On baryons, critical endpoints and the QCD phase diagram

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Austria

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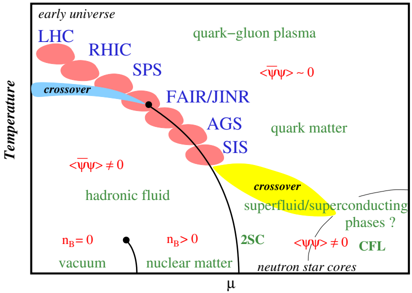
Der Wissenschaftsfonds.

Outline

- **Motivation: QCD Phase Diagram**
- **Why QCD for $N_c = 2$?**
- **(Polyakov-)Quark-Meson-Diquark ((P)QMD) Model**
- **RG versus Mean-field approximation**
- **Results: Phase diagrams etc.**

The conjectured QCD Phase Diagram for $\mathbf{N_c = 3}$



At densities/temperatures of interest **only model calculations** available

➜ can one improve the model calculations?

➜ remove model parameter dependency?

Open issues: (selection)

related to chiral & deconfinement transition

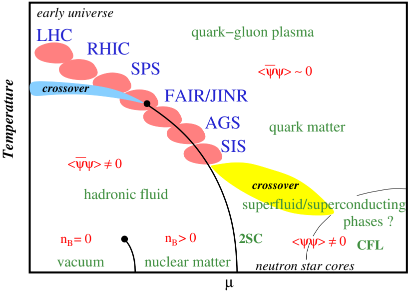
- ▷ existence/location of CEP? How many? Additional CEPs?
- ▷ coincidence of both transitions at $\mu = 0$ and $\mu > 0$ (quarkyonic phase)?
- ▷ relation between chiral and deconfinement? chiral CEP/deconfinement CEP?
- ▷ finite volume effects? $\rightarrow$ lattice comparison
- ▷ so far mostly MFA results **effects of fluctuations are important!** e.g. size of crit. region
- ▷ What are good exp. signatures? $\rightarrow$ higher moments more sensitive
- ▷ $\mu > \mathbf{0}$: **role of baryonic d.o.f.?**

non-perturbative functional methods (FunMethods)

$\rightarrow$ complementary to lattice

• no sign problem $\mu > 0$ • chiral symmetry/fermions (small masses/chiral limit) etc...

The conjectured QCD Phase Diagram for $\mathbf{N_c = 3}$



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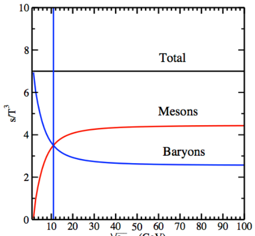
non-perturbative functional methods (FunMethods)

Method of choice: **Funtional Renormalization Group Method (FRG)**
one needs a truncation: e.g. (Polyakov)-quark-meson model

- good description for chiral sector
- implementation of gauge dynamics (deconfinement sector)

Why deforming QCD to $\mathbf{N_c = 2}$?



[J. Cleymann et al.]

- ▷ QC$_2$D becomes simpler: no sign problem
  lattice simulations $\Longleftrightarrow$ functional methods
- ▷ baryonic d.o.f. more and more important for $\mu > 0$
  for $\mathbf{N_c = 2}$ inclusion of baryonic dof simpler:
  scalar diquarks play a dual role as **bosonic baryons**
- ▷ QC$_2$D: playground for a deeper understanding of baryonic dof
- ▷ Relativistic analog of models for ultracold quantum gases

Properties of QC$_2$D

- ■ fund. rep. of $SU(2)$ pseudoreal (**2 = 2***) $\Rightarrow$ Dirac op. D has antiunitary symmetry

$\Rightarrow$ color-neutral bound states of two quarks (**bosonic (anti)diquarks**)

$\Rightarrow$ enlarged flavor symmetry: $SU(4) \cong SO(6)$ $(\mu = 0)$ here $N_f = 2$

replaces usual chiral $SU(2)_L \times SU(2)_R \times U(1)_B$

- ■ Symmetry breaking: $SU(2N_f) \to Sp(N_f)$ [or $SO(6) \to SO(5)$]
  ➜ 5 Goldstone bosons: 3 pions and 2 (anti)diquarks

Quark-Meson-Diquark (QMD) Model

Chiral effective model:

