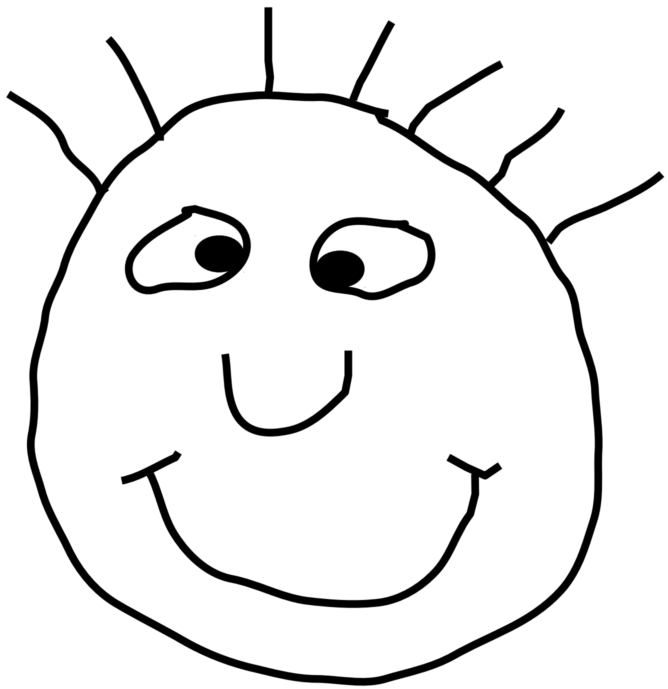
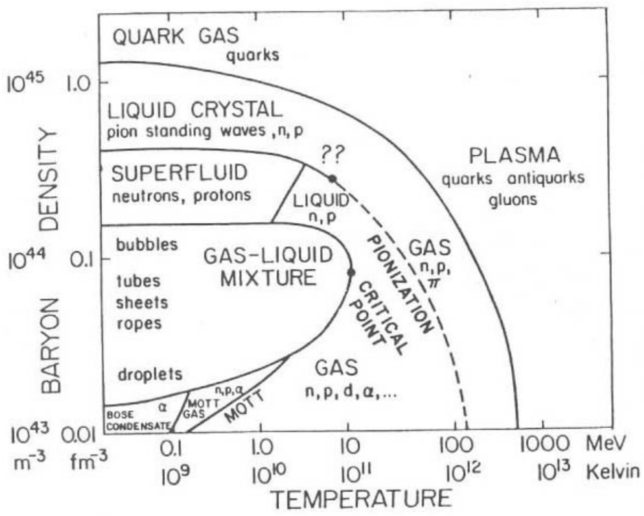
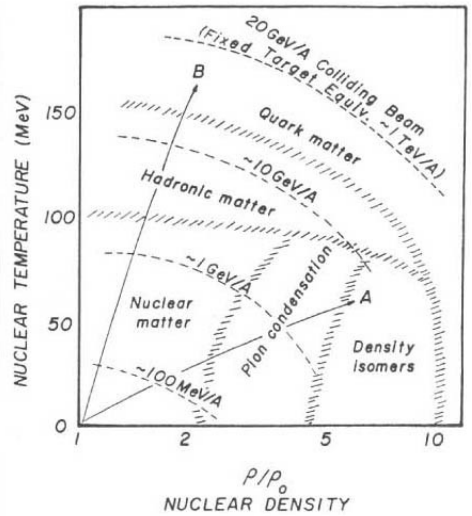
The QCD Face Diagram

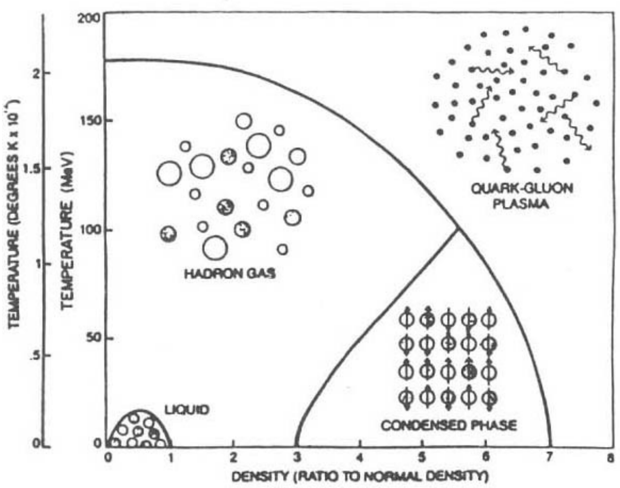


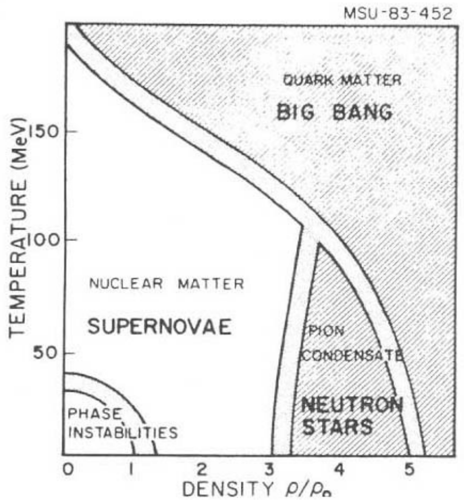
Courtesy M. Nahrgang

A long story....

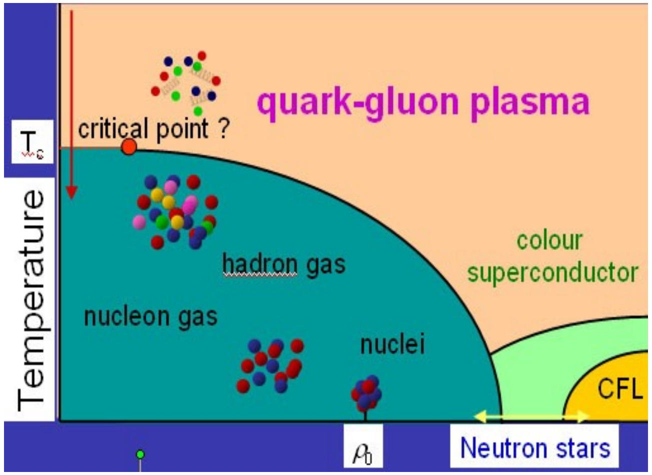

QUARK GAS
quarks
LIQUID CRYSTAL
pion standing waves, n, p
SUPERFLUID
neutrons, protons
??
LIQUID
n, p
PLASMA
quarks antiquarks
gluons
bubbles
tubes
sheets
ropes
droplets
GAS-LIQUID
MIXTURE
CRITICAL POINT
PIONIZATION
GAS
n, p, π
GAS
n, p, d, α, ...
BOSE CONDENSATE
α
MOTT GAS
n, p, α
MOTT
BARYON DENSITY
TEMPERATURE
MeV
Kelvin

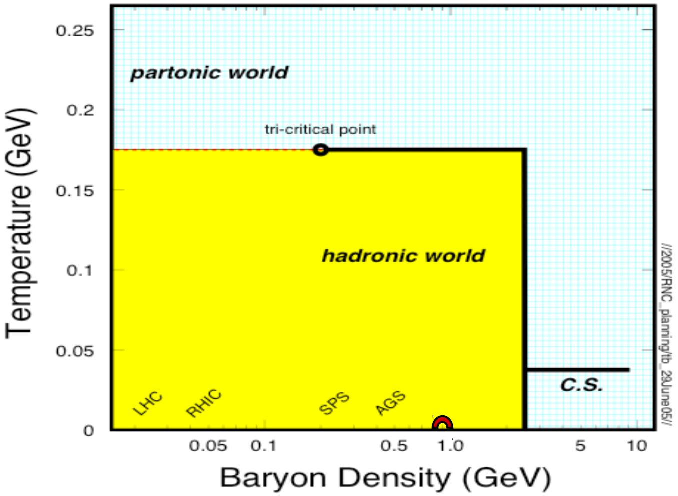

20GeV/A Colliding Beam
(Fixed Target Equiv. ~1 TeV/A)
Quark matter
~10GeV/A
Hadronic matter
~1 GeV/A
Nuclear matter
Pion condensation
Density isomers
~100MeV/A
NUCLEAR TEMPERATURE (MeV)
ρ/ρ₀
NUCLEAR DENSITY

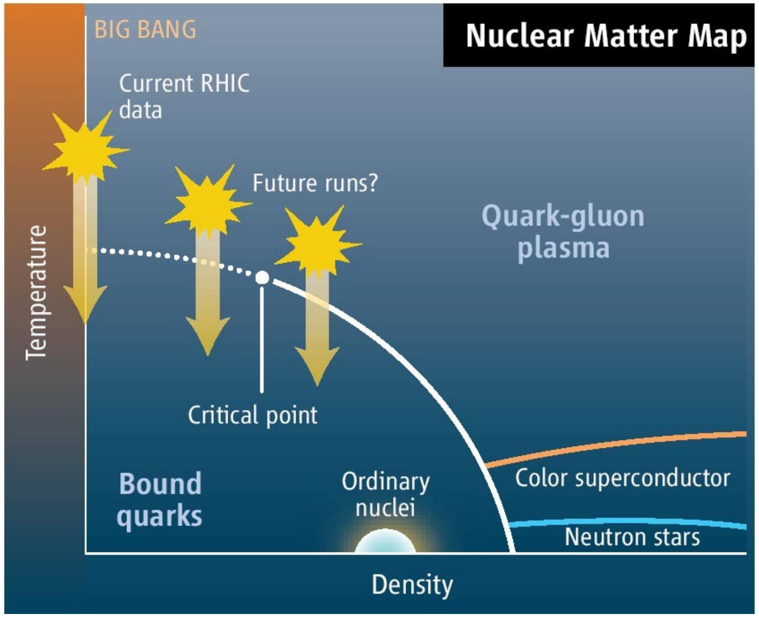

QUARK-GLUON PLASMA
HADRON GAS
CONDENSED PHASE
LIQUID
TEMPERATURE (MeV)
DENSITY (RATIO TO NORMAL DENSITY)

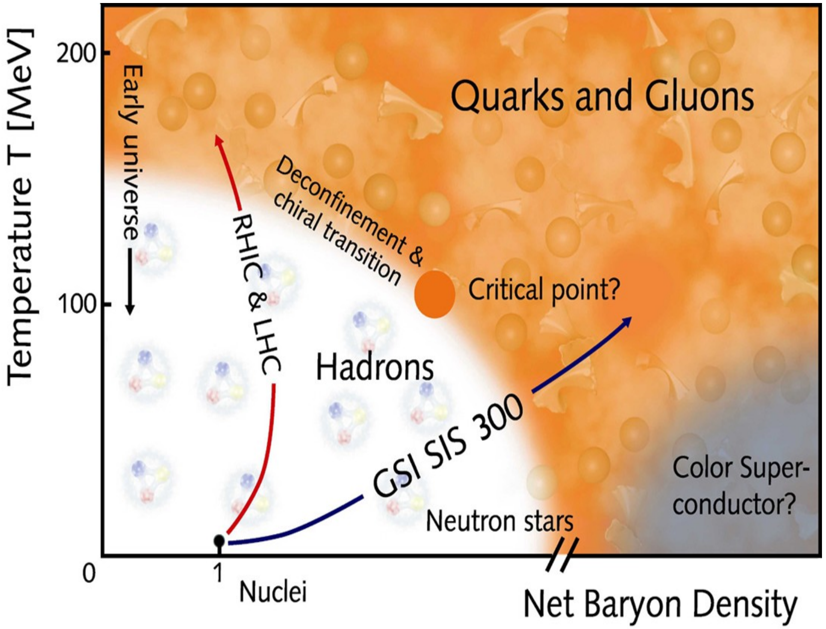

MSU-83-452
QUARK MATTER
BIG BANG
NUCLEAR MATTER
SUPERNOVAE
PION CONDENSATE
NEUTRON STARS
PHASE INSTABILITIES
TEMPERATURE (MeV)
DENSITY ρ/ρ₀

gets more colorful ...

