Yang-Mills Thermodynamics from the Gribov Region

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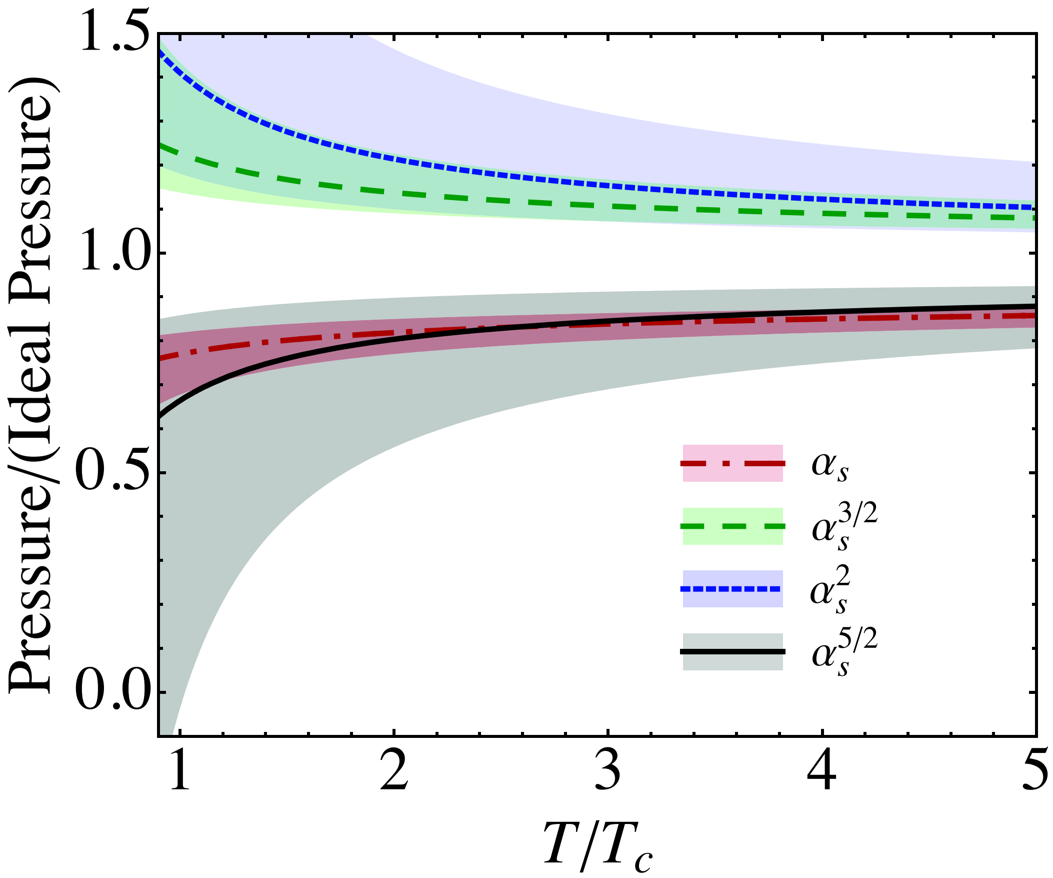
Introduction - Perturbative Yang-Mills free energy



Perturbative YM free energy with vs temperature.

$\pi T \le \mu \le 4\pi T$ and $\alpha_s = g^2/4\pi$

- Weak-coupling expansion of YM free energy is known to $\alpha_s^3 \log \alpha_s$.[1–7]
- At the RHIC energy $T \sim 0.35\,\mathrm{GeV}$, $\alpha_s^{\mathrm{PT}} \sim 0.3$ or $g^{\mathrm{PT}} \sim 2$.
- Convergence is reached when $\alpha_s \lesssim 1/20 \rightarrow T \gtrsim 10^5\ \mathrm{GeV}$...
- RESUMMATION IS NEEDED!

[1] Shuryak, 78.

[2] Kapusta, 79.

[3] Toimela, 85.

