CONFINEMENT FROM CORRELATION FUNCTIONS

Leonard Fister
NUI Maynooth

NUI MAYNOOTH
Ollscoil na hÉireann Má Nuad

science foundation ireland
fondúireacht eolaíochta éireann

EMMI

LF, J.M. Pawlowski,

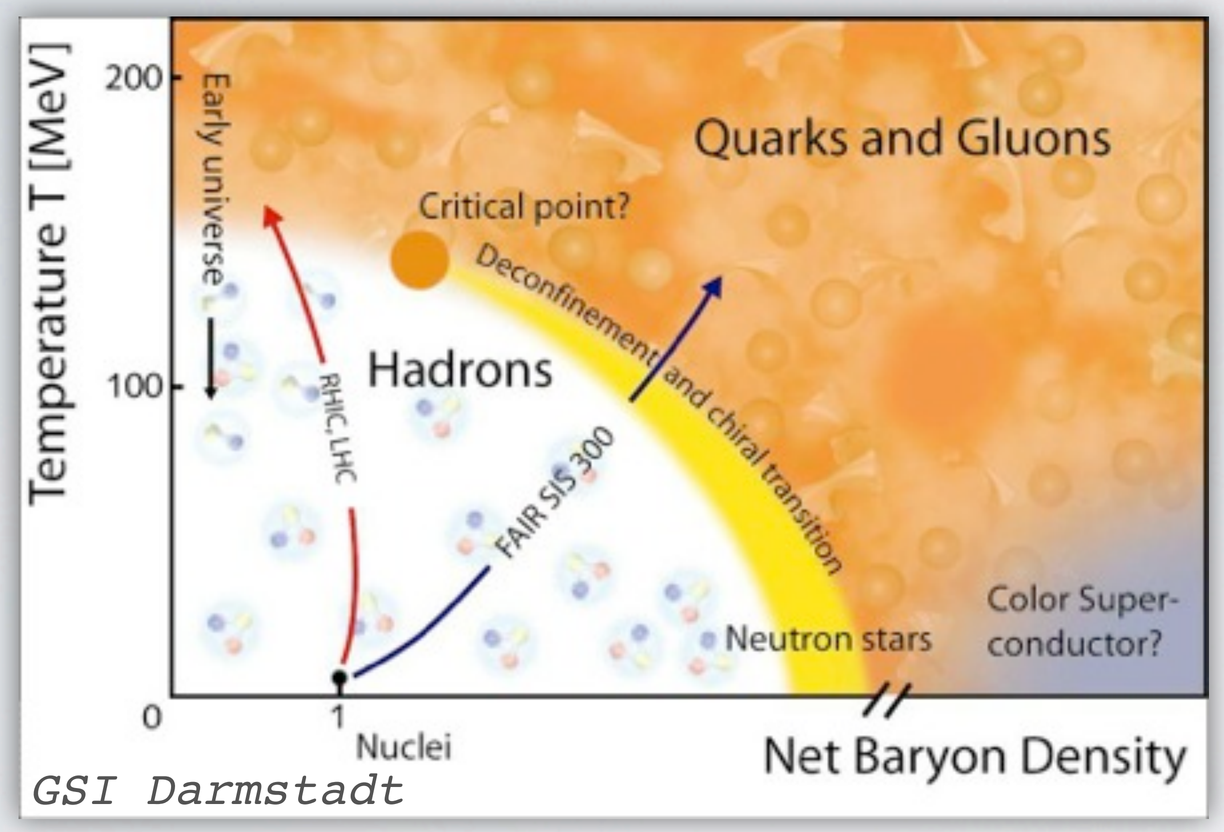
arXiv: 1301.4163 [hep-ph], 1302.1373 [hep-ph],

arXiv: 1112.5440 [hep-ph], PoS QCD-TNT-II2011 (2011) 021 (arXiv: 1112.5429 [hep-ph]).

Quarks, Gluons, and Hadronic Matter under Extreme Conditions

St. Goar, March 18, 2013

Motivation: QCD Phase Diagram



characteristic features at low energies

- confinement
- dynamical chiral symmetry breaking

non-perturbative computation of physical observables from microscopic dynamics

here: study aspects of the phase diagram with non-perturbative **functional continuum methods**

→ static quark confinement via the Polyakov loop potential

phase transition order, phase transition temperature, confinement criterion via infrared behaviour of propagators

→ thermodynamics of pure gluodynamics (Yang–Mills theory)

pressure at temperatures around the phase transition

OUTLOOK

- Motivation
- (Thermal) Yang–Mills Propagators
- Quark Confinement
- Thermodynamics of Yang–Mills Theory

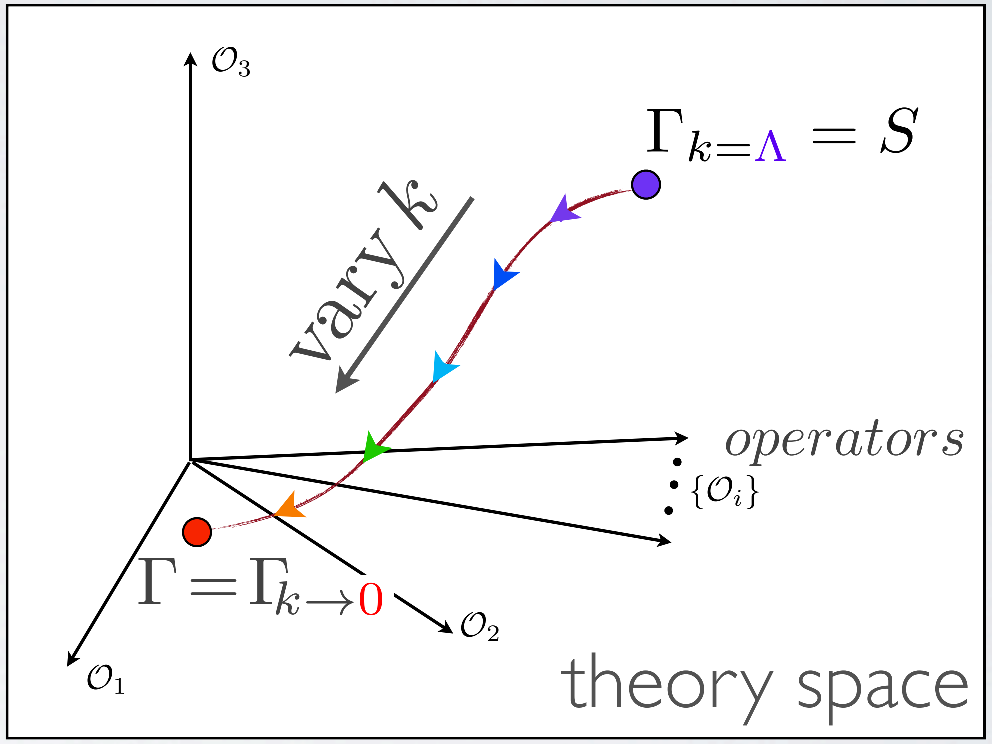
FUNCTIONAL RENORMALISATION GROUP (FRG)

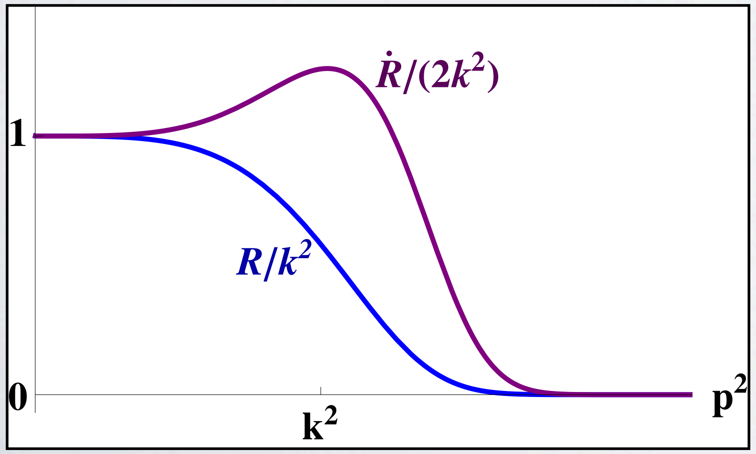
Wetterich, Phys. Lett. B301 (1993) 90-94.

⟶ equation for the **free energy** (= effective action Γ)

k ... **energy scale:**
integrate **fluctuations shell-wise** from UV to IR

'flow' along in **theory space**
... spanned by (all) operators
... start with classical action S
... full quantum theory Γ at $k \to 0$





$$S \to S + \frac{1}{2}\int_p \varphi(-p) R_k(p) \varphi(p)$$

suppression of infrared fluctuations,
the ultraviolet is unchanged

FRG FOR YANG–MILLS

$$S_{\rm YM} = \int d^4x \left(-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

Landau gauge

'flow' equation

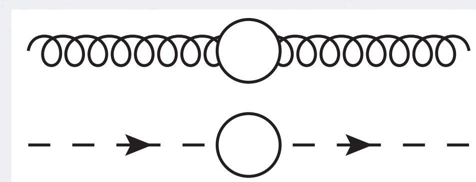
$$\underbrace{k\,\partial_k}_{\partial_t} \Gamma_k[A,\bar{c},c] = \frac{1}{2} \mathrm{Tr} \left\{ \frac{1}{\Gamma^{(2)}[A,\bar{c},c] + R_k} \partial_t R_k \right\} - \partial_t C_k$$