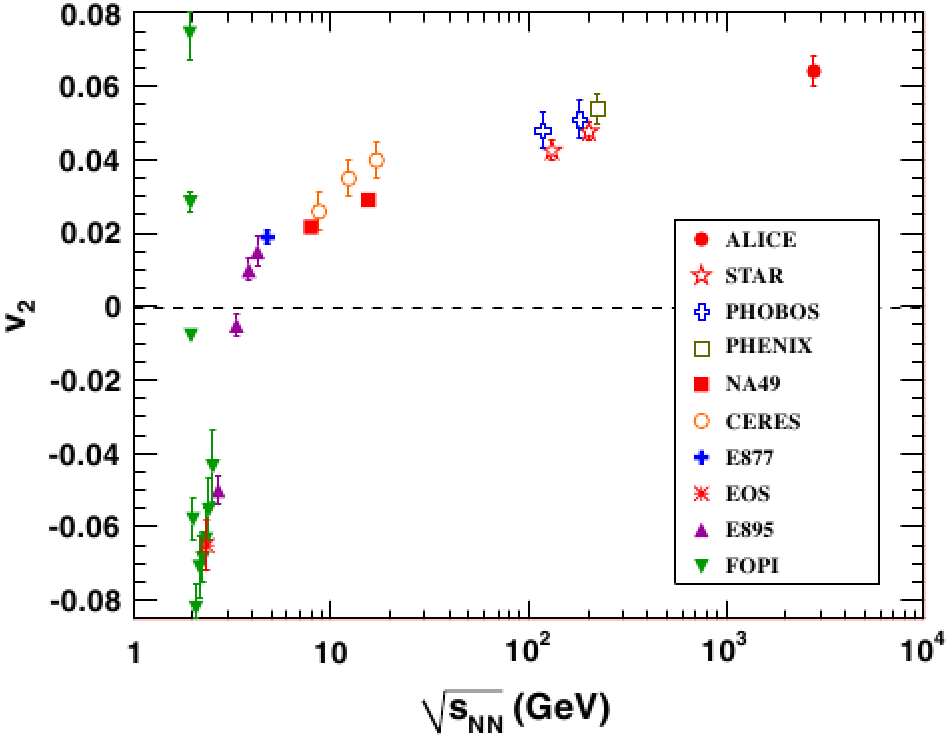




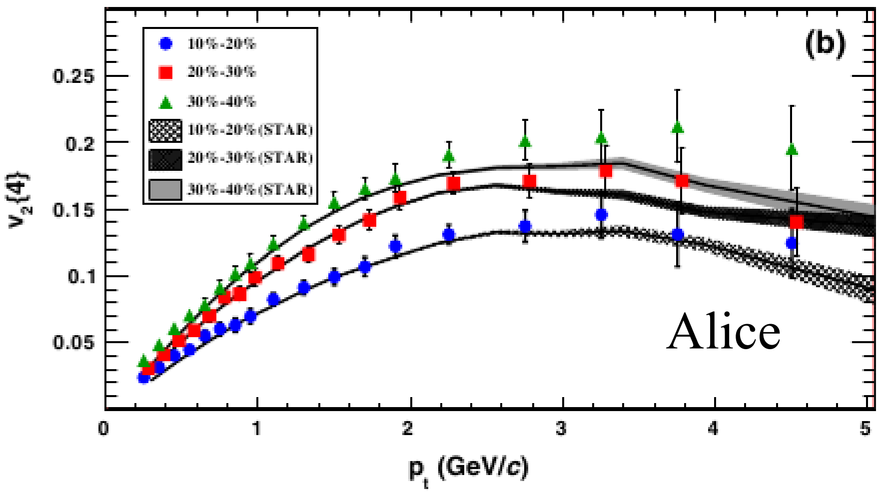




RHIC AND LHC

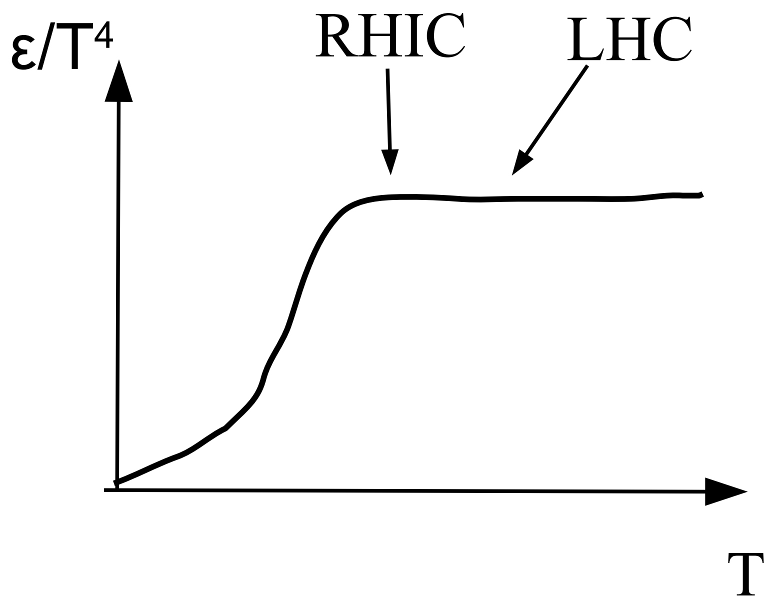


0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
v_2
1
10
10^2
10^3
10^4
$\sqrt{s_{NN}}$ (GeV)
ALICE
STAR
PHOBOS
PHENIX
NA49
CERES
E877
EOS
E895
FOPI



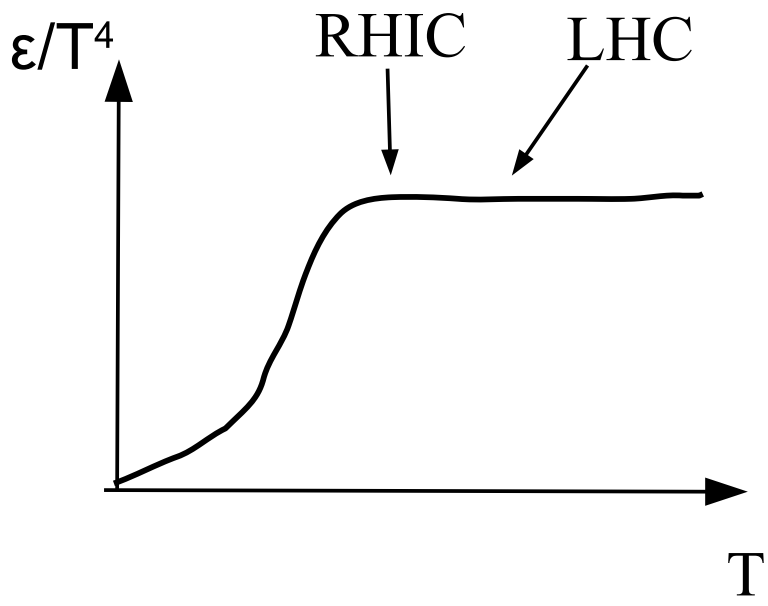
(b)
10%-20%
20%-30%
30%-40%
10%-20%(STAR)
20%-30%(STAR)
30%-40%(STAR)
0.25
0.2
0.15
0.1
0.05
$v_2\{4\}$
0
1
2
3
4
5
p_t (GeV/c)
Alice

The Paradigm



Seems in good shape

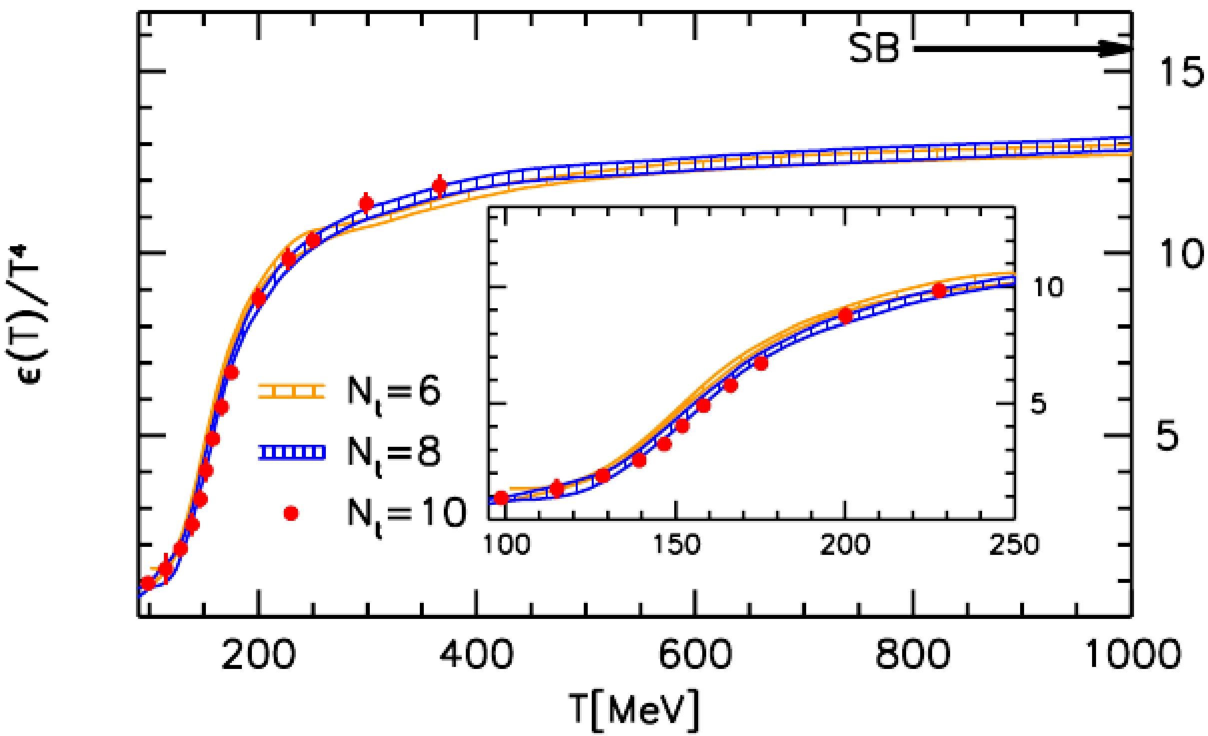
So how about the other side?



Can we establish that there is indeed a transition?