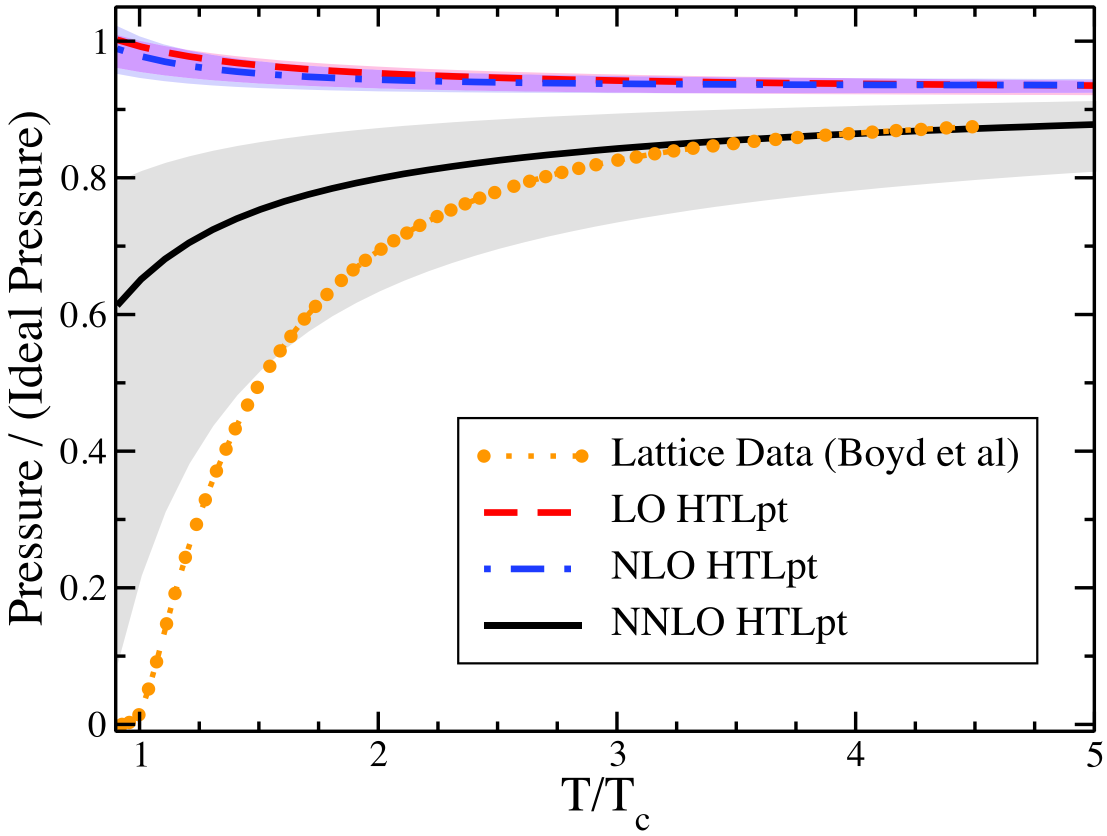
[4] Arnold, Zhai, 94/95.

[5] Kastening, Zhai, 95.

[6] Braaten, Nieto, 96.

[7] Kajantie, Laine, Rummukainen, Schröder, 02.

Introduction - HTLpt Yang-Mills pressure



HTLpt YM free energy with vs temperature.

$\pi T \leq \mu \leq 4\pi T$ and $\alpha_s = g^2/4\pi$

- Resummation of electric gluons $\sim gT$.
- HTLpt is one candidate. [1,2,3]. 3-loop free energy agrees with lattice down to $3\,T_c$.
- Similar findings from DR [4] and DRSPT [5].
- Due to Linde problem, PT breaks down at 4 loops!
- MAGNETIC GLUONS ($\sim g^2T$) ARE MISSING!

[1] Andersen, Braaten, Strickland, 99/00.

[2] Andersen, Braaten, Strickland, 02.

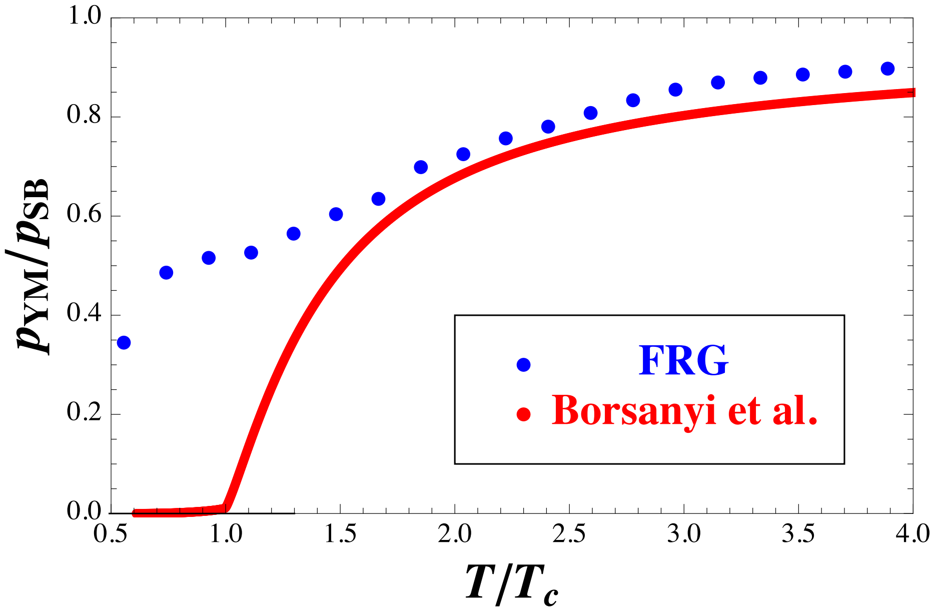
[3] Andersen, Strickland, NS, 09/10.