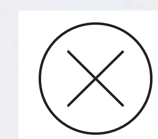
full propagator

regulator

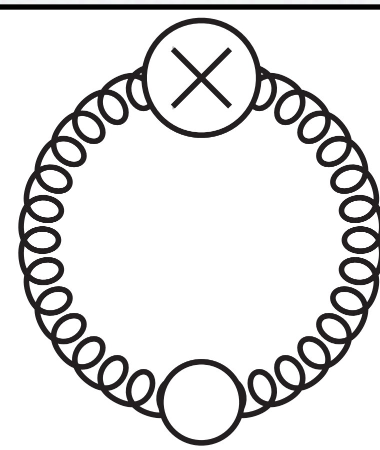
gluon

Faddeev–Popov ghosts

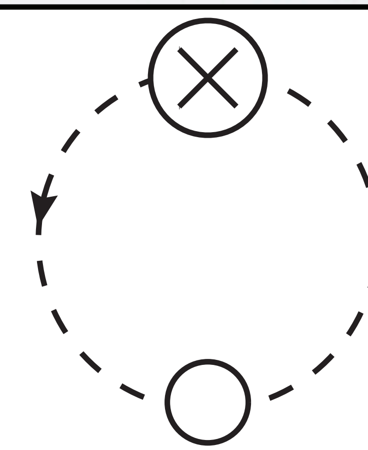




$$\partial_t \Gamma_k[A,\bar{c},c] \;=\; \frac{1}{2}$$

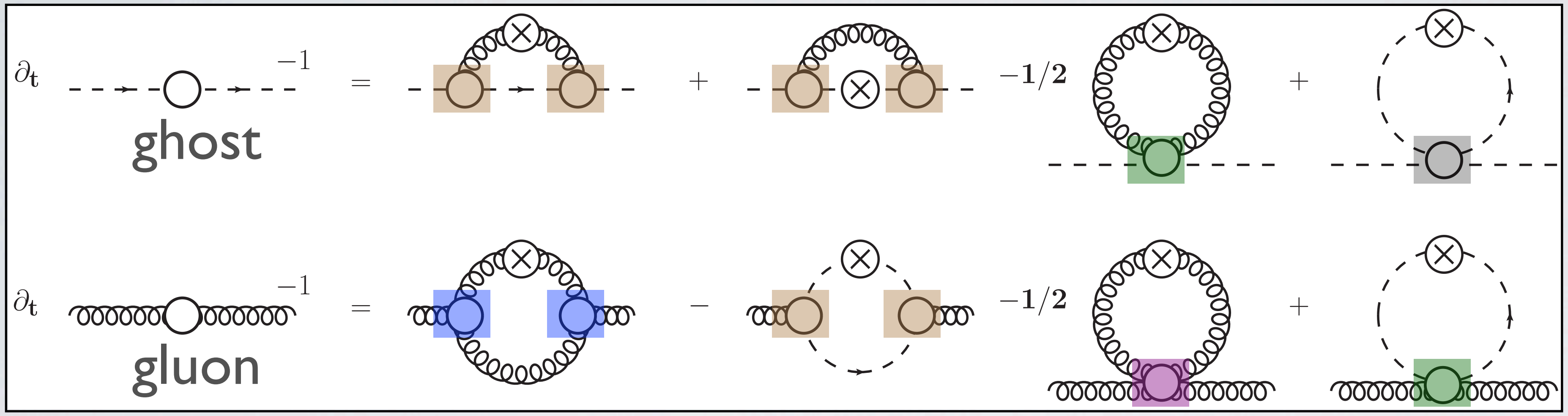


$$-$$



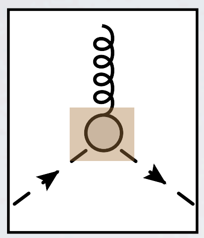
... and flow equations for n-point functions via functional derivatives wrt fields

YANG–MILLS PROPAGATORS



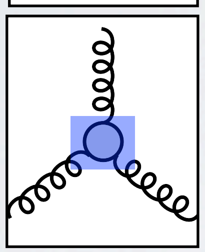
... FRG equations

truncation based on



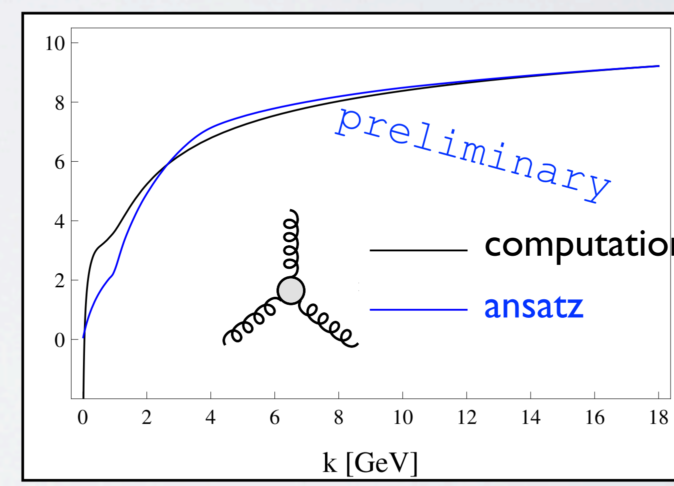
FRG LF, Pawlowski, arXiv: 1112.5440 [hep-ph].

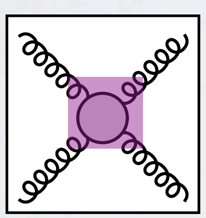
DSE Huber, Maas, von Smekal, JHEP 1211 (2012) 035.
Huber, von Smekal, arXiv: 1211.6092 [hep-th].



FRG LF, Pawlowski, in preparation.

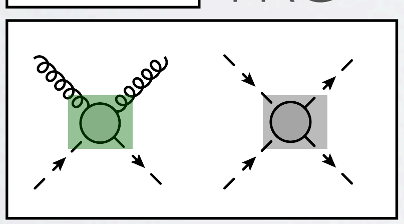
DSE Huber, von Smekal, *cf. talk M. Huber* .





DSE Kellermann, Fischer,
Phys.Rev.D78, 025015 (2008).

FRG LF, Pawlowski, in preparation.



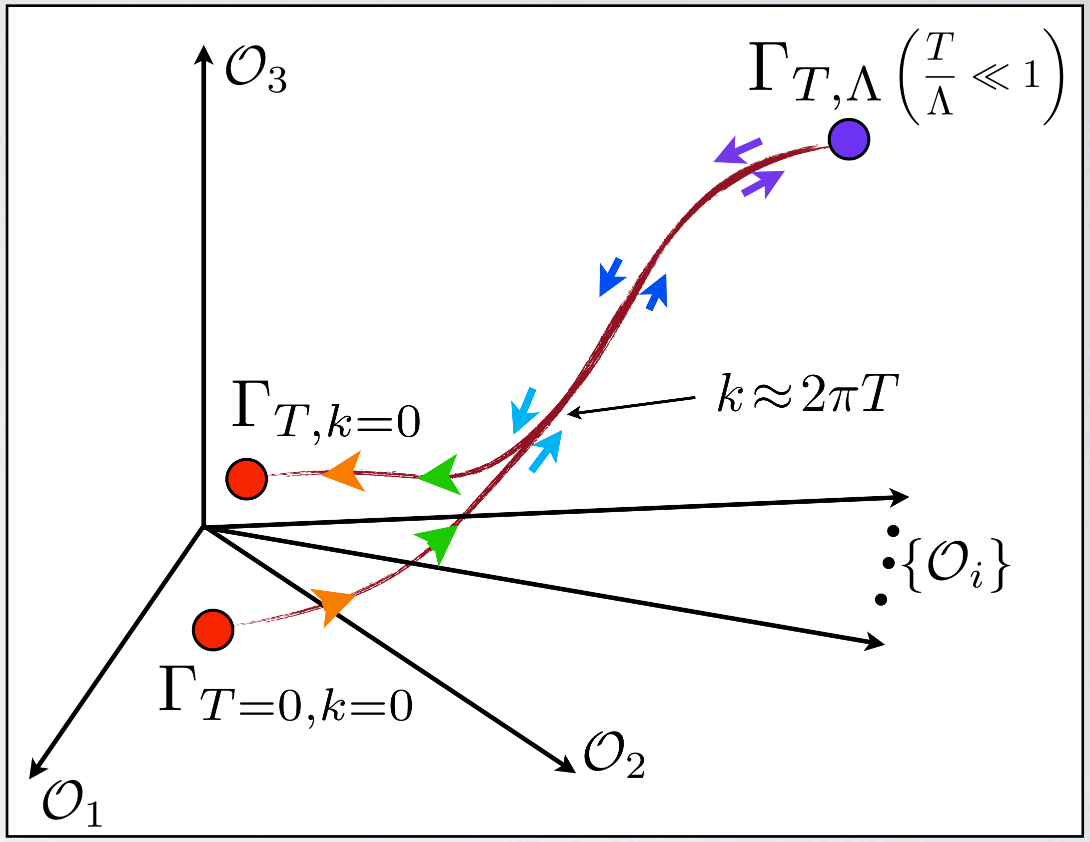
resummations LF, Pawlowski, arXiv: 1112.5440 [hep-ph].

Propagators have non-trivial **temperature** and **momentum dependence**, both are **indispensable**(, in particular **for thermodynamics**).

THERMAL FRG

two-step procedure:

1. start with full quantum theory
2. ‘add‘ thermal fluctuations to quantum theory



temperature effects restricted to infrared $k \lesssim 2\pi T$

practical advantages:

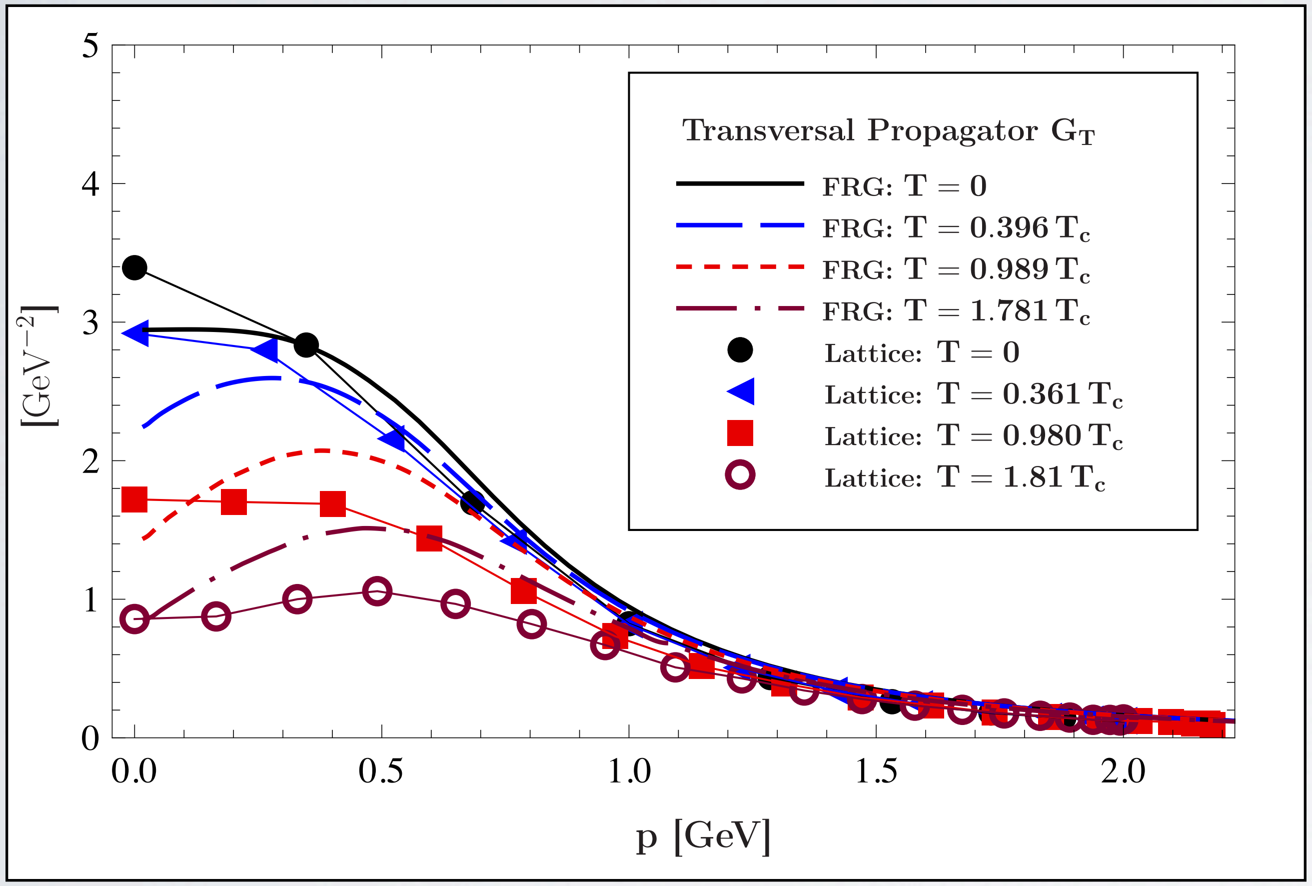
- quantum theory can be taken from any method, i.e. also from lattice gauge theory
- truncation errors affect only the infrared

