't Hooft Determinant at Finite Temperature with Fluctuations

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In collaboration with:

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Chiral Symmetry and Chiral Anomaly

- symmetry of Quantum Chromodynamics (QCD) with N_f massless flavors:

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η'-meson is no pseudo-Goldstone boson

- $N_f^2 - 1$ $+1$ broken generators
- experiment $N_f = 2$ ($N_f = 3$): 3 pions (+4 kaons and 1 η-meson)
- $U(1)_A$ broken by chiral anomaly [Adler, Bell, Jackiw, 1969], [Fujikawa, 1979]

$U(1)_A$ at Finite Temperature

- Witten-Veneziano Relation at $T = 0$: $m^2_{\eta'} + m^2_\eta - 2m^2_K \propto \frac{\chi_{YM}}{f^2_\pi}$:
  anomalous $m'_\eta \longleftrightarrow$ topological gauge configurations

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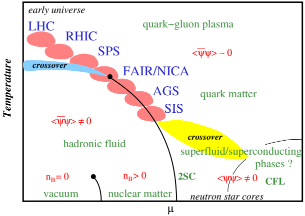
Recent experimental results (PHENIX, STAR):
[Csörgö et al., 2010; Vértesi et al., 2011]

- drop $\delta m_{\eta'} \gtrsim 200$ MeV at chiral transition
- (partial) $U(1)_A$-symmetry restoration already at chiral crossover?



[CBM Physics Book, 2011]

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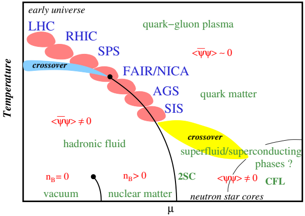


[CBM Physics Book, 2011]

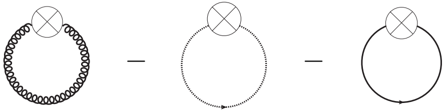
Effects of $U(1)_A$ at chiral transition:

- χ limit: order or transition [Pisarski, Wilczek, 1983]
- critical scaling
- curvature of transition line [Braun, 2009]

QCD with the Functional Renormalization Group

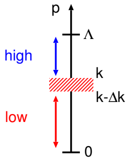
QCD flow equation:

$$\partial_k \Gamma_k = \frac{1}{2}$$



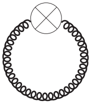
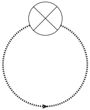
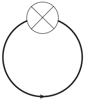
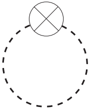
$$\Gamma_{k\to\Lambda\to\infty} = S_{QCD}$$

$$\Gamma_{k\to 0} = \Gamma_{QCD}$$



QCD with the Functional Renormalization Group

QCD flow with mesons:

$\partial_k \Gamma_k = \frac{1}{2}$  $-$  $-$  $+\ \frac{1}{2}$ 

$$\Gamma_{k\to\Lambda\to\infty} = S_{QCD}\ ,$$
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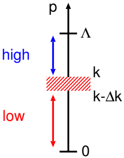
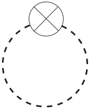
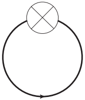
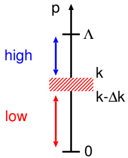


QCD with the Functional Renormalization Group

Low Energy truncation (Quark-Meson model):

$$\partial_k \Gamma_k = \frac{1}{2}$$  $$-$$ 

$$\Gamma_{k \to \Lambda \to 1\text{GeV}} = S_{QM} \quad \longleftrightarrow \quad \Gamma_{QCD,\Lambda} \, ,$$
$$\Gamma_{k \to 0} = \Gamma_{QM} \, ,$$



Low Energy Effective Description

The Quark-Meson Model (general N_f) [Ellwanger, Wetterich, 1994; Jungnickel, Wetterich, 1996]

$$\mathcal{L}_{QM} = \bar{q}\left(\mathrm{i}\partial_\mu\gamma_\mu + \mathrm{i}ht^a\left(\sigma^a + \mathrm{i}\gamma_5\pi^a\right)\right)q + \mathcal{L}_M$$

$$\mathcal{L}_M = \mathrm{tr}\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right] + U\left(\{\rho_i\}, \xi\right) - \mathrm{tr}\left(C\left(\Sigma + \Sigma^\dagger\right)\right)$$

$$\Sigma = t^a\left(\sigma^a + \mathrm{i}\pi^a\right) \,, \quad t^a : \text{ generators of } U(N_f)$$

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$U(1)_A$ anomaly in QM-Model

