Thermalization of Hawking Radiation in AdS_5

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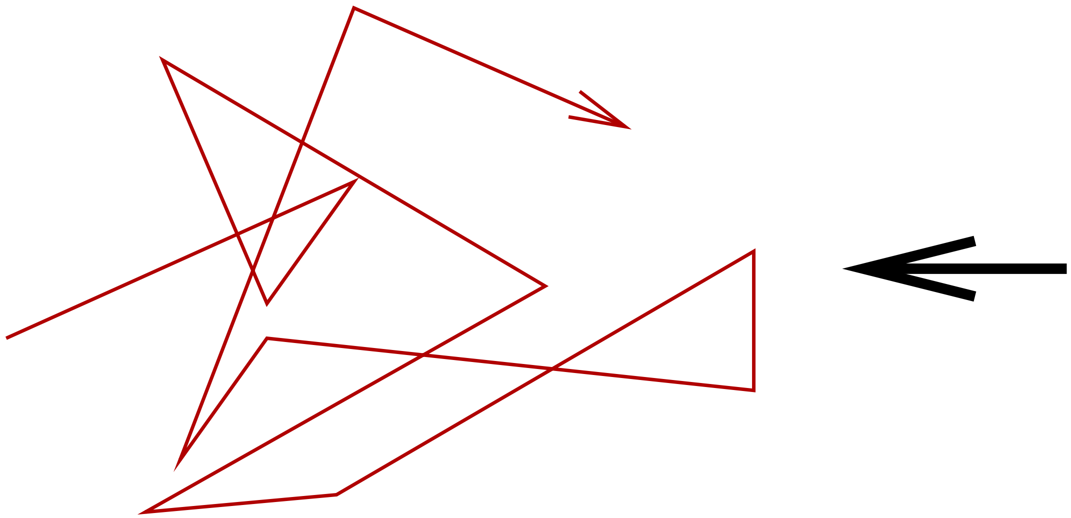
- Dam T. Son, DT; JHEP. arXiv:0901.2338
- Simon Caron-Huot, DT, Paul Chesler; arXiv:1102.1073

Brownian Motion and Equilibrium

$$M\frac{d^2\mathbf{x}}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{\mathbf{x}} + \underbrace{\xi}_{\text{Noise}}$$



"Artist's" conception of Brownian Motion

1. Equilibrium is a state constant fluctuations

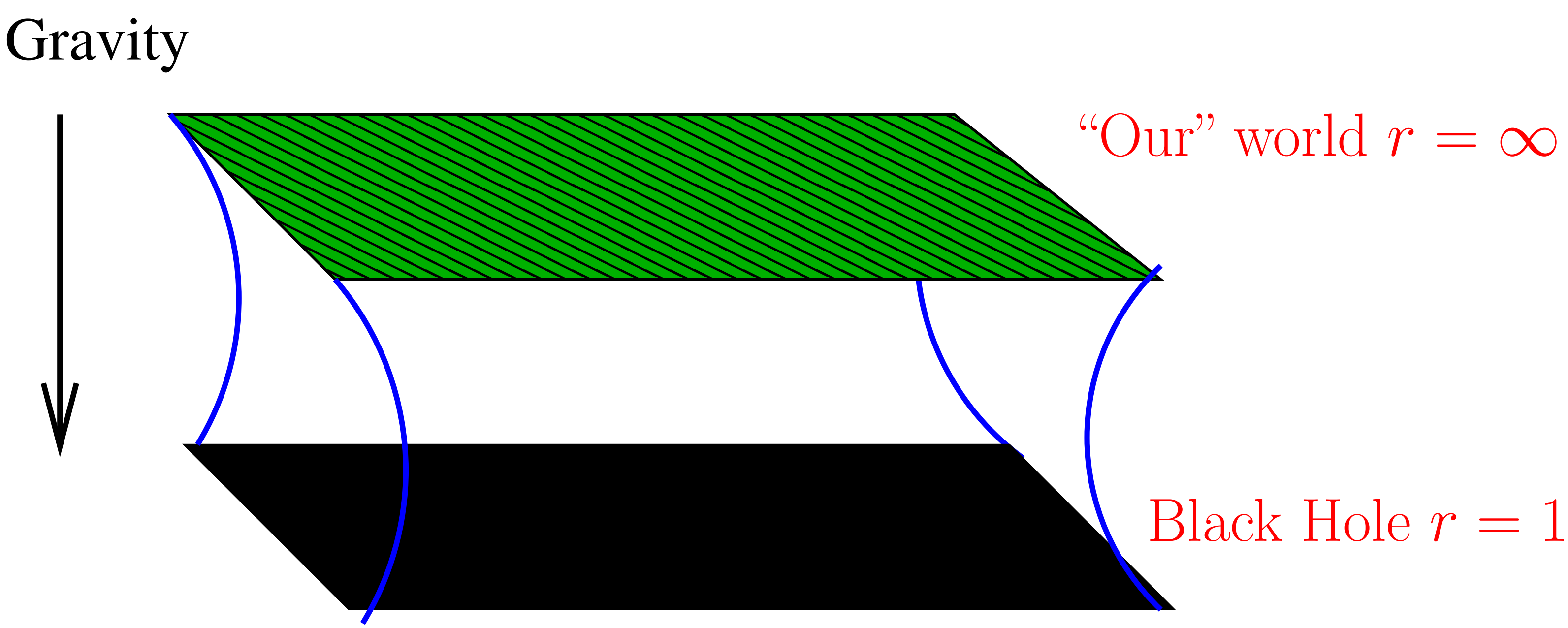
2. Equilibrium is a perpetual competition between drag and noise