YANG–MILLS PROPAGATORS

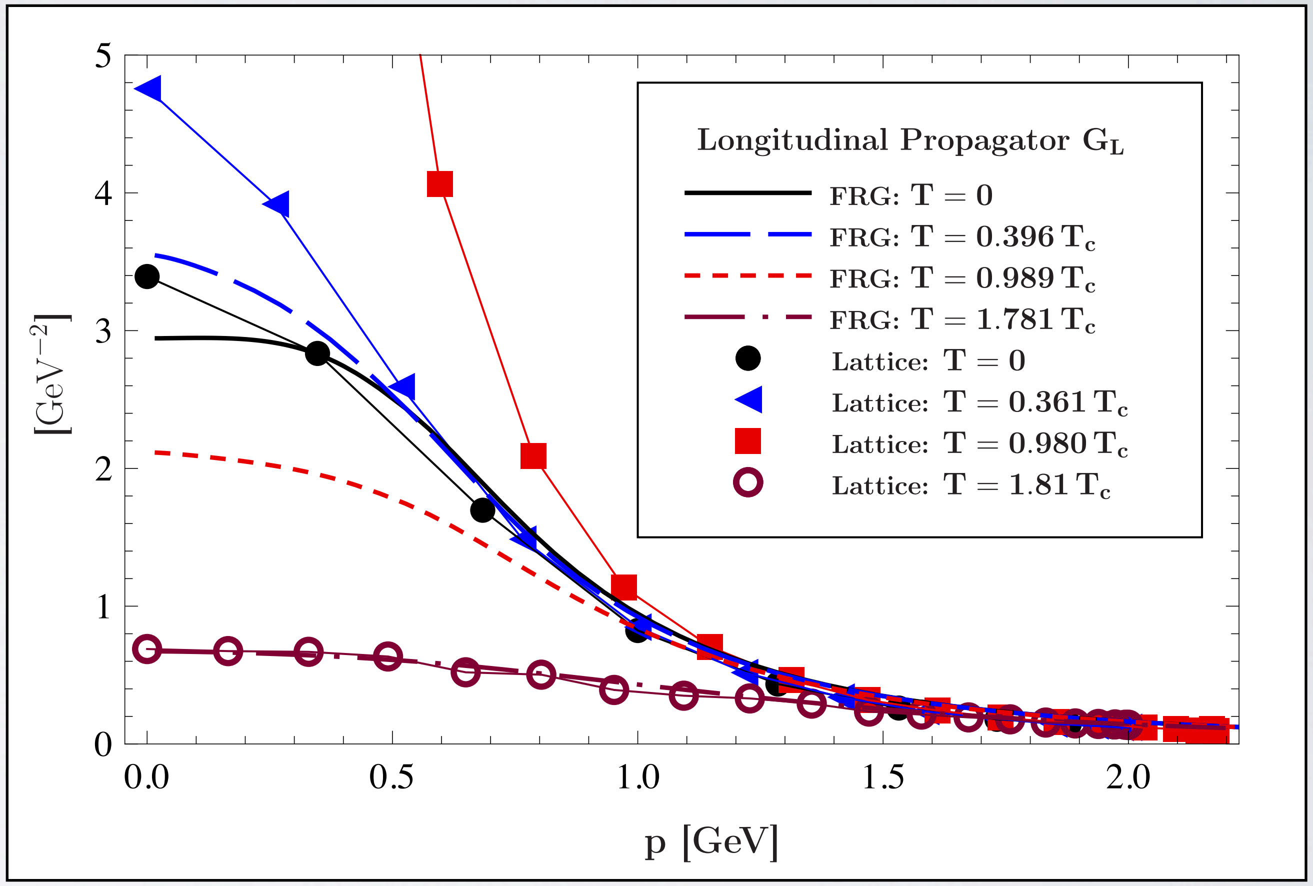
momentum dependence at **non-zero temperature**

at finite temperature: the gluon propagator has two projections (⊥, ∥) wrt the heatbath

chromomagnetic (= transversal) **gluon**



chromoelectric (= longitudinal) **gluon**



FRG results:

LF, Pawlowski, arXiv: 1112.5440 [hep-ph].

LF, Pawlowski, PoS QCD-TNT-II2011 (2011) 021 [arXiv: 1112.5429 [hep-ph]].

lattice data:

Maas, Pawlowski, von Smekal, Spielmann, Phys. Rev. D85 (2011) 034037.

Quark Confinement

POLYAKOV LOOP POTENTIAL

cf. talk H. Reinhardt for the Hamiltonian approach

The expectation value of the **Polyakov loop**, $\langle L[A_0]\rangle$, relates to the free energy F_q of a single quark.

→ order parameter for static quark confinement

$$e^{-F_q/T} \sim \langle L[A_0]\rangle = \left\langle \frac{1}{N_c}\mathcal{P} \; e^{ig\int_0^{1/T} dt A_0} \right\rangle \begin{cases} = 0 \ldots \text{ confinement} \\ > 0 \ldots \text{ deconfinement} \end{cases}$$

Also

$$L[\langle A_0\rangle] \begin{cases} = 0 & \text{if } \langle L\rangle = 0 \\ \geq \langle L\rangle & \text{if } \langle L\rangle > 0 \end{cases}$$

is an order parameter.

Braun, Gies, Pawlowski, Phys. Lett. B684, 262 (2010).
Marhauser, Pawlowski, arXiv: 0812.1144 [hep-ph].

$L[\langle A_0\rangle]$ accessible in **background field formalism:**

$\langle A_0\rangle$ minimum of **effective potential** $V[A_0]$ of constant background field A_0.

→ in FRG, DSE, 2PI, ...

POLYAKOV POTENTIAL - REPRESENTATIONS

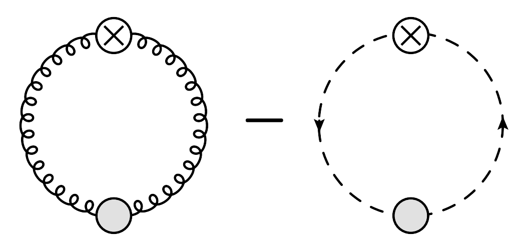
eff. potential $\quad V[A_0] = \dfrac{T}{\text{volume}} \Gamma[A_0; a=0]$

background field method:
gluon $A = A_0 + a$
(temporal) background ↑ ↑ fluctutation about background

↓

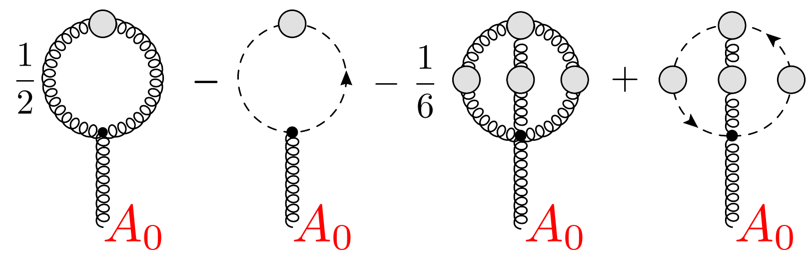
Confinement is immanent, if **minima of $V[A_0]$ at confining values,**
i.e. at these $\langle A_0 \rangle_{\text{conf}} : \; L[\langle A_0 \rangle_{\text{conf}}] = 0$.

FRG:

$k\partial_k \Gamma_k [A_0; a, \bar{c}, c] = \frac{1}{2}$ 

Braun, Gies, Pawlowski,
Phys. Lett. B684, 262 (2010).
Braun, Eichhorn, Gies, Pawlowski,
Eur. Phys. J. C70, 689 (2010).
LF, Pawlowski, arXiv: 1301.4163 [hep-ph],
1302.1373 [hep-ph].

DSEs:

$\dfrac{\delta(\Gamma - S)}{\delta A_0} =$ 

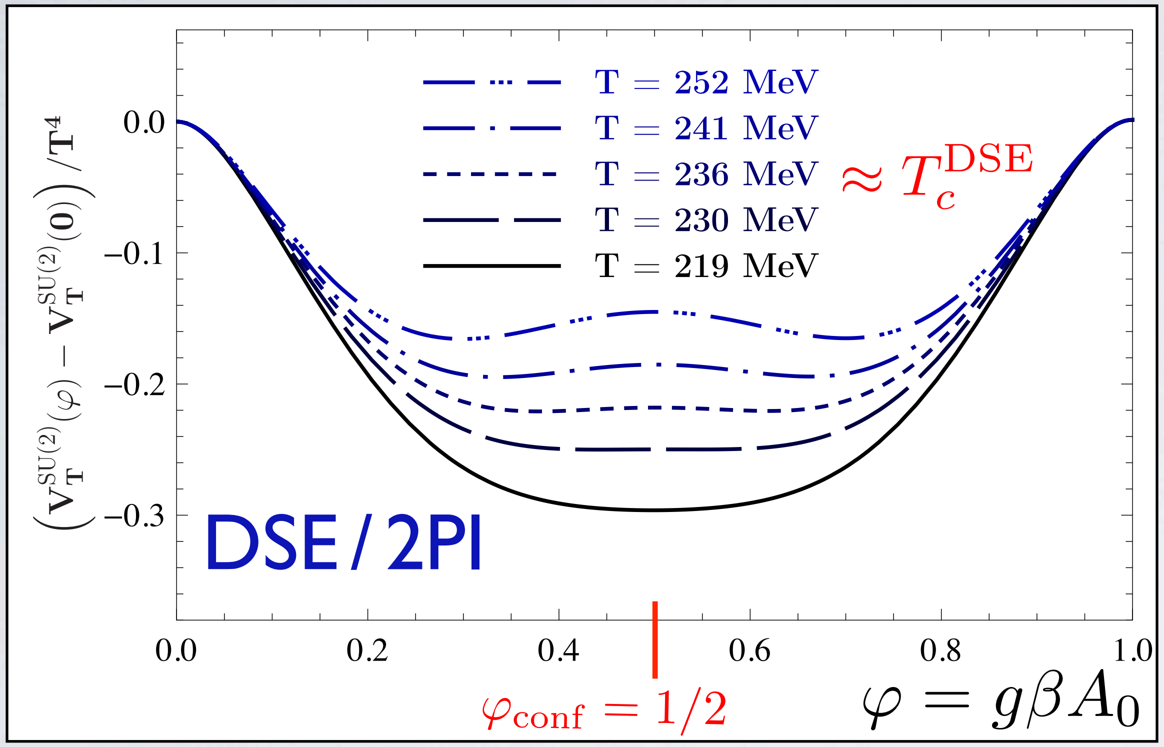
LF, Pawlowski, arXiv: 1301.4163 [hep-ph],
1302.1373 [hep-ph].