[4] Hietanen, Kajantie, Laine, Rummukainen, Schröder, 08.

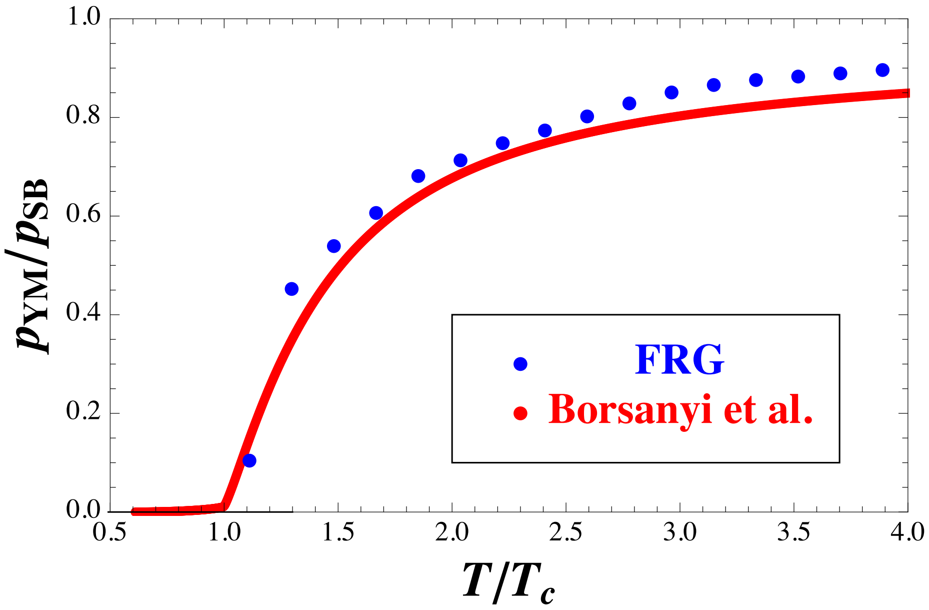
[5] Blaizot, Iancu, Rebhan, 03.

Introduction: Our quest!

FRG pressure w/o PL



FRG pressure w PL



Fister and Pawlowski, this workshop

Polyakov loop only affects the phase transition regime till $\sim 1.7\, T_c$!

Can we achieve similar goal with resummed PT when PL is irrelevant?

Gauge fixing - Is Faddeev-Popov enough?

- In gauge theories, only gauge inequivalent configurations should be counted in path integral.
- For QED, gauge fixing is archived by Faddeev-Popov method
  - Picking up a gauge condition by constructing

$$1 = \int \mathcal{D}\xi(x)\, \delta\left(G(A^{\xi})\right) \det\left(\frac{\delta G(A^{\xi})}{\delta\xi}\right) \quad \text{with} \quad A_{\mu}^{\xi} = A_{\mu} + \frac{1}{e}\partial_{\mu}\xi$$

  - Inserting the constructed “1” to path integral

$$Z = \left(\int \mathcal{D}\xi(x)\right) \int \mathcal{D}A(x)\, \delta\left(G(A^{\xi})\right) \det\left(\frac{\delta G(A^{\xi})}{\delta\xi}\right) e^{-S_{\mathrm{YM}}}$$

- YM is non-Abelian, is FP good enough in gauge fixing? NO!!!

