Nonperturbative heavy quarks and confinement in Coulomb gauge

Peter Watson

Institut für Theoretische Physik
Universität Tübingen

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Collaborators: C. Popovici and H. Reinhardt

PRD81, 105011; PRD83, 025013 (and references therein)

Outline

- Motivation
- Some features of Coulomb gauge
- Nonperturbative (not effective) heavy quarks
- Bound states (color singlet?)
- Summary

Motivation

How can we set the physical scale in Yang-Mills theory?
– there's no experimental probe!
Or, is there a simple connection between the gluon propagator and the hadron spectrum?

- lattice: Wilson loops and the string tension
- covariant gauges: perturbation theory
- continuum Coulomb gauge?
  (perturbation theory and Wilson loops are difficult)

$\Rightarrow$ heavy quarks

R. Sommer, Nucl.Phys.B411,839(1994); M. Pak, H. Reinhardt, PRD80,125022(2009).

Coulomb gauge

- $\vec{\nabla} \cdot \vec{A} = 0$, noncovariant ($\vec{A}$ and A_0 parts different)
- Gauss' law: total color charge is conserved and vanishing.
- Temporal gluon propagator

$$W_{00}^{ab}(k) = \imath\delta^{ab}\frac{D_{00}(\vec{k}^2)}{\vec{k}^2}$$

  - (lattice) independent of energy
  - (lattice) infrared enhanced, consistent with $W \sim 1/\vec{k}^4$

H. Reinhardt, PW, PRD79,045013(2009); M. Quandt *et al.* Proc. Conf8, arXiv:0812.3842;

A. Cucchieri *et al.* Mod.Phys.Lett A22,2429(2007).

Nonperturbative heavy quarks

Generating functional (quark part)

$$Z[\overline{\chi},\chi] = \int \mathcal{D}\Phi \exp\left\{\imath\mathcal{S} + \imath \int [\overline{\chi}q + \overline{q}\chi]\right\},$$
$$\mathcal{S} = \int \overline{q}\left[\imath\gamma^0 D_0 + \imath\vec{\gamma}\cdot\vec{D} - m\right] q$$

and decompose using $P_\pm = (1 \pm \gamma^0)/2$

$$q = e^{-\imath m x^0}[h + H], \quad h = e^{\imath m x^0} P_+ q, \quad H = e^{\imath m x^0} P_- q.$$

(Coulomb gauge) specific choice of heavy quark decomposition.

Nonperturbative heavy quarks

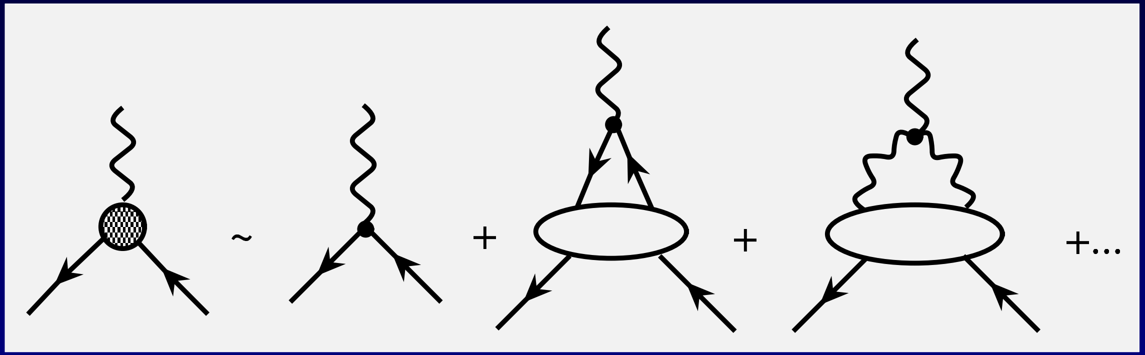
- H-dep: $\mathcal{S} \sim \overline{H}[2m + \imath D_0]H,\ \overline{h}[\imath\vec{\gamma}\cdot\vec{D}]H$
- Integrate out H, and restrict to leading order in $1/m$, *but*: keep full sources $\overline{\chi}, \chi$ (unlike HQET).
- Functional formalism (DSes, STids) preserved but with different tree-level expressions!
- $\mathcal{S} = \int \overline{h}[\imath\partial_0 + gT^a A_0^a]h$

$$\Rightarrow \underline{\Gamma^{(0)}_{\bar{q}q0} = gT^a}, \Gamma^{(0)}_{\bar{q}qA} = \mathcal{O}(1/m).$$