What we know from the Lattice

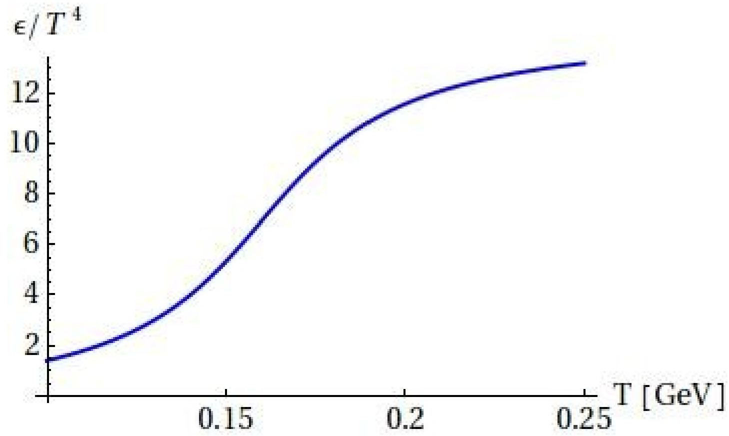
μ=0 only

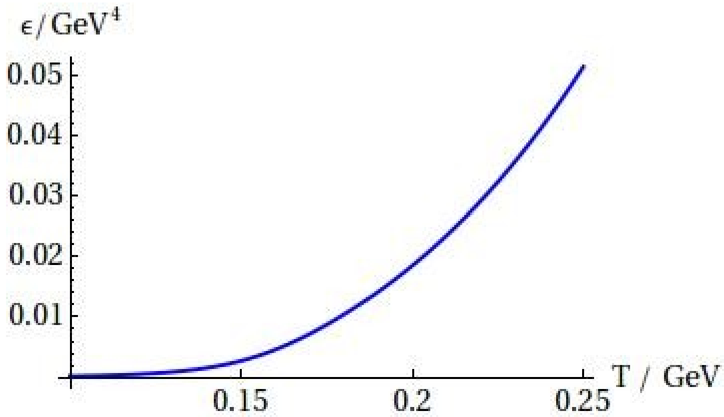


Wuppertal/Budpest

+ Cross over transition at zero net baryon density

The Lattice EOS



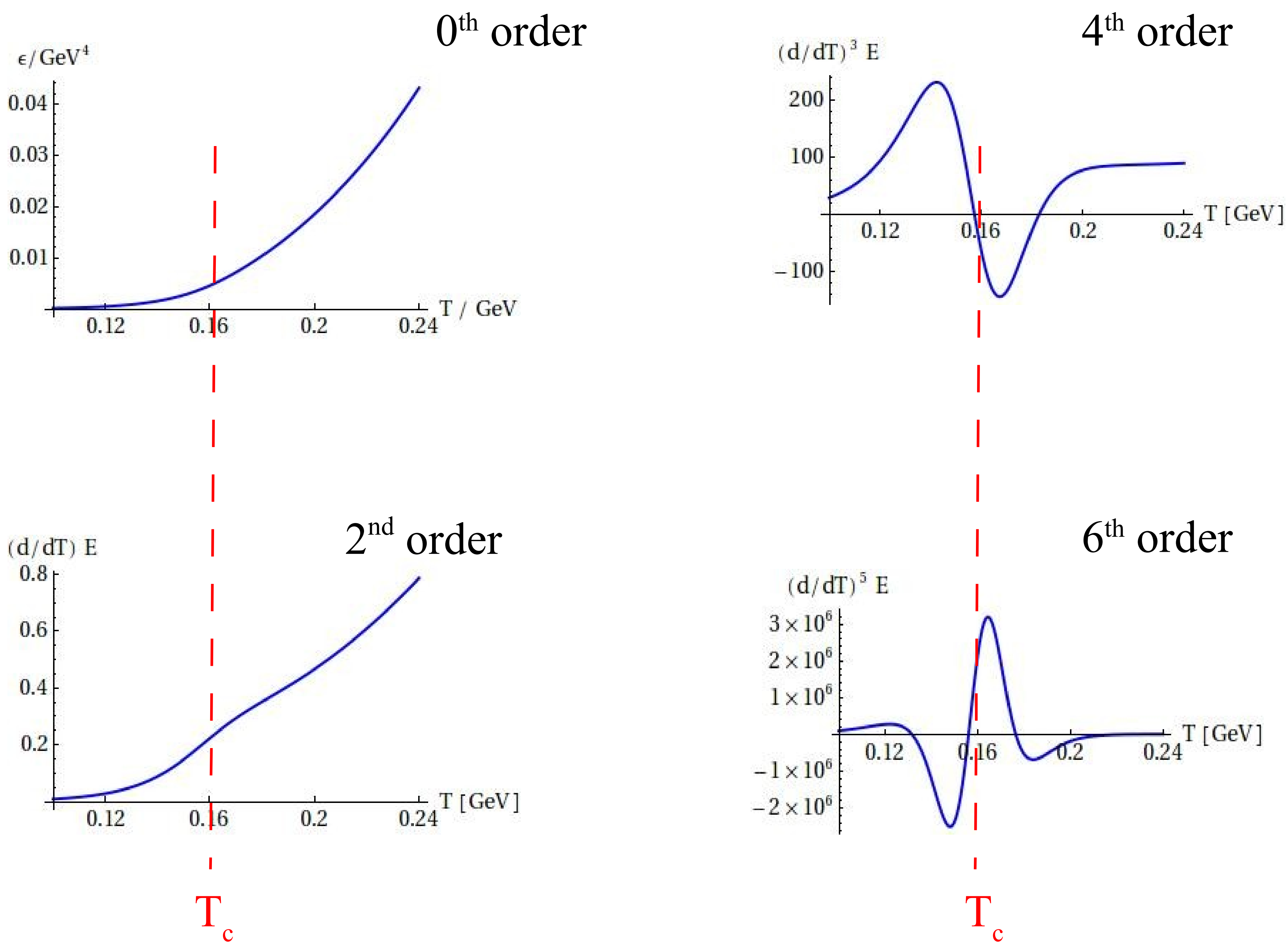


What we always see....

What it really means....

“T_c” ~ 160 MeV

Derivatives



How to measure derivatives

$$Z = tr\, e^{-\hat{E}/T + \mu/T\, \hat{N}_B}$$

At μ = 0:

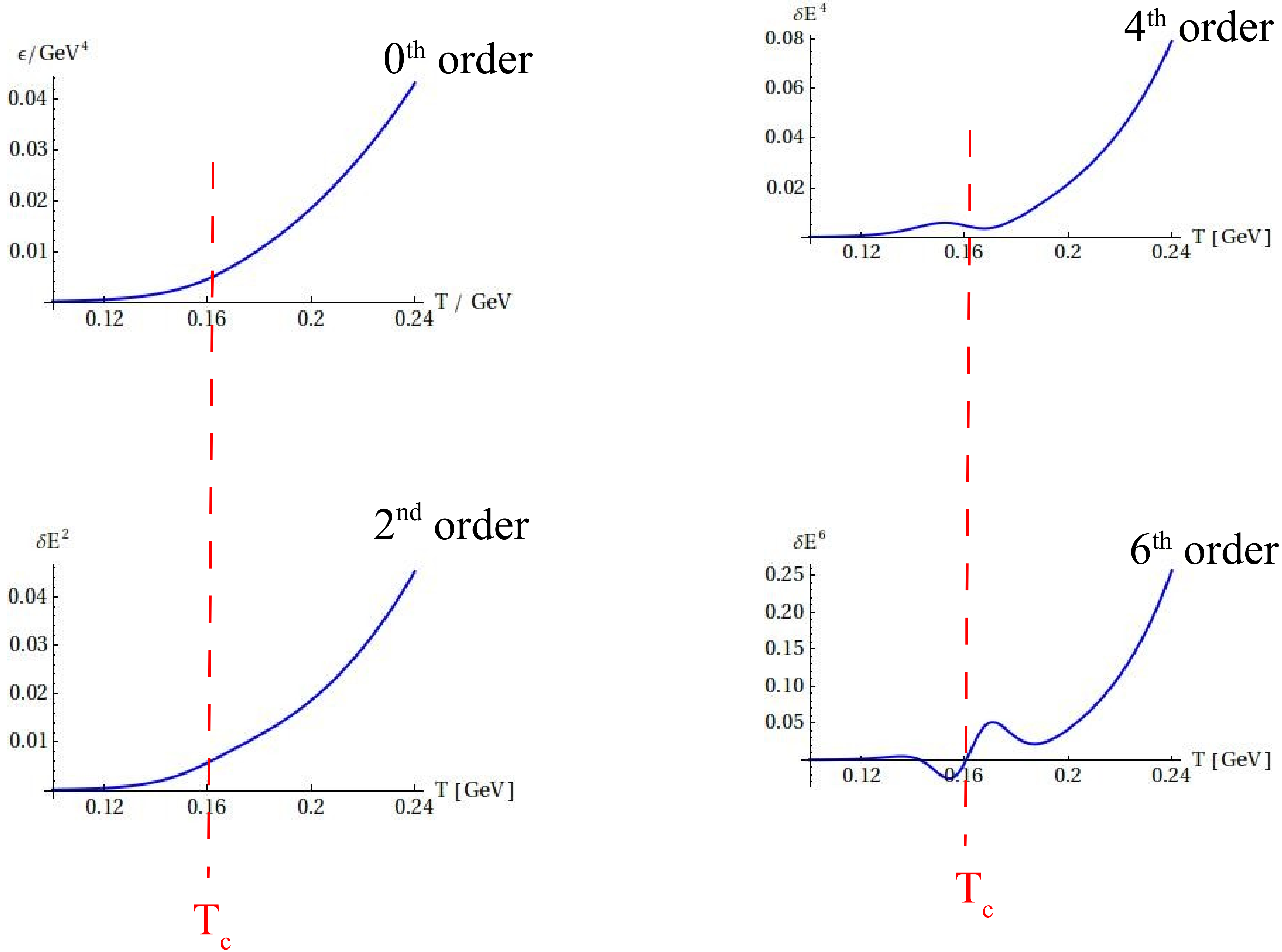
$$\langle E \rangle = \frac{1}{Z} tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T\, \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

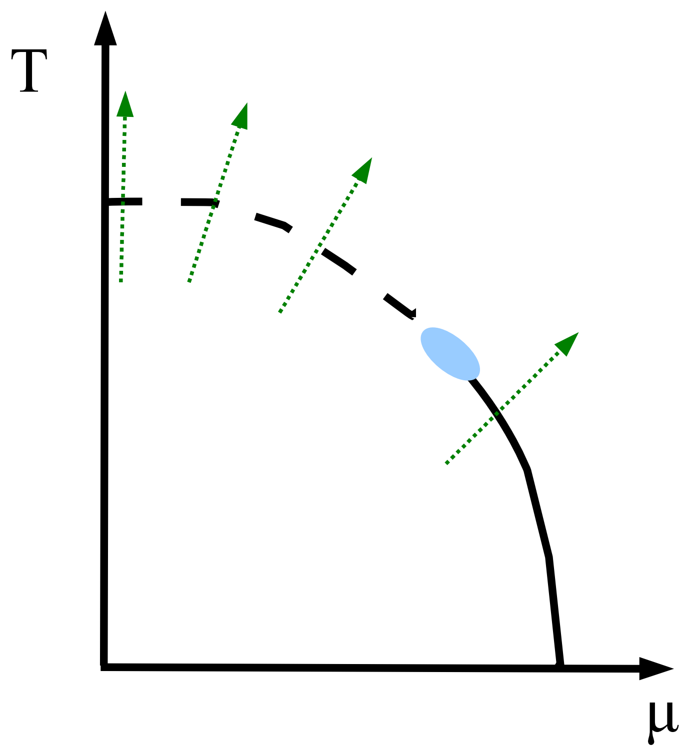
$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

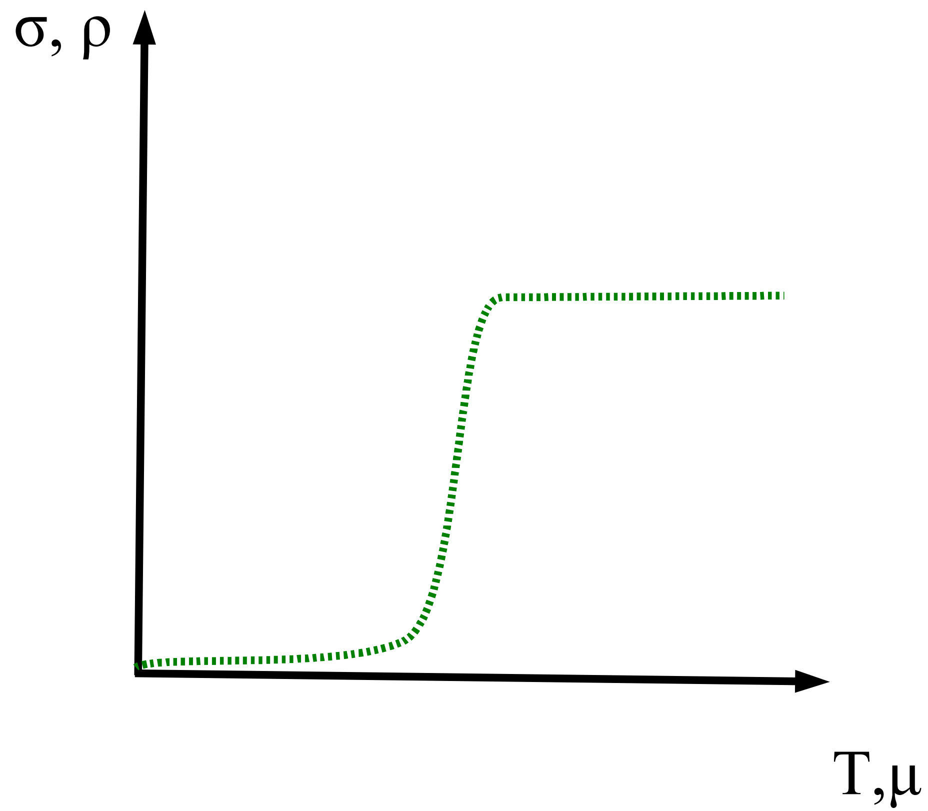
Cumulants of Energy measure the derivatives of the EOS

Fluctuations / Cumulants



Generic Phase Diagram





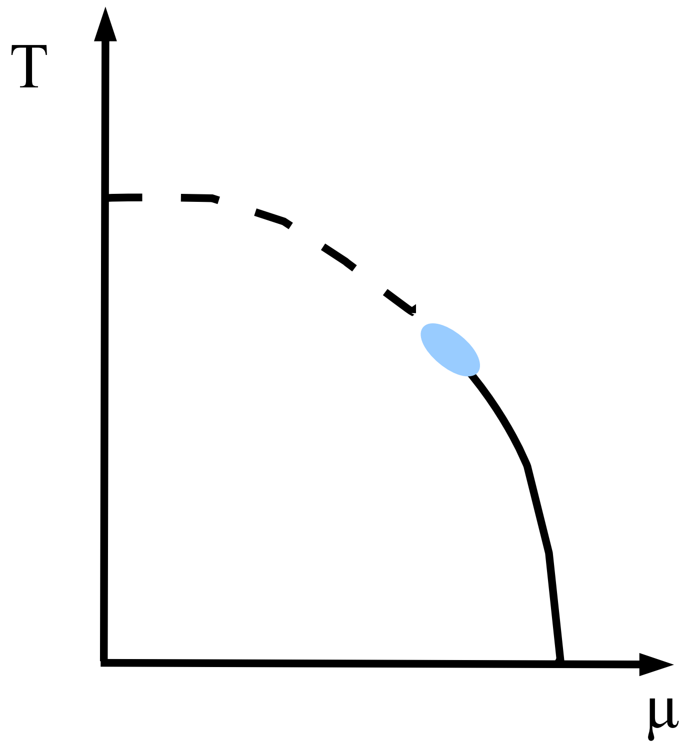
"Simply" use appropriate combination of T and μ

Requires: $\langle(\delta E)^n\rangle$ $\langle(\delta N_B)^n\rangle$ $\langle(\delta E)^m(\delta N_B)^n\rangle$ Mixed cumulants!