Gauge fixing - Gribov copies (Gribov, 78)

- For YM, after FP there are still residue gauge transformations, i.e. Gribov copies.
- String theory is similar: After the first gauge condition, a second one is needed to kill residue gauge transformations.
- YM functional integral should be restricted to Gribov region

$$\Omega \equiv \{\ A : \partial_i A_i = 0,\ \text{and}\ \ -D_i(A)\partial_i \geq 0\ \}$$

- With m_G the Gribov mass which breaks conformal symmetry,

$$E_G(\mathbf{p}) \ = \ \sqrt{\mathbf{p}^2 + \frac{m_G^4}{\mathbf{p}^2}} \quad \text{(Gribov dispersion relation)}$$

- Reduction of physical state space in IR as a feature of confinement mechanism. (Gribov 78; Feynman, 81; Zwanziger 97)

Propagators from Gribov region in Landau gauge

Gribov's scenario has confinement features built in:

- Gluons are IR suppressed

$$D_A = \left\langle A^a_\mu(P) A^b_\nu(-P) \right\rangle = \delta^{ab} \frac{P^2}{P^4 + m_G^4} \left(\delta_{\mu\nu} - \frac{P_\mu P_\nu}{P^2} \right)$$

- Ghosts are IR enhanced

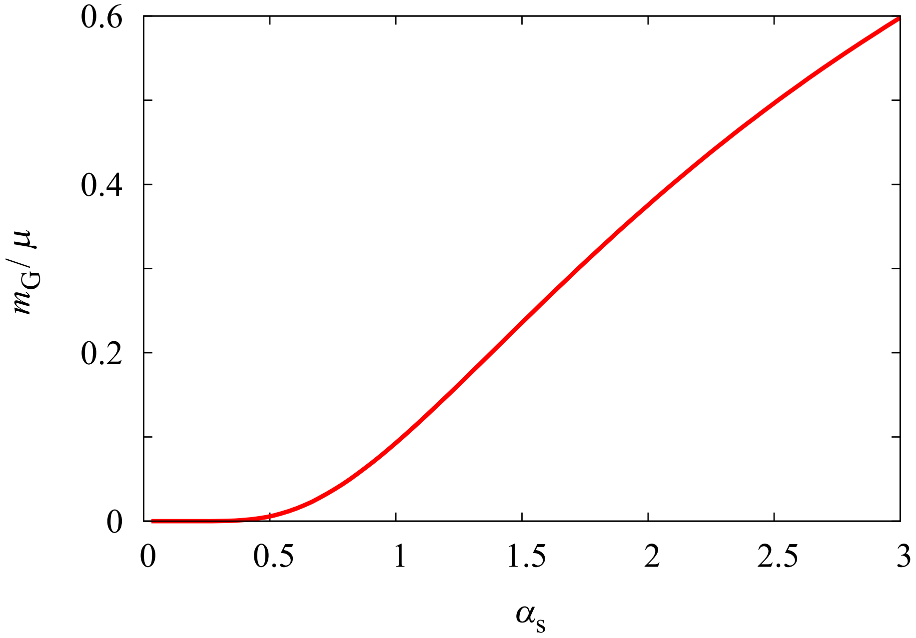
$$D_c = \left\langle \bar{c}^a(P) c^b(-P) \right\rangle = \frac{\delta^{ab}}{P^2} \frac{1}{1 - \sigma(P)} \approx \frac{\delta^{ab}}{P^4} \frac{128\pi m_G^2}{N_c g^2} , \quad (P \to 0)$$

- In line with lattice and functional methods.
- m_G is solved from gap eq: $\frac{d}{d+1} N_c g^2 \int_P \frac{1}{P^4 + m_G^4} = 1$
- Gap eq at $T \to \infty$: $m_G \sim g^2 T$ (Zwanziger, 07)