- quarks: ψ
- mesons: σ, $\vec{\pi}$
- diquarks (baryons): $\mathrm{Re}\Delta, \mathrm{Im}\Delta$
- gauge fields: A^a_μ in $D_\mu = \partial_\mu + iA_\mu \to$ Polyakov-loop extended (PQMD) model

QMD Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QMD}} &= \bar{\psi}\left(\not{D} + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \mu\gamma^0\right)\psi \\
&+ \frac{g}{2}\left(\Delta^*(\psi^T C\gamma^5\tau_2 S\psi) + \Delta(\psi^\dagger C\gamma^5\tau_2 S\psi^*)\right) \\
&+ \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V(\vec{\phi}) \\
&+ \frac{1}{2}\big((\partial_\mu - 2\mu\,\delta^0_\mu)\Delta\big)(\partial_\mu + 2\mu\,\delta^0_\mu)\Delta^*
\end{aligned}
$$

[N. Strodthoff, BJS, L. von Smekal; 2012]

Mean-Field Approximation (MFA)

- Integration of quarks, neglect bosonic fluctuations:

Grand potential

$$\Omega(T,\mu;\sigma,d^2 \equiv |\Delta|^2) = \Omega_{\mathsf{vac}} + \Omega_{\mathsf{T}} + V_{\mathsf{MF}}(\sigma,d^2) \qquad (+\mathcal{U}_{\mathsf{Poly}}(\Phi)\ )$$

vacuum term: sharp three-momentum cutoff Λ

$$\Omega_{\mathsf{vac}}(\Lambda) = -4\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\left\{E_p^- + E_p^+\right\}$$

$$E_p^{\pm} = \sqrt{g^2d^2 + (\epsilon_p \pm \mu)^2}$$

$$\epsilon_p = \sqrt{\vec{p}^2 + g^2\sigma^2}$$

for each Λ: adjust model parameters f_π, m_σ, m_π

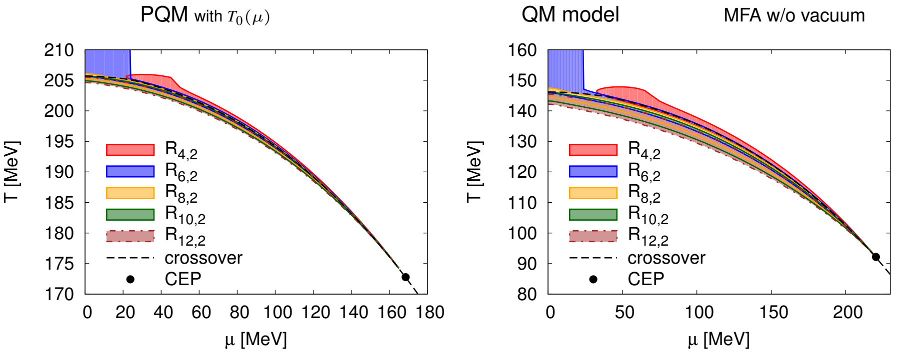
role of vacuum term: example for (P)QM models **[BJS, M. Wagner, 2012]**

[Skokov et al. 2010]

Role of vacuum term in (P)QM models

Fluctuations of higher moments exhibit **strong variation from HRG model**

- $\rightarrow$ turn negative — Karsch, Redlich, Friman, Koch et al.; 2011
- higher moments: $R^q_{n,m} = c_n/c_m$
- regions where $R_{n,2}$ are negative along crossover line in the phase diagram



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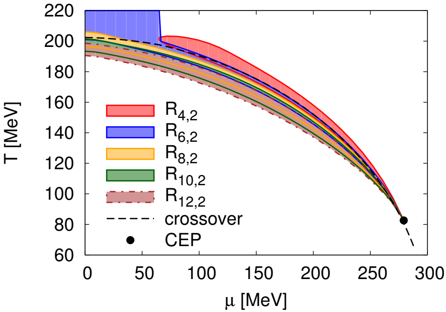
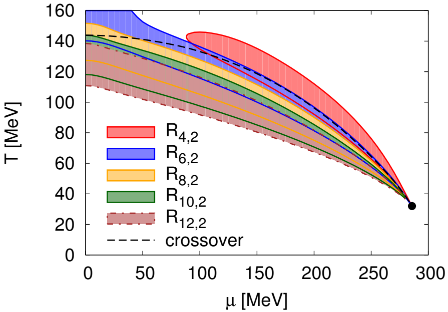
renormalized PQM with $T_0(\mu)$