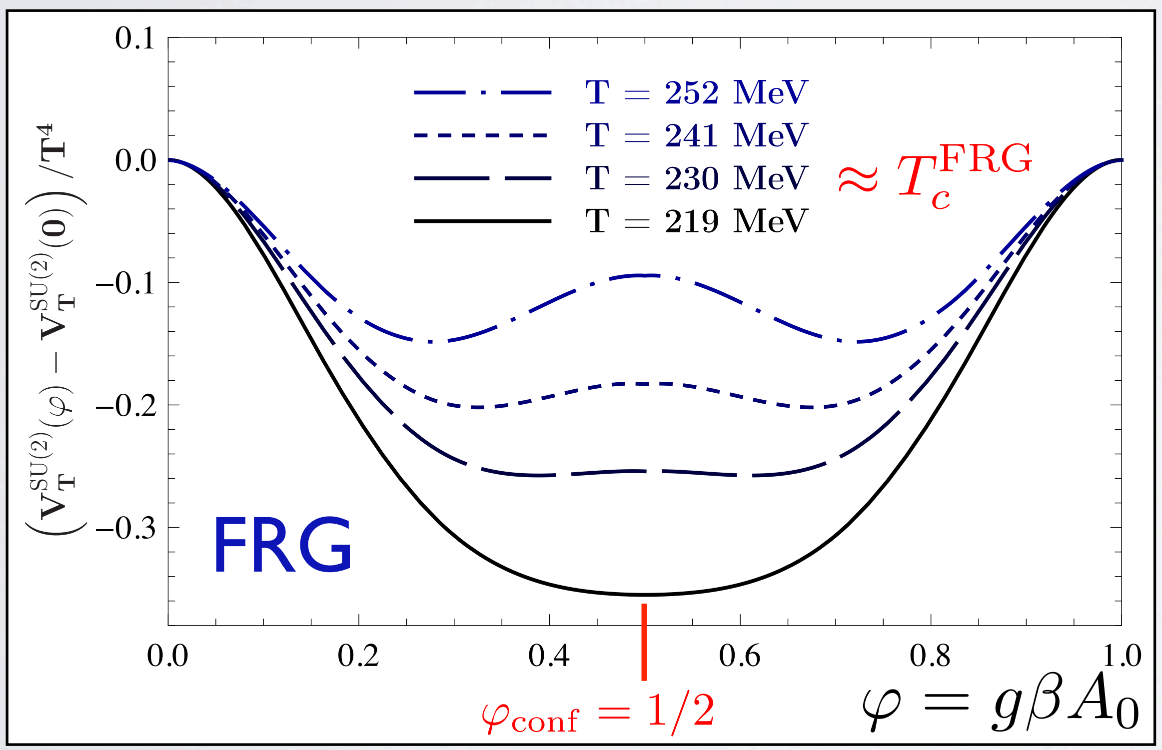
2PI:

for the purpose presented here it is equivalent to the DSE

LF, Pawlowski, arXiv: 1301.4163 [hep-ph], 1302.1373 [hep-ph].

POLYAKOV POTENTIAL - SU(2)



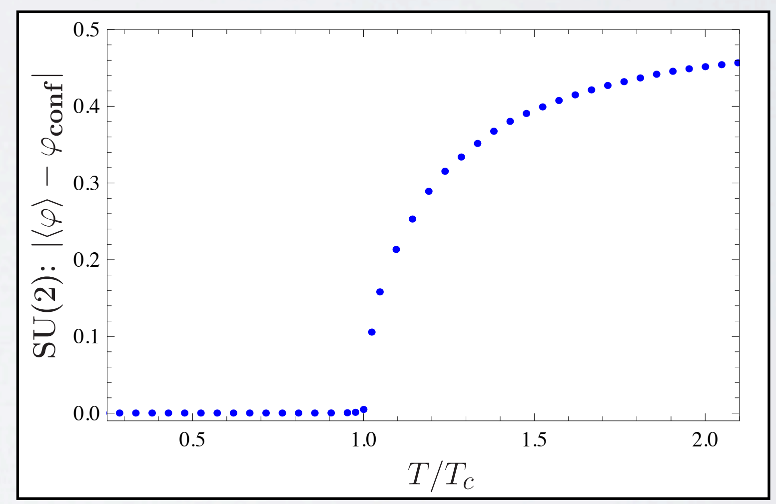


DSE / 2PI	$T_c^{\mathrm{DSE}}/\sqrt{\sigma} \approx 0.56$
FRG	$T_c^{\mathrm{FRG}}/\sqrt{\sigma} \approx 0.548$
lattice gauge theory	$T_c^{\mathrm{latt.}}/\sqrt{\sigma} \approx 0.709$

mismatch due to backcoupling of Polyakov potential to propagators

Spallek, Diploma Thesis (2010), U. Heidelberg.

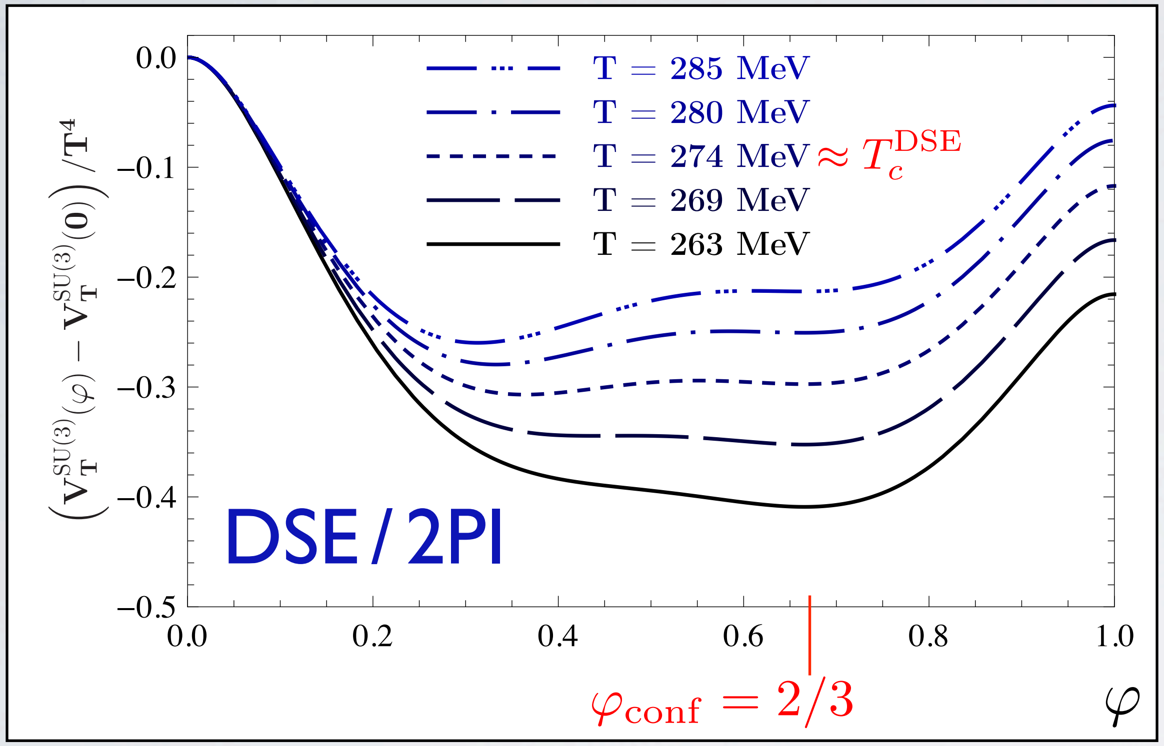
- order parameter signals second order transition

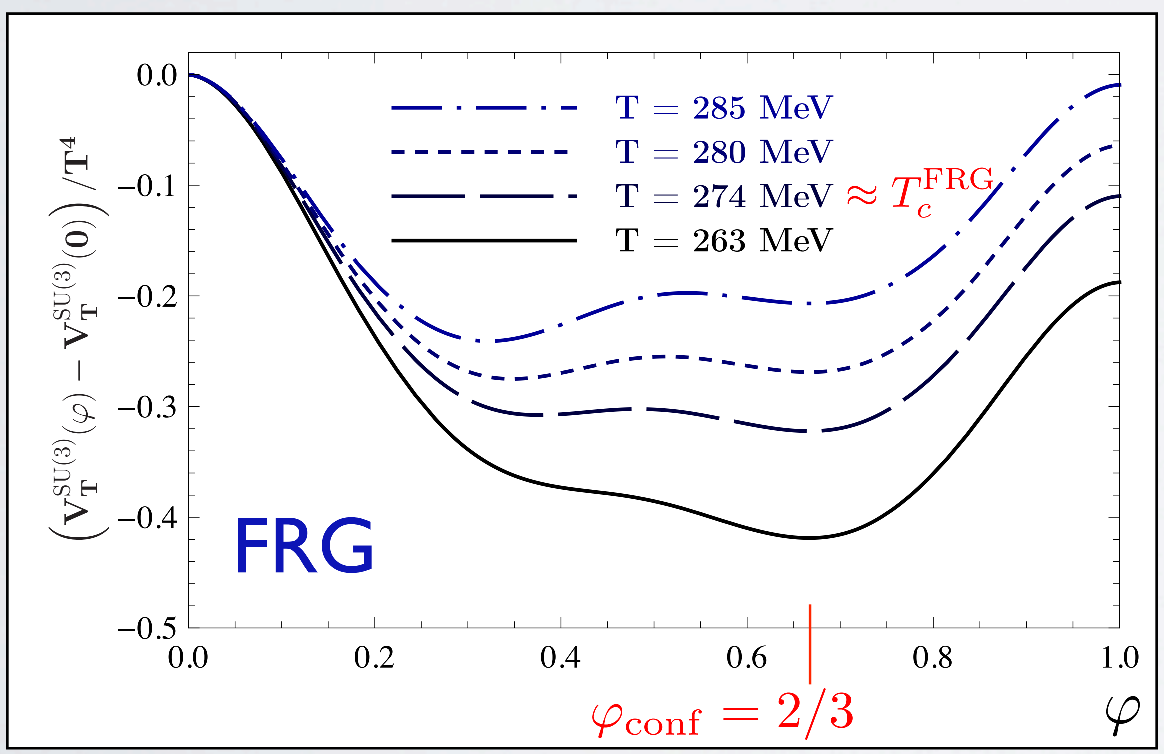


- amplitude of potential relevant for models (PQM, PNJL) ... agrees for different functional methods

LF, Pawlowski, arXiv: 1301.4163 [hep-ph], 1302.1373 [hep-ph].
Lattice data: Lucini, Teper, Wenger, JHEP 01, 061 (2004).