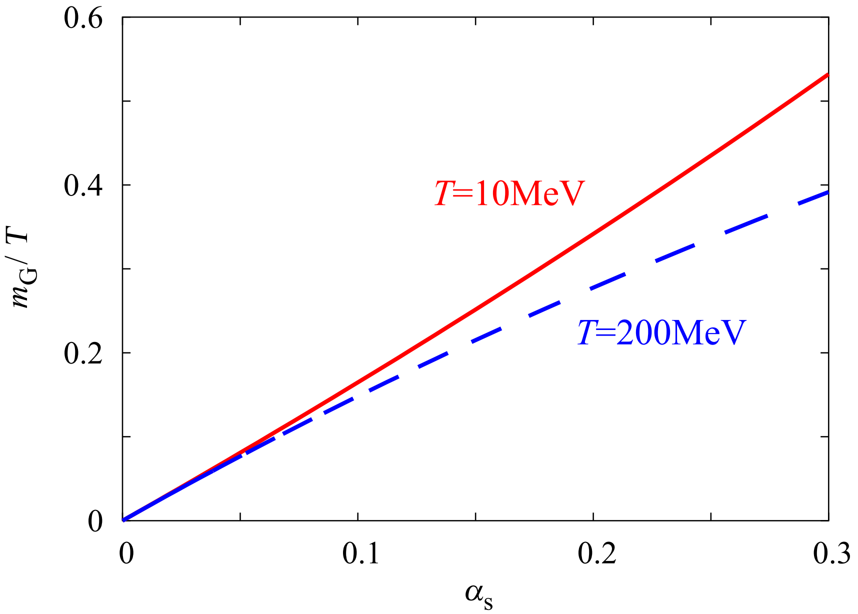
Gap eq at finite T

- dim-reg with $\overline{\mathrm{MS}}$: $\frac{3N_c g^2}{64\pi^2}\left[\frac{5}{6}-\ln\left(\frac{m_G^2}{\mu^2}\right)+\frac{8}{m_G^2}\int_0^\infty dp\, p^2 \mathrm{Im}\frac{n_\mathrm{B}(\omega_-)}{\omega_-}\right]=1$
- $T=0$: $m_G^2=\mu^2\,\exp\left(\frac{5}{6}-\frac{64\pi^2}{3N_c g^2}\right)$ & $T\to\infty$: $m_G\to\frac{d}{d+1}\frac{N_c}{4\sqrt{2}\pi}g^2 T$

$T=0$

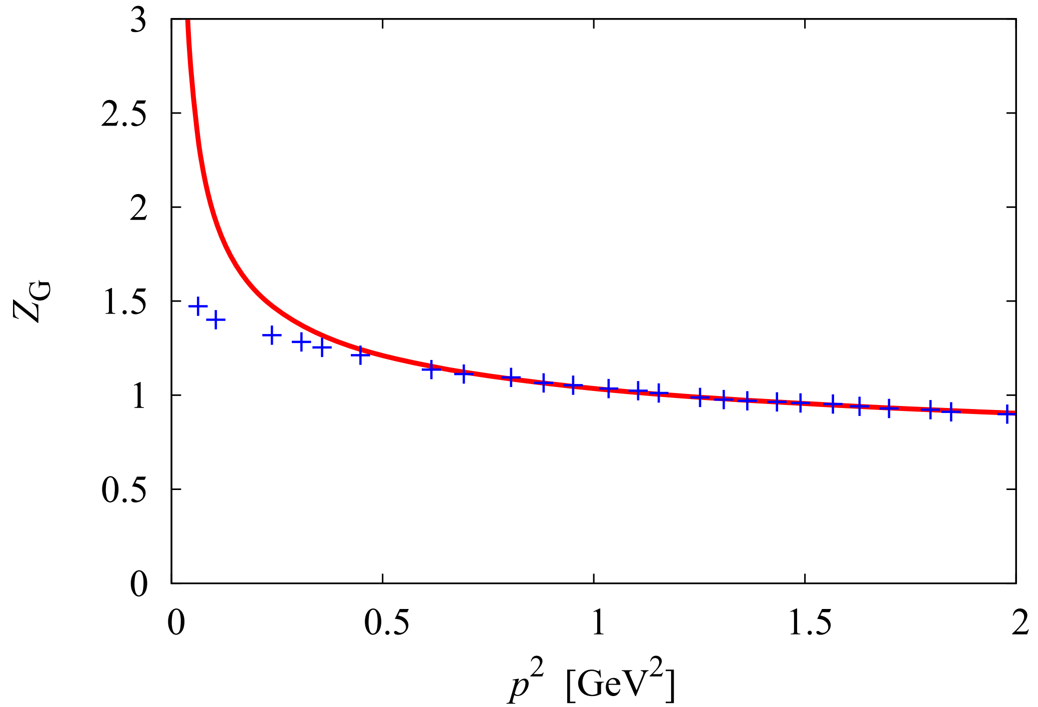


Finite T



Renormalization

- Assuming ghosts T indep. (obser. from lat.), $[1-\sigma(P)]_{T=0}$ reads $\frac{N_c g^2}{128\pi^2}\left\{\pi\frac{P^2}{m_G^2}-5+\left(3-\frac{m_G^4}{P^4}\right)\log\left[1+\frac{P^4}{m_G^4}\right]+2\left(3-\frac{P^4}{m_G^4}\right)\frac{m_G^2}{P^2}\arctan\frac{P^2}{m_G^2}\right\}$
- With $\alpha_{s0}=1.14$ and $m_{G0}^2=0.0053\,\mathrm{GeV}^2$, $\mu^2=0.31\,\mathrm{GeV}^2$ fixed!



Lattice (Blue): Aouane, *et al.*, arXiv:1108.1735.