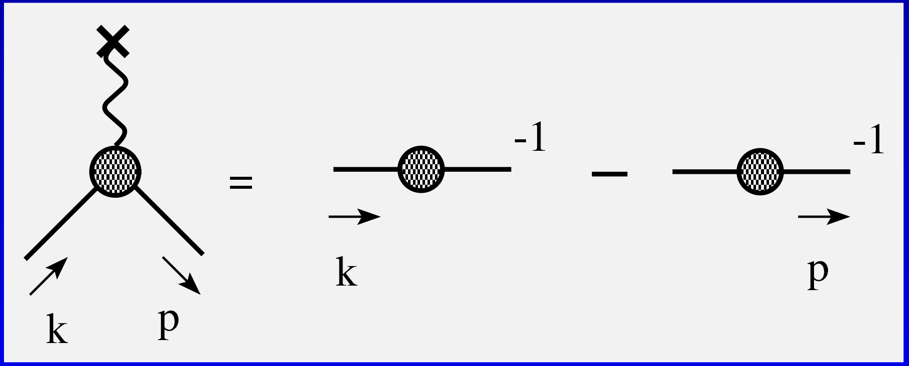
- Only temporal components survive, spin decouples (hint: Gauss' law and charge conservation).

Nonperturbative heavy quarks

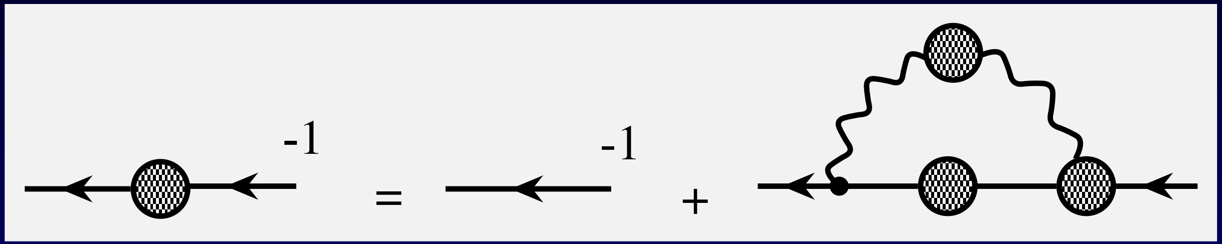
Spatial quark-gluon vertex DSe



Set (truncate) YM vertices to zero: $\Gamma_{\overline{q}qA} = \mathcal{O}(1/m)$ (NB $\Gamma_{000} = 0$ at tree-level). Temporal quark-gluon vertex STid reduces:



Nonperturbative heavy quarks



$$W_{\bar{q}q}(k) = \frac{-\imath}{[k_0 - m - I_r + \imath\varepsilon]}, \quad I_r = \frac{g^2 C_F}{2} \int_r \frac{\vec{d}\,\vec{\omega}\, D_{00}(\vec{\omega}^2)}{\vec{\omega}^2}$$

- single pole (cf HQET)
- an IR enhanced D_{00}, gives a divergent I_r

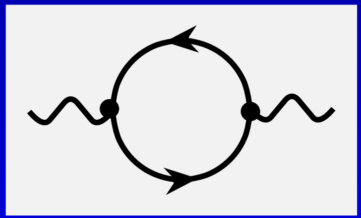
  $\Rightarrow$ quark self-energy diverges (no finite pole position)

- $\Gamma_{\bar{q}q0} = \Gamma^{(0)}_{\bar{q}q0}$: rainbow truncation is exact

Nonperturbative heavy quarks

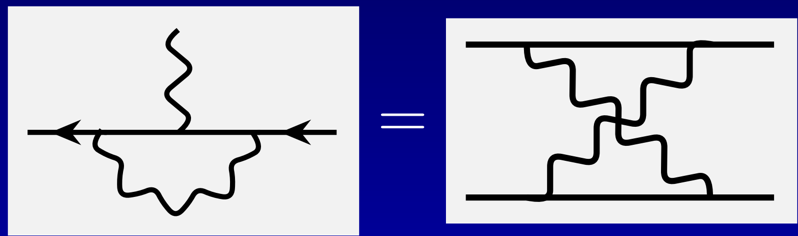
- antiquark:

$$W_{q\bar{q}}(k) = \frac{-\imath}{[k_0 + m - I_r + \imath\varepsilon]}$$

- heavy quark decomposition splits the four spinor states (two are $1/m$ suppressed, the other two have single poles)
- the Feynman prescription is important
  – all energy integrals reduce to $2\pi\imath$ or zero, depending on the sign!
- eg.  $= 0$ (heavy theory is quenched)