$$\langle \xi(t)\xi(t')\rangle = 2T\eta\,\delta(t-t') \qquad \text{to reach equilibrium} \qquad P(\mathbf{p}) \propto e^{-\frac{\mathbf{p}^2}{2MT}}$$

AdS/CFT

- Classical solutions in curved spacetime = CFT for nonzero temperature

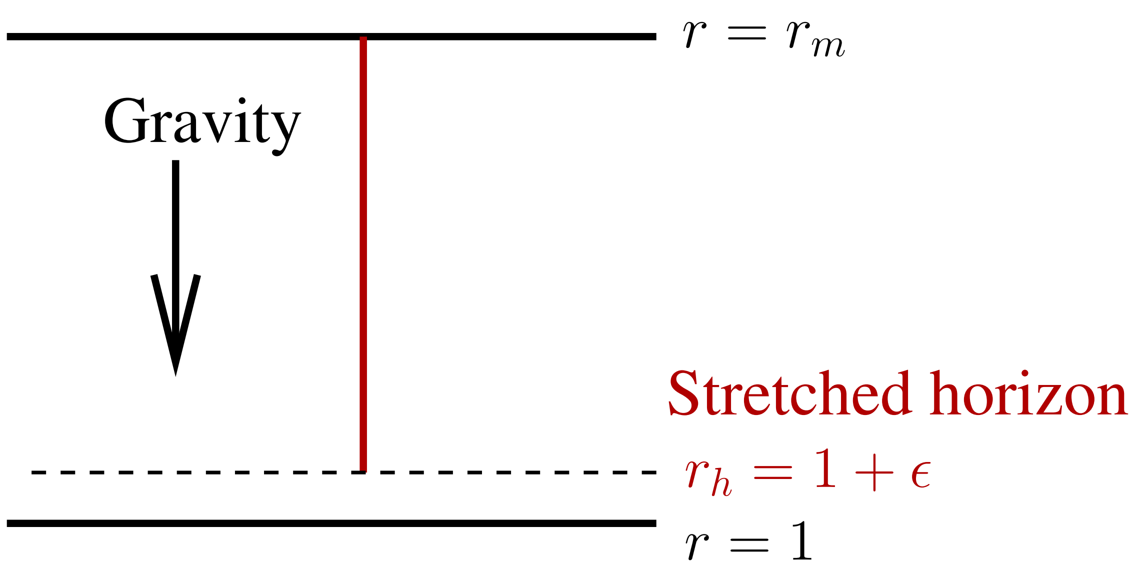
$$\mathrm{d}s^2 = (\pi T)^2 r^2 \left[-f(r)\mathrm{d}t^2 + \mathrm{d}x^2\right] + \frac{\mathrm{d}r^2}{r^2 f(r)} \qquad\qquad f(r) = 1 - \frac{1}{r^4}$$



How can a static metric be dual to equilibrium=constant fluctuations ?

A heavy quark in AdS/CFT

- Solve classical string (Nambu-Goto) EOM and find:



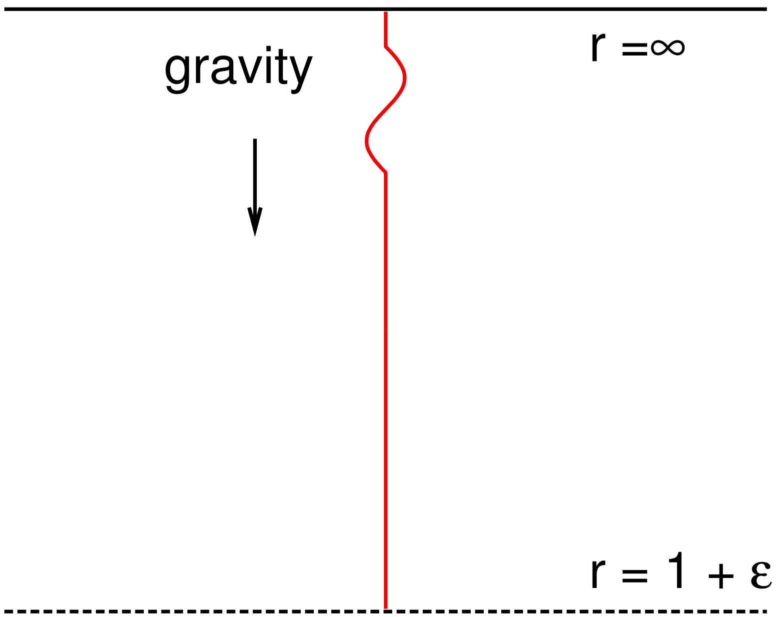
Not the dual of an equilibrated quark!