Gribov pressure in a nutshell

- LO 2PI (quasiparticle approximation)

$$\Gamma = \frac{1}{2}\mathrm{tr}\log G^{-1} - \frac{1}{2}\mathrm{tr}\log(G^{-1} - G_0^{-1})G + \Gamma_2[G]$$

- Gluon loop

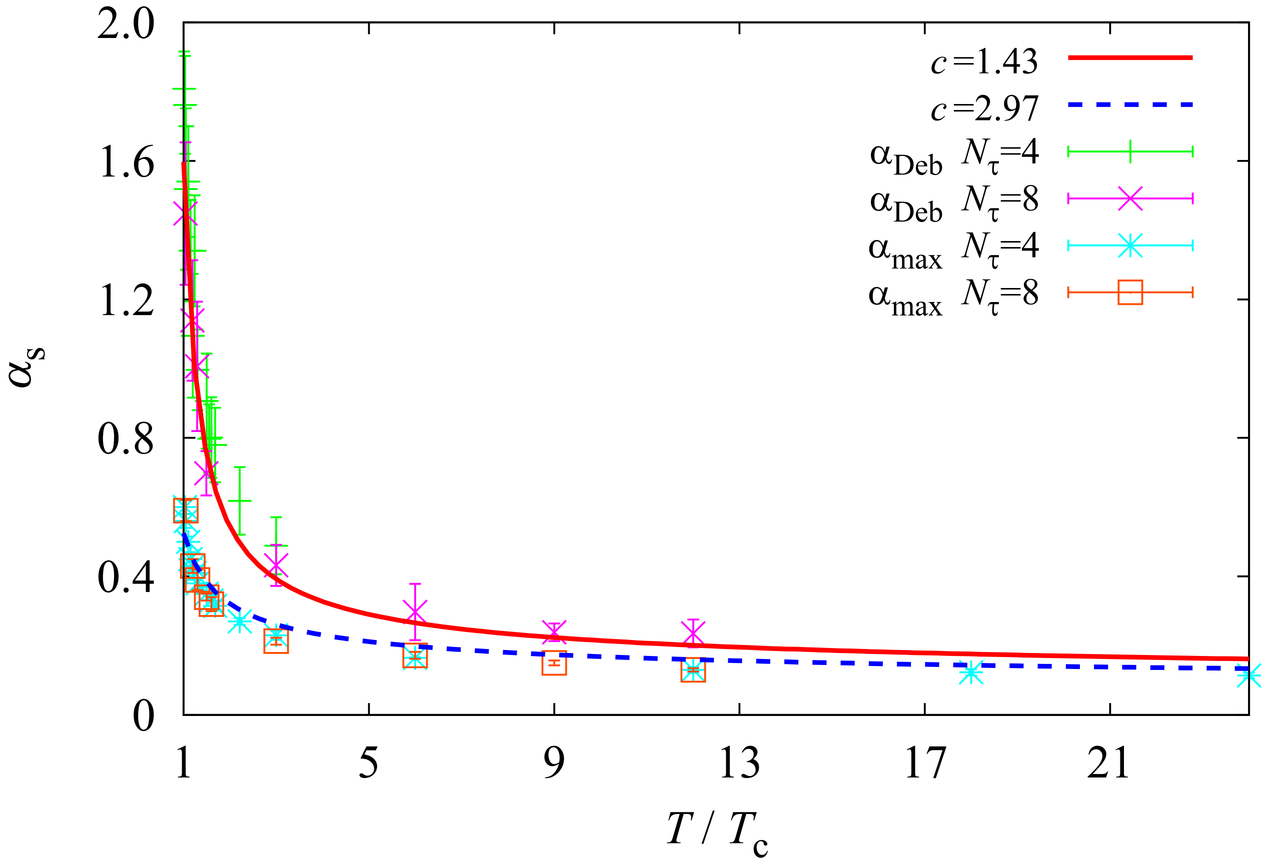
$$\frac{1}{2}\mathrm{tr}\log D_A^{-1} = \frac{1}{2}(N_c^2 - 1)\sum\!\!\!\!\!\!\!\int_P \left\{3\log\frac{P^4 + m_G^4}{P^2} + \ln P^2\right\}$$

- Ghost loop

$$-\mathrm{tr}\log D_c^{-1} = -(N_c^2 - 1)\sum\!\!\!\!\!\!\!\int_P \left\{\log P^2 + \log\left[1 - \sigma(P)\right]_{T=0}\right\}$$

