Non-perturbative results for two- and three-point functions of Landau gauge Yang-Mills theory

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Quarks, Gluons, and Hadronic Matter under Extreme Conditions
St. Goar

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Landau gauge Green functions from functional methods

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- Information about confinement
- Input for phenomenological calculations, e.g., bound states, QCD phase diagram

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QCD phase diagram with functional equations:

\+ no sign problem at non-zero chemical potential

\+ physical quark masses easy

\- infinitely large system $\rightarrow$ influence of higher Green functions?
$\Rightarrow$ tests of truncations

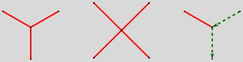
Landau Gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

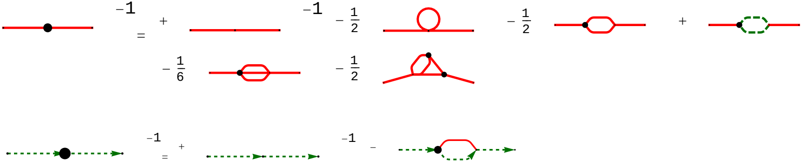
Propagators and vertices are gauge dependent
$\rightarrow$ choose any gauge, ideally one that is convenient.

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \rightarrow 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}}\,(-\Box + g\,\boldsymbol{A}\,\times)\,\boldsymbol{c}$
- 2 fields: 3 vertices: 

Dyson-Schwinger equations: Propagators

Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:



- Equations of motion of correlation functions:
  Describe how fields propagate and interact non-perturbatively!
- Infinite tower of coupled integral equations.
- Derivation straightforward, but tedious
  $\longrightarrow$ automated derivation with *DoFun* [MQH, Braun, CPC183 (2012)].
- Contain three-point and four-point functions:
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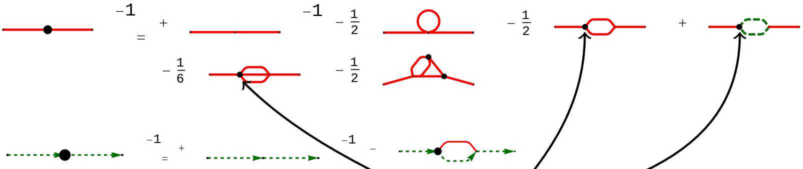

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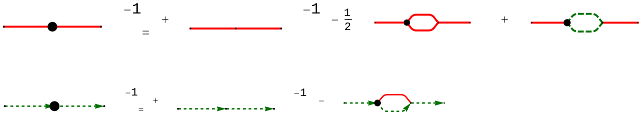
Truncated propagator Dyson-Schwinger equations

Standard truncation:



using bare ghost-gluon vertex and three-gluon vertex model

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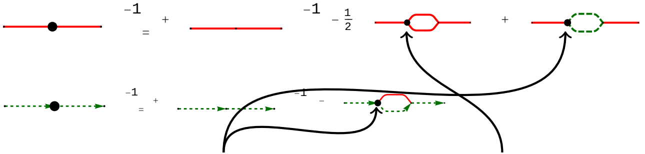

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Truncating Dyson-Schwinger equations

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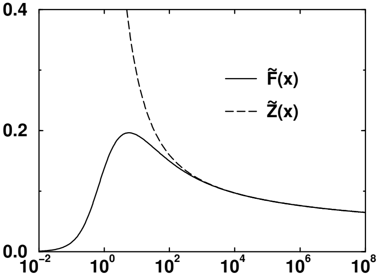

[Hauck, von Smekal, Alkofer, CPC 112 (1998)]

$$D_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{\tilde{Z}(p^2)}{p^2}$$

- gluon dressing $\tilde{Z}(p^2)$ IR divergent $\rightarrow$ IR slavery

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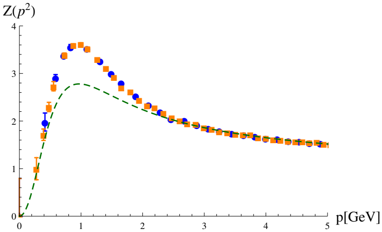

[MQH, von Smekal, 1211.6092; Sternbeck, hep-lat/0609016]

$$D_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

$$D_{gh}(p) = -\frac{G(p^2)}{p^2}$$

- gluon dressing $Z(p^2)$ IR vanishing
- deviations from lattice results in mid-momentum regime