Dissipation in classical black hole dynamics

Herzog et al; DT J. Casalderrey-Solana; Gubser

$$M\frac{d^2x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x}^o$$

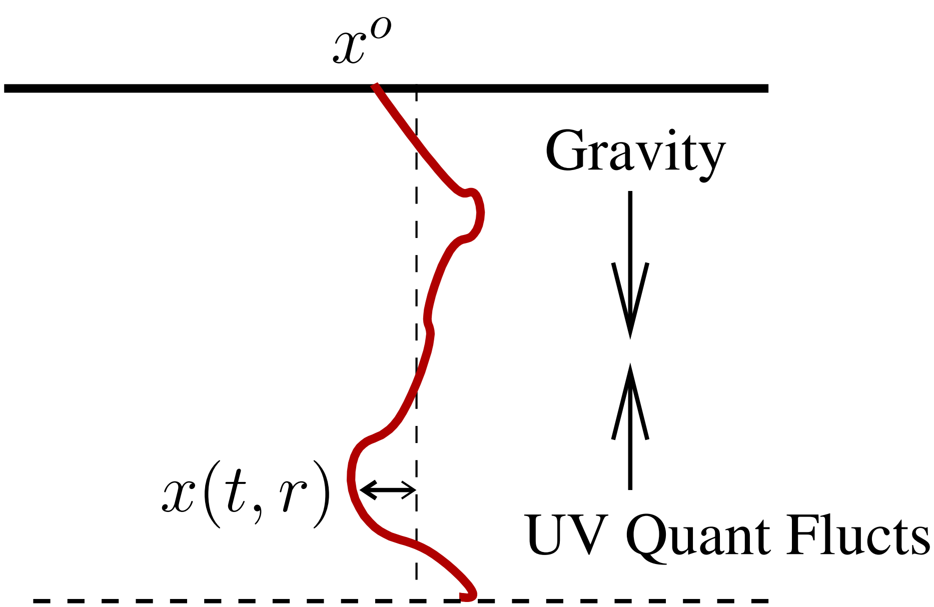
$$\underbrace{\eta = \frac{\sqrt{\lambda}}{2\pi}g_{xx}(r_h) = \frac{\sqrt{\lambda}}{2\pi}(\pi T)^2}_{\text{Coupling of string to near horizon metric}}$$



Classical dissipation determines drag

Detailed Balance and Hawking Radiation:

$$M\frac{d^2x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x}^o + \underbrace{\xi}_{\text{Noise}}$$



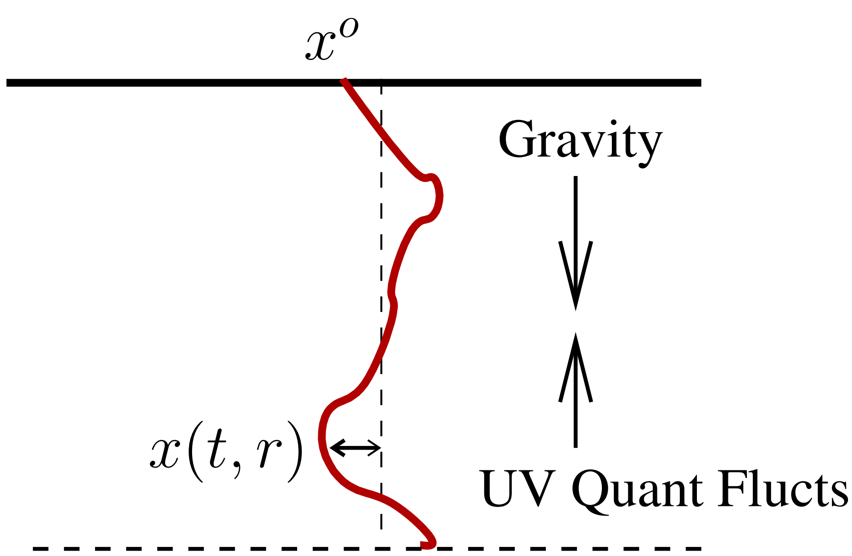
Evolves to Classical

Prob. Dist ?:

$$P[x,\pi_x] \propto e^{-\beta H[x,\pi_x]}$$

Hawking Radiation Balanced by Gravity?