Another way

$$F = F(r), \qquad r = \sqrt{T^2 + a\mu^2}$$

a~ curvature of critical line

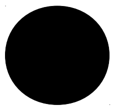


$$\partial_\mu^2 F(T,\mu)_{\mu=0} = \frac{a}{T}\partial_T F(T,0)$$

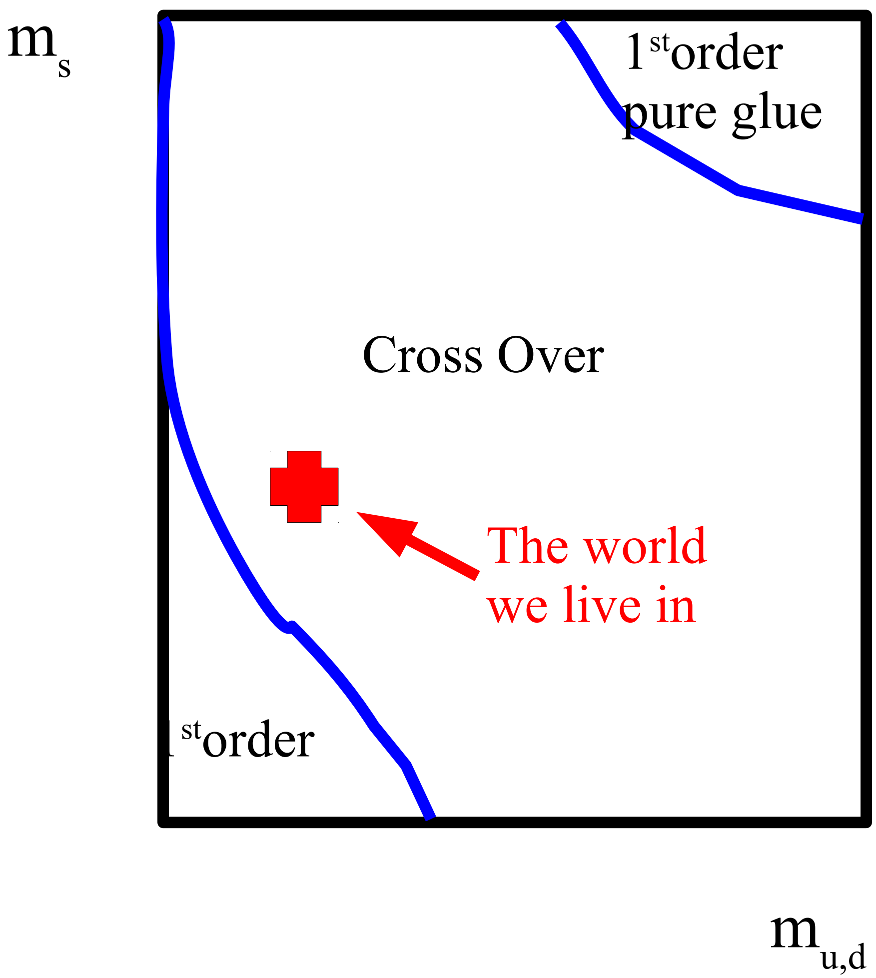
$$\partial_\mu^4 F(T,\mu)_{\mu=0} = 3\frac{a^2}{T}\left(T\partial_T^2 - \partial_T\right)F(T,0)$$

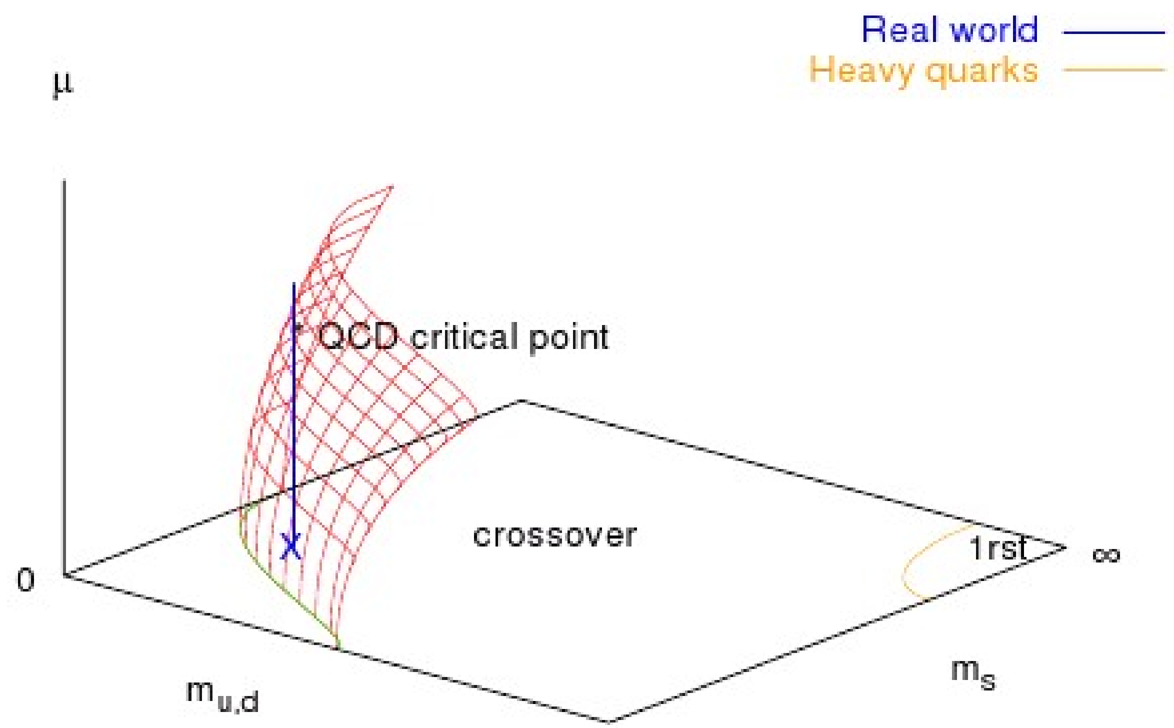
Baryon number cumulants give same info. Less problem with flow etc.

The critical Point



Is there a critical point?



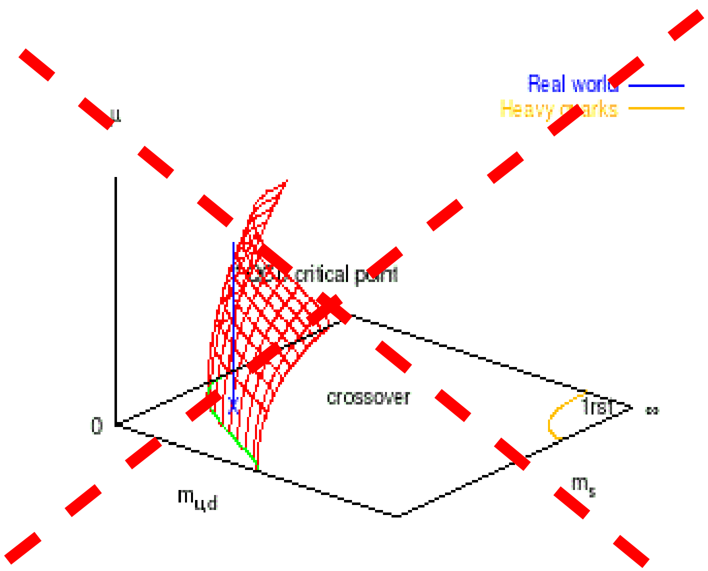


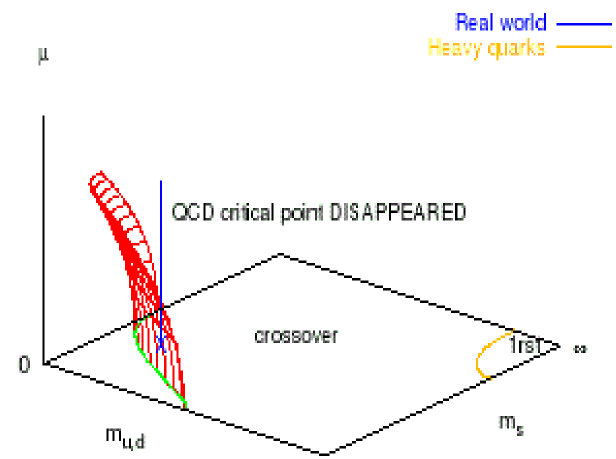
Lattice and the critical point

Forcrand, Philipsen

A non-standard scenario: no critical point?

sign of $c_1 = \frac{dm_c(\mu)}{d\mu^2}\Big|_{\mu=0}$



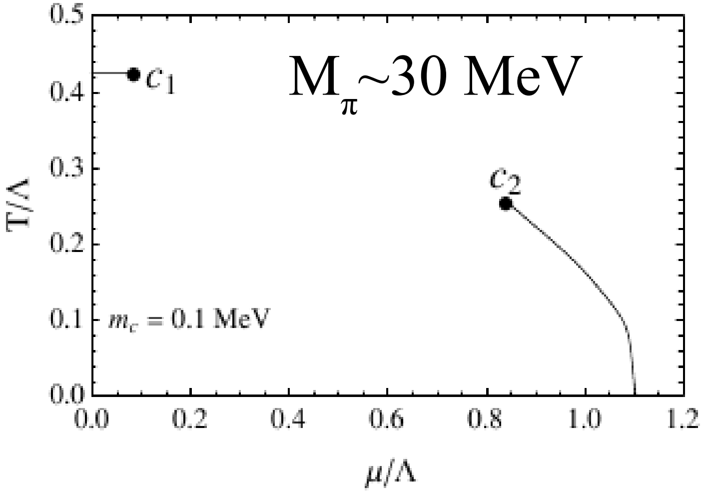


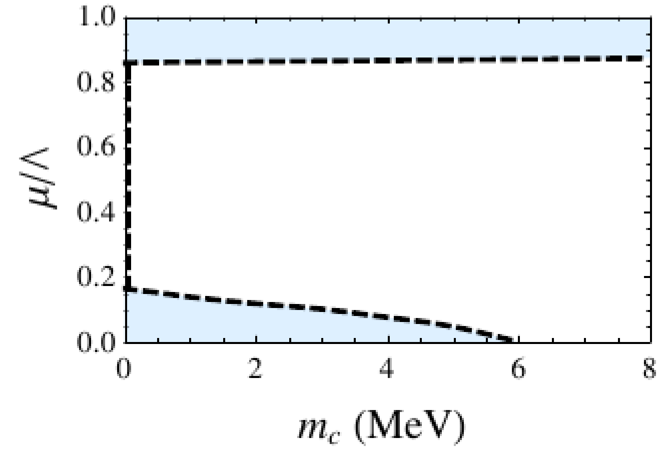
Favored by Lattice QCD

Note: Surface may bend back!!!

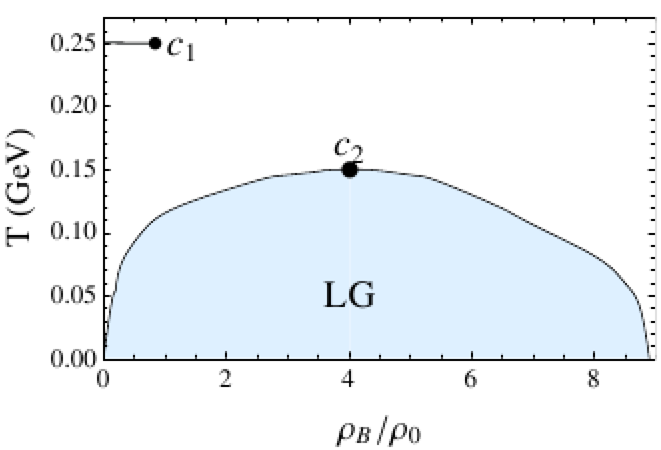
Two Critical Points ?!

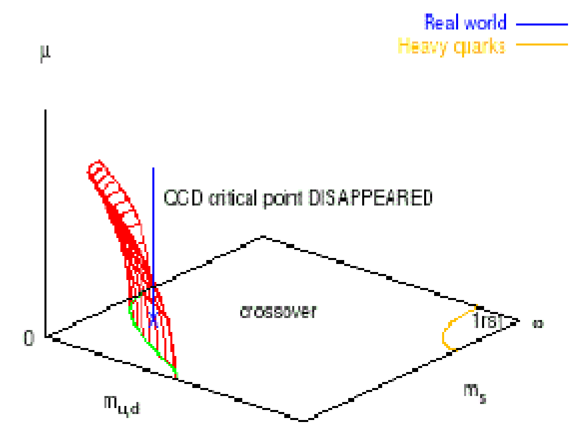
M. Pinto et al, Phys.Rev. C82 (2010) 055205





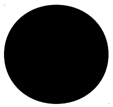
Two flavors





Seen in both Nambu and Linear Sigma Model

Measuring



Higher moments (cumulants) and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments (connected) of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\, \sigma(x)$:

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT\,\xi^2\,; \qquad \kappa_3 = \langle \sigma_V^3 \rangle = 2VT^2\,\lambda_3\,\xi^6\,;$$

$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8\,.$$

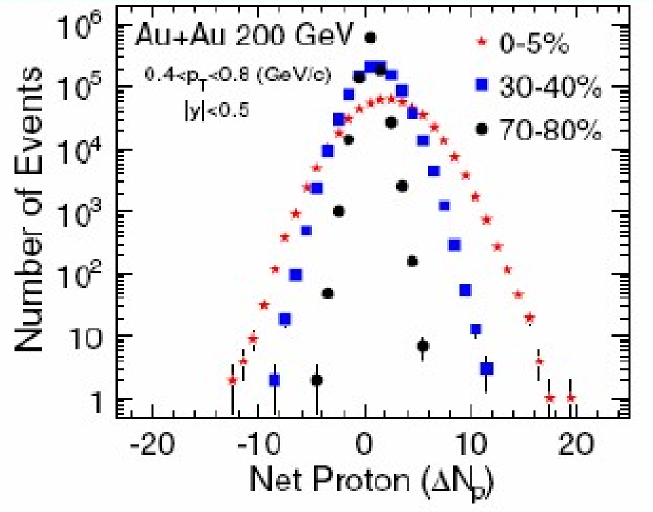
- Tree graphs. Each propagator gives ξ^2.