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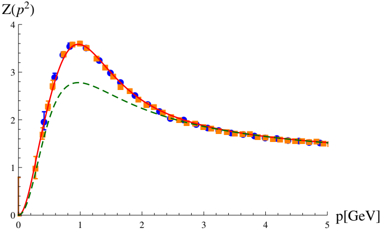

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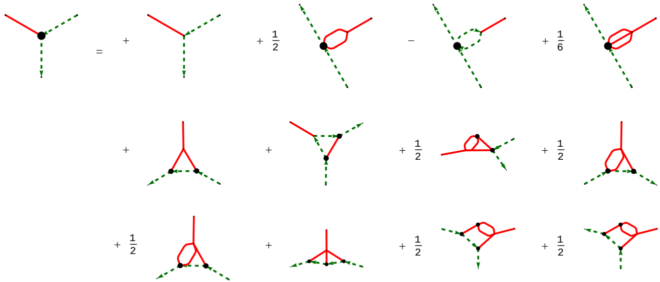
- gluon dressing $Z(p^2)$ IR vanishing
- improved mid-momentum behavior

Improved truncations necessary for quantitative results and also extensions, e.g., non-zero temperature [Fister, Pawlowski, 1112.5440].

Ghost-gluon vertex DSE

Full DSE:



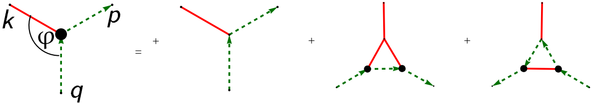
- Lattice results [Cucchieri, Maas, Mendes, PRD77 (2008); Ilgenfritz et al., BJP37 (2007)]
- OPE analysis [Boucaud et al., JHEP 1112 (2011)]
- Modeling via ghost DSE [Dudal, Oliveira, Rodriguez-Quintero, PRD86 (2012)]
- Semi-perturbative DSE analysis [Schleifenbaum et al., PRD72 (2005)]
- FRG [Fister, Pawlowski, 1112.5440]

Ghost-gluon vertex

$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\left(p_\mu A(k;p,q) + k_\mu B(k;p,q)\right)$$

Note:
$B(k;p,q)$ is irrelevant in Landau gauge (but it is not the pure longitudinal part).
Taylor argument applies only to longitudinal part (it's an STI).

IR and UV consistent truncation:



System of eqs. to solve:
gluon and ghost propagators + ghost-gluon vertex

Only unfixed quantity in present truncation: three-gluon vertex.

Three-gluon vertex: Ultraviolet

Bose symmetric version:

$$D^{A^3,UV}(x,y,z) = G\left(\frac{x+y+z}{2}\right)^{\alpha} Z\left(\frac{x+y+z}{2}\right)^{\beta}$$

Fix α and β:

1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex?

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1. UV behavior of three-gluon vertex
2. IR behavior of three-gluon vertex $\rightarrow$ yes, but ...

Three-gluon vertex: Infrared

Three-gluon vertex might have a zero crossing.

$d = 2, 3$: seen on lattice [Cucchieri, Maas, Mendes, PRD77 (2008); Maas, PRD75 (2007)],

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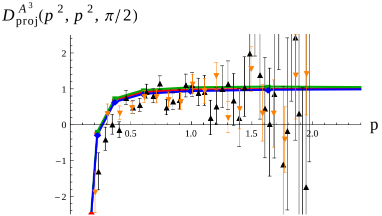

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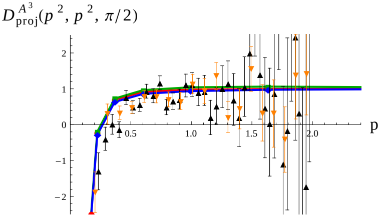
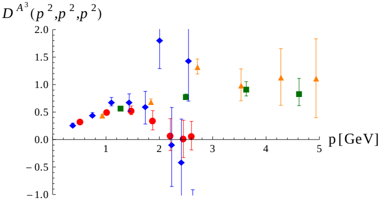


$d = 4$:

[Cucchieri, Maas, Mendes, PRD77 (2008)]