Detailed Balance and Hawking Radiation (Technical Discussion)



1. Fluctuations:

$$G_{rr} \equiv \frac{1}{2} \langle \{ \hat{x}(t_1, r_1), \hat{x}(t_2, r_2) \} \rangle \ ,$$

2. Dissipation (Spectral Density)

$$\rho_{ra-ar} \equiv \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle \ .$$

- Equilibrium $\equiv$ Fluctuation Dissipation Theorem

$$G_{rr}(\omega, r_1, r_2) = \left(\frac{1}{2} + n_B(\omega) \right) \rho_{ra-ar}(\omega, r_1, r_2) \qquad n(\omega) \equiv \frac{1}{e^{\omega/T} - 1}$$

Establish in FDT with Hawking Radiation ? Non-equilibrium ?

Outline

1. Give a different derivation of Hawking radiation
   - Similar to 2PI formalism (collisions not needed)
2. Show how Hawking radiation gives Brownian motion in 5D.
3. Study non-equilibrium and how FDT is established
4. Unusual features of thermalization in AdS

Hawking Radiation

1. Fluctuations
2. Dissipation

Formulas

- Action for string fluctuations, $h^{\mu\nu}$ = string metric

$$S = \frac{\sqrt{\lambda}}{2\pi} \int \mathrm{d}t\mathrm{d}r\, g_{xx} \left[-\frac{1}{2} \sqrt{h} h^{\mu\nu} \partial_\mu x \partial_\nu x \right] ,$$

- $h^{\mu\nu}$ is the string metric

$$h_{\mu\nu} \mathrm{d}\sigma^\mu \mathrm{d}\sigma^\nu = -(\pi T)^2 r^2 f(r) dt^2 + \frac{dr^2}{f(r) r^2} ,$$

