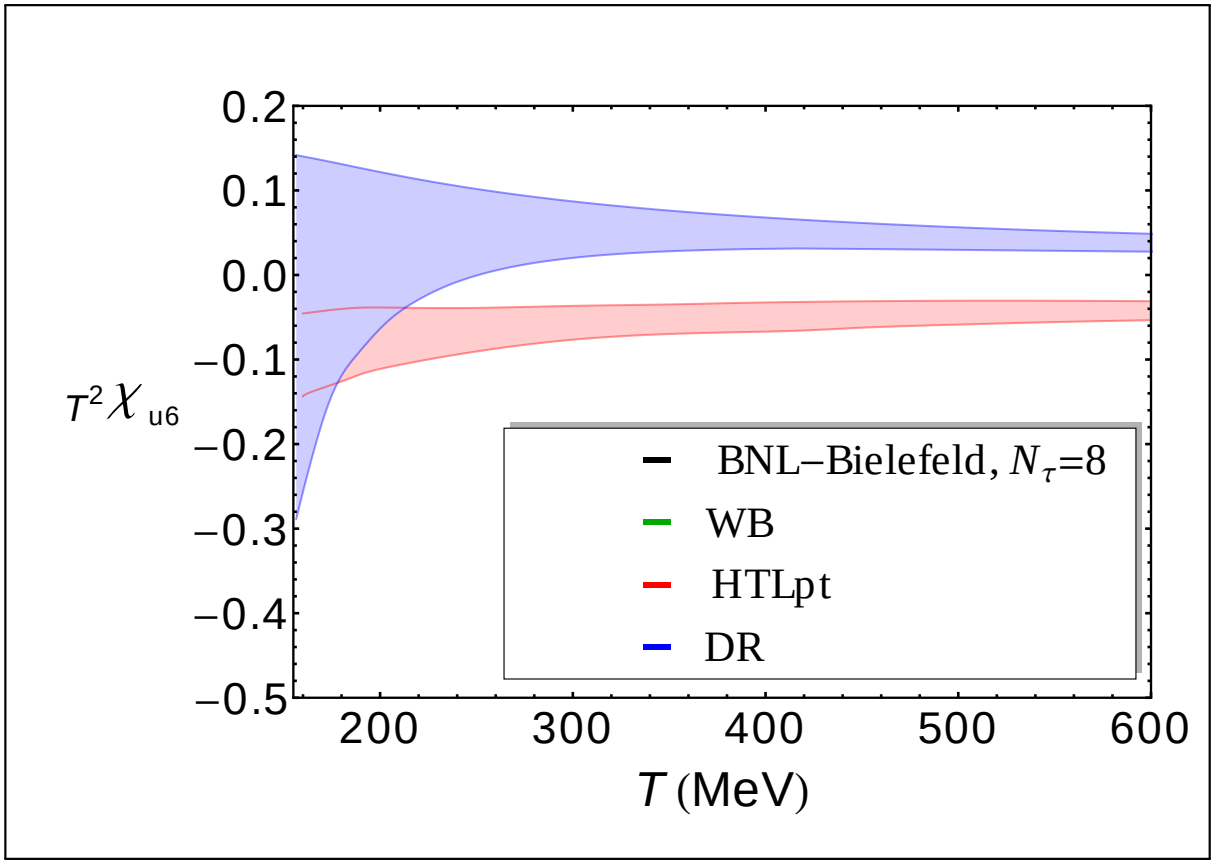
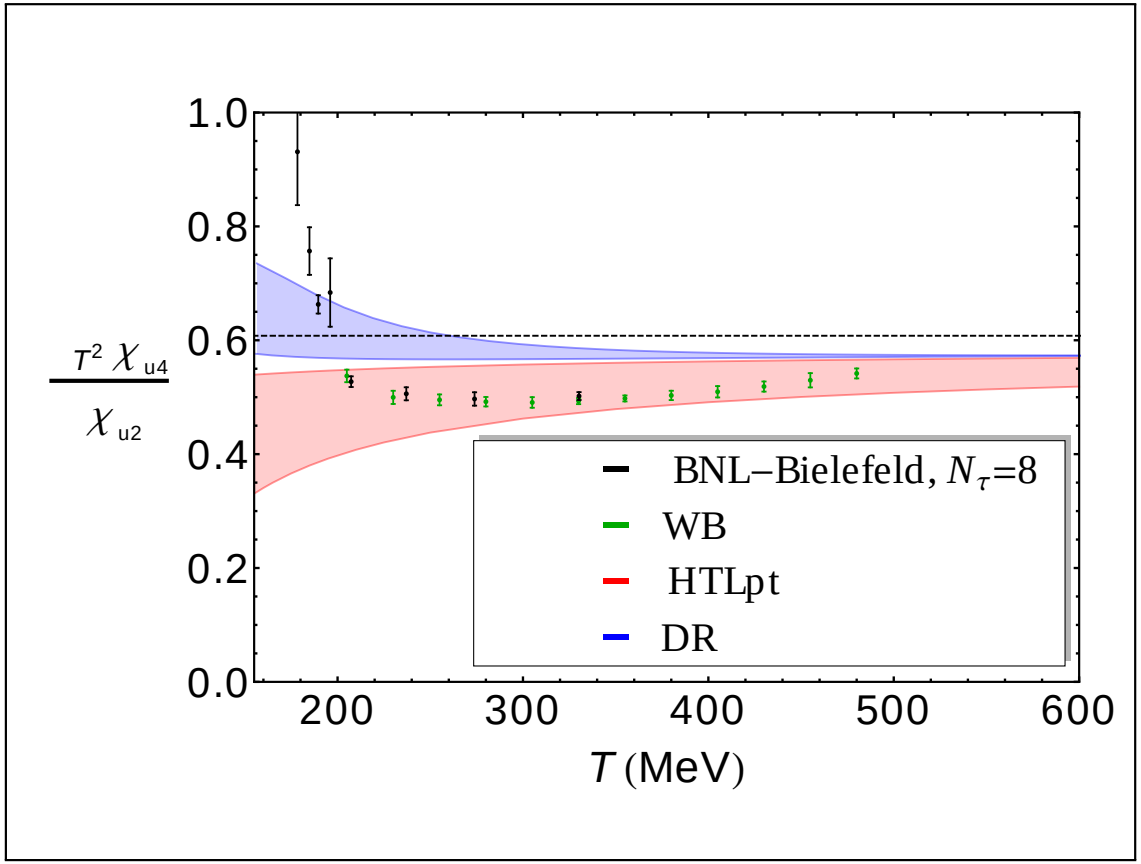