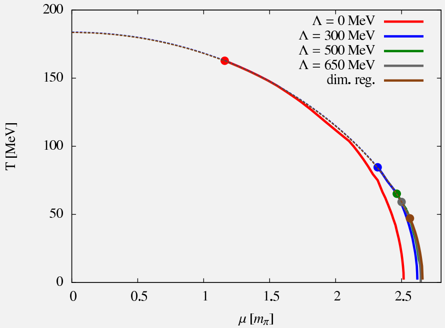
renormalized QM model



BJS, M.Wagner; 2012

Phase diagram in MFA

Influence of Ω_{vac} on CEP for various Λ's compared to dimensional regularization



[N. Strodthoff, BJS, L. von Smekal; 2012]

- no diquark condensation ($d = 0$)
- $O(6)$-symmetric potential
  $V_{\text{MF}} = V_{\text{MF}}(\phi^2)$ where
  $\vec{\phi} = (\sigma, \vec{\pi}, \text{Re}\Delta, \text{Im}\Delta)$
  ➜ 1-dim. field variable

- 1^{st} order chiral transition
- CEP around $\mu \approx 2.5 m_\pi$

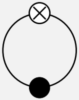
Functional RG Approach

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) Wetterich 1993

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



- Ansatz for Γ_k: (LO derivative expansion $\rightarrow$ arbitrary potential V_k)

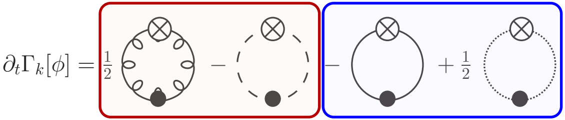
$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

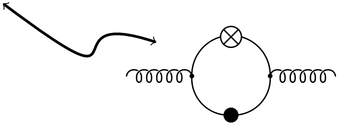
FRG and QCD

full dynamical QCD FRG flow: fluctuations of
gluon, ghost, quark and meson (via hadronization) fluctuations

Braun, Haas, Marhauser, Pawlowski; 2009



in presence of dynamical quarks:
gluon propagator modified:



$\Rightarrow$ pure Yang Mills flow $+$ matter back-coupling

pure Yang Mills flow
replaced by eff. Polyakov-loop potential $\mathcal{U}_{\text{Pol}}$:
(fit to lattice YM thermodynamics)

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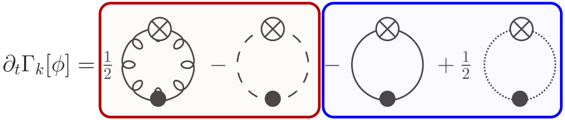

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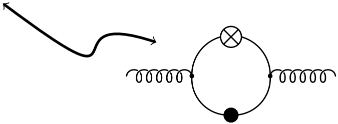

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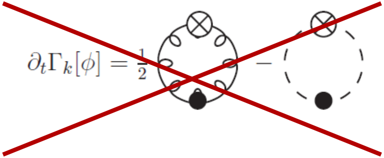

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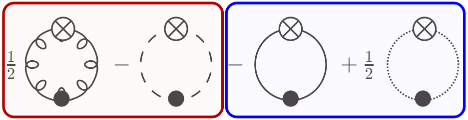
$\partial_t \Gamma_k[\phi] =$



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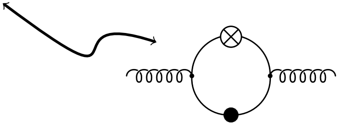

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$$\partial_t \Gamma_k[\phi] \quad \Rightarrow \quad \mathcal{U}_{\text{Pol}}(\Phi, \bar{\Phi})$$

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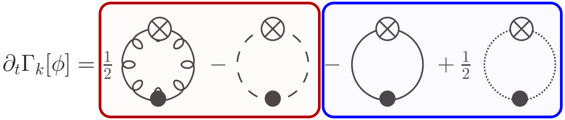

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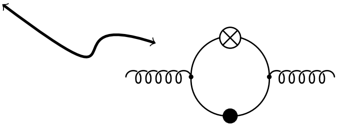