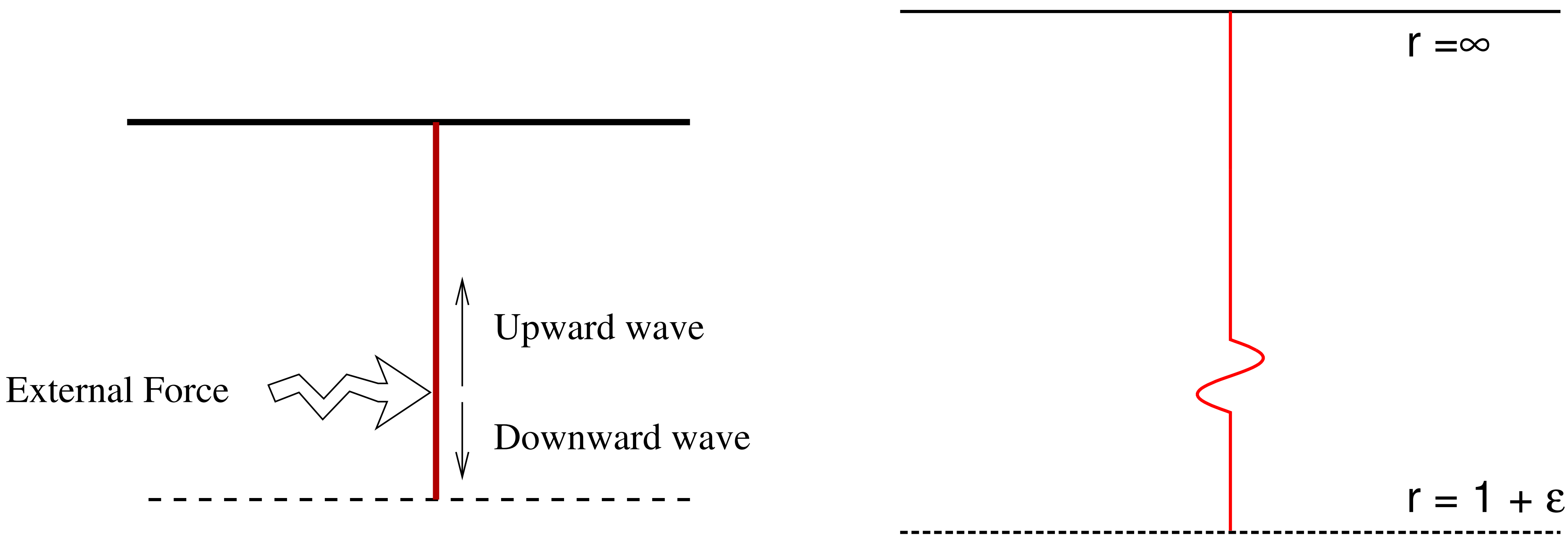
- Retarded Green Function

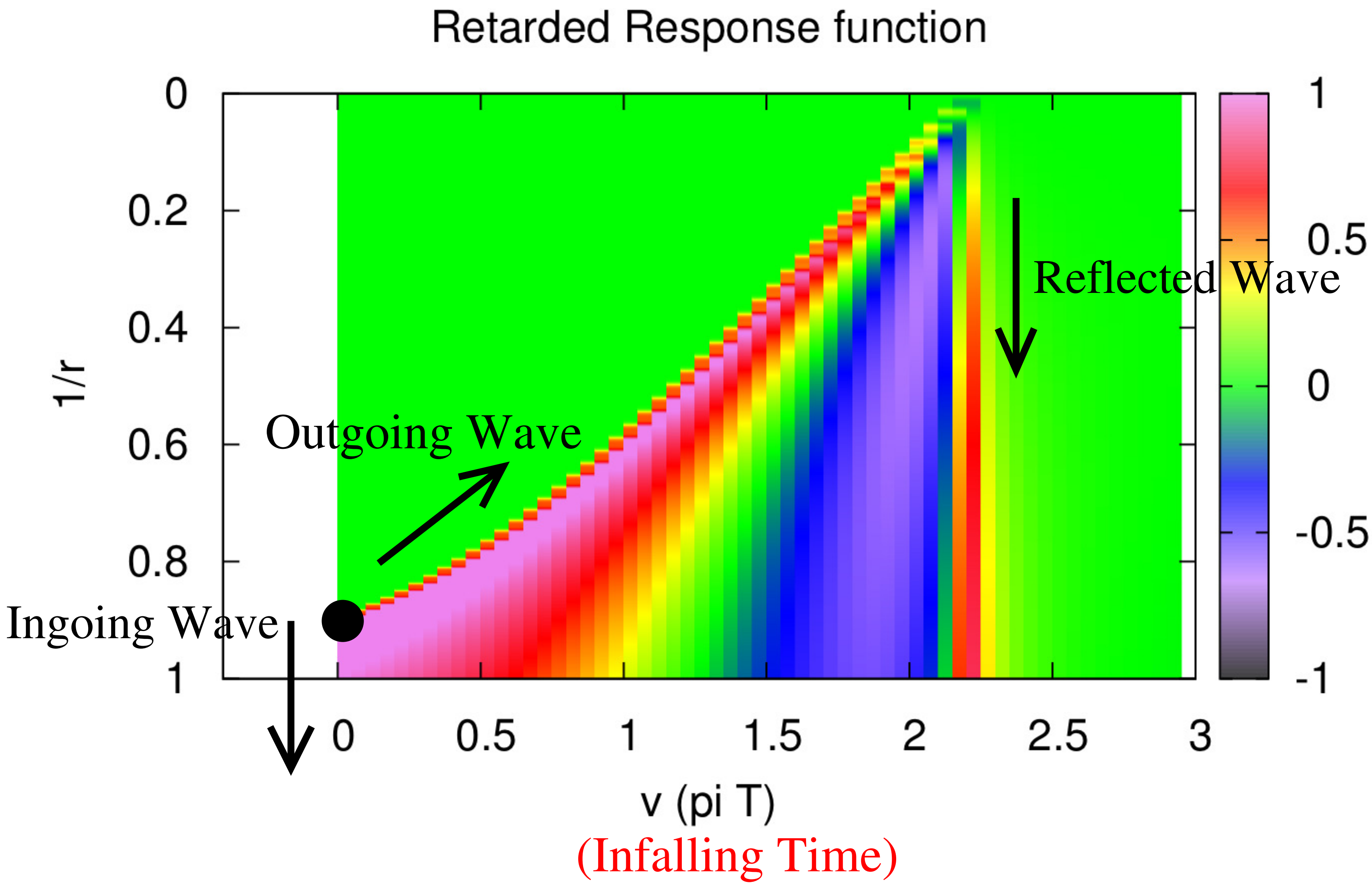
$$iG_{ra}(t_1 r_1 | t_2 r_2) \equiv \theta(t - t') \left\langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \right\rangle ,$$

$G_{ra}(t_1 r_1 | t_2 r_2)$ is the classical response to a force at $t_2 r_2$

$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu\, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] G_{ra}(t_1 r_1 | t_2 r_2) = \delta(t_1 - t_2) \delta(r_1 - r_2) ,$$

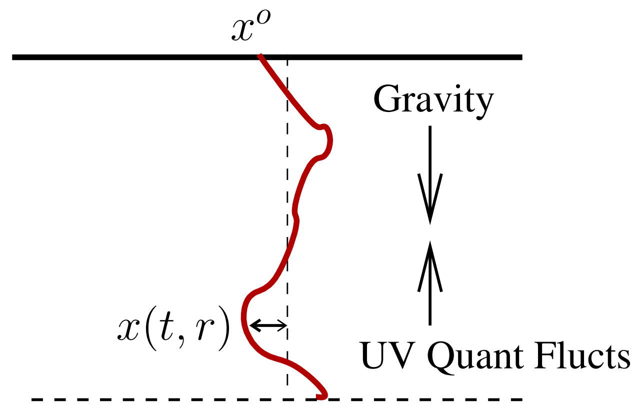
Classical Green Function (Typical of AdS)





$$v = t - \frac{1}{2\pi T}\left[\tan^{-1}(r) + \tanh^{-1}(r)\right] \qquad v = \text{Eddington time}$$

