Equilibrium Thermodynamics and Nucleation in the $N_f = 2 + 1$ Polyakov-Quark-Meson Model[1]

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QGHXM

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Outlook

Outlook

Degrees of Freedom

Gluons	effectively described by	Polyakov loop Φ
Mesons		(Pseudo-)scalar fields
Quarks		Interaction between Φ and mesons

PQM Lagrangian ($N_f = 2+1$)

- Let us start with the tree-level lagrangian

$$\mathcal{L}_0 = \mathcal{L}_{mesons} + \mathcal{L}_{quarks} + \mathcal{L}_{gauge} + \mathcal{L}_{int}$$

where $M = \lambda_a(\sigma_a + i\pi_a) = \vec{\lambda}\cdot\vec{M}_{scalar} + i\vec{\lambda}\cdot\vec{M}_{pseudo}$

$$\begin{aligned}
\mathcal{L}_{mesons} &= \mathrm{Tr}(\partial_\mu M \partial^\mu M^\dagger) - m^2 \mathrm{Tr}(M^\dagger M) - \lambda_1[\mathrm{Tr}(M^\dagger M)]^2 - \\
&\quad -\lambda_2 \mathrm{Tr}(M^\dagger M)^2 + c[\det(M) + \det(M^\dagger)] + \\
&\quad +\mathrm{Tr}[H(M + M^\dagger)] \\
\mathcal{L}_{quarks} &= \bar{q}\,(i\gamma^\mu\partial_\mu)\,q \\
\mathcal{L}_{gauge} &= -\frac{1}{4}\mathrm{Tr}\ F_{\mu\nu}F^{\mu\nu} \\
\mathcal{L}_{int} &= \mathcal{L}_{Yukawa} + \mathcal{L}_{min.\,coupl.} \\
\mathcal{L}_{Yukawa} &= -g\bar{q}(\vec{\lambda}\cdot\vec{M}_{scalar})q - ig\bar{q}(\gamma^5\vec{\lambda}\cdot\vec{M}_{pseudo})q \\
\mathcal{L}_{min.\,coupl.} &= -g_s\bar{q}\gamma^\mu A_\mu q
\end{aligned}$$

PQM in-medium effective potential

- Finally (?) the in-medium effective potential of the PQM model is

$$\Omega = U + \mathcal{U} + \Omega_{q\bar{q}}$$

$$U(\sigma_x,\sigma_y) = \frac{m^2}{2}(\sigma_x^2+\sigma_y^2) + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1+\lambda_2)\sigma_x^4 + \\ + \frac{1}{4}(\lambda_1+\lambda_2)\sigma_y^4 - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y - h_x\sigma_x - h_y\sigma_y$$

$$\frac{\mathcal{U}_{\log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}A(T)\bar{\Phi}\Phi + B(T)\ln\left[1-6\bar{\Phi}\Phi+4(\Phi^3+\bar{\Phi}^3)-3(\bar{\Phi}\Phi)^2\right]$$

$$\Omega_{q\bar{q}}\,(\sigma_x,\sigma_y,\Phi,\bar{\Phi};T,\mu) = -2T\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\times \\ \times\left\{\ln\left[1+3(\Phi+\bar{\Phi}e^{-(E_{q,f}-\mu_f)/T})e^{-(E_{q,f}-\mu_f)/T}) + e^{-3(E_{q,f}-\mu_f)/T}\right] + \\ + \ln\left[1+3(\bar{\Phi}+\Phi e^{-(E_{q,f}+\mu_f)/T})e^{-(E_{q,f}+\mu_f)/T} + e^{-3(E_{q,f}+\mu_f)/T}\right]\right\}$$

A Sign Problem in Effective Models Coupled to the Polyakov Loop

- The effective potential Ω at finite T and μ is a complex function of the complex variables Φ and $\bar{\Phi}$.
- This can be seen explicitly by defining new (real) variables

$$\begin{aligned}\alpha &:= \mathrm{Re}\,\Phi = \frac{\Phi + \bar{\Phi}}{2} \\ \beta &:= \mathrm{Im}\,\Phi = \frac{\Phi - \bar{\Phi}}{2i}\end{aligned}$$