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$$\partial_t \Gamma_k[\phi] \Rightarrow \mathcal{U}_{\text{Pol}}(\Phi, \bar{\Phi})$$

Flow for the QMD model truncation

2 condensates: Chiral and diquark [Strodthoff, BJS, von Smekal; 2012]

$$\partial_t U_k(\sigma, d^2) = \frac{k^5}{12\pi^2} \left\{ \frac{3}{E_k^\pi} \coth\left(\frac{E_k^\pi}{2T}\right) + \sum_{i=1}^{3} \frac{\alpha_2 z_i^4 - \alpha_1 z_i^2 + \alpha_0}{(z_{i+1}^2 - z_i^2)(z_{i+2}^2 - z_i^2)} \frac{1}{z_i} \coth\left(\frac{z_i}{2T}\right) - \sum_{\pm} \frac{8}{E_k^\pm} \left(1 \pm \frac{\mu}{\sqrt{k^2 + g^2\rho^2}}\right) \left(1 - 2N_q(E_k^\pm; T)\right) \right\}$$

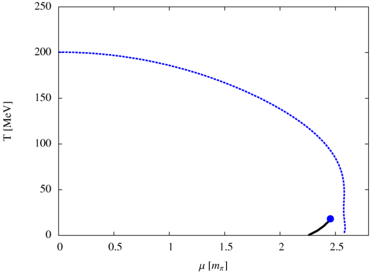
$E_k^\pi = \sqrt{k^2 + 2U_{k,\rho}}$; α_i, z_i : diquarks-sigma mixing; N_q : quark occupation numbers

Chiral condensate only ($SO(6)$-symmetric flow)

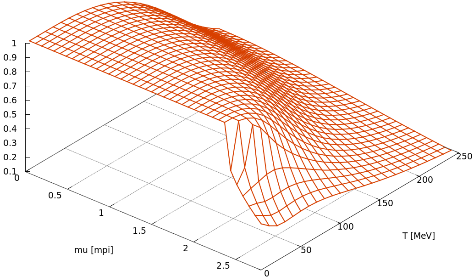
$$\partial_t U_k(\phi) = \frac{k^5}{12\pi^2} \left\{ \frac{3}{E_k^\pi} \coth\left(\frac{E_k^\pi}{2T}\right) + \frac{1}{E_k^\sigma} \coth\left(\frac{E_k^\sigma}{2T}\right) + \frac{1}{E_k^\pi} \coth\left(\frac{E_k^\pi - 2\mu}{2T}\right) + \frac{1}{E_k^\pi} \coth\left(\frac{E_k^\pi + 2\mu}{2T}\right) - \frac{16}{\epsilon_k}\left[1 - N_q\left(\epsilon_k - \mu; T\right) - N_q(\epsilon_k + \mu; T) \right] \right\}$$

Phase diagram with FRG

- no diquark condensation:
- $O(6)$-symmetric potential $U_k = U_k(\phi^2)$



- chiral condensate $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$



▷ "typical" RG phase diagram
back-bending 1st order line with a CEP

[Strodthoff, BJS, von Smekal; 2012]

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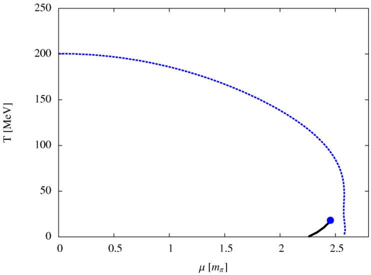

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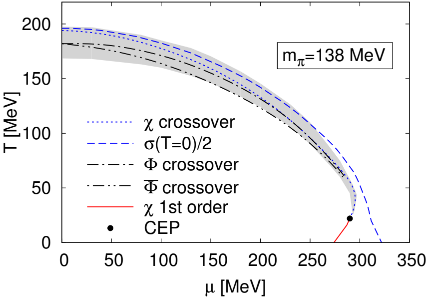
- FRG phase diagram

Polyakov-Quark-Meson model **with** matter back-reaction to YM system



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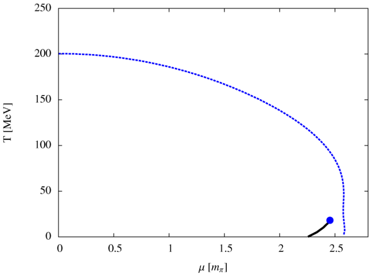

