Flavor hierarchy in the QCD deconfinement transition

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Motivation

✧ We live in a very exciting era to understand the fundamental constituents of matter and the evolution of the Universe

✧ We can create the deconfined phase of QCD in the laboratory

✧ Lattice QCD simulations have reached unprecedented levels of accuracy
  ➞ physical quark masses
  ➞ several lattice spacings → continuum limit

✧ The joint information between theory and experiment can help us to shed light on QCD
The **deconfined phase** of QCD can be reached in the laboratory

Need for **unambiguous observables** to identify the phase transition

- susceptibilities of conserved charges (baryon number, electric charge, strangeness)
  

A rapid change of these observables in the vicinity of $T_c$ provides an unambiguous signal for deconfinement

These observables are sensitive to the **microscopic structure of the matter**

- non-diagonal correlators give information about **presence of bound states** in the QGP

They can be measured **on the lattice** as combinations of **quark number susceptibilities**
The observables under study

✦ The chemical potentials are related:

\[
\begin{align*}
\mu_u &= \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q; \\
\mu_d &= \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q; \\
\mu_s &= \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S.
\end{align*}
\]

✦ Susceptibilities are defined as follows:

\[
\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}.
\]

✦ Here we concentrate on the quadratic susceptibilities

\[
\chi^X_2 = \frac{1}{VT^3} \langle N^2_X \rangle
\]

✦ and on the correlators between different charges

\[
\chi^{XY}_{11} = \frac{1}{VT^3} \langle N_X N_Y \rangle.
\]
Physical meaning

♦ Diagonal susceptibilities measure the response of quark densities to an infinitesimal change in the chemical potential

\[ \chi^X_2 = \frac{\partial^2 p/T^4}{\partial (\mu_X/T)^2} = \frac{\partial}{\partial (\mu_X/T)} \left( n_X / T^3 \right) \]

➤ A rapid increase of these observables in a certain temperature range signals a phase transition

♦ Non-diagonal susceptibilities measure the correlation between different quark flavors

\[ \chi^{XY}_{11} = \frac{\partial^2 p/T^4}{\partial (\mu_X/T) \partial (\mu_Y/T)} = \frac{\partial}{\partial (\mu_Y/T)} \left( n_X / T^3 \right) \]

➤ They can provide information about bound-state survival above the phase transition
susceptibilities of conserved charges

\[ N_f = 2 + 1 \] dynamical quark flavors

\[ m_s / m_{u,d} \approx 28 \]

WB collaboration, JHEP 2012
Results: light quark susceptibilities

\[ \chi_u = \left. \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_u \partial \mu_u} \right|_{\mu_i=0} \]

✧ quark number susceptibilities exhibit a rapid rise close to \( T_c \)

✧ at large \( T \) they reach \( \sim 90\% \) of the ideal gas limit
Results: nondiagonal susceptibilities

\[ \chi^{us}_{11} = \chi^{ds}_{11} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_u \partial \mu_s} \bigg|_{\mu_i=0} \]

- non-diagonal susceptibilities look at the linkage between different flavors
- they exhibit a strong dip in the vicinity of \( T_c \)
- they vanish in the QGP phase at large temperatures
Results: strange quark susceptibilities

\[ \chi_2^s = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_s^2} \bigg|_{\mu_i=0} \]

✧ quark number susceptibilities exhibit a rapid rise close to \( T_c \)

✧ at large \( T \) they reach \( \sim 90\% \) of the ideal gas limit
strange quark susceptibilities have their rapid rise at larger temperatures compared to the light quark ones

they rise more slowly as a function of $T$

There is a difference of $\sim 15 - 20$ MeV between the inflection points of the two curves
Simple experimental verification

- Yields of strange particles should be *enhanced* relative to yields of non-strange particles.
- Yields of strange particles should result in a *higher temperature* than yields of non-strange particles when fitted with a statistical hadronization model (SHM).

![Graph showing particle ratios and data from ALICE experiment.](image_url)

R. Preghenella
for ALICE
SQM 2012
arXiv:1111.7080
Caveats

❖ Lattice results for susceptibilities are first-principle calculations

➢ However, $T_c$ cannot be univocally defined

❖ The experimental results are fitted by means of the Statistical Hadronization Model

❖ It would be nice to have a direct comparison between first-principle calculations and experimental results
Higher order susceptibilities and ratios

- susceptibilities are defined as follows:

$$\chi_{BSQ}^{l mn} = \frac{\partial^{l+m+n} p / T^4}{\partial (\mu_B/T)^{l} \partial (\mu_S/T)^{m} \partial (\mu_Q/T)^{n}}.$$ 

- we are now interested in fourth order susceptibilities ($\chi_4$) and in particular in ratios $\chi_4 / \chi_2$

- Ratios have a very peculiar shape which allows to unambiguously spot the transition

- They can be directly related to an experimental measurement: no need for model interpretation!
Relating lattice results to experimental measurement

- the first four cumulants are:

\[
\begin{align*}
\chi_1 &= \langle(\delta x)\rangle \\
\chi_2 &= \langle(\delta x)^2\rangle \\
\chi_3 &= \langle(\delta x)^3\rangle \\
\chi_4 &= \langle(\delta x)^4\rangle - 3\langle(\delta x)^4\rangle^2
\end{align*}
\]

- we can relate them to higher moments of multiplicity distributions:

\[
\begin{align*}
\text{variance} : \sigma^2 &= \chi_2 \\
\text{standard deviation} : \sigma &= \sqrt{\chi_2} \\
\text{skewness} : S &= \frac{\chi_3}{\chi_2^{3/2}} \\
\text{kurtosis} : \kappa &= \frac{\chi_4}{\chi_2^2} \\
S\sigma &= \chi_3/\chi_2 \\
\kappa\sigma^2 &= \chi_4/\chi_2
\end{align*}
\]