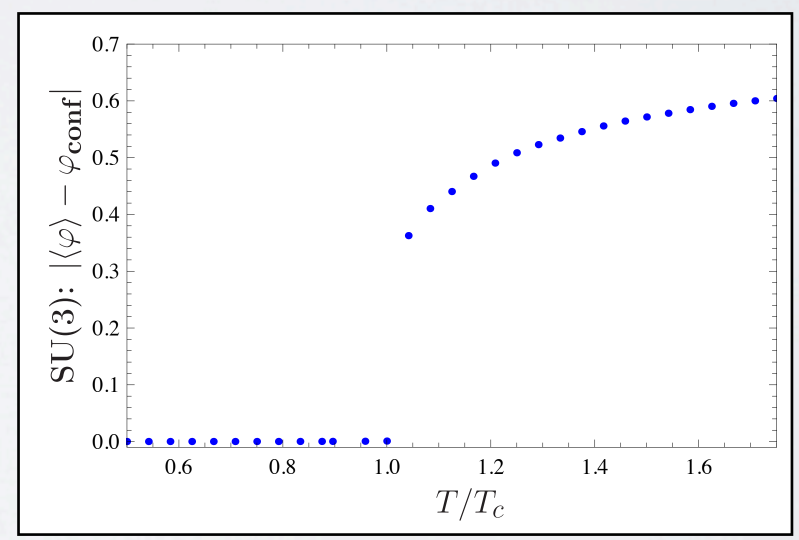
POLYAKOV POTENTIAL - SU(3)





DSE / 2PI	$T_c^{\mathrm{DSE}}/\sqrt{\sigma} \approx 0.651$
FRG	$T_c^{\mathrm{FRG}}/\sqrt{\sigma} \approx 0.655$
lattice gauge theory	$T_c^{\mathrm{latt.}}/\sqrt{\sigma} \approx 0.643$

- order parameter signals first order transition

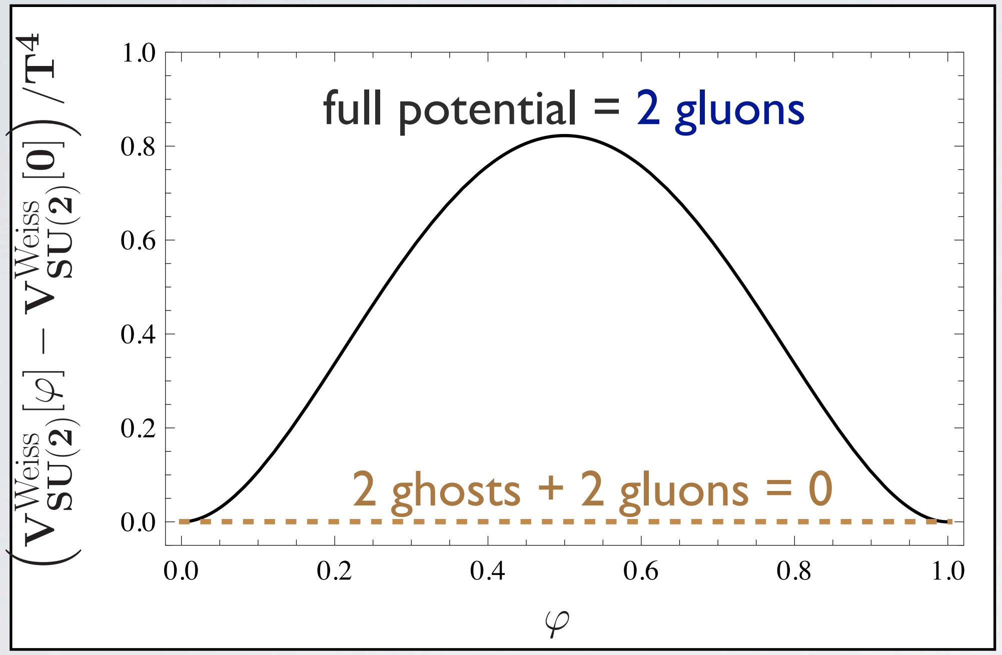


LF, Pawlowski, arXiv: 1301.4163 [hep-ph], 1302.1373 [hep-ph].
Lattice data: Lucini, Teper, Wenger, JHEP 01, 061 (2004).

CONFINEMENT CRITERION

perturbation theory → **Weiss potential**

```
Weiss, Phys. Rev. D24, 475 (1981).
Gross, Pisarski, Yaffe,
        Rev. Mod. Phys. 53, 43 (1981).
```

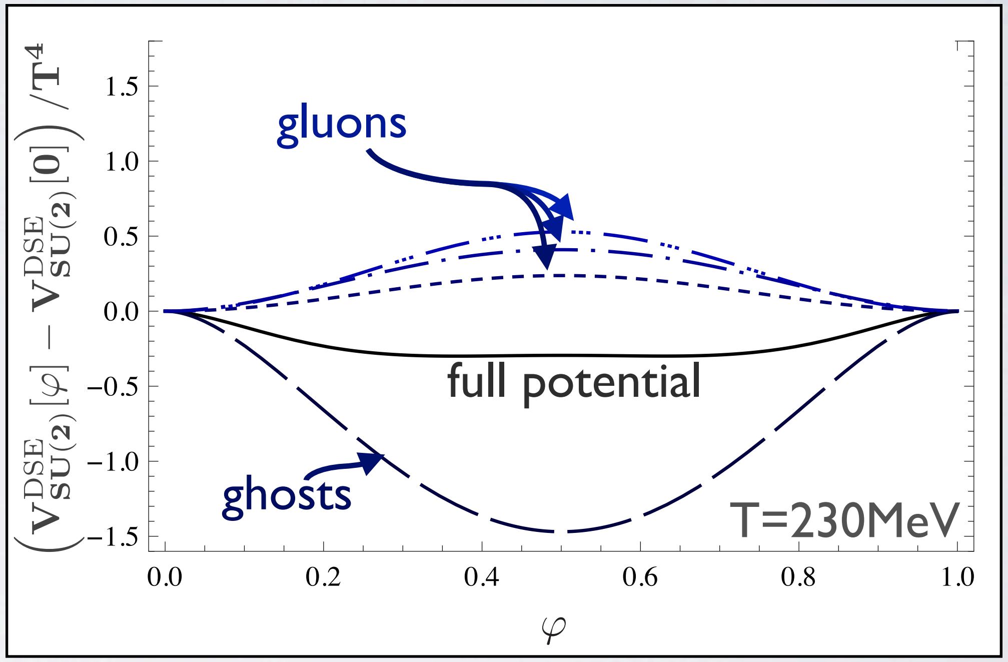


two (transversal) gluonic modes, others cancel exactly

minima at **integer** values of $\varphi = g\beta A_0$
confining value of φ at 1/2,
⤳ **no confinement in perturbation theory**

non-perturbatively

gluon suppression
ghost enhancement



gluon modes have **positive** contributions,
ghost modes have **negative** contributions,

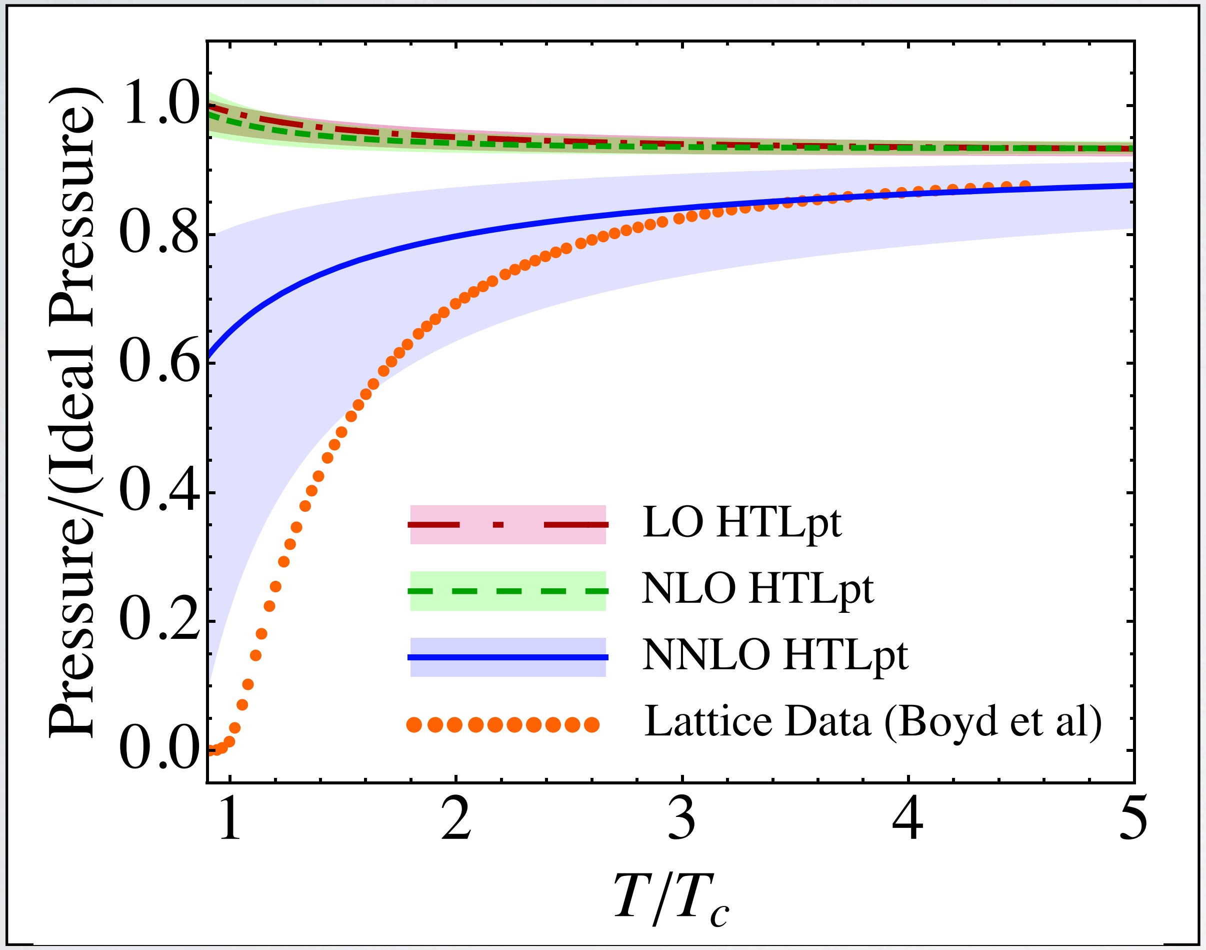
no exact cancellation of modes,
ghosts dominate at small temperatures
⤳ **confinement at small temperatures**

confinement criterion:
infrared suppressed gluons but non-suppressed ghosts → confinement