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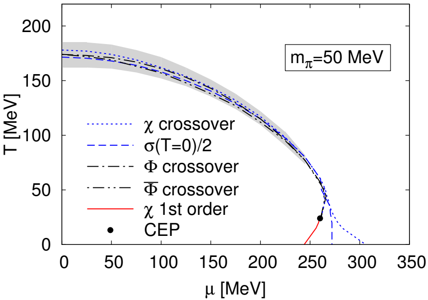

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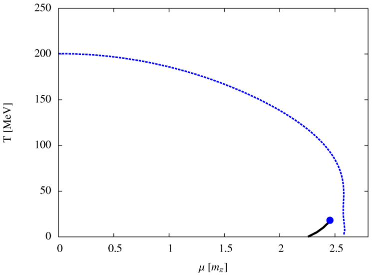

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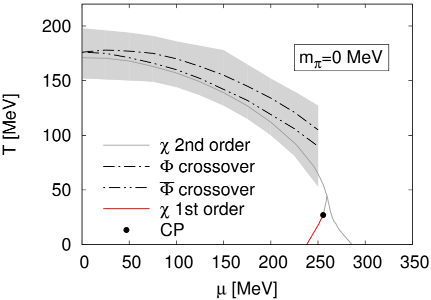

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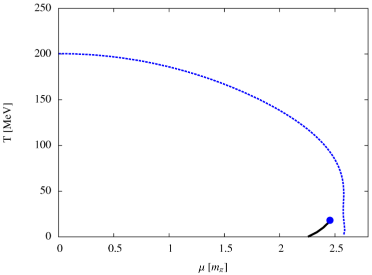

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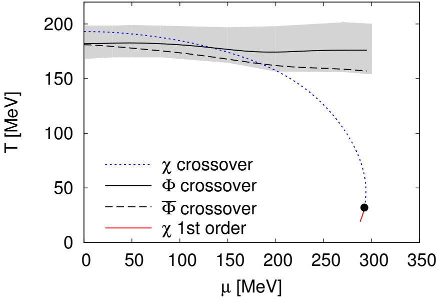
Polyakov-Quark-Meson model **without** matter back-reaction to YM system



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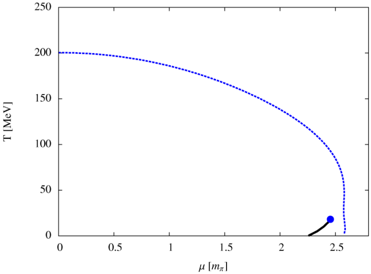

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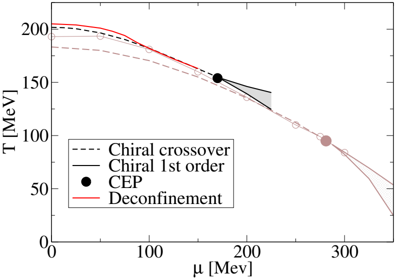
- DSE phase diagram

with matter back-reaction to YM system via HTL/improv. quark-Prop.



[C.S. Fischer, J. Luecker, J.A. Mueller; 2011/2012]

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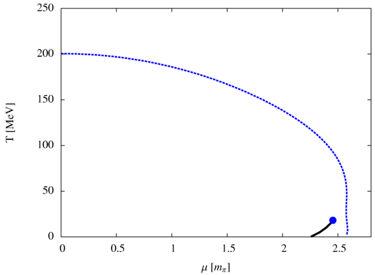

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- MFA phase diagrams $N_f = 2 + 1$

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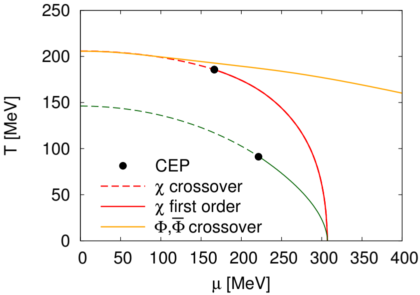

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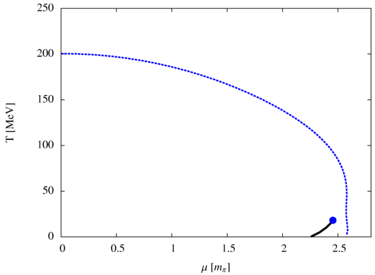