- "integrate instantons" in QCD Lagrangian ['t Hooft, 1976]
  $\Rightarrow$ contribution to Lagrangian proportional (for QCD with $\theta = 0$)

$$\det_f\left(\bar{q}_R q_L\right) + \det_f\left(\bar{q}_L q_R\right)$$

- axial anomaly in QM-model: $\xi = \det(\Sigma) + \det(\Sigma^\dagger)$

$N_f = 2$

- Functional RG for effective potential ($\sigma_0 \leftrightarrow \bar{u}u + \bar{d}d$, $\sigma_3 \leftrightarrow \bar{d}d - \bar{u}u$):

$$U_k(\Sigma) = \tilde{U}_k(\rho_1, \xi) - c_0\sigma_0 - c_3\sigma_3 \ ,$$

$\Rightarrow$ scale and temperature dependence of $U(1)_A$ violating terms

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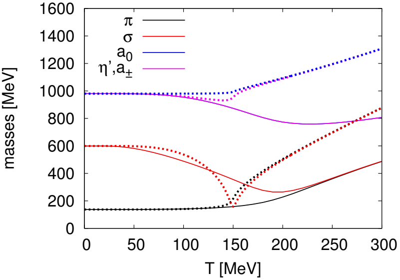
- $\Gamma_{\Lambda\approx 1\text{GeV}}$ via m_π, f_π, $m_d + m_u$, $m_{\eta'}$ and m_σ at $k \to 0$ and $T = 0$

Mesonic Masses

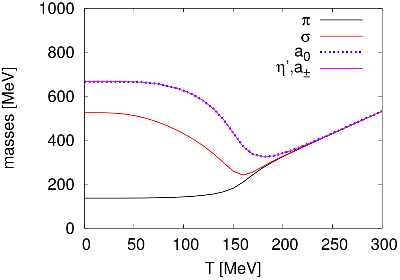
- realistic $(N_f = 3)$ $m_{\eta'}$:

  $m_{\eta'}(k \to 0) = 980$ MeV



- ultraviolet: $\det(k = \Lambda) \approx 0$,

  $m_{\eta'}(k = \Lambda) \approx m_\pi(k = \Lambda)$

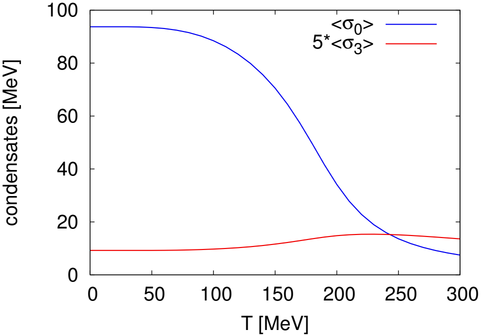


solid: RG, short dashed: MF

[MM, Schaefer, Strodthoff, von Smekal, in preparation 2013]

Condensates

- realistic ($N_f = 3$) $m_{\eta'}$: $m_{\eta'}(k \to 0) = 980$ MeV
- explicit breaking: $c_3 = c_0$



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Explicit Symmetry Breaking and Vacuum Alignment

- $\rho_1 \propto \sigma_0^2 + \sigma_3^2$ respects rotations in σ_0, σ_3 plane
- absence of determinant terms ξ: $(\langle\sigma_0\rangle, \langle\sigma_3\rangle) \propto (c_0, c_3)$

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- but: $\langle\sigma_0\rangle \gg \langle\sigma_3\rangle$ (or $\langle\bar{d}d + \bar{u}u\rangle \gg \langle\bar{d}d - \bar{u}u\rangle$)

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- suppression of $\langle\sigma_3\rangle$ requires generation of other operator ρ_1, e.g. ξ

New Phase transition: Z_2 Universality Class

$\xi^2 = \left(\frac{\sigma_0^2 - \sigma_3^2}{2}\right)^2$:
symmetry $\sigma_0 \leftrightarrow \sigma_3$

$c_0 = c_3$ (one bare quark massless):

- $c_0\sigma_0 + c_3\sigma_3$: symmetry $\sigma_0 \leftrightarrow \sigma_3$

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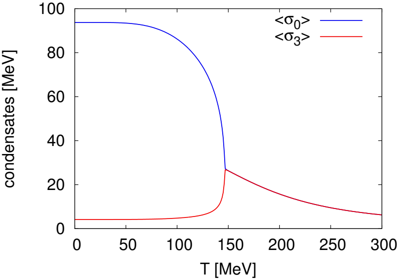
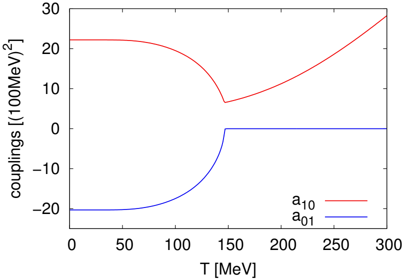
- ultraviolet: $\det(k = \Lambda) \approx 0$, $m_{\eta'}(k = \Lambda) \approx m_\pi(k = \Lambda)$
- $m_\pi^2 \propto a_{10} + a_{01}$
- $m_{\eta'}^2 \propto a_{10} - a_{01}$