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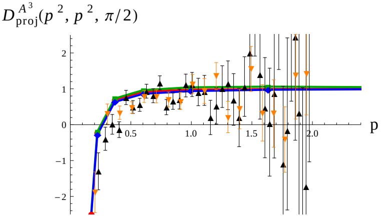

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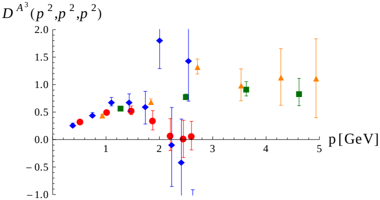
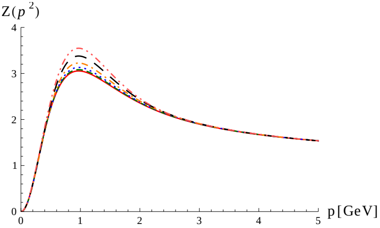
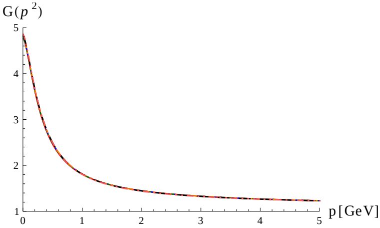


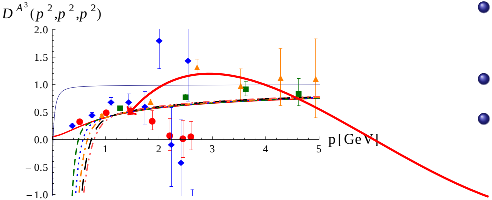
$$D^{A^3,IR}(x,y,z) = h_{IR}\, G(x+y+z)^3 \left(f^{3g}(x) f^{3g}(y) f^{3g}(z)\right)^4$$

IR damping function $f^{3g}(x) := \dfrac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$

Influence of the three-gluon vertex





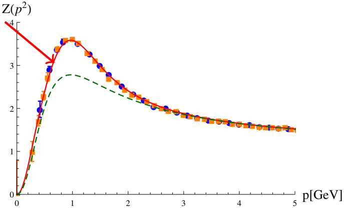


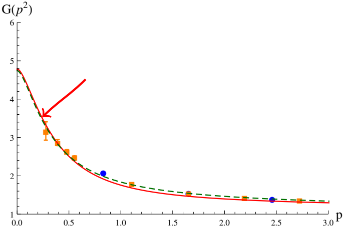
- Vary Λ_{3g} $\longrightarrow$ vary mid-momentum strength
- Ghost almost unaffected
- Thin line: Leading IR order DSE calculation for three-gluon vertex $\Rightarrow$ zero crossing

Optimized effective three-gluon vertex:
Choose Λ_{3g} where gluon dressing has best agreement with lattice results.
[MQH, von Smekal, 1211.6092]

Dynamic ghost-gluon vertex: Propagator results

Dynamic ghost-gluon vertex, opt. eff.
three-gluon vertex [MQH, von Smekal, 1211.6092]





Good quantitative agreement for ghost and gluon dressings.

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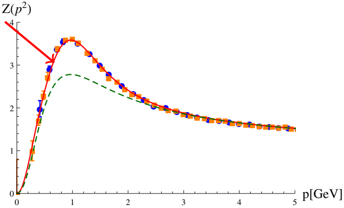
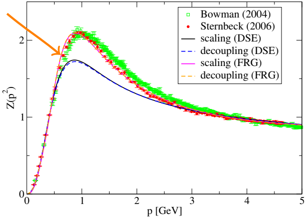
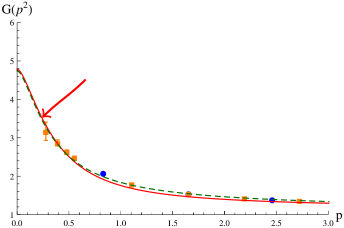
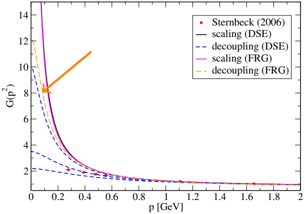
FRG results
[Fischer, Maas, Pawlowski, AP324 (2009)]









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Ghost-gluon vertex: Selected configurations (decoupling)