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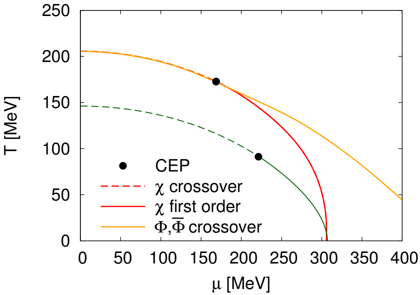

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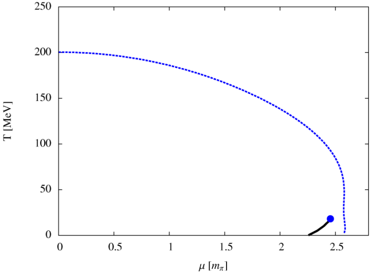

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phase diagrams $N_f = 2 + 1$

beyond MFA

next talk by Mario Mitter

Including diquarks

Symmetry breaking patterns for $N_f = 2$:

	$\mu = 0$	$\mu > 0$
$m_q = 0$	$SU(4) \cong SO(6)$	$SU(2)_L \times SU(2)_R \times U(1)_B$ $\cong$ $SO(4) \times SO(2)$
$m_q > 0$	$Sp(2) \cong SO(5)$	$SO(3) \times SO(2)$

$\Rightarrow$ need 2 condensates:

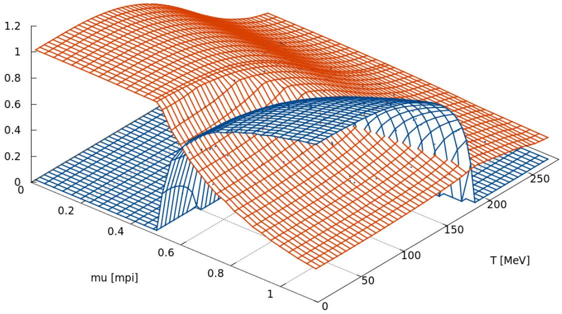
chiral condensate $\langle \bar{q}q \rangle (\equiv \sigma)$ and diquark condensate $d^2 = |\Delta|^2$

$\Rightarrow$ effective potential $U_k = U_k(\rho^2, d^2)$ with $\rho^2 = \sigma^2 + \vec{\pi}^2$

$\Rightarrow$ solution of flow eqs on **2-dim grid in field space** (first time!)

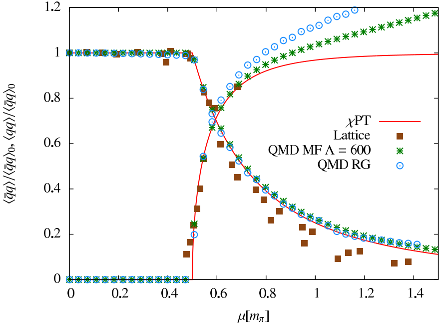
Including diquarks

chiral $\langle \bar{q}q \rangle$ and diquark condensates $d^2 = |\Delta|^2$



Diquark condensation at $\mathbf{T = 0}$

RG and MFA



[Strodthoff, BJS, von Smekal; 2012]

lattice data: **[Hands et al. '00]**

LO χPT **[Kogut, Stephanov et al. '00]**

➜ diquark condensation: $\mu_c = m_B/N_c$

model independent result

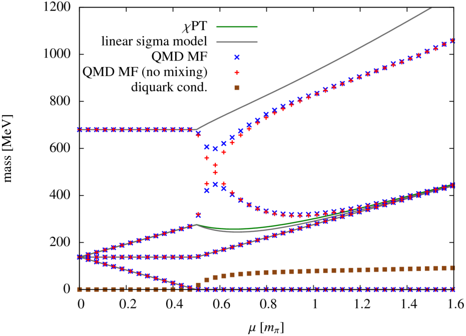
➜ $\langle qq\rangle$ in χPT:

$$\langle qq\rangle = \sqrt{1 - 1/x^4} \text{ with } x = 2\mu/m_\pi$$

$$\langle\bar{q}q\rangle = 1/x^2$$

Diquark condensation at $T = 0$

(pole) mass spectrum



[Strodthoff, BJS, von Smekal; 2012]

lattice data: **[Hands et al. '00]**

PNJL model: **[Brauner, Fukushima, Hidaka, '09]**

(LO)χPT: **[Kogut, Stephanov et al. '00]**

➜ diquark condensation: $\mu_c = m_B / N_c$

➜ distinguish between pole and screening masses

pole masses:

$T = 0$: $m_{\pm} = m_\pi \pm 2\mu$

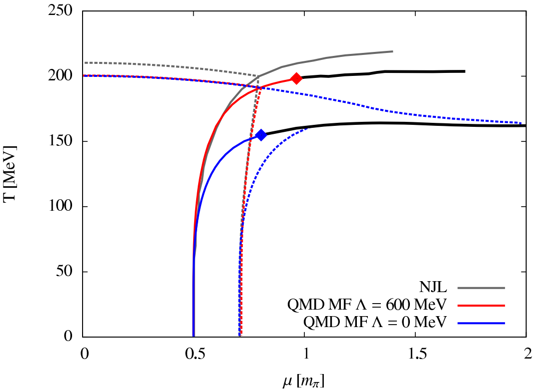
(according to $B = \pm 1$)

pion: $B = 0 \Rightarrow \mu$-indep.

Phase diagrams

NJL and QMD model phase diagrams

MFA with & w/o vacuum term



[Strodthoff, BJS, von Smekal; 2012]

Findings:

NJL: continuous $\langle qq \rangle$ condensation

QMD: MFA 2^{nd} order and TCP

➜ (NLO)χPT predicts also TCP

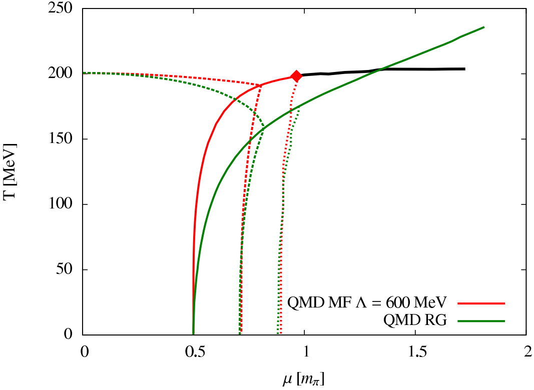
[Splittorff, Toublan, Verbaarschot; '02]

- BUT this is a MFA artifact!

Phase diagrams

QMD phase diagrams

RG vs. MFA



[Strodthoff, BJS, von Smekal; 2012]

Findings:

QMD: MFA 2nd order and TCP

➜ (NLO)χPT predicts also TCP

[Splittorff, Toublan, Verbaarschot; '02]

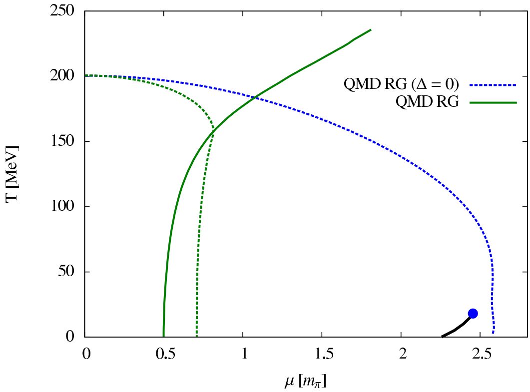
- BUT this is a MFA artifact!

  QMD: RG no endpoint

Phase diagrams

QMD phase diagrams

comparison with and
without baryonic fluctuations ($\Delta = 0$)



Findings:

QMD: RG no endpoint

- No 1st order low T!
- No CEP $\mu \approx 2.5 m_\pi$

➜ more tests are needed

- first step towards covariant quark-diquark description with a quark-meson-baryon model for QCD

[Strodthoff, BJS, von Smekal; 2012]

Summary and Outlook

- chiral (Polyakov)-quark-meson-diquarks ((P)QMD model study (two flavor)

➜ QC_2D as playground for $N_c = 3$ QCD

➜ towards understanding of baryons

➜ influence of baryonic dof's and fluctuations on existence of the CEP

➜ $N_c = 2$ importance of baryonic dof's

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functional approaches (such as the FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

➜ FunMethods guide the way towards full QCD