[MM, Schaefer, Strodthoff, von Smekal, in preparation 2013]

- $\beta = 0.37$ (Z_2 universality class: 0.326)

$N_f = 2 + 1$

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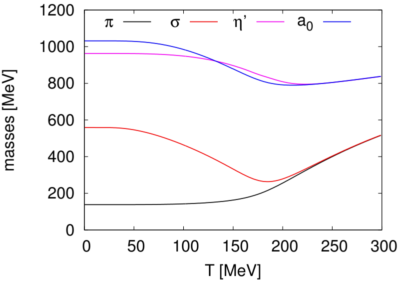
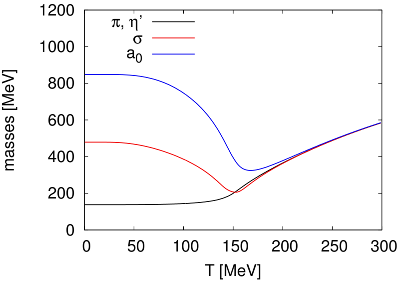
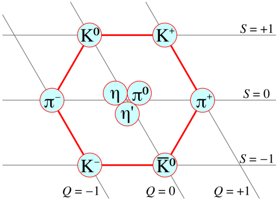
Mesonic Masses I at Physical Mass Point

- with determinant $\propto \Sigma^3$
- without determinant







[MM, Schaefer, in preparation 2013]

+ scalar nonet

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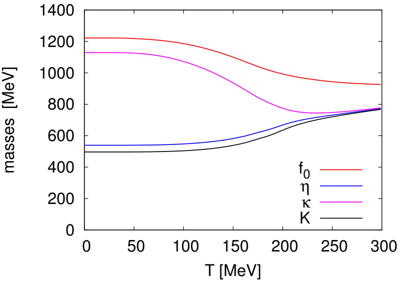

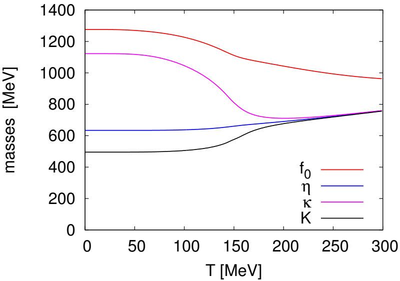

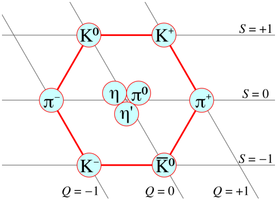

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Chiral Condensates at Physical Mass Point

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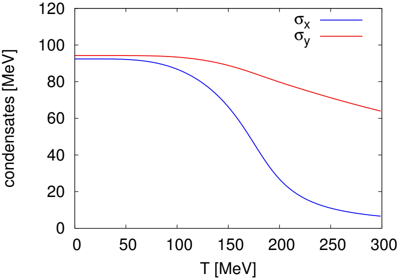

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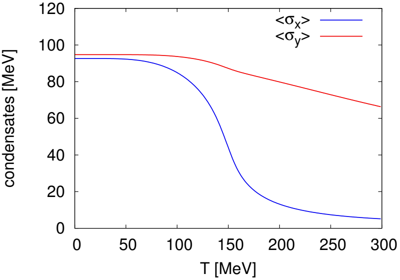


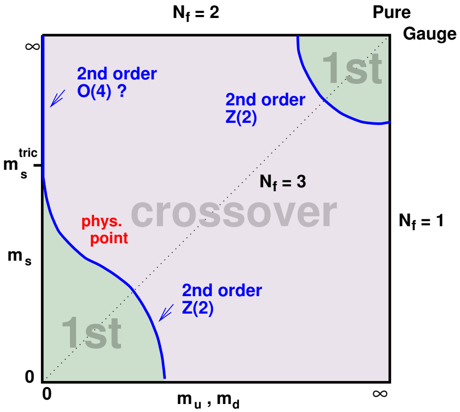
[MM, Schaefer, in preparation 2013]

Columbia Plot and Chiral Anomaly

Order of transition in chiral limit:

[Pisarski, Wilczek, 1984]

- $N_f > 2$: 1^{st} order



[Laermann, Philipsen, 2003]