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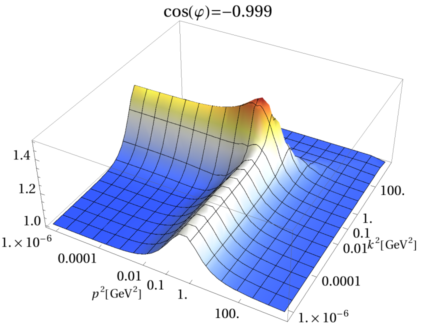
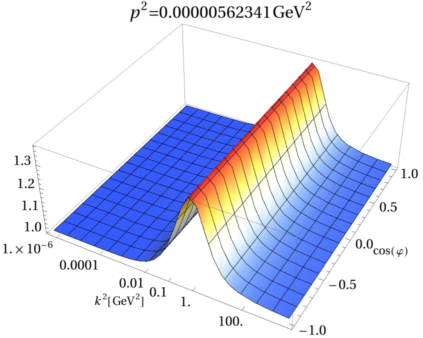
Fixed angle:



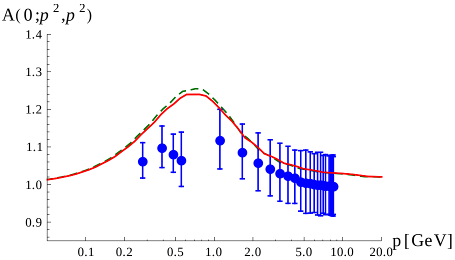
[MQH, von Smekal, 1211.6092]

Fixed anti-ghost momentum:



Ghost-gluon vertex: Comparison with lattice data

Orthogonal configuration $k^2 = 0$, $q^2 = p^2$:



- constant in the IR
- relatively insensitive to changes of the three-gluon vertex (red/green lines: different three-gluon vertex models)

DSE calculation: [MQH, von Smekal, 1211.6092]
lattice data: [Sternbeck, hep-lat/0609016]

Towards the phase diagram of QCD with DSEs

- Lattice results helpful
  - as input

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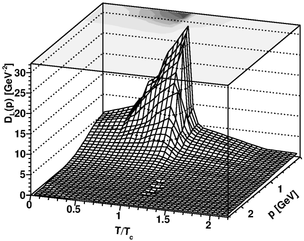
- Self-consistent functional RG calculation of correlation functions:
  [Fister, Pawlowski, 1112.5440; Fister, PhD thesis, 2012]
- DSE calculations:
  [Maas, Wambach, Alkofer, EPJC42 (2005); Cucchieri, Maas, Mendes, PRD75 (2007)]

Ghost propagator

First steps towards full system: Take some lattice input.

Gluon propagator: lattice based fits [Fischer, Maas, Müller, EPJC68 (2010)]



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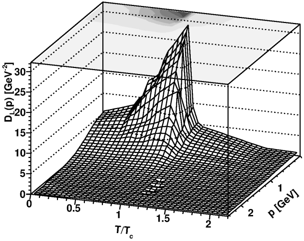
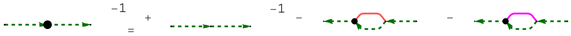


Ghost propagator [preliminary]:



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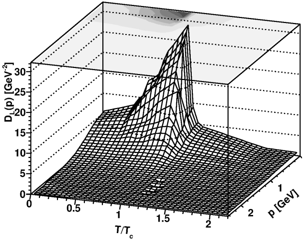

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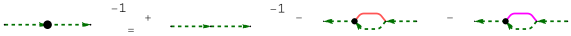
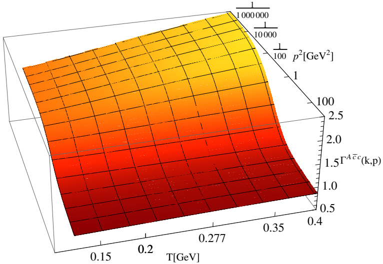


lattice: [Maas, Pawlowski, Spielmann, von Smekal, PRD85 (2012)]



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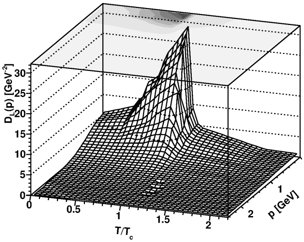

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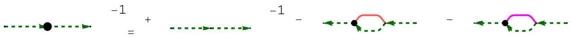