- The effective potential can be written in terms of α and β.
- The Polyakov loop potentials are manifestly real functions of the new real variables. For example,

$$\frac{\bar{\mathcal{U}}_{poly}(\alpha,\beta)}{T^4} = -\frac{b_2(T)}{2}(\alpha^2+\beta^2) - \frac{b_3}{3}(\alpha^3 - 3\alpha\beta^2) + \frac{b_4}{4}(\alpha^2+\beta^2)^2$$

The Sign Problem in Effective Models Coupled to the Polyakov Loop

- The quark contribution $\Omega_{q\bar{q}}$ can be put in a manifestly complex form

$$\Omega_{q\bar{q}} = \Omega_{q\bar{q}}^{R} + i\Omega_{q\bar{q}}^{I}$$

with

$$\Omega_{q\bar{q}}^{R} := -2T\sum_{f}\int \frac{d^3p}{(2\pi)^3}\log[\sqrt{R^2+I^2}]$$

$$\Omega_{q\bar{q}}^{I} := -2T\sum_{f}\int \frac{d^3p}{(2\pi)^3}\arctan\left(\frac{I}{R}\right)$$

The Sign Problem in Effective Models Coupled to the Polyakov Loop

$$\Omega_{q\bar{q}} = \Omega_{q\bar{q}}^R + i\Omega_{q\bar{q}}^I$$

with the ($\beta-$even) real part

$$\begin{aligned} R := 1 &+ e^{-3(E-\mu)/T} + e^{-3(E+\mu)/T} + e^{-6E/T} + \\ &+6\alpha e^{-E/T}\left[\cosh\left(\frac{\mu}{T}\right) + e^{-E/T}\cosh\left(\frac{2\mu}{T}\right)\right] + \\ &+6\alpha e^{-4E/T}\left[\cosh\left(\frac{2\mu}{T}\right) + e^{-E/T}\cosh\left(\frac{\mu}{T}\right)\right] + \\ &+9(\alpha^2+\beta^2)(1+e^{-2E/T})e^{-2E/T} + 18(\alpha^2-\beta^2)e^{-3E/T}\cosh\left(\frac{\mu}{T}\right) \end{aligned}$$

and the ($\beta-$odd) imaginary part

$$\begin{aligned} I := \;& 6\beta e^{-E/T}\left[\sinh\left(\frac{\mu}{T}\right) - e^{-E/T}\sinh\left(\frac{2\mu}{T}\right)\right] + 6\beta e^{-4E/T}\left[e^{-E/T}\sinh\left(\frac{\mu}{T}\right) - \sinh\left(\frac{2\mu}{T}\right)\right] - \\ &-36\alpha\beta\sinh\left(\frac{\mu}{T}\right)e^{-3E/T} \end{aligned}$$

The Sign Problem in Effective Models Coupled to the Polyakov Loop

- The PQM grand partition function (sum over all field states)

$$\mathcal{Z}_{PQM} = \int [D\sigma_x][D\sigma_y][D\alpha][D\beta]\, e^{-S_{PQM}[\sigma_x,\sigma_y,\alpha,\beta]}$$

- In full equilibrium (mean-field approximation)

$$S_{PQM} = \frac{V}{T}\Omega = \frac{V}{T}\left(U_m + \mathcal{U}_P + \Omega_{q\bar{q}}^{(R)} + i\Omega_{q\bar{q}}^{(I)}\right)$$

- It can be split in a $\beta-$even part and an imaginary $\beta-$odd term

$$S_{PQM} = \frac{V}{T}\left(\Omega_{\beta-even} + i\Omega_{\beta-odd}\right)$$

The Sign Problem in Effective Models Coupled to the Polyakov Loop

- The mean-field PQM "partition function" (sum over homogeneous states)