- and therefore:

\[
\kappa_B\sigma_B^2 \equiv \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^T} = \frac{\chi_4^B(T)}{\chi_2^T} \left[ 1 + \frac{1}{2} \frac{\chi_6^B(T)}{\chi_4^B(T)} \left( \mu_B/T \right)^2 + \ldots \right] \]

F. Karsch (2012)
We plot the $V$-independent ratio $\kappa \sigma^2 = c_4 / c_2$. 

Electric charge kurtosis
Proposal: measure the flavor-specific kurtosis

- light and strange quark $\chi_4/\chi_2$
- comparison to experimental measurement will give the flavor-specific freeze-out temperatures
Defining the experimental measurement

✧ Problem: we need to go from the $B, Q, S$ basis to the basis of $u, d, s$ quark flavors

✧ In principle we need to measure all light and strange quark final states

⇒ Experimentally impossible, most resonances are too rare and cannot be reconstructed

✧ Most resonances decay one to one to their ground state

✧ Strange weak decays need to be reconstructed

\[
\kappa_S \sigma_s^2 = \kappa \sigma^2 (K, K^0, \Lambda, \Xi, \Omega \text{ incl. } K^*, \Lambda^*, \Sigma, \Xi^*) \\
\kappa_u \sigma_u^2 = \kappa \sigma^2 (\pi, p \text{ incl. } \rho, \omega, \Delta, N^*)
\]

Ongoing project with P. Alba, W. M. Alberico, R. Bellwied, M. Bluhm, D. Chinellato and M. Weber

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Conclusions

✦ High precision (continuum limit) lattice QCD predicts flavor separation in the crossover from the partonic to the hadronic matter.

✦ this could lead to a short mixed phase of degrees of freedom in which strange particle formation is dominant

✦ this should lead to measurable effects in the strange hadron yields (evidence from ALICE)

✦ new model-independent comparison between theory and experiment: $\chi_4/\chi_2$

✦ work in progress: define a meaningful experimental measurement
Backup slides
Alternate explanation: non-equilibrium proton annihilation after hadronization

Idea based on enhanced in-medium annihilation cross sections in hadronic transport codes, e.g. UrQMD

Steinheimer, Aichelin, Bleicher, arXiv:1203.5302
Karpenko, Sinyukov, Werner, arXiv:1204.5351
Becattini et al., arXiv:1201.6349

A hydro-based model which modifies the chemical freeze-out temperatures in a species dependent way due to viscous corrections exists as well (but slightly over predicts proton yield and under predicts multi strange yield).


Method to exclude in-medium proton suppression (annihilation models): Compare Npart scaled proton yield as a function of centrality in PbPb and in pp. If proton is suppressed in medium then scaled yield has to drop with centrality.
SHM yield fit

- Simultaneous fit gives $T = 152$ MeV

![Graph showing particle yields](image)

L. Milano for ALICE, QM 2012

- However, protons are **overestimated** and strange baryons are **underestimated**
Fitting them separately, a hierarchy is evident

R. Bellwied (2012)
At RHIC the proton is not suppressed but rather slightly enhanced

H. Caines for STAR
SQM 2006
nucl-ex/0608008
Strangeness enhancement in ALICE

B. Hippolyte
for ALICE
SQM 2011
arXiv:1112.5803
HRG calculations are very sensitive to particle composition
Flavor hierarchy in the QCD deconfinement transition

Statistics and continuum extrapolation

-$\star$ all observables are continuum extrapolated

- obtained by measuring them at several $a \sim 1/N_t$ and taking the limit $a \to 0$

-$\star$ parabolic fit is performed
There are evidences for deviations from statistical model predictions at the LHC - J/ψ production -

Data: ALICE / PHENIX (forward rapidity) - QM 2011
Data: ALICE / ATLAS / CMS (mid rapidity) - QM 2011

Prediction: Braun-Munzinger, Stachel arXiv:0901.2500

Conclusion:
All datasets (forward and mid-rapidity, low and high pT) show significant J/ψ suppression in central collisions in contradiction to statistical model predictions: possibly no common freeze-out surface or no strong partonic recombination?
All path approach

✧ Our goal:
  ➟ determine the equation of state for several pion masses
  ➟ reduce the uncertainty related to the choice of $\beta^0$

✧ conventional path: A, though B, C or any other paths are possible
✧ generalize: take all paths into account
Finite volume and discretization effects

- finite $V: N_s/N_t = 3$ and 6 (8 times larger volume): no sizable difference

- finite $a$: improvement program of lattice QCD (action observables)
  - tree-level improvement for $p$ (thermodynamic relations fix the others)
  - trace anomaly for three $T$-s: high $T$, transition $T$, low $T$
  - continuum limit $N_t = 6, 8, 10, 12$: same with or without improvement

- improvement strongly reduces cutoff effects: slope $\simeq 0$ (1 − 2$\sigma$ level)
Flavor hierarchy in the QCD deconfinement transition

Pseudo-scalar mesons in staggered formulation

✦ Staggered formulation: four degenerate quark flavors (‘tastes’) in the continuum limit

✦ Rooting procedure: replace fermion determinant in the partition function by its fourth root

✦ At finite lattice spacing the four tastes are not degenerate

➤ each pion is split into 16

➤ the sixteen pseudo-scalar mesons have unequal masses

➤ only one of them has vanishing mass in the chiral limit

![Graph](image)

✦ Scaling starts for $N_t \geq 8$. 

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