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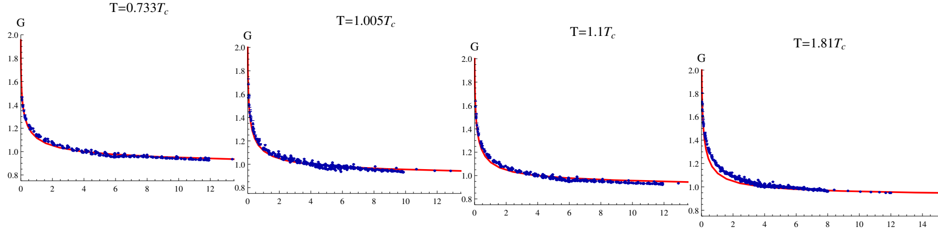

Ghost-gluon vertex

Remember: Relevance of ghost-gluon vertex for non-zero temperature known from functional RG [Fister, Pawlowski, 1112.5440]!

Simple approximation:

Fully iterated ghost propagator
Gluon propagator from the lattice
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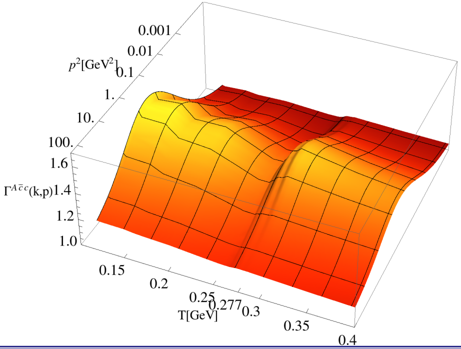

[preliminary]

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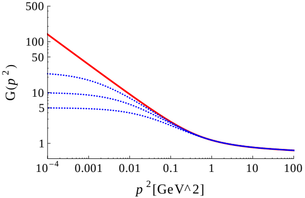
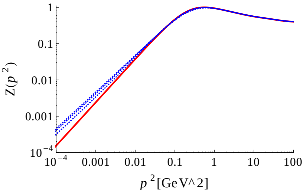
  - ☐ Phase transitions

Thank you for your attention.

Decoupling and scaling solutions

DSEs: Vary ghost boundary condition [Fischer, Maas, Pawlowski, AP324 (2009)]





Realization on lattice?

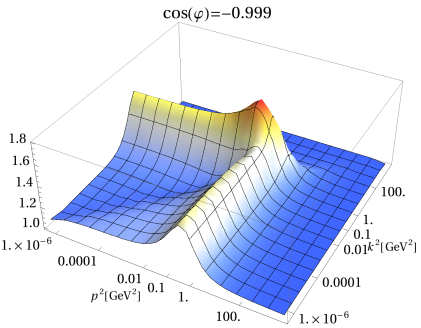
- Dependence of propagators on Gribov copies, e.g., [Bogolubsky, Burgio, Müller-Preussker, Mitrjushkin, PRD 74 (2006); Maas, PR524 (2013)]
- First hints from [Sternbeck, Müller-Preussker, 1211.3057]:
  choosing Gribov copies by the lowest eigenvalue of the Faddeev-Popov operator
- $d = 2$: [Maas, PRD75 (2007); Maas et al., EPJC68 (2010); Cucchieri et al., PRD85 (2012)]

Continuum $d = 2$

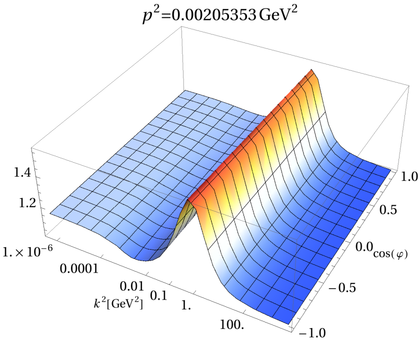
Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85 (2012); MQH, Maas, von Smekal, JHEP1211 (2012)] as well as from analysis of Gribov region [Zwanziger, 1209.1974].

Ghost-gluon vertex and scaling solution

Fixed angle:



Fixed momentum:



- Dressing not 1 in the IR $\longleftarrow$ Contributions from loop corrections (for decoupling they are suppressed)
- Scaling/decoupling also seen in ghost-gluon vertex