$$
\begin{aligned}
\mathcal{Z}_{PQM}^{(MF)} &= \int [D\vec{\sigma}][D\alpha] \int_{-\infty}^{\infty} d\beta \, \exp\left[-\frac{V}{T}\Omega\right] \\
&= \int [D\vec{\sigma}][D\alpha] \int_{-\infty}^{\infty} d\beta \, e^{-\frac{V}{T}\Omega_{\beta-even}} e^{-i\frac{V}{T}\Omega_{\beta-odd}} \\
&= \int [D\vec{\sigma}][D\alpha] \int_{0}^{\infty} d\beta \, e^{-\frac{V}{T}\Omega_{\beta-even}} \left[e^{-i\frac{V}{T}\Omega_{\beta-odd}} + e^{+i\frac{V}{T}\Omega_{\beta-odd}}\right] \\
&= \int [D\vec{\sigma}][D\alpha][D\beta] \, e^{-\frac{V}{T}\Omega_{\beta-even}} \cos\left[\frac{V}{T}\Omega_{q\bar{q}}^{(I)}\right]
\end{aligned}
$$

Integrand is not $\geq 0 \rightarrow$ Sign Problem!

The Sign Problem in Effective Models Coupled to the Polyakov Loop

- If we insist to define an effective potential, the 1-loop quark contribution

$$\tilde{\Omega}_{q\bar{q}} = \Omega^{(R)}_{q\bar{q}} + \frac{T}{V}\log\left[\cos\left(\frac{V}{T}\Omega^{(I)}_{q\bar{q}}\right)\right]$$

- Thermodynamical limit ($V \to \infty$) poorly defined!
- Tentative solution: $\Omega_{q\bar{q}} \to 0$, i.e.,

$$e^{-\frac{V}{T}\Omega} = e^{-\frac{V}{T}\left(\Omega^{(R)}+i\Omega^{(I)}_{q\bar{q}}\right)} \to \left|e^{-\frac{V}{T}\Omega}\right| = \left|e^{-\frac{V}{T}\Omega^{(R)}}\right|$$

- This is analogous to

$$\mathrm{det}D(A,m,\mu,T) \to |\mathrm{det}D(A,m,\mu,T)|$$

States of Thermodynamical Equilibrium

- Now that $\Omega \equiv \mathcal{E}$ is a real function of real variables, it makes sense to write

$$\mathcal{Z} = \int [D\vec{\Psi}] e^{-\frac{V}{T}\mathcal{E}[\vec{\Psi}]}$$

- Equilibrium $\Leftrightarrow$ **Minimum** of S_{eff} (impossible if $S_{eff}[\vec{\Psi}] \in \mathbb{C}$!)

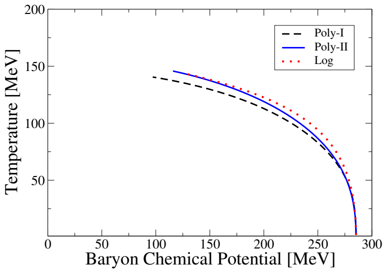
$$\frac{\partial \mathcal{E}}{\partial \sigma_x} = \frac{\partial \mathcal{E}}{\partial \sigma_y} = \frac{\partial \mathcal{E}}{\partial \alpha} = \frac{\partial \mathcal{E}}{\partial \beta} = 0$$

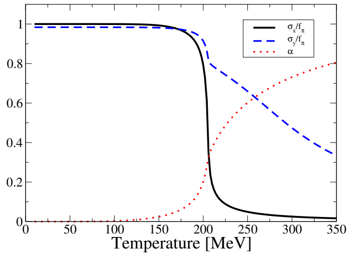
and the 4 eigenvalues of

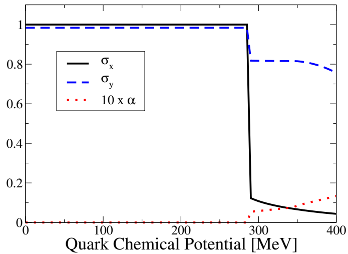
$$\mathcal{H}_{ij} = \frac{\partial^2 \mathcal{E}}{\partial X_i \partial X_j}$$

$(X_i = \sigma_x, \sigma_y, \alpha, \beta)$ are all positive.