Statistical Correlator



$$G_{rr} = \frac{1}{2}\left\langle \{x(t_1,r_1), x(t_2,r_2)\}\right\rangle$$

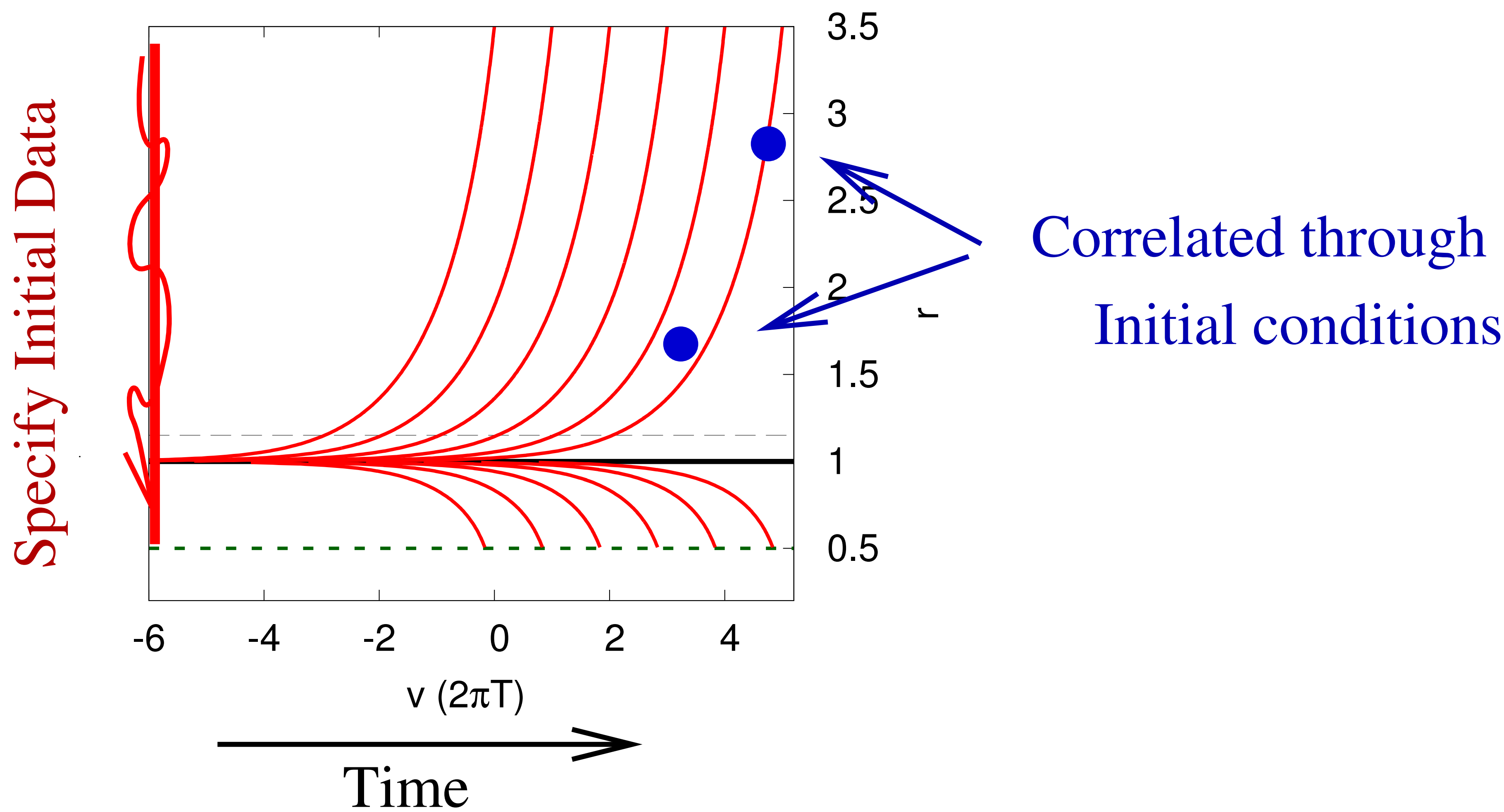
- The statistical correlator obeys the homogeneous EOM

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G_{rr}(t_1 r_1|t_2 r_2) = 0$$