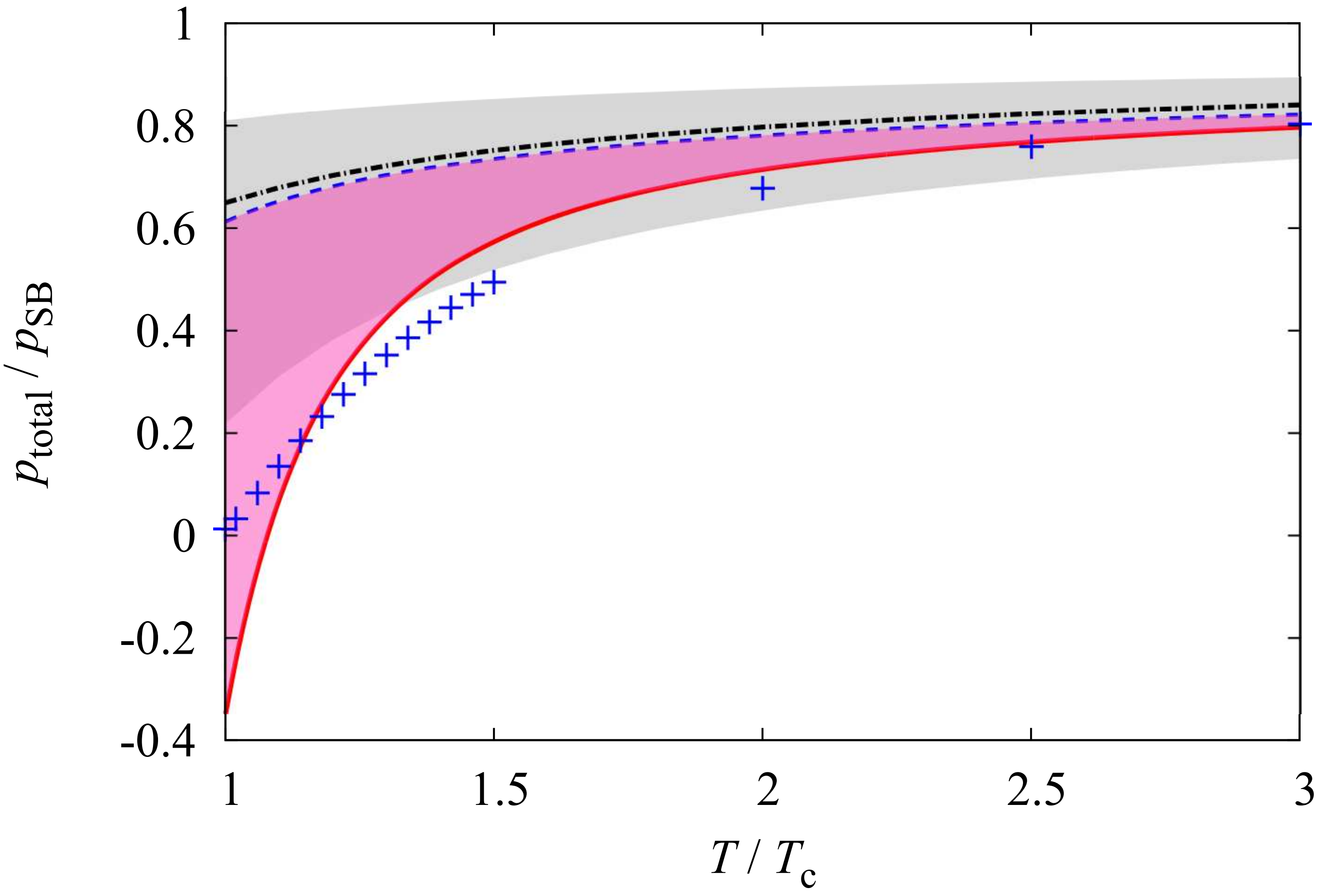
Running coupling

- Pert. running is not applicable near T_c!
- $\alpha_s(T)$ from lattice (fit: $\alpha_s(T/T_c) = \frac{6\pi}{11N_c \log[c(T/Tc)]}$)