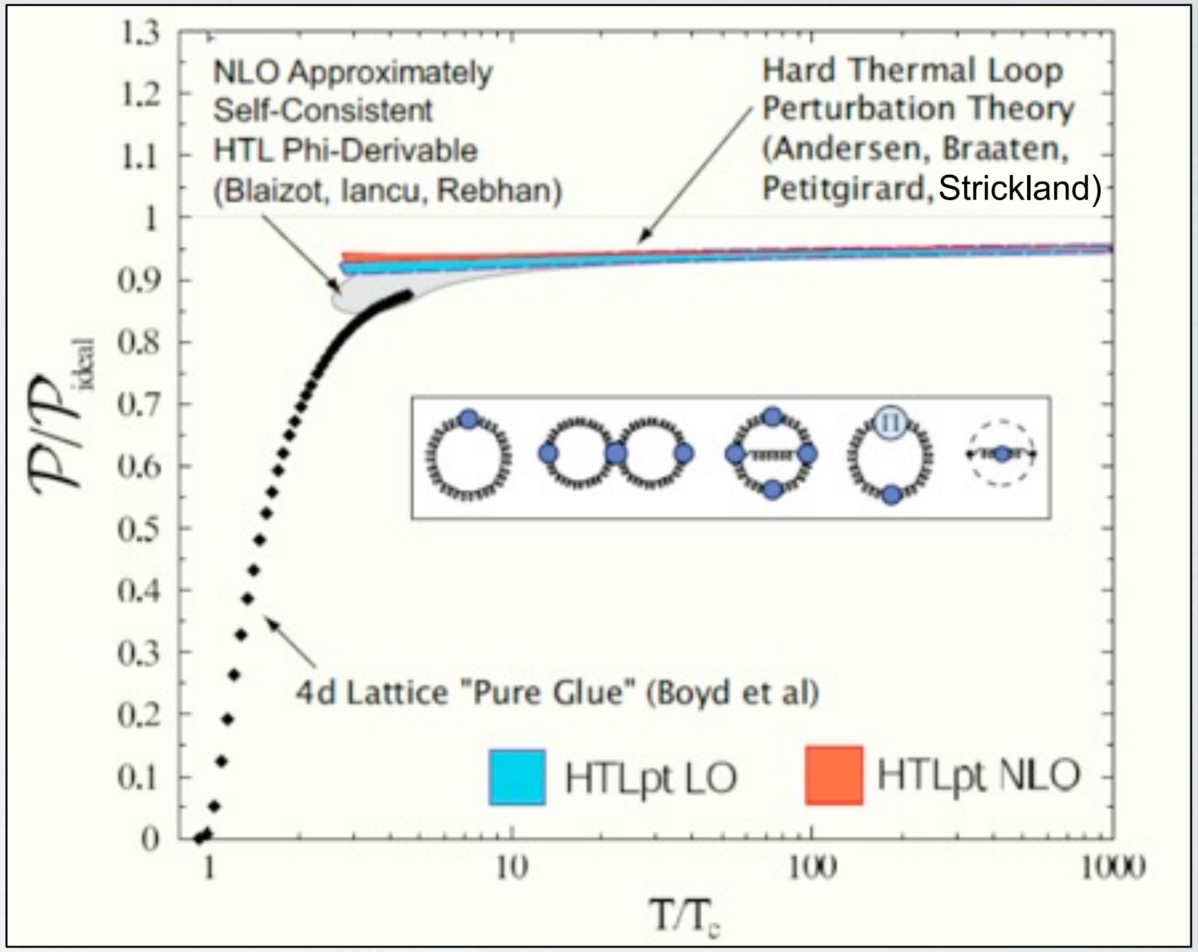
Yang–Mills Thermodynamics

PRESSURE FROM OTHER METHODS

cf. talk N. Su



Andersen, Strickland, Su,
Phys. Rev. Lett. 104 (2010).



from a talk by Strickland

perturbative methods fail for $T \lesssim 3T_c$

PRESSURE FROM THE FRG

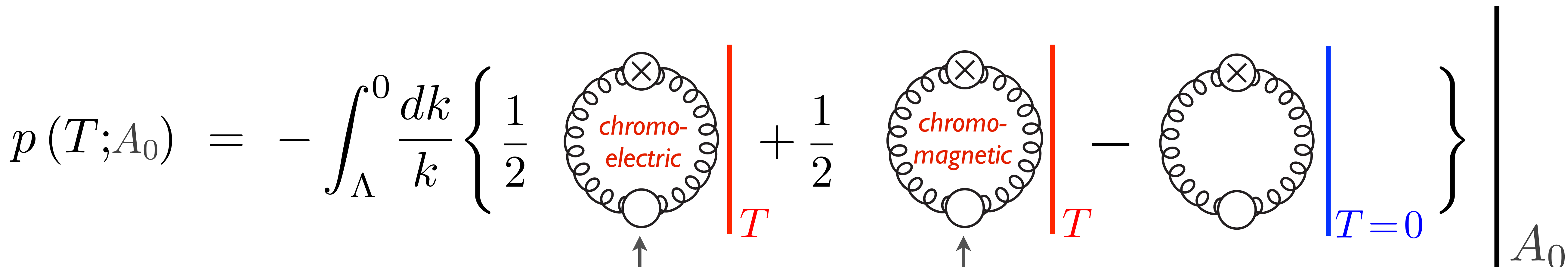
LF, Pawlowski, preliminary

The thermal pressure p is the effective action evaluated on the EoM, normalised in the vacuum.

$$p_k\,(T;A_0) = -\Delta\Gamma_{k,T}(A_0) \quad = -\left(\Gamma_{k,T} - \Gamma_{k,T=0}\right)$$

projection onto ‘physical‘ subspace

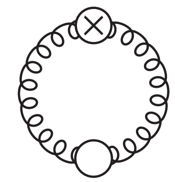
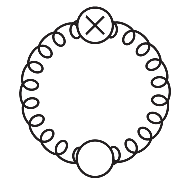
- one chromoelectric +
- one chromomagnetic mode



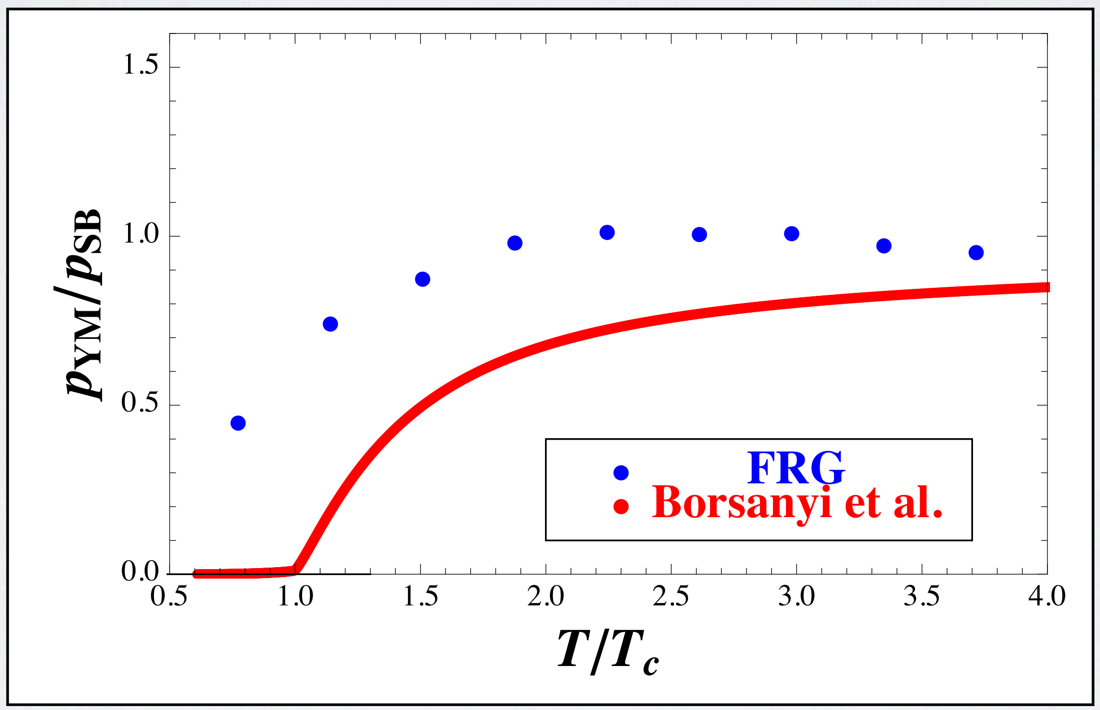
Polyakov loop potential is crucial for the critical physics.
Implicit temperature dependence of the propagators for quantitative accuracy.

PRESSURE WITH T-INDEP. PROPAGATORS, WITHOUT POLYAKOV LOOP

LF, Pawlowski, preliminary

$$-p(T) = \int_\Lambda^0 \frac{dk}{k} \Big\{$$  $$\Big|_T -$$  $$\Big|_{T=0} \Big\}$$

$$\sum\!\!\!\!\!\!\int_p G_{\mathbf{T=0},k}\, \partial_t R_k \qquad \int_p G_{T=0,k}\, \partial_t R_k$$

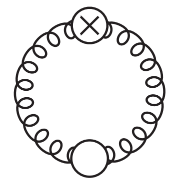
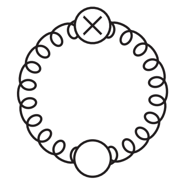


Temperature dependence of the propagators is important!