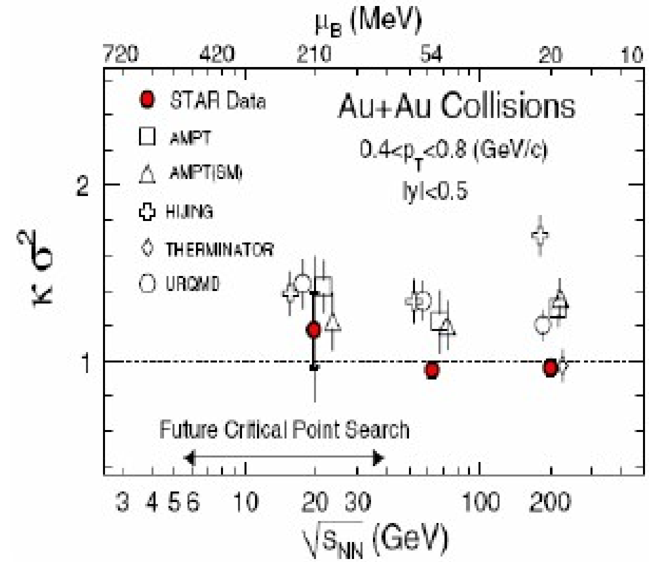
- Scaling requires "running": $\lambda_3 = \tilde\lambda_3 T(T\xi)^{-3/2}$ and $\lambda_4 = \tilde\lambda_4 (T\xi)^{-1}$, i.e.,

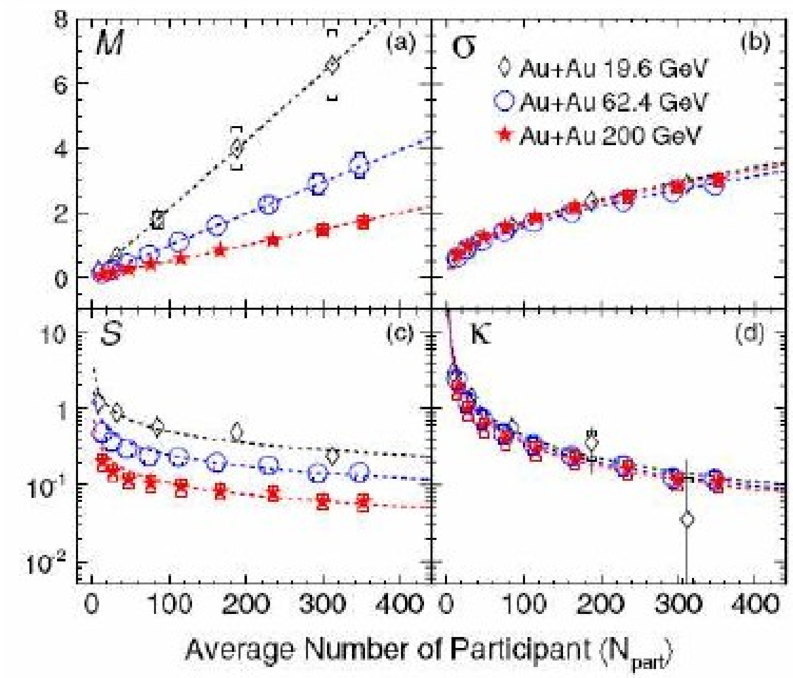
$$\kappa_3 = \langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}\,; \quad \kappa_4 = 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

Non-gaussian fluctuations at the QCD critical point – p. 7/14

STAR First result on higher moments of net-proton







STAR Collaboration, PRL 105 (2010) 022302.

p+pbar = const

- STAR first results on higher moments analysis are up to fourth order.
- Using ratios used to establish base line measurements for the QCD critical point search.
- **This talk: C_6 / C_2 and C_4 / C_2.**

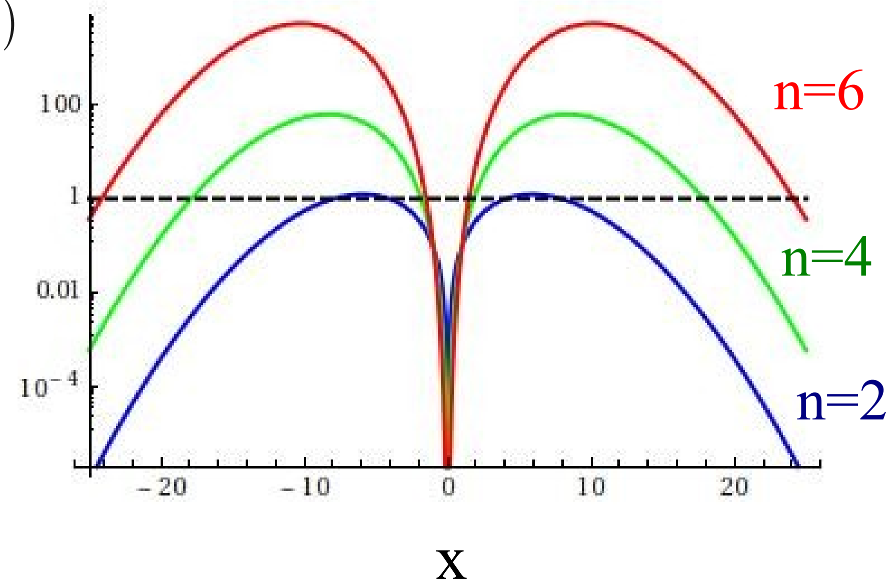
Lizhu Chen Fluctuations, Correlations and RHIC Low Energy Runs, Oct 3-5, BNL 8

Higher cumulants are promising!

D-

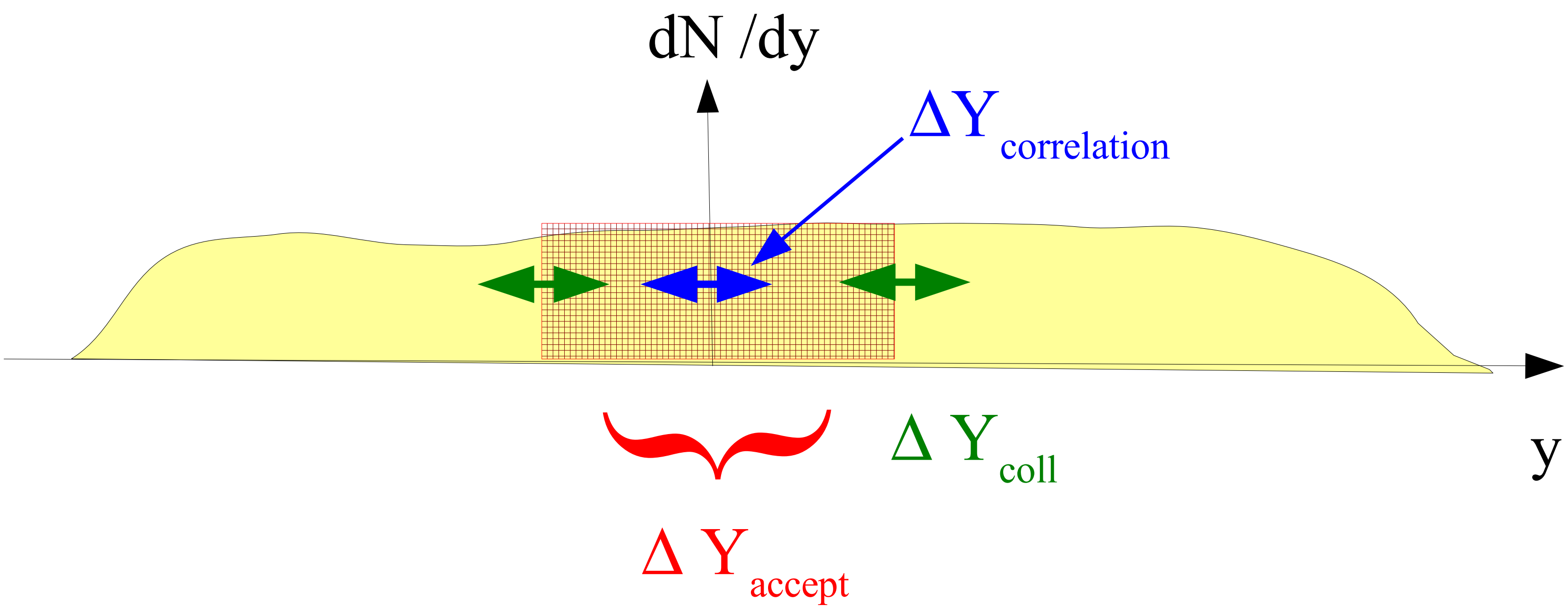
Higher Moments probe the tails

$$P_n(x) = x^n \, Prob(x)$$



Gaussian with width = 4.2,
corresponds roughly to STAR net proton distribution at 200 GeV

"Charge" fluctuations

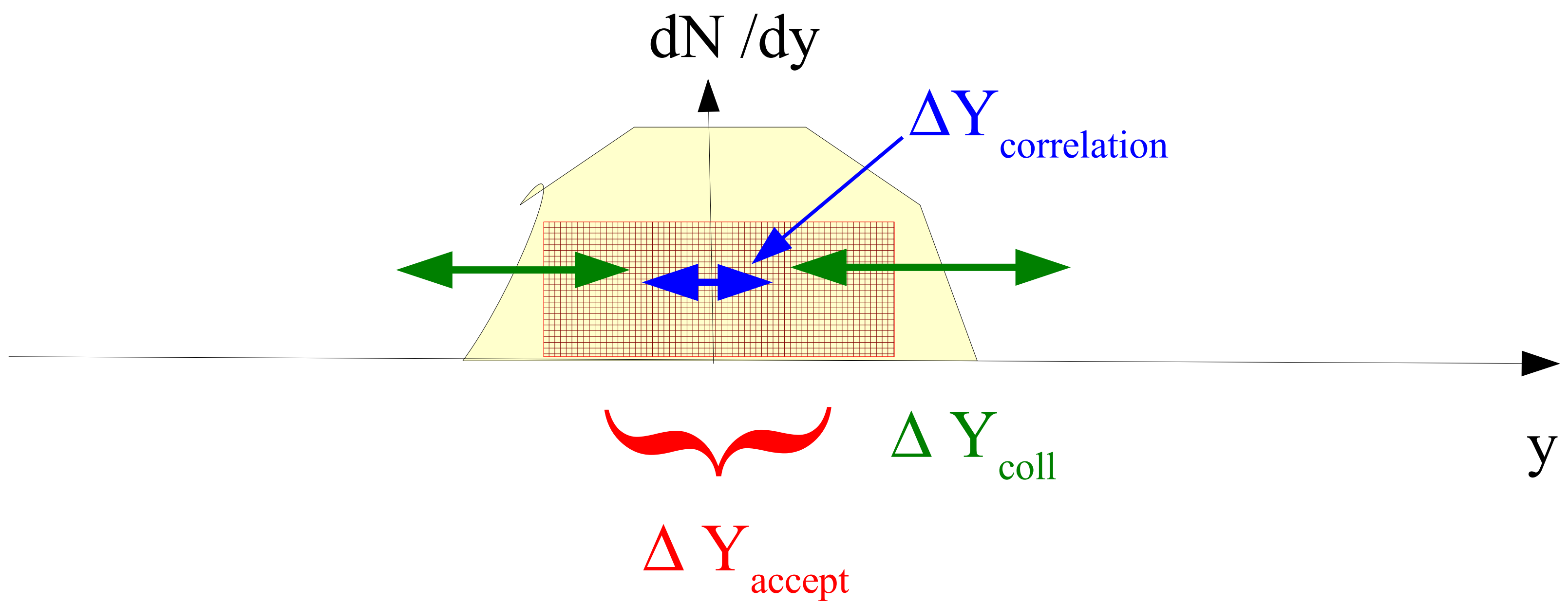


Conditions for "charge" fluctuations:

0) $\Delta Y_{corrrelation} << \Delta Y_{accept}$ **(catch the physics)**

2) $\Delta Y_{total} >> \Delta Y_{accept} >> \Delta Y_{coll}$ **(keep the physics)**

“Charge” fluctuations at SPS and below



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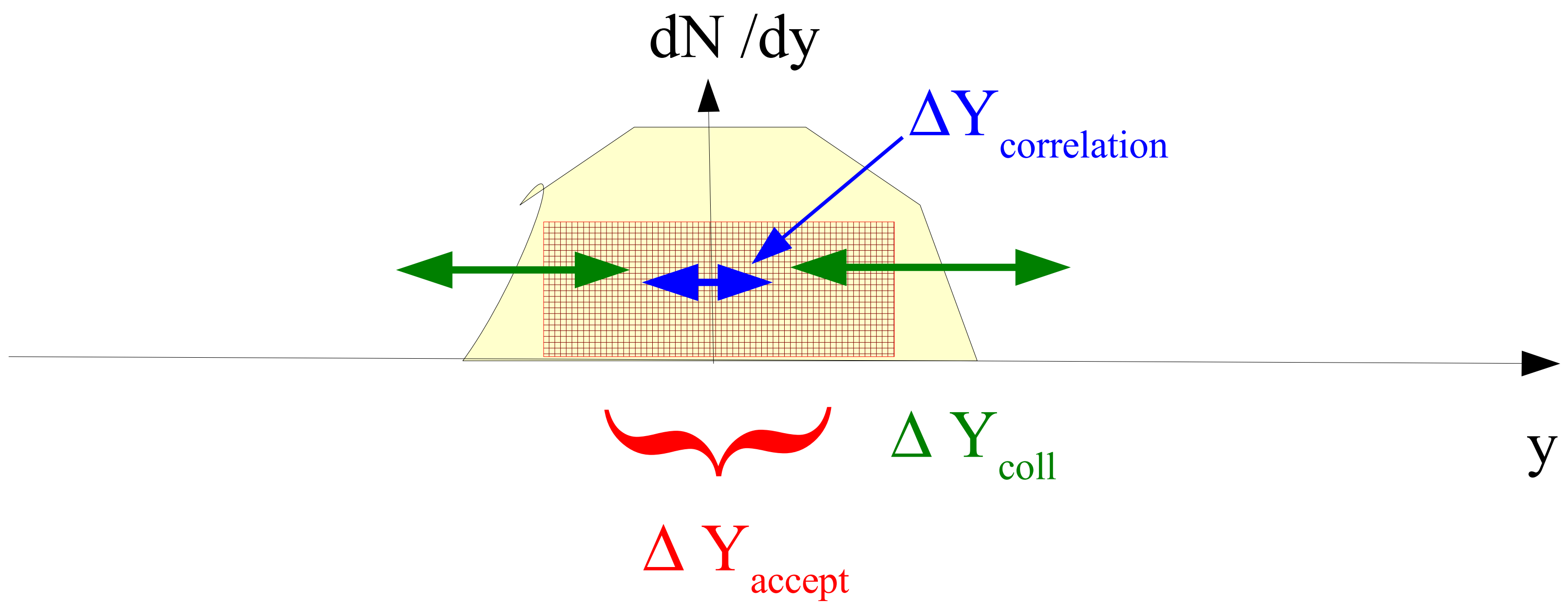
“Charge” fluctuations at SPS and below



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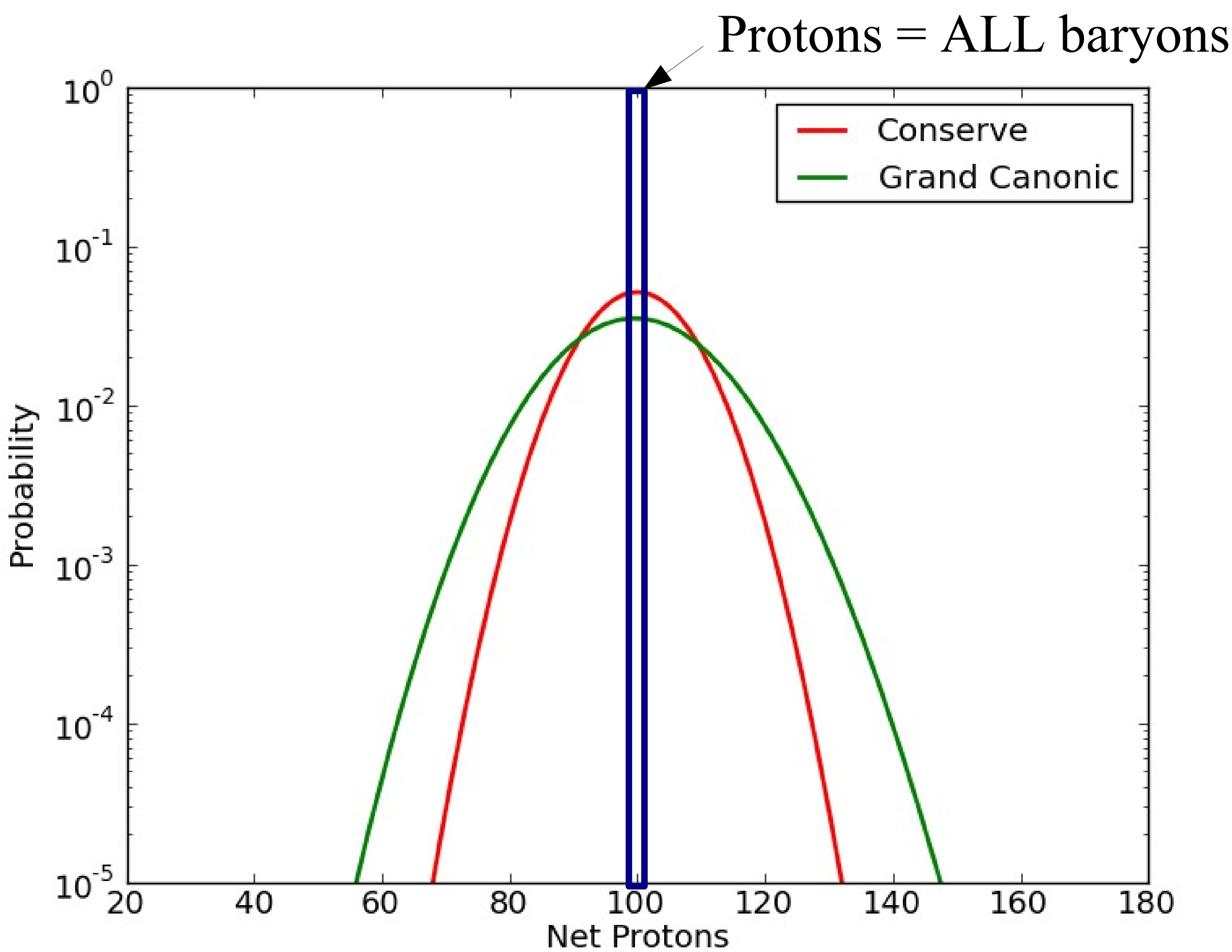
Baryon number (+ Charge) conservation!



B=200, Q=100, Z_1(nucleons)=40, Z_1(pion)=500

Baryon Number Conservation

A. Bzdak, V. Skokov, VK, Phys.Rev. C87 (2013) 014901

p_B, p_{B_bar}

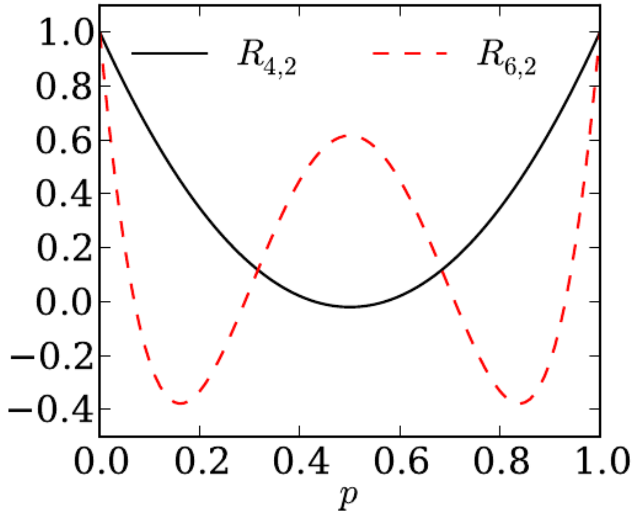
$$P_B(n) = \left(\frac{p_B}{p_{\bar{B}}}\right)^{n/2} \left(\frac{1-p_B}{1-p_{\bar{B}}}\right)^{(B-n)/2} \quad (7)$$

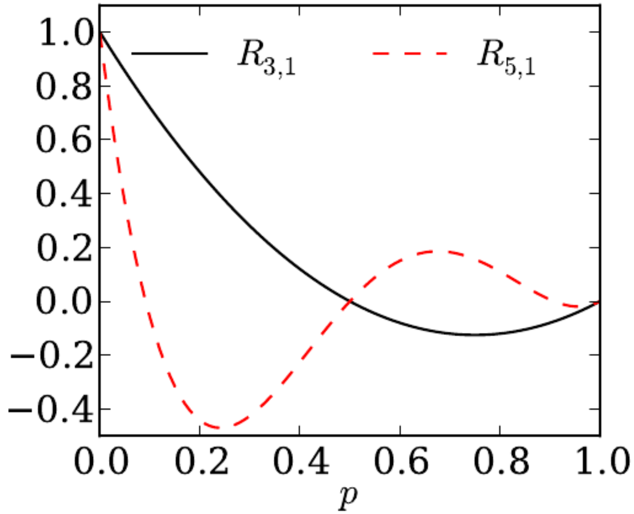
$$\times \frac{I_n\left(2z\sqrt{p_B p_{\bar{B}}}\right) I_{B-n}\left(2z\sqrt{(1-p_B)(1-p_{\bar{B}})}\right)}{I_B(2z)},$$

$$z = \sqrt{\langle N_B \rangle \langle N_{\bar{B}} \rangle}.$$

Protons only: $p_B = \frac{\langle n_B \rangle}{\langle N_B \rangle} \to \frac{\langle n_p \rangle}{\langle N_B \rangle}$, $< 1/2$

"Avoid" conservation Laws

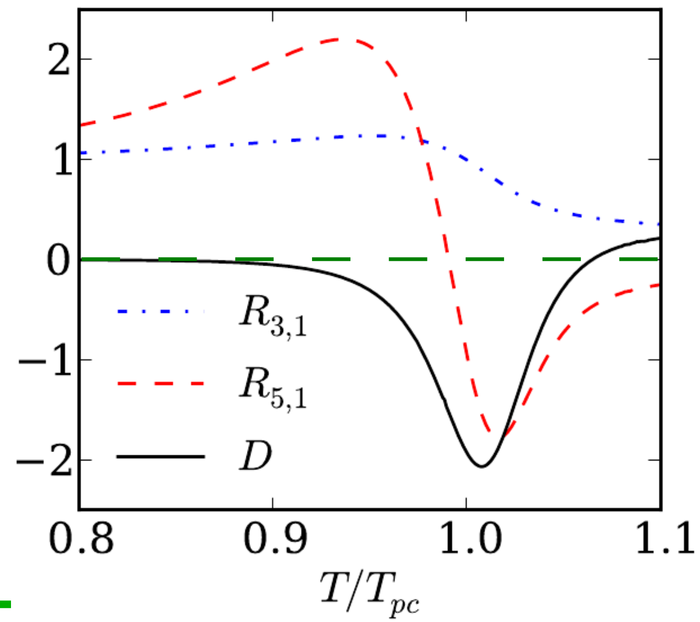




Define: $R_{m,n} = \frac{C_m}{C_n}$

$$D = R_{5,1} - R_{3,1}\left[1 - \frac{3}{4}(1+\gamma)(3-\gamma)\right] \qquad \gamma = \sqrt{1+8R_{3,1}}$$

D = 0 for ALL values of p, B, …
in ABSENCE of any correlations



No physics

A.Bzdak,VK,V.Skokov arXiv:1203.4529

Finite Acceptance

A. Bzdak, VK; Phys.Rev. C86 (2012) 044904

Effect of conservation laws get reduced with finite acceptance:
"Equilibration via ignorance"

Model with binomial distribution: $p_{1,2}$ = probability to see particle, antiparticle

True distribution

$$p(n_1, n_2) = \sum_{N_1=n_1}^{\infty} \sum_{N_2=n_2}^{\infty} P(N_1, N_2) \frac{N_1!}{n_1!(N_1-n_1)!} p_1^{n_1} (1-p_1)^{N_1-n_1} \times \frac{N_2!}{n_2!(N_2-n_2)!} p_2^{n_2} (1-p_2)^{N_2-n_2}.$$