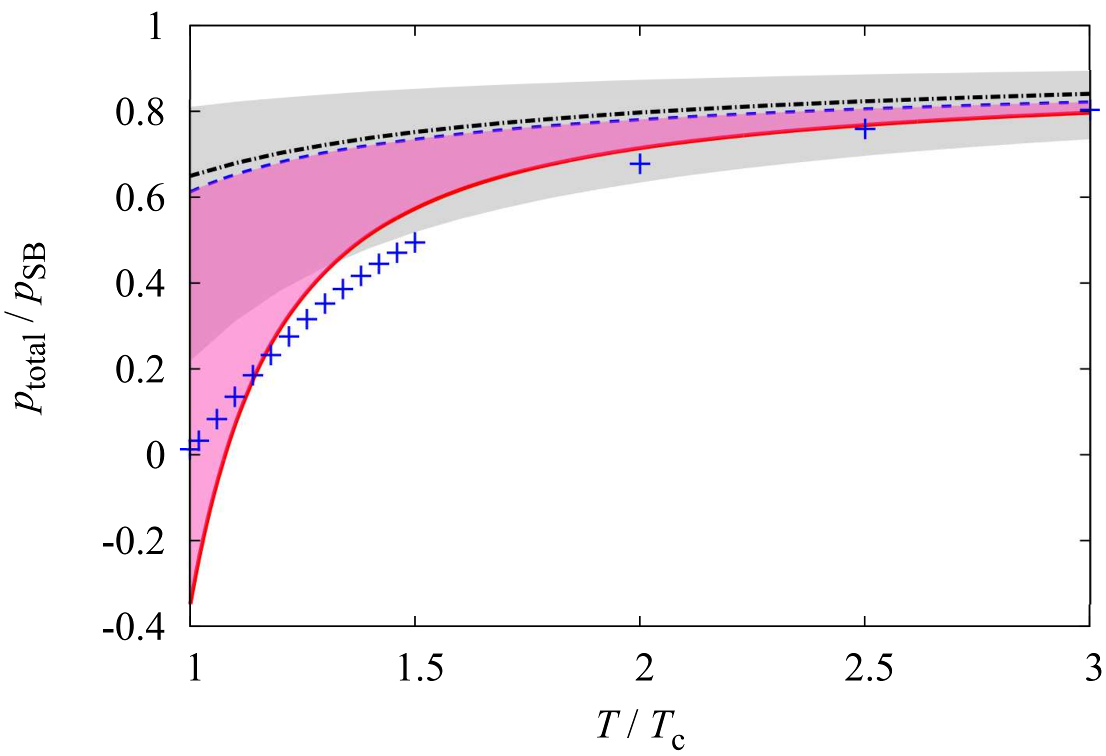
Lattice: Kaczmarek *et al.*, hep-lat/0406036

Gribov pressure (preliminary)



Lattice (Blue): W-B, arXiv:1204.6184; HTLpt (grey): Andersen, Strickland, NS, arXiv:0911.0676

Discussions



Lattice (Blue): W-B, arXiv:1204.6184

HTLpt (grey): Andersen, Strickland, NS, arXiv:0911.0676

- In good agreement with FRG pressure w/o PL! (Accident or not?)
- Lattice uncertainty highly suppressed above $2\,T_c$, ROBUST!
- Contributions to pressure:
  - T: $\sim T^4$
  - g^2T: $\sim m_G^3 T \sim g^6 T^4$ nonpert.!
  - T & g^2T mixed...
- "Resummed PT" works till $\sim 2\,T_c$(?)

Outlook

- Gribov's scenario provides a scheme to tackle the nonpert. magnetic scale analytically (partially?).
- Systematic finite T studies of the local renormalizable Gribov-Zwanziger action is need!
- What is m_G? Any relation to magnetic mass / glueball / $Z(N_c)$?
- Electric and magnetic screening masses from $\Pi^{\mu\nu}$. ($m_{\rm mag} \neq 0?$)
- Including Polyakov loop to attack phase transition? (Fukushima, Kashiwa, arXiv:1206.0685)
- Impact on realtime dynamics: no soft/collinear divergences?...
- How about the electroweak sector? (Sorella et al., arXiv:1210.4734, arXiv:1212.1003)

Backup

Linde problem - A missing ingredient in PT

(Linde, 80; Gross, Pisarski and Yaffe, 81)

- Contribution of zero Matsubara mode to a vacuum diagram with N 4-g vertices in the thermodynamic potential

$$\sim \; g^{2N} \left[T \int_p \frac{1}{p^2+\Lambda^2} \right]^2 \left[T \int_p \frac{1}{(p^2+\Lambda^2)^2} \right]^{N-1}$$

$$\sim \; g^{2N} (T\Lambda)^2 \left(\frac{T}{\Lambda} \right)^{N-1}$$

$$\sim \; g^6 T^4 \left(\frac{g^2 T}{\Lambda} \right)^{N-3} , \qquad \text{with } \Lambda \text{ an IR cutoff.}$$

- $\Pi^{00}(0, \mathbf{p} \to 0) \sim g^2 T^2$: Screening of the static electric fields generates the scale gT (soft), perturbative.
- $\Pi^{ij}(0, \mathbf{p} \to 0) = 0$: Absence of Λ greater than $g^2 T$ (ultrasoft) for the static magnetic fields makes that scale nonperturbative!