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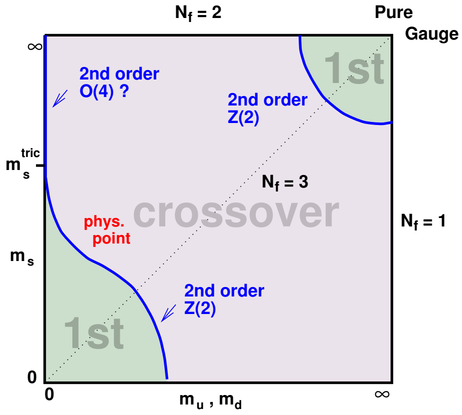

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- $N_f = 2$: 1^{st} order
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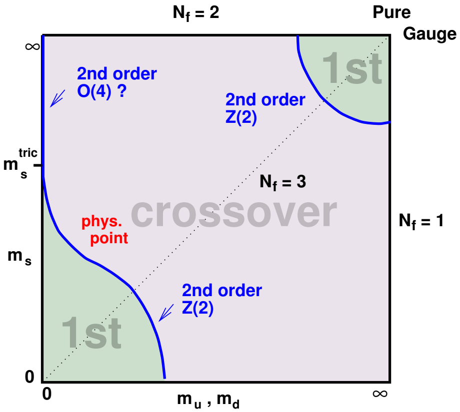
- $N_f = 2$: 2^{nd} order
  with $c\xi$ insensitive to $T < T_c$

Analysis with mesonic fluctuations



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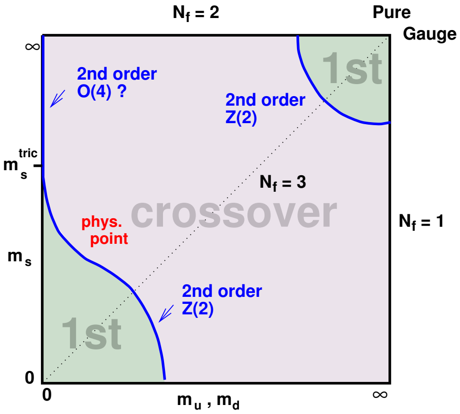
Analysis with mesonic fluctuations

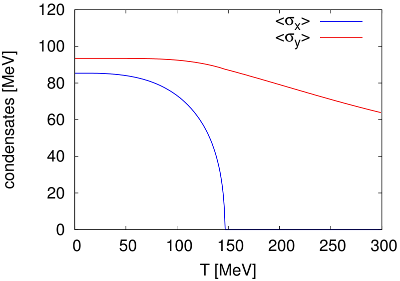
Order at $m_u = m_d = 0$?



[Laermann, Philipsen, 2003]

MF vs. RG: Chiral Condensates in Light Chiral Limit

- with determinant



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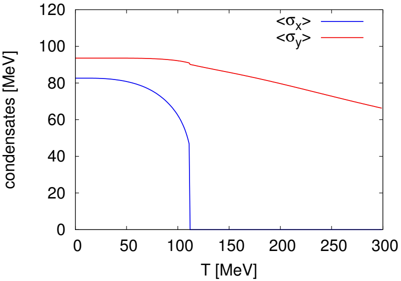
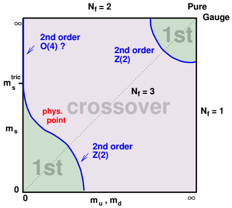
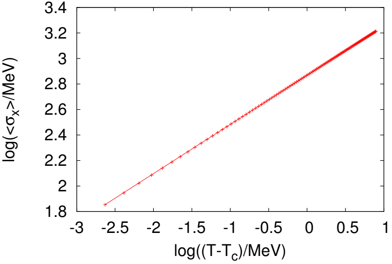


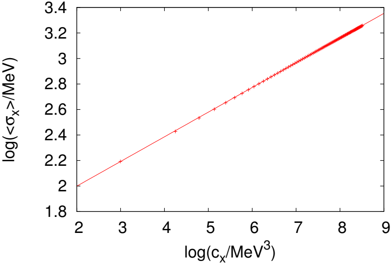


[MM, Schaefer, in preparation 2013]

Critical Exponents in Light Chiral Limit

- $\beta = 0.39$
- $\delta = 5.2$





[MM, Schaefer, in preparation 2013]

- compare well with $O(4)$ exponents in leading order derivative expansion
- indistinguishable from our results for $O(4)$ QM-model

Summary and Outlook

- temperature dependence: $m_{\eta'}$

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- 2 flavor:
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Connect to QCD:

- initial values directly from QCD
- dynamical hadronization
- effect of 't Hooft term on curvature of transition line
- Polyakov loop potential