Phase Diagram and Evolution of Order Parameters







Bubble Nucleation in 1^{st} Order Transitions

- Phase transitions need a dynamical underlying mechanism.
- 1^{st} order PhT, small metastability $\rightarrow$ bubble nucleation.
- Timescale for PhT $\gtrsim$ Timescale for bubble nucleation.
- Average time for first seed (critical) bubble (in a unit volume)

$$t_{nucl} \sim \exp\left[S_{bub}\right] \sim \exp\left[\frac{\Delta F_{bub}}{T}\right]$$

- Free energy shift due to the nucleation of a bubble

$$\Delta F_{bub} = F_{bubble} - F_{metastable}$$

- Thin-wall approximation (Σ: quark-hadron surface tension)

$$F_{bubble} = -\frac{4\pi}{3} R_b^3 \Delta\mathcal{E} + 4\pi R_b^2 \Sigma$$

The Importance of the Surface Tension

- Average time for first seed (critical) bubble (in a unit volume)

$$t_{nucl} \sim \exp\left[\frac{\Delta F_{bub}}{T}\right]$$

- Again in the thin-wall approximation (good close to coexistence)

$$\Delta F_{bub} = \frac{16\pi}{3}\frac{\Sigma^3}{(\Delta\mathcal{E})^2}$$

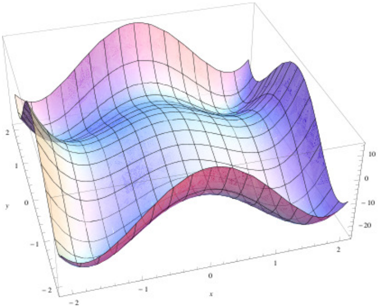
- The surface tension dominates the nucleation time

$$t_{nucl} \sim \exp\left[\Sigma^3\right]$$

- Σ can be calculated from the effective model: we do it!

Surface Tension with more than 1 Order Parameter

- The situation can be quite complicated with 2 or more order parameter: saddle-points + many coupled EL equation.



The Straight Line Ansatz Gives an Overestimate of Σ

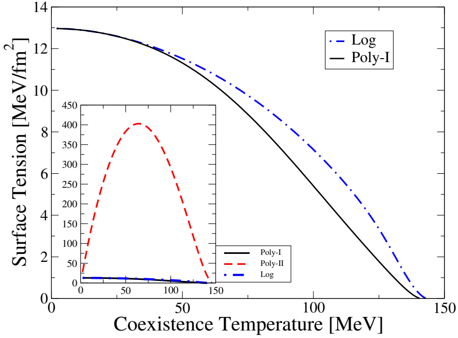
- True bubble (solution of coupled EL equations): $\vec{\Psi}_{true}(r)$.
- Any other function $\vec{\Psi}_f(r)$ that connects the minima has $S[\vec{\Psi}_f] > S[\vec{\Psi}_{true}]$.
- But $S \sim \Delta F \sim \Sigma^3$...
- Therefore,
  the straight line gives an overestimate of the surface tension!

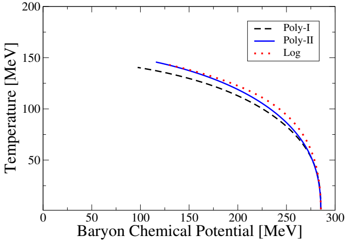
$$\Sigma = h \int_0^1 d\xi \sqrt{2\tilde{\mathcal{E}}(\xi)}$$

$$h := \sqrt{(\Delta\sigma_x)^2 + (\Delta\sigma_y)^2 + (\kappa\Delta\alpha)^2 + (\kappa\Delta\beta)^2}$$

Overestimate of the Surface Tension in the PQM Model

- Surface tension along the coexistence (1^{st} order) line of the phase diagram.
- Poly-I and Log Parametrizations: $\Sigma \lesssim 15$MeV/fm^2 (low value!).
- Poly-II parametrization: strange behavior...





CONCLUSIONS

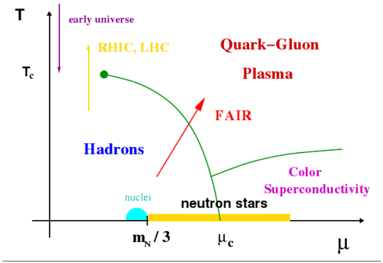
Summary and Conclusions

- The fermion sign problem is also present in chiral effective models coupled to the Polyakov loop, even at mean-field level.
- The fermion sign problem in the PQM and PNJL models is related to an imaginary part of the *equilibrium* effective potential $\rightarrow$ a complex equilibrium effective potential has no physical meaning.
- The ad-hoc elimination of $\Omega_{q\bar{q}}^{(I)}$ allows the existence of minima
  - Thermodynamically consistent determination of equilibrium properties.
  - Determination of quasi-equilibrium properties of the system (e.g., Σ).
- Our overestimate $\Sigma \lesssim 15$ MeV/fm^2: **low value**, in line with previous results with chiral models [Palhares/Fraga (2010), Pinto/Koch/Randrup (2012)].