Figure : No lattice data yet.

FULL 4th/2nd light QNS, PRELIMINARY



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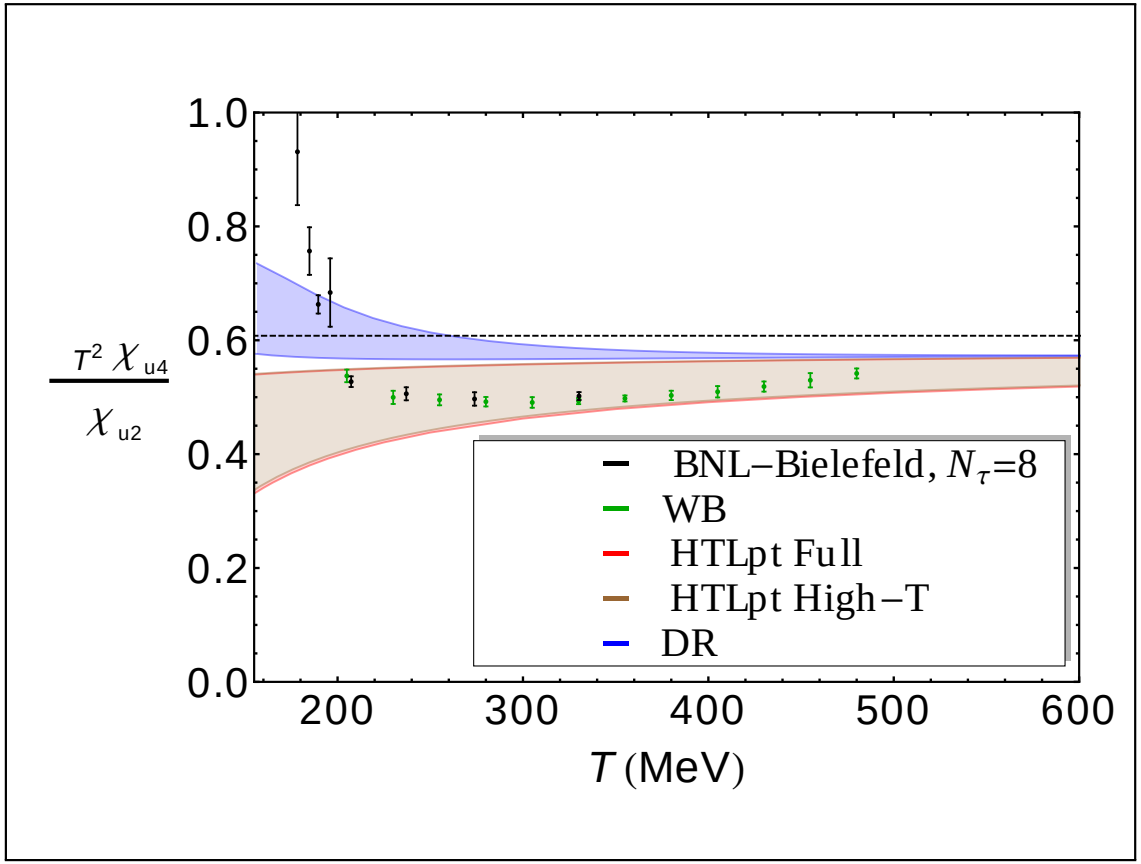