Bound states

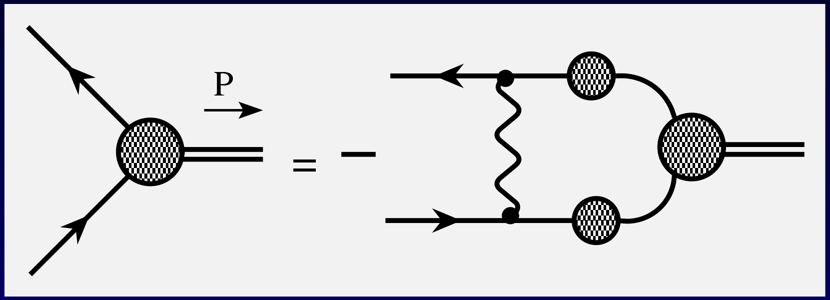
- Question: complete the following phrase "rainbow-?"
  – this is a result of the AXWTI, or charge conservation (Gauss' law)
- Here, because of the Feynman prescription



$= 0$

- AXWTI is preserved, BSe is in ladder truncation!

Bound states



- Quark-antiquark channel, pole energy as a function of separation:

$$P_0(x) = g^2 \int_r \frac{\bar{d}\,\vec{\omega} D_{00}(\vec{\omega}^2)}{\vec{\omega}^2} \left[C_F - e^{\imath\vec{\omega}\cdot\vec{x}} C_M\right]$$

- IR finite if $C_M = C_F$ or *color singlet*!
- if $D_{00} = X/\vec{\omega}^2$ (note, $g^2 D_{00}$ is RG invariant),

$$P_0(x) = \frac{g^2 C_F X}{8\pi} x$$

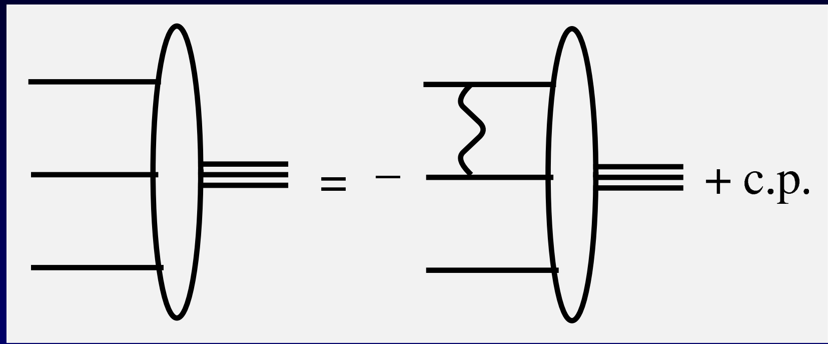
Bound states

- Diquark channel

$$P_0(x) = 2m + g^2 \int\limits_r \frac{\vec{d}\,\vec{\omega} D_{00}(\vec{\omega}^2)}{\vec{\omega}^2} \left[C_F + e^{\imath \vec{\omega}\cdot\vec{x}} C_D \right]$$

- IR finite if $C_D = -C_F$ or $N_c = 2$.
- The SU(2) baryon!

Bound states



- Only 2-body interactions, equal spacing

$$P_0(x) = 3m + \frac{3}{2}g^2 \int_r \frac{\vec{d}\,\vec{\omega} D_{00}(\vec{\omega}^2)}{\vec{\omega}^2} \left[C_F - 2C_B e^{\imath\vec{\omega}\cdot\vec{x}}\right]$$

- IR finite if $C_B = C_F/2$ or $N_c = 3$.
- Note the factor 3/2.

Summary

- Nonperturbative heavy quarks in Coulomb gauge.
- Leading order mass expansion, YM truncation.
- The temporal gluon propagator and vertex are prominent.
- Gap and BS equations solved analytically (rainbow-ladder is *exact*).
- Only color singlet states have finite pole energies (Gauss' law: total color charge is conserved and vanishing).
- There appears to be a connection between the temporal gluon propagator and the string tension.