EXTRA SLIDES

Phase Transitions of Strongly Interacting Matter

- QCD is relatively well understood only at high energies (perturbation theory): asymptotic freedom and chiral symmetry restoration.
- However, "our"(i.e., nuclear) regime: very low energies.



[Fraga/Schaffner-Bielich/Pisarski]

Chiral Symmetry

- Chiral transformation on a fermion field:
- $m_q \neq 0$: chiral symmetry broken. E.g.: in the vacuum.
- $m_q = 0$: chiral symmetry restored. E.g.: high temperatures.

$$\mathcal{L} = \bar{\psi} i\gamma^\mu \partial_\mu \psi - g\bar{\psi}\psi\sigma + \frac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma)$$

  - Integrate over constant σ field.
  - Mean field approximation: $\sigma \to \langle\sigma\rangle$.

$$\mathcal{L}_{eff} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_{eff})\psi + U_{eff}(\langle\sigma\rangle),$$

where $m_{eff} = g\,\langle\sigma\rangle$.

- Chiral symmetry of quarks broken by $\langle\sigma\rangle \neq 0$ (controlled by U_{eff}).

Confinement

- Quarks are confined inside hadrons. Why? How?
- One (debatable) criterion: Wilson area law for static $q\bar{q}$ pair.

$$U_P = \left\langle \mathcal{P} \exp \left[ig \oint_P dx_\mu A^\mu \right] \right\rangle = \exp\left[iE(R)T \right]$$

$E(R) = \alpha R$: confinement.

- At finite temperature, $\mathcal{M}^4 \to \mathbb{R}^3 \times S^1$.
- Polyakov loop: go around (compact) imaginary time direction $\tau \in [0, \beta)$.

$$\Phi = \frac{1}{N_c} \mathrm{Tr} \mathcal{P} \exp \left[\mathrm{ig} \int_0^\beta \mathrm{d}\tau \mathrm{A}^0 \right] = \exp\left[-\beta \Delta \mathrm{F} \right]$$

$\Phi \to 0 \Leftrightarrow \Delta F \to \infty$: confinement.
$\Phi \to 1 \Leftrightarrow \Delta F \to 0$: free quarks.

PQM in-medium effective potential

- Grand partition function (fixed T, μ):

$$\mathcal{Z} = \int [DA][D\bar{q}][Dq][DM] \exp\left\{ - \int_0^\beta d\tau \int d^3x \left[\mathcal{L} - \mu \bar{q} \gamma_0 q \right] \right\}$$

- 1^{st} step: Mean-Field Approximation

$$\begin{aligned} M &\rightarrow \langle M \rangle \\ A &\rightarrow \langle A \rangle \rightarrow \Phi \\ -\frac{1}{4}\mathrm{Tr}F^2 &\rightarrow \frac{\kappa^2}{2}(\partial_\mu \bar{\Phi} \partial^\mu \Phi) + \mathcal{U}(\Phi, \bar{\Phi}, T) \end{aligned}$$

PQM in-medium effective potential

- 2^{nd} step:
  - Explicit symmetry breaking: $H_0, H_3, H_8 \neq 0 \rightarrow$ "flavorless"vacuum.
  - VEV of pseudoscalars: $\langle \pi_a \rangle = 0$.
  - $SU(2)$ isospin symmetry $(N_f = 2+1)$: $H_3 = 0 \Rightarrow \langle \sigma_3 \rangle = \langle \pi_3 \rangle = 0$.
  - Diagonalization [Schaefer/Wagner, 2009]:

$$\sigma_x := \frac{\sqrt{2}\sigma_0 + \sigma_8}{\sqrt{3}}$$

$$\sigma_y := \frac{\sigma_0 - \sqrt{2}\sigma_8}{\sqrt{3}}$$

- Final form of tree-level meson potential

$$U(\sigma_x, \sigma_y) = \frac{m^2}{2}(\sigma_x^2 + \sigma_y^2) + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1 + \lambda_2)\sigma_x^4 + \\ + \frac{1}{4}(\lambda_1 + \lambda_2)\sigma_y^4 - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y - h_x\sigma_x - h_y\sigma_y$$