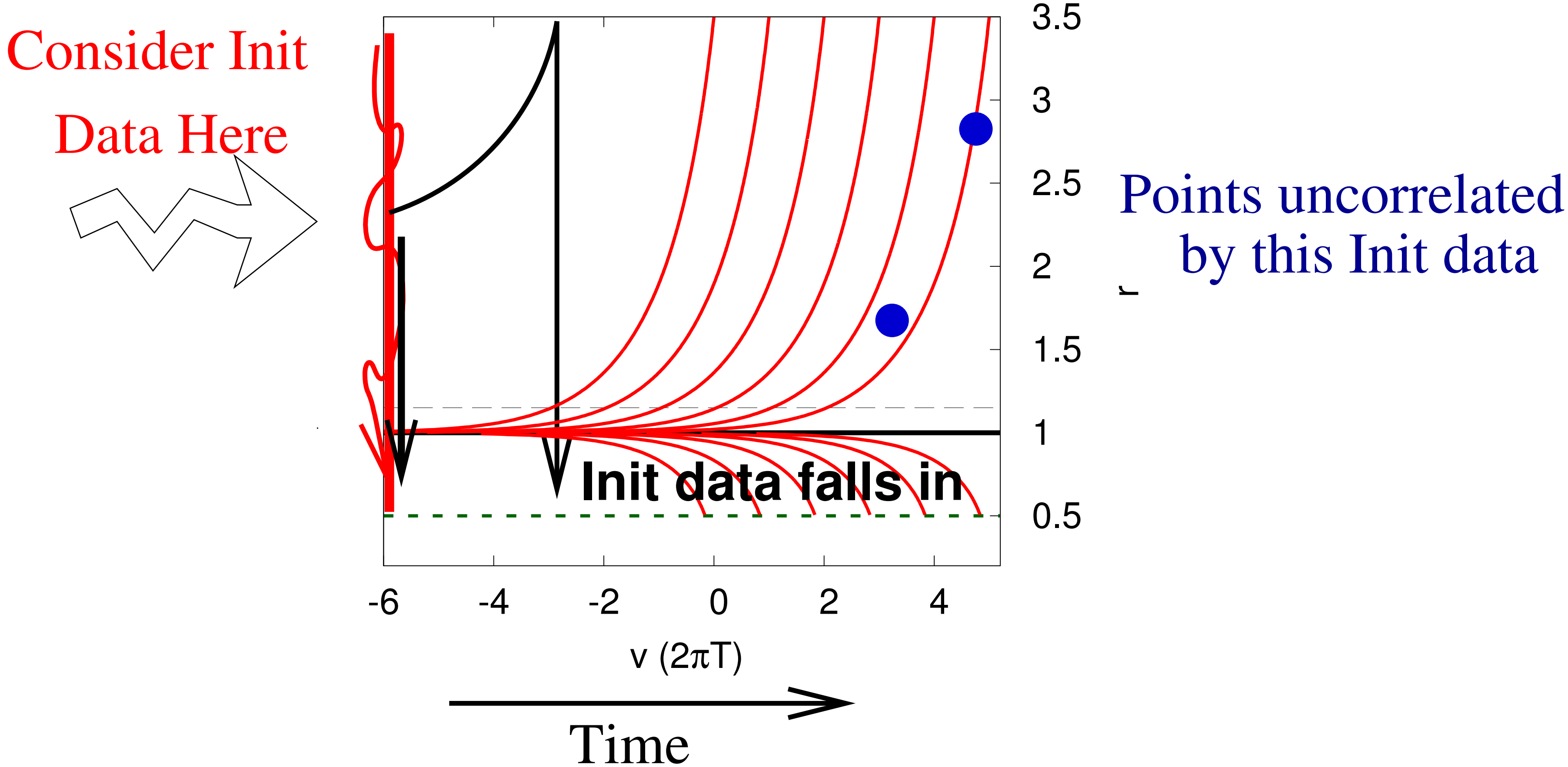
- So:

1. Specify the correlations (density matrix) in the past
2. Final state fluctuations are correlated only through initial conditions