Finite Acceptance

True $$F_{ik} \equiv \left\langle \frac{N_1!}{(N_1-i)!}\frac{N_2!}{(N_2-k)!} \right\rangle = \sum_{N_1=i}^{\infty}\sum_{N_2=k}^{\infty} P(N_1,N_2)\frac{N_1!}{(N_1-i)!}\frac{N_2!}{(N_2-k)!},$$

Measured $$f_{ik} \equiv \left\langle \frac{n_1!}{(n_1-i)!}\frac{n_2!}{(n_2-k)!} \right\rangle = \sum_{n_1=i}^{\infty}\sum_{n_2=k}^{\infty} p(n_1,n_2)\frac{n_1!}{(n_1-i)!}\frac{n_2!}{(n_2-k)!}.$$

$$f_{ik} = p_1^i \cdot p_2^k \cdot F_{ik}.$$

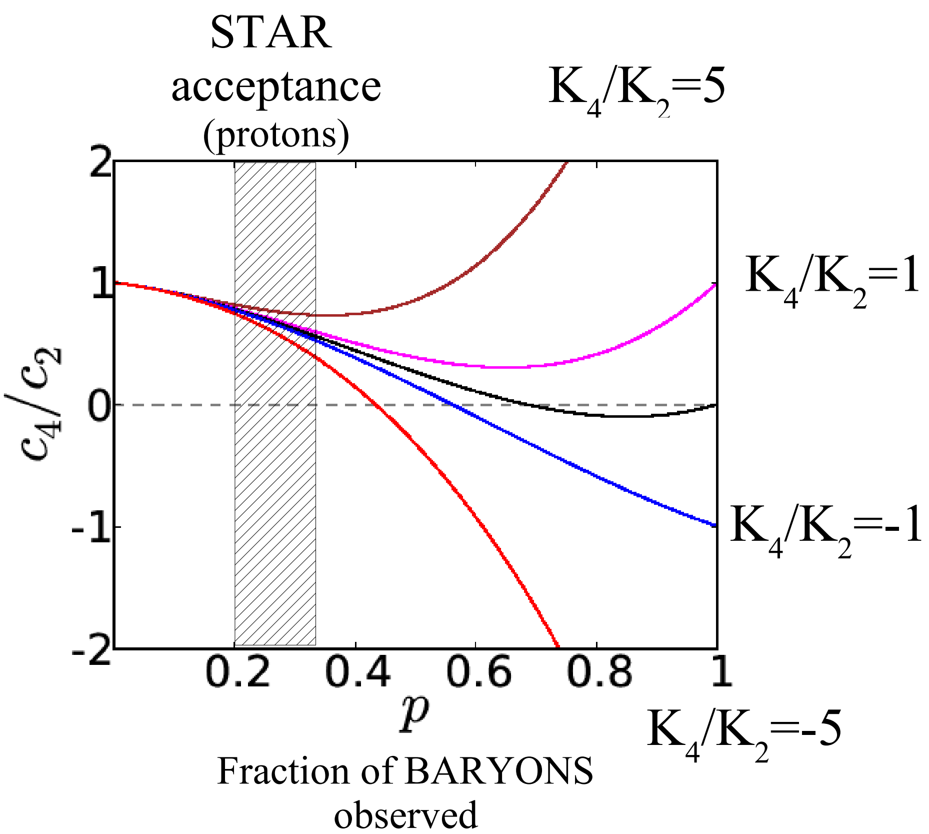
$$\begin{aligned} c_1 &= pK_1, \\ c_2 &= p\,(1-p)\,N + p^2 K_2, \\ c_3 &= p(1-p^2)K_1 + 3p^2(1-p)\,(F_{20} - F_{02} - NK_1) + p^3 K_3, \end{aligned}$$

$$N = N_1 + N_2$$

$$\begin{aligned} c_4 \;=\; & Np(1-p) - 3N^2p^2(1-p)^2 + 6p^2(1-p)(F_{02}+F_{20}) - 12K_1p^3(1-p)(F_{20}-F_{02}) \\ & +6Np^3(1-p)(K_1^2 - K_2) + p^2(1-p^2)(K_2 - 3K_1^2) \\ & +6p^3(1-p)(F_{03} - F_{12} + F_{02} + F_{20} - F_{21} + F_{30}) + p^4 K_4. \end{aligned}$$

Due to “acceptance” not only Cumulants of the the true distribution enter

Finite Acceptance



Spoils the fun for Baryon cumulants

Electric Charge cumulants better.
BUT issue with separation of (rapidity scales) at low energies

Finite Acceptance

Unfolding?

Can be done (in principle)

$$
\begin{aligned}
pK_1 &= c_1, \\
p^2K_2 &= c_2 - n(1-p), \\
p^3K_3 &= c_3 - c_1(1-p^2) - 3(1-p)(f_{20} - f_{02} - nc_1),
\end{aligned}
$$

$$
\begin{aligned}
p^4K_4 \;=\; & c_4 - np^2(1-p) - 3n^2(1-p)^2 - 6p(1-p)(f_{20} + f_{02}) + 12c_1(1-p)(f_{20} - f_{02}) \\
& -(1-p^2)(c_2 - 3c_1^2) - 6n(1-p)(c_1^2 - c_2) \\
& -6(1-p)(f_{03} - f_{12} + f_{02} + f_{20} - f_{21} + f_{30}).
\end{aligned}
$$

Requires measurement of factorial moments, f_{20}, f_{02}, f_{21},.... with GOOD precision

Help from Theory?

$$c_4 = Np(1-p) - 3N^2p^2(1-p)^2 + 6p^2(1-p)(F_{02}+F_{20}) - 12K_1p^3(1-p)(F_{20}-F_{02})$$
$$+6Np^3(1-p)(K_1^2-K_2) + p^2(1-p^2)(K_2-3K_1^2)$$
$$+6p^3(1-p)(F_{03}-F_{12}+F_{02}+F_{20}-F_{21}+F_{30}) + p^4K_4.$$

Can we calculate the necessary factorial moments?

Simple example: $\langle B+\bar{B} \rangle$ or $\langle p+\bar{p} \rangle$

Lattice: NO

Maybe some Model can do that?

Help from Theory?

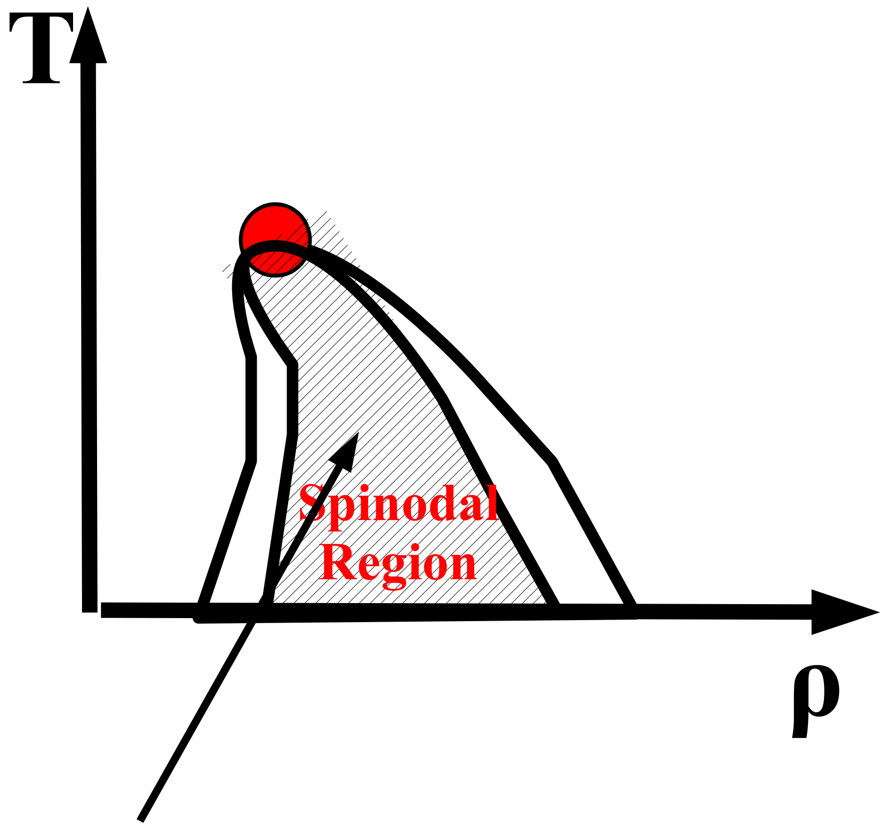
No or smaller and better controlable acceptance effects:
N-particle densities

$$\frac{dN}{dp_1\, dp_2 \ldots dp_n} \sim \langle a^{+}_{p_n} \ldots a^{+}_{p_2} a^{+}_{p_1}\ a_{p_n} \ldots a_{p_2} a_{p_1} \rangle$$

Can we calculate this in the various models?

Can we establish that these are sensitive to critical phenomena?

Co-existence region



System should spent long time
in spinodal region

Spinodal instability:
Mechanical instability

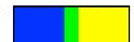
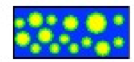
$$\frac{\partial p}{\partial \epsilon} < 0$$

Exponential growth of clumping

Non-equilibrium phenomenon!

Phase-transition dynamics: Density clumping

Phase transition =>

- Phase coexistence: *tension* 
- Phase separation: *instabilities* 

Introduce a ***gradient*** term:

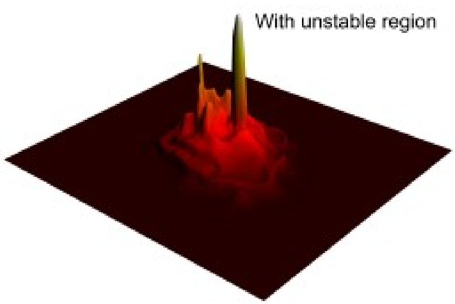
$$p(\boldsymbol{r}) = p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - C\rho(\boldsymbol{r})\boldsymbol{\nabla}^2\rho(\boldsymbol{r})$$

Insert the modified pressure into existing ideal finite-density fluid dynamics code

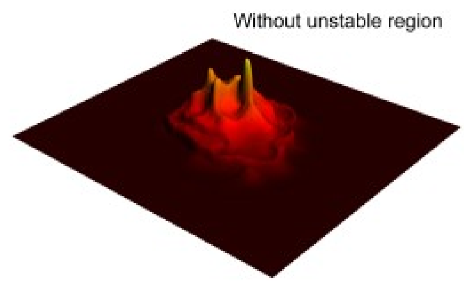
Use UrQMD for pre-equilibrium stage to obtain fluctuating initial conditions

Simulate central Pb+Pb collisions at ≈3 GeV/A beam kinetic energy on fixed target, using an Equation of State either ***with*** a phase transition ***without*** (Maxwell partner):

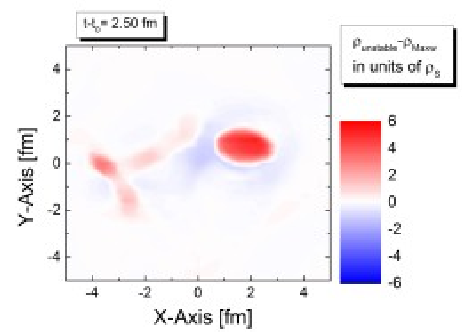
With phase transition:

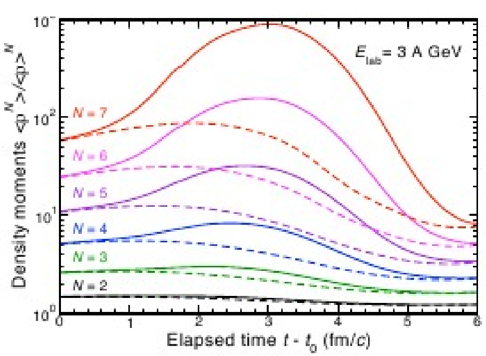


Without phase transition:



Density enhancement:

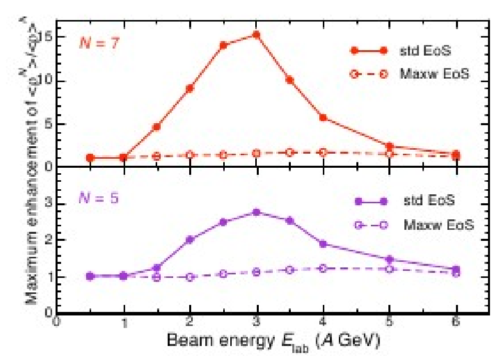




Evolution of density moments

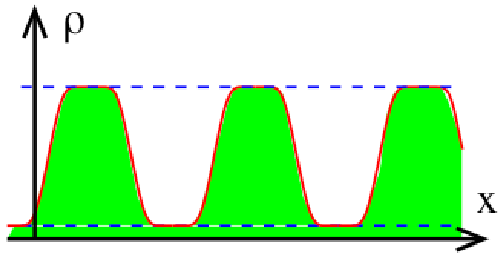
$$\langle \rho^N \rangle \equiv \frac{1}{A}\int \rho(\boldsymbol{r})^N \rho(\boldsymbol{r}) d^3\boldsymbol{r}$$