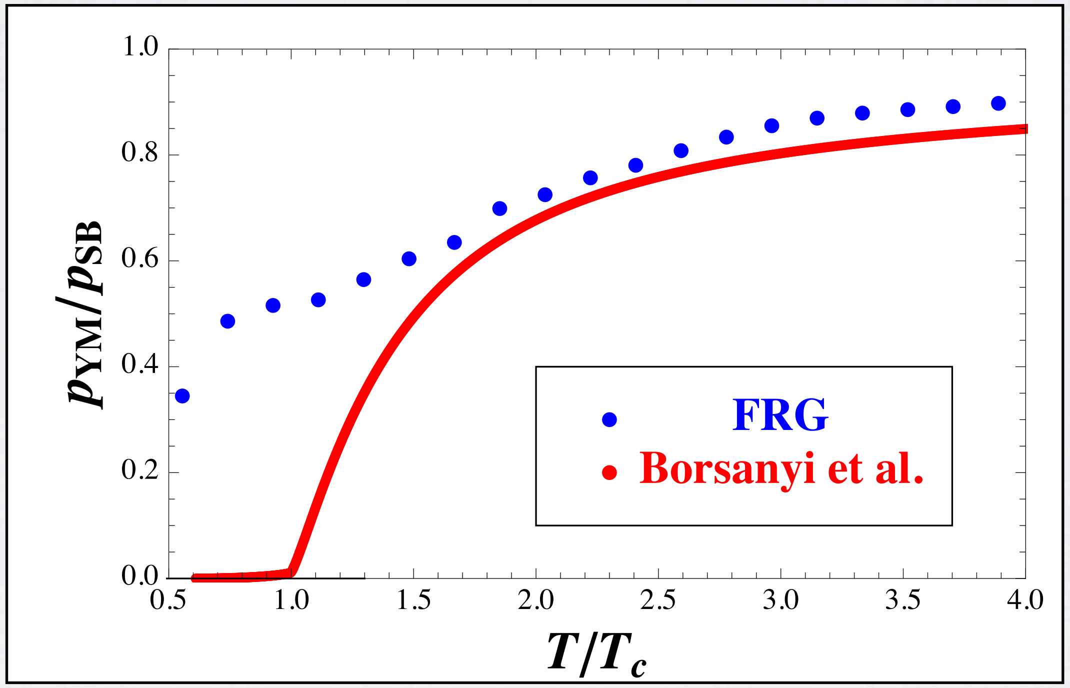
lattice data:
Borsanyi, Endrodi, Fodor, Katz and Szabo, JHEP 1207, 056 (2012).

PRESSURE WITHOUT POLYAKOV LOOP

LF, Pawlowski, preliminary

$$-p(T) = \int_\Lambda^0 \frac{dk}{k} \Big\{$$  $$\Big|_T -$$  $$\Big|_{T=0} \Big\}$$

$$\sum\!\!\!\!\!\!\int_p G_{T,k}\, \partial_t R_k \qquad \int_p G_{T=0,k}\, \partial_t R_k$$

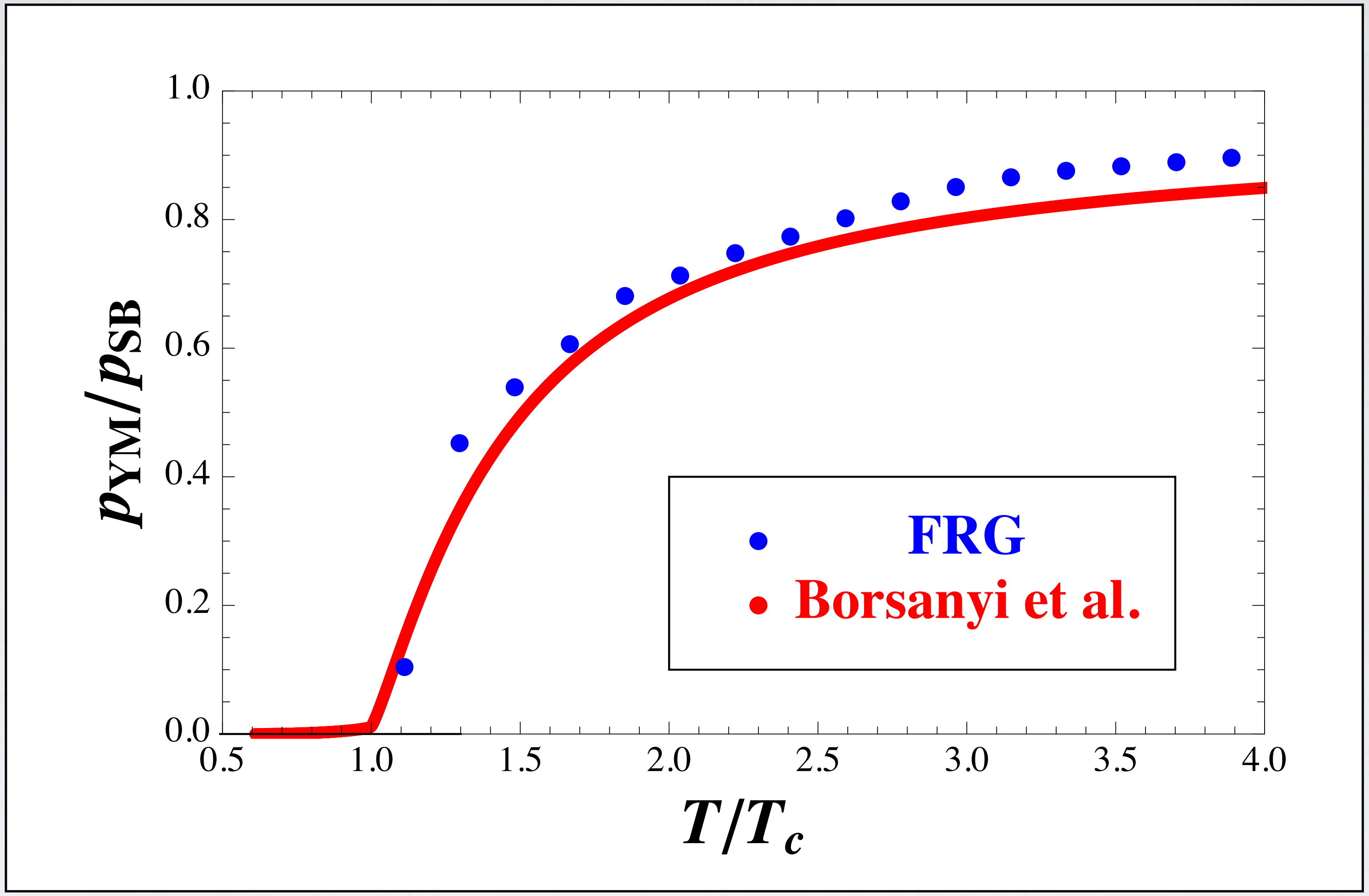


Polyakov loop potential crucial for critical physics.

lattice data:
Borsanyi, Endrodi, Fodor, Katz and Szabo, JHEP 1207, 056 (2012).

PRESSURE

LF, Pawlowski, preliminary



Polyakov loop potential is crucial for the critical physics.
Implicit temperature dependence of the propagators for quantitative accuracy.

lattice data:
Borsanyi, Endrodi, Fodor, Katz and Szabo, JHEP 1207, 056 (2012).

CONCLUSIONS

- **Thermal Propagators**
  - Temperature dependence is crucial for confinement and thermodynamics.
- **Confinement**
  - critical temperatures $\frac{T_c^{\mathrm{FRG/DSE}}}{T_c^{\mathrm{latt.GT}}} \approx \begin{cases} 0.78 & \mathrm{SU}(2),\ \text{second order} \\ 1.01 & \mathrm{SU}(3),\ \text{first order} \end{cases}$
  - Criterion for confinement: gluons must be IR suppressed, while ghosts must *not.*
- **Yang–Mills Thermodynamics**
  - Pressure at all temperatures, in particular for $T \lesssim 3\,T_c$.

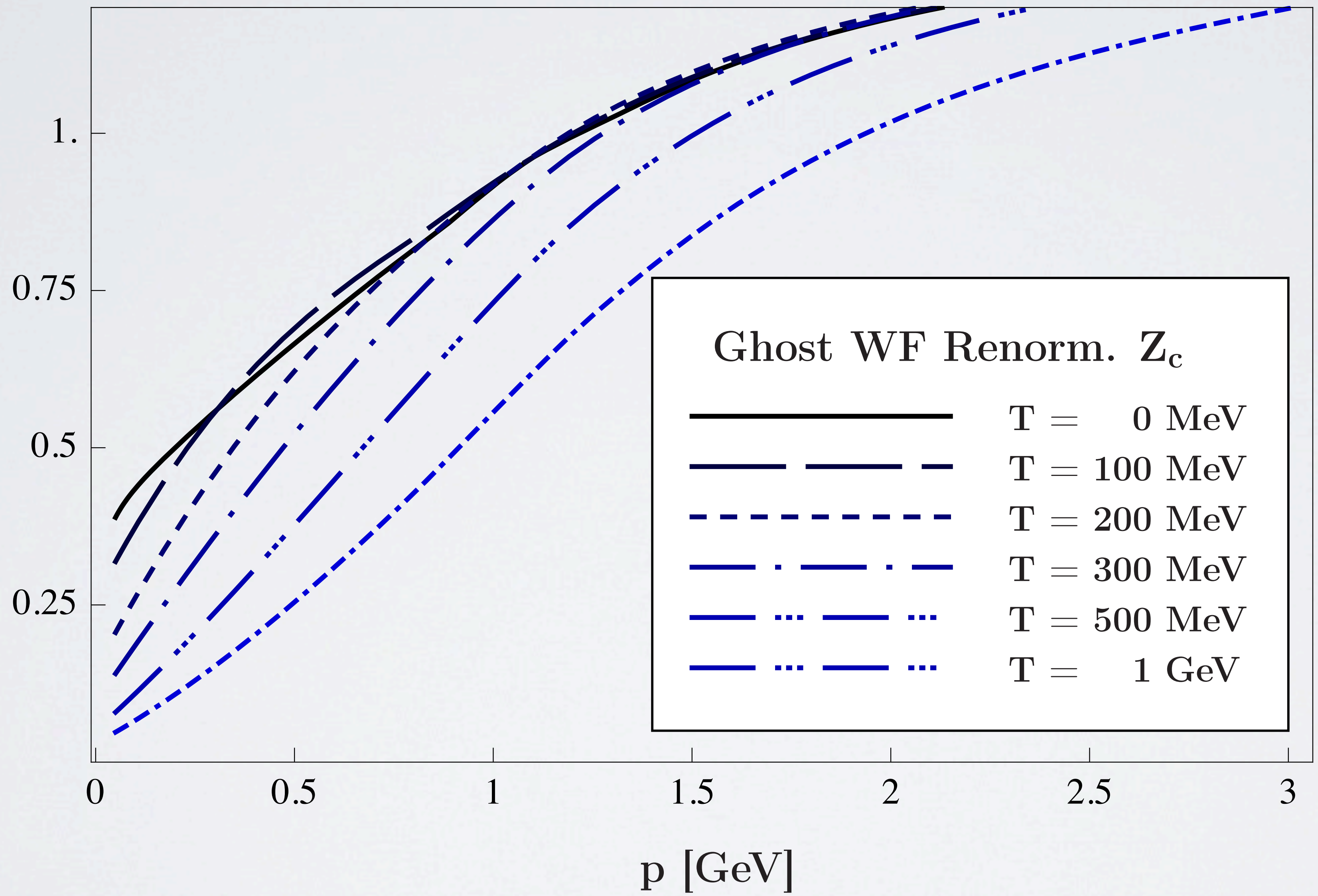
OUTLOOK

- **Truncation** *cf. talk M. Huber, P. Silva, A. Sternbeck*
- **Yang–Mills Thermodynamics** *cf. talk N. Su*
- **QCD**
  - Phase diagram at non-zero temperature and chemical potential.
- **QC_2D** *cf. talks S. Hands, L. von Smekal*
  - Direct comparison of propagators at non-zero chemical potential from functional and lattice methods.