PQM in-medium effective potential

- 3^{rd} step (equal for PQM, PNJL):
    - Integrate out the quarks (1-loop)

$$\Omega_{q\bar{q}}(\sigma_x, \sigma_y, \Phi, \bar{\Phi}; T, \mu) = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \times$$
$$\times \left\{ \ln \left[1 + 3(\Phi + \bar{\Phi} e^{-(E_{q,f}-\mu_f)/T}) e^{-(E_{q,f}-\mu_f)/T}) + e^{-3(E_{q,f}-\mu_f)/T} \right] + \right.$$
$$\left. + \ln \left[1 + 3(\bar{\Phi} + \Phi e^{-(E_{q,f}+\mu_f)/T}) e^{-(E_{q,f}+\mu_f)/T} + e^{-3(E_{q,f}+\mu_f)/T} \right] \right\}$$

- Constituent quarks are massive quasiparticles

$$E_f := \sqrt{p^2 + m_f^2}$$
$$m_u = m_d = \frac{g\sigma_x}{2} \qquad m_s = \frac{g\sigma_y}{\sqrt{2}}$$

PQM in-medium effective potential

- 4^{th} step: The Polyakov loop potential
  - Ad-hoc potential.
  - Parameters fitted from thermodynamics (pure gauge lattice).
  - Polynomial parametrization

$$\frac{\mathcal{U}_{\text{poly}}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

  - Logarithmic parametrization

$$\frac{\mathcal{U}_{\log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}A(T)\bar{\Phi}\Phi + B(T)\ln\left[1 - 6\bar{\Phi}\Phi + 4(\Phi^3 + \bar{\Phi}^3) - 3(\bar{\Phi}\Phi)^2\right]$$

The Fermion Sign Problem in QCD (in a Nutshell)

- The QCD Lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{q}(i\gamma^\mu\partial_\mu - m + g\gamma^\mu A_\mu)q$$

- The grand partition function is (gauge fixing omitted)

$$\mathcal{Z}_{QCD} = \int [DA][D\bar{q}][Dq]\exp\left[-\int_0^\beta d\tau \int d^3x\, (\mathcal{L}_{QCD} - \mu\bar{q}\gamma_0 q)\right]$$

- Formally performing the fermionic integrals yields

$$\mathcal{Z}_{QCD} = \int [DA]\, e^{-S_{eff}}\, \mathrm{det}D(A,m,\mu,T)$$

- The Dirac determinant $\mathrm{det}D(A,m,\mu,T)$ is complex for real $\mu \neq 0$.
- Therefore, the would-be Boltzmann weight is not positive-defined $\rightarrow$ sign problem.

Calculating the Surface Tension (1 order parameter)

- Assume an effective potential V and only one order parameter ϕ.
- At coexistence, the in-medium effective potential has two degenerate minima: $V(\phi_1) = V(\phi_2) = 0$.
- The surface tension is defined by [ϕ_b: solution of minimum action]

$$\Sigma := \int_0^\infty dr \left(\frac{d\phi_b(r)}{dr} \right)^2$$

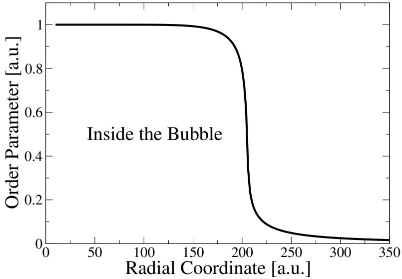
- In the thin-wall approximation (good close to coexistence)

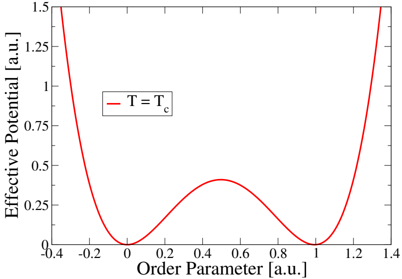
$$\Sigma = \int_{\phi_1}^{\phi_2} d\phi \sqrt{2V(\phi)}$$

- Knowledge of potential between minima is necessary.