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- Lattice data agree with full 1-loop HTLpt for:
  The 2nd, 4th order light QNS, hence for the kurtosis...
  ...over a wide range of temperatures, down to $T \approx 200$ MeV!

However,

1. 2-loop truncated correction seems to ruin the agreement
2. $\Rightarrow$ need of the 3-loop correction to conclude

- Slight but visible discrepancy, between lattice data and 4-loop DR...
  ...below $T \approx 500$ MeV

However,

1. Possible artefact of the resummation which drastically reduce the theoretical uncertainty
2. Possible sizable effect of the unknown $\mathcal{O}\left(g^6\right)$ term in the pressure...

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- THANKS FOR YOUR ATTENTION!

- Type of full 1-loop sum-integral to compute, via branch cuts:

$$
\begin{aligned}
p_{\mathrm{q_f}}(T,\boldsymbol{\mu}) &= 2\sum\!\!\!\!\!\!\int_{\widetilde{K}_f} \log\left[A_{\mathrm{S}}^2(\widetilde{K}_f) - A_0^2(\widetilde{K}_f)\right] \\
A_0(\widetilde{K}_f) &\equiv i\,\widetilde{\omega}_n + \mu_f - \frac{m_{q_f}^2}{i\,\widetilde{\omega}_n + \mu_f}\;\widetilde{\mathcal{T}}_{\mathrm{K}}(i\,\widetilde{\omega}_n + \mu_f, k) \\
A_{\mathrm{S}}(\widetilde{K}_f) &\equiv k + \frac{m_{q_f}^2}{k}\left[1 - \widetilde{\mathcal{T}}_{\mathrm{K}}(i\,\widetilde{\omega}_n + \mu_f, k)\right] \\
\widetilde{\mathcal{T}}_{\mathrm{K}} &\equiv \frac{\Gamma\left(\frac{3}{2}-\epsilon\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma(1-\epsilon)}\int_0^1 \mathrm{d}c\,(1-c^2)^{-\epsilon}\,\frac{(i\,\widetilde{\omega}_n+\mu_f)^2}{(i\,\widetilde{\omega}_n+\mu_f)^2 - k^2c^2} \\
&= {}_2F_1\left(\frac{1}{2},1;\frac{3}{2}-\epsilon;\frac{k^2}{(i\,\widetilde{\omega}_n+\mu_f)^2}\right)
\end{aligned}
$$

where:

$$\widetilde{\omega}_n \equiv (2n+1)\pi T$$

- 1-loop HTLpt mass parameters prescription:
  Identification to Debye and quark thermal mass weak coupling values

$$
\begin{aligned}
m_D^2 &\equiv \frac{g^2}{3}\left[\left(N_c+\frac{N_f}{2}\right)T^2+\frac{3}{2\pi^2}\sum_g^{N_f}\mu_g^2\right] \\
m_{q_f}^2 &\equiv \frac{g^2}{4}\,\frac{N_c^2-1}{4\,N_c}\left(T^2+\frac{\mu_f^2}{\pi^2}\right)
\end{aligned}
$$