Boz, Cotter, LF, Mehta, Skullerud, arXiv: 1303.3223 [hep-lat].

. . . supplementary material

Ghost Wave-Function Renormalisation



$$Z_c = \frac{1}{p^2\, G_c}$$

Gluonic Vertices — Ansätze vs. Computation

LF, Pawlowski, preliminary

ansatz

$$\Gamma^{(3)}_{A^3} \sim S^{(3)}_{A^3}\Big|_{g=1} \sqrt{4\pi\alpha_s}\, (\mathbf{Z_A})^{\frac{3}{2}}$$

$$\Gamma^{(4)}_{A^4} \sim S^{(4)}_{A^4}\Big|_{g=1} 4\pi\alpha_s\, (\mathbf{Z_A})^{2}$$

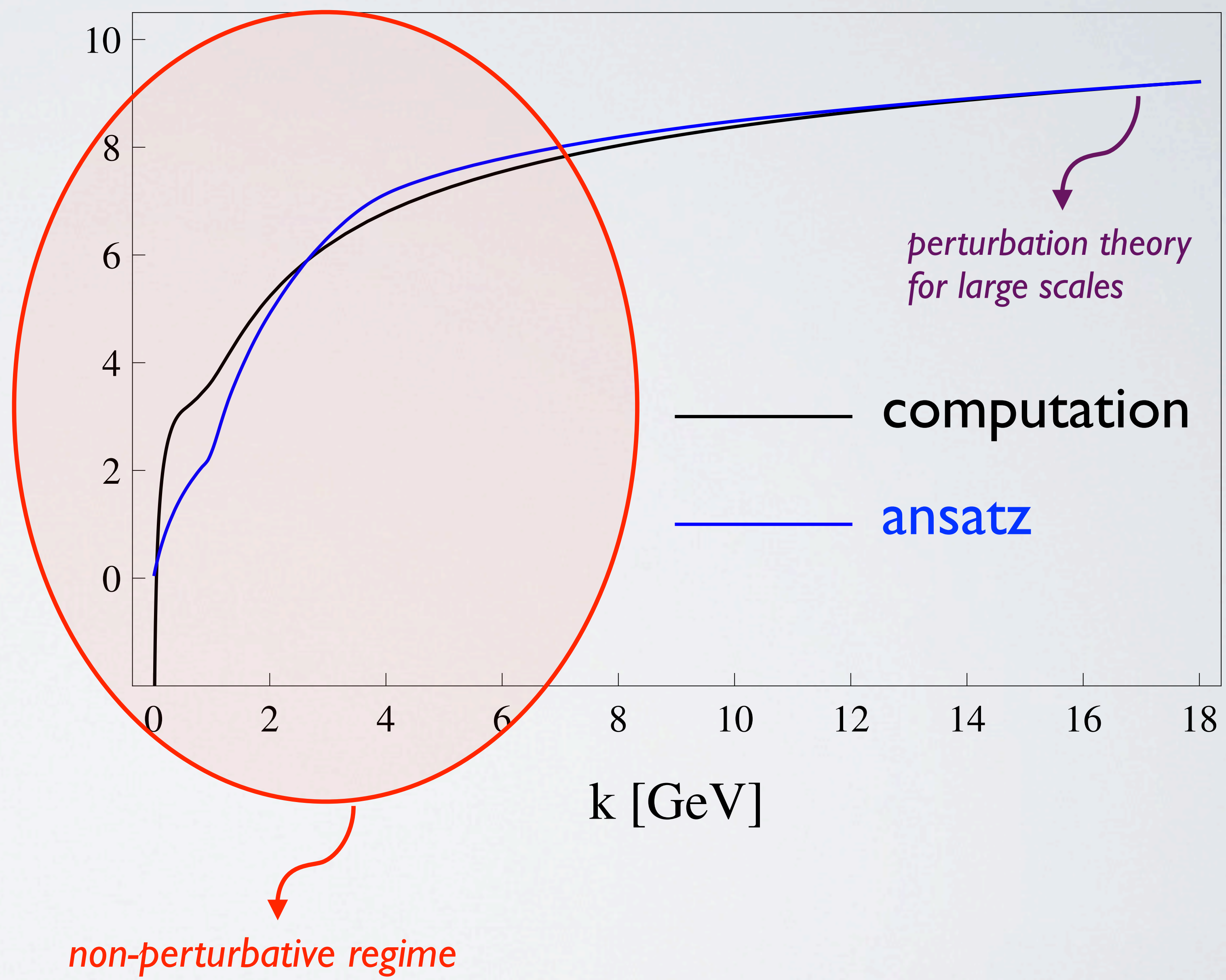
vertex $\sim$ class. tensor–struct.

$\times$ non–pert. runn. coupl.

$\times$ **RG running**

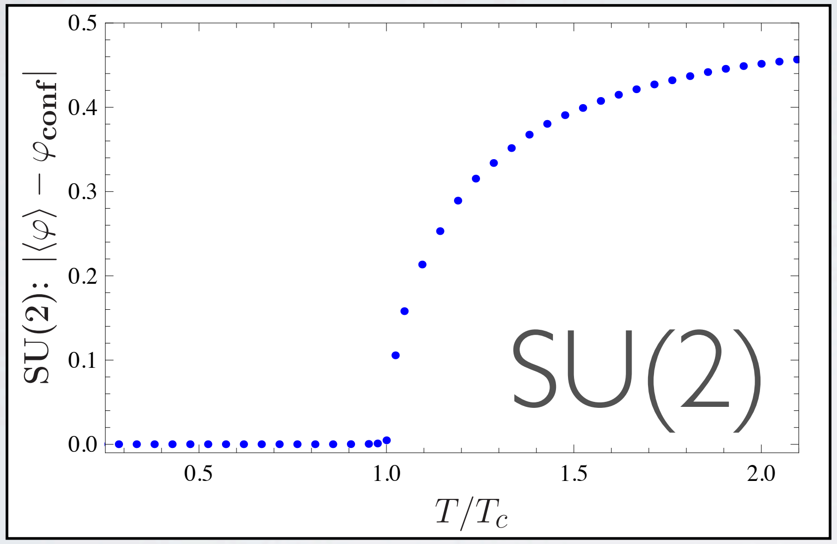
trigluon vertex $\Gamma^{(3)}_{A^3}$

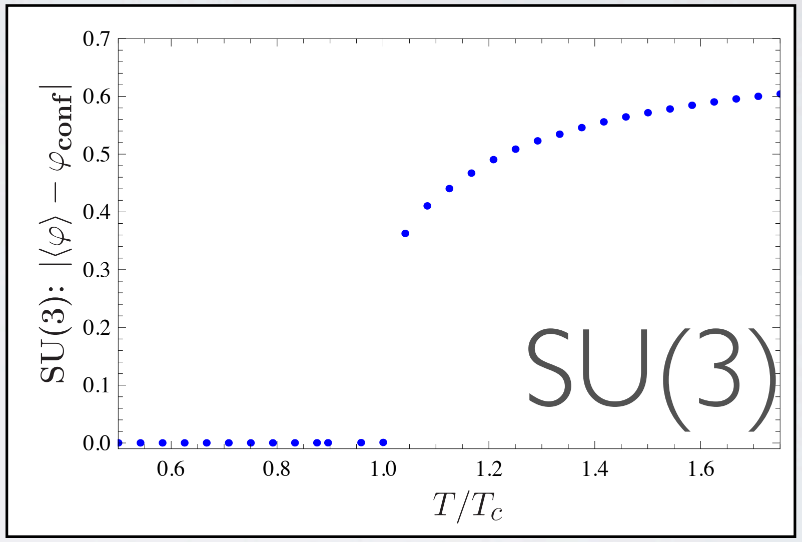
(Ansatz vs. Computation)



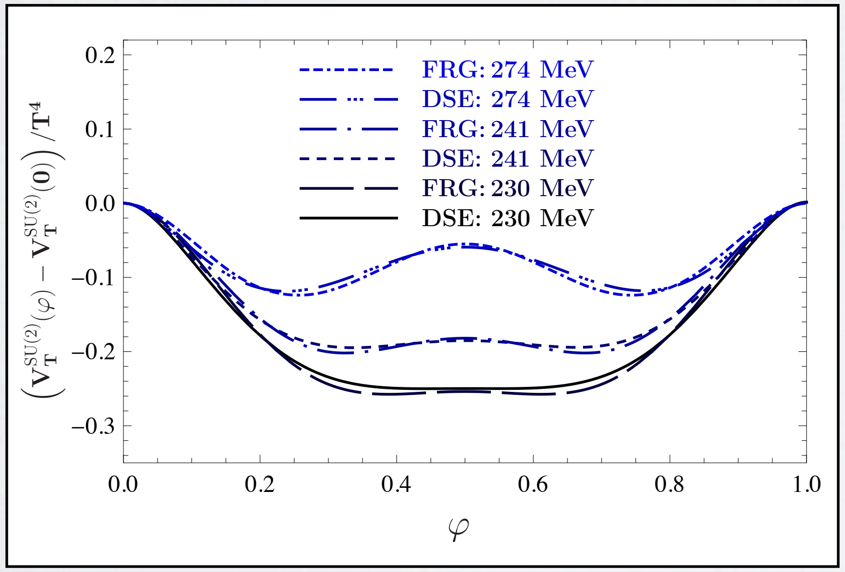
POLYAKOV POTENTIAL COMMENTS

- minimum moves smoothly away from $\varphi = \varphi_{\text{confining}}$: **second order** phase transition for SU(**2**)
  minimum jumps away from $\varphi = \varphi_{\text{confining}}$: **first order** phase transition for SU(**3**)





- **amplitude** of potential relevant for models (PQM, PNJL), agree for different functional methods




```
recently addressed in
  Haas, Stiele, Braun,
  Pawlowski, Schaffner-Bielich,
        arXiv: 1302.1993 [hep-ph].
```

```
LF, Pawlowski, arXiv: 1301.4163 [hep-ph].
```

- implicit temperature dependence of propagators has a 10% effect
- not sensitive to scaling/decoupling