Calculating the Surface Tension (1 order parameter)

$$\Sigma := \int_0^\infty dr \left(\frac{d\phi_b(r)}{dr}\right)^2 = \int_{\phi_1}^{\phi_2} d\phi \sqrt{2V(\phi)}$$



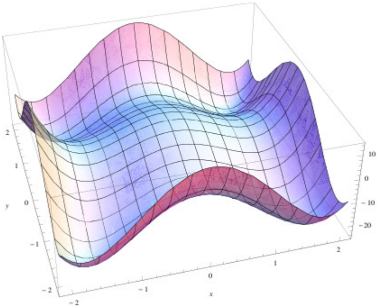


- Bubble equation (inverted potential)

$$\frac{d^2\phi_b(r)}{dr^2} + \frac{2}{r}\frac{d\phi_b(r)}{dr} = V'[\phi_b(r)]$$

Surface Tension with more than 1 Order Parameter

- The situation can be quite complicated with 2 or more order parameter: saddle-points + many coupled EL equation.



Surface Tension with more than 1 Order Parameter

- One assumption near coexistence: only two minima $\vec{\Psi}_1$ and $\vec{\Psi}_2$

$$\vec{\Psi}_i = (\sigma_x^{(i)}, \sigma_y^{(i)}, \alpha^{(i)}, \beta^{(i)})$$

- Nucleation time

$$t_{nucl} \sim \exp\left[S(\vec{\Psi}_{bubble})\right]$$

- $\vec{\Psi}_{bubble}$: solution of 4 Euler-Lagrange equations that connects minima.
- $S(\vec{\Psi}_{bubble})$: minimum action.
- However, no general method to solve the coupled EL equations...

The Straight Line Ansatz

- Let us make the problem simpler!
- Project the potential over a straight line that connects the minima.
- The line is parametrized by $\xi(r) \in [0,1]$.

$$\begin{aligned}
\sigma_x &= (1-\xi)\sigma_x^{(1)} + \xi\sigma_x^{(2)} \\
\sigma_y &= (1-\xi)\sigma_y^{(1)} + \xi\sigma_y^{(2)} \\
\alpha &= (1-\xi)\alpha^{(1)} + \xi\alpha^{(2)} \\
\beta &= (1-\xi)\beta^{(1)} + \xi\beta^{(2)}
\end{aligned}$$

- The problem is now effectively 1-dimensional.
- Okay... but we must have lost something...

The Straight Line Ansatz Gives an Overestimate of Σ

- True bubble (solution of coupled EL equations): $\vec{\Psi}_{true}(r)$.
- Any other function $\vec{\Psi}_f(r)$ that connects the minima has $S[\vec{\Psi}_f] > S[\vec{\Psi}_{true}]$.
- But $S \sim \Delta F \sim \Sigma^3$...
- Therefore,
  the straight line gives an overestimate of the surface tension!

$$\Sigma = h \int_0^1 d\xi \sqrt{2\tilde{\mathcal{E}}(\xi)}$$

$$h := \sqrt{(\Delta\sigma_x)^2 + (\Delta\sigma_y)^2 + (\kappa\Delta\alpha)^2 + (\kappa\Delta\beta)^2}$$

The Position of the CEP

- The position of the CEP is different with the two approaches...

Parametrization	T_0 [MeV]	(T_C, μ_C)[MeV]	$(T_C, \mu_C)^S$[MeV]
Log [Roessner: 07]	182	$(143, 129)$	$(143, 128)$
	270	$(192, 88)$	$(192, 84)$
Poly-I [Scavenius: 02]	182	$(139, 99)$	$(140, 92)$
	270	$(171, 103)$	$(175, 83)$
Poly-II [Ratti: 06]	182	$(146, 115)$	$(152, 80)$
	270	$(176, 129)$	$(184, 103)$

Table: $m_\sigma = 500$ MeV and $T_0 = 182$ MeV (with effective screening) or $T_0 = 270$ MeV (pure gauge).