J. Steinheimer & J. Randrup, PRL **109**



Different approach

Even if total baryon <u>number</u> does not fluctuate the baryon **density** does



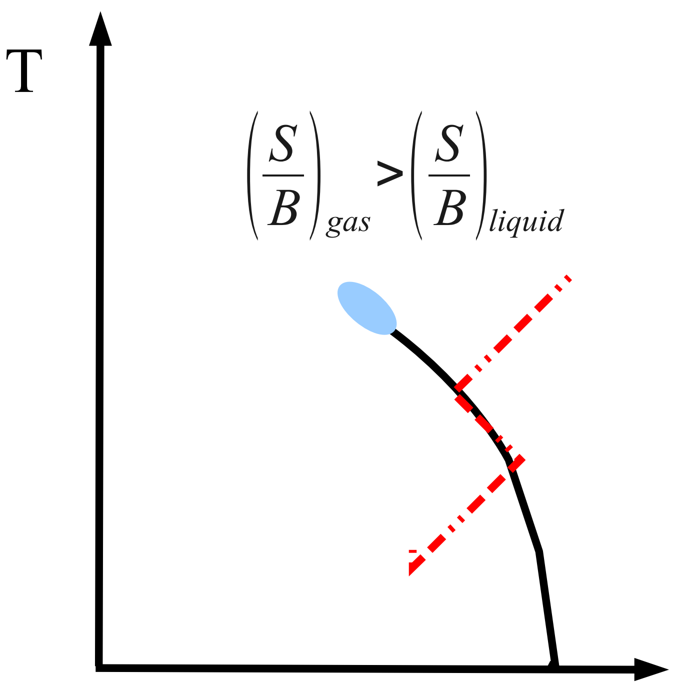
Therefore measure production of NUCLEI: d, ^{3}He, ^{4}He, ^{7}Li....

$$\langle d \rangle \sim \langle \rho_B^2 \rangle \qquad \langle {}^3He \rangle \sim \langle \rho_B^3 \rangle \qquad \langle {}^7He \rangle \sim \langle \rho_B^7 \rangle$$

Extracts higher moments of the baryon **density** at freeze out

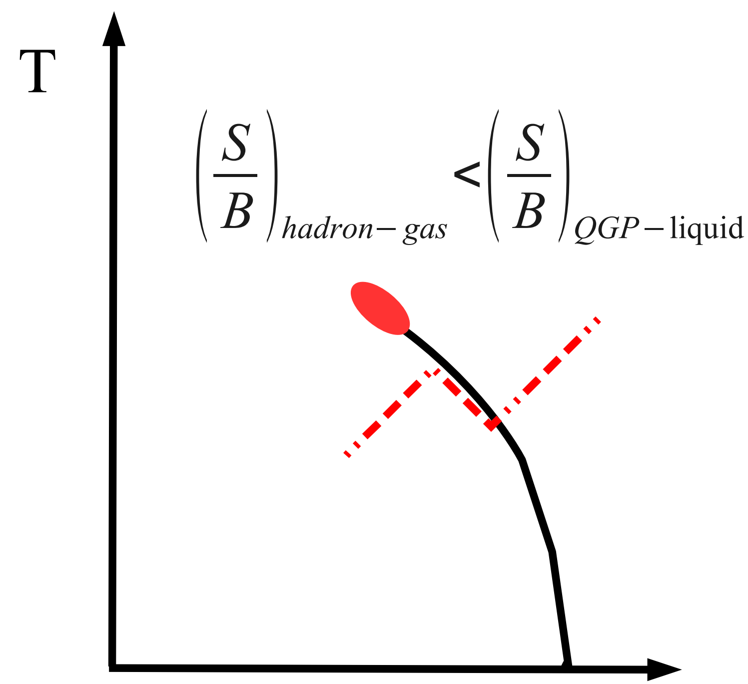
Nice Idea, but does not seem to work......

A note on the Phasediagram



$P(T)_{\text{co-exist}} = 0$

Droplets are stable in vacuum

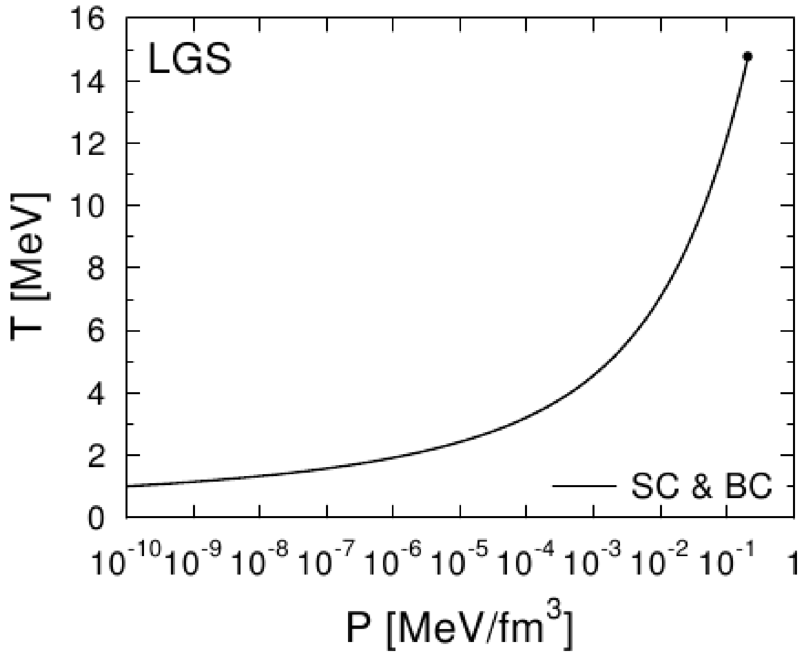


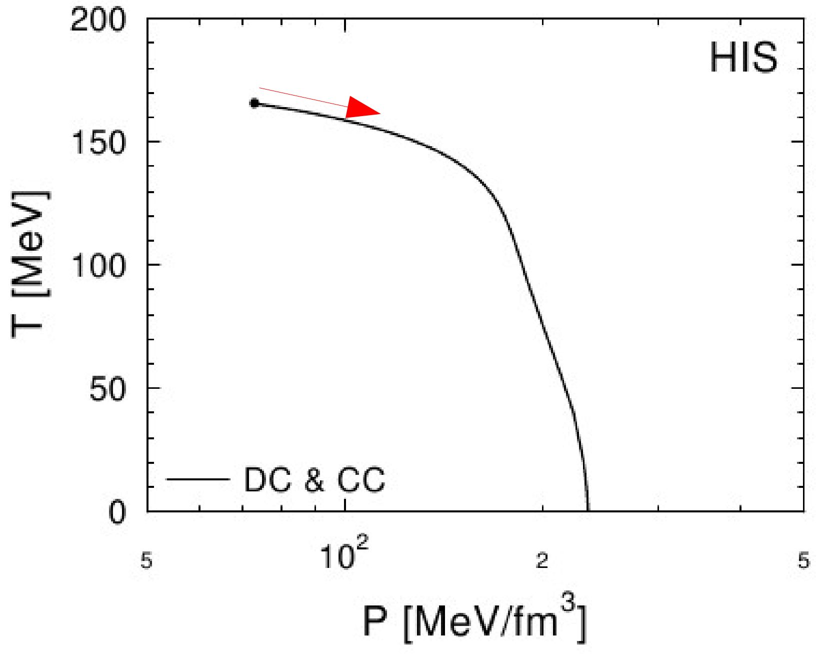
$P(T)_{\text{co-exist}} > 0$!!!

No stable droplets in vacuum

Difference between Liquid Gas and QCD PT

Dexheimer et al, arXiv:1302.2835





One check on models: Does pressure increase along the critical line as one goes away from critical point?

Issues

- Impact parameter (volume) fluctuations
  - C(4)/C(2) is independent of volume but NOT of volume fluctuations (see Skokov et al)
- Initial state fluctuations
  - At high energy large fraction of baryons/charges not part of the system
  - Can be addressed with transport (in principle)
- Can we unfold the effects of charge conservation?
  - Maybe.... vary acceptance (rapidity)

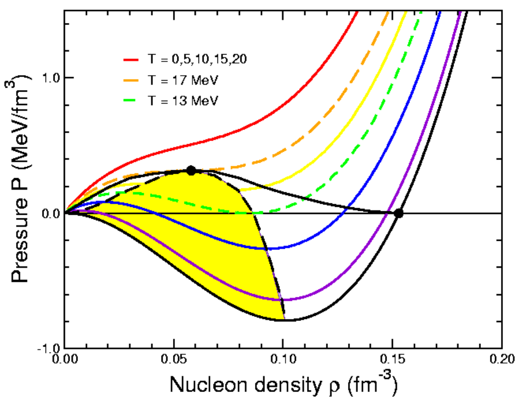
Summary

- Need to measure cumulants (or equivalent) in order to really say something about phase boundary
  - Higher cumulants are very sensitive to trivial effects
    - Charge conservation
    - Efficiency fluctuations
    - Initial state fluctuations
  - Proton cumulants are not a good proxy for baryon cumulants. Need to measure additional fact. moments.
- Can we help from theory?
  - Calculate factorial moments
  - Calculate n-particle densities
- Verify the EOS is not of liquid gas type!!!

BACKUP

Spinodal Multifragmentation

Nuclear EoS:

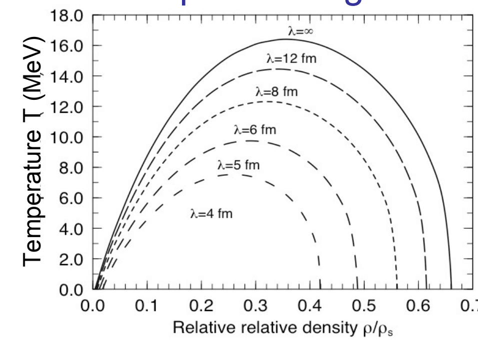


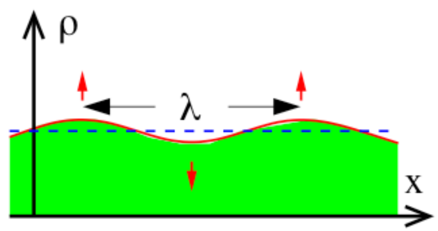
1st order phase transtion

⇕

Spinodal instability

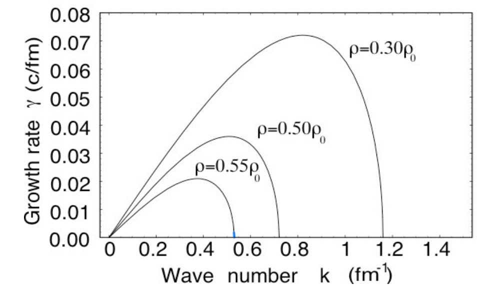
Spinodal region:



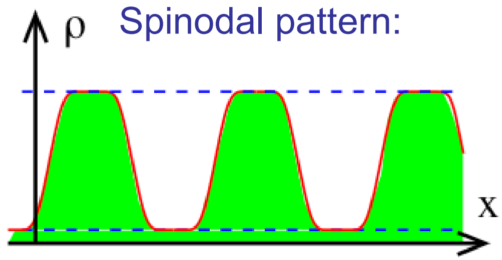


Density undulations
may be amplified

Growth rates:



Spinodal pattern:



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263

⇨

Fragments
≈ *equal!*

★ Highly non-statistical => Good candidate signature

CLUMPING of Baryon Density

J. Randrup

Input required for realistic estimate of conservation effects

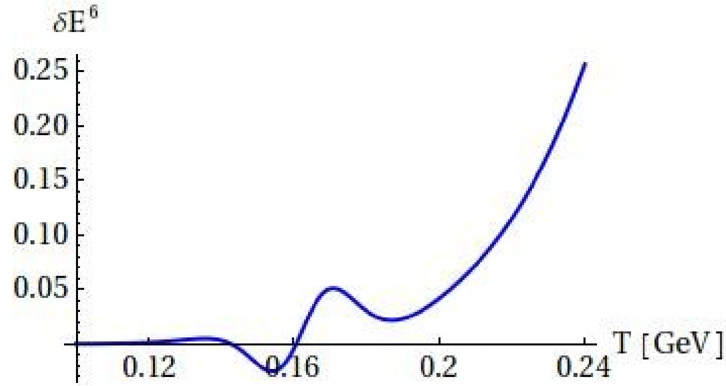
Note: This is likely only to work at lower energies where we have baryon stopping
Note: at low energies anti-protons likely to be irrelevant

Need:
- Total number of protons and (anti-protons) (4 Pi)
- Number of protons and (anti-protons) actually measured
- Total number of charged particles

Big Question: Over what rapidity range are the various charges conserved?
- Balance Functions? Only averages!

QCD vs HRG

QCD



"HRG"