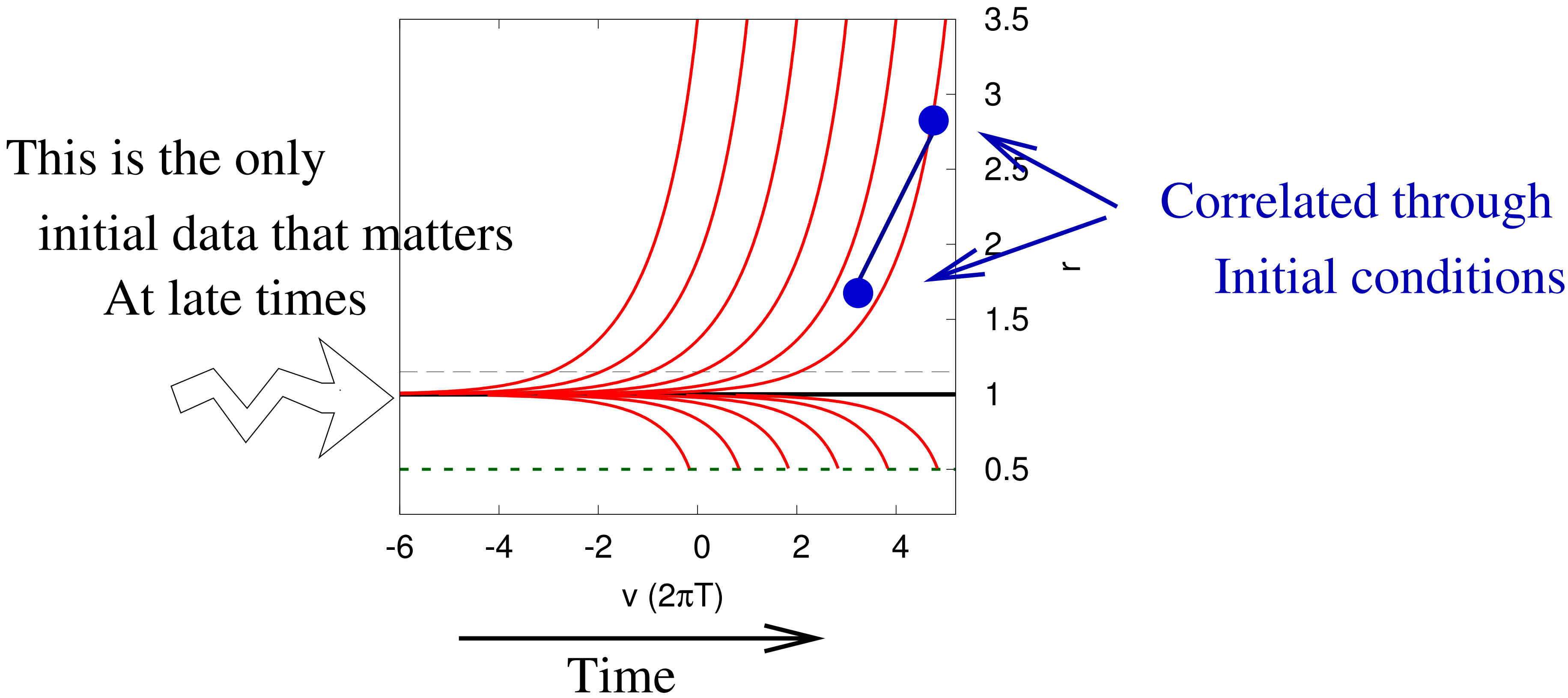
Correlations through Initial conditions



Correlations through Initial conditions



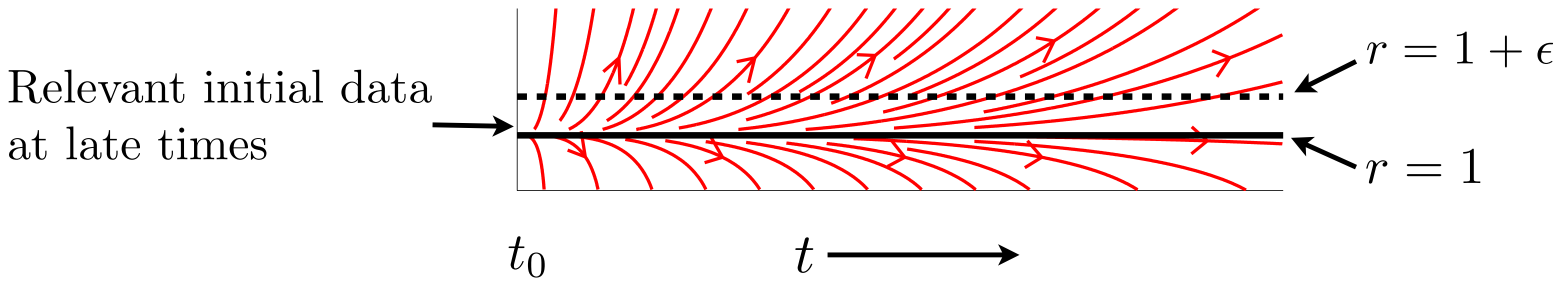
Correlations through Initial conditions



1. Final correlation come from correlated initial data near horizon
2. Initial data is inflated by near horizon geometry

Initial Data from Quantum Fluctuations

1. Initial data is determined at short distance = Flat Space Physics



2. Scalar Field in 1+1D flat space

$$\frac{1}{2}\langle\{\phi(X_1), \phi(X_2)\}\rangle = -\frac{1}{4\pi K}\log|\mu\,\eta_{\mu\nu}\Delta X^{\mu}\Delta X^{\nu}|$$

K=norm of action

3. String flucts in near horizon geometry

$$S^{\text{near}-\text{horizon}} = \eta\int \mathrm{d}t\mathrm{d}r\left[-\frac{1}{2}\sqrt{h}h^{\mu\nu}\partial_{\mu}x\partial_{\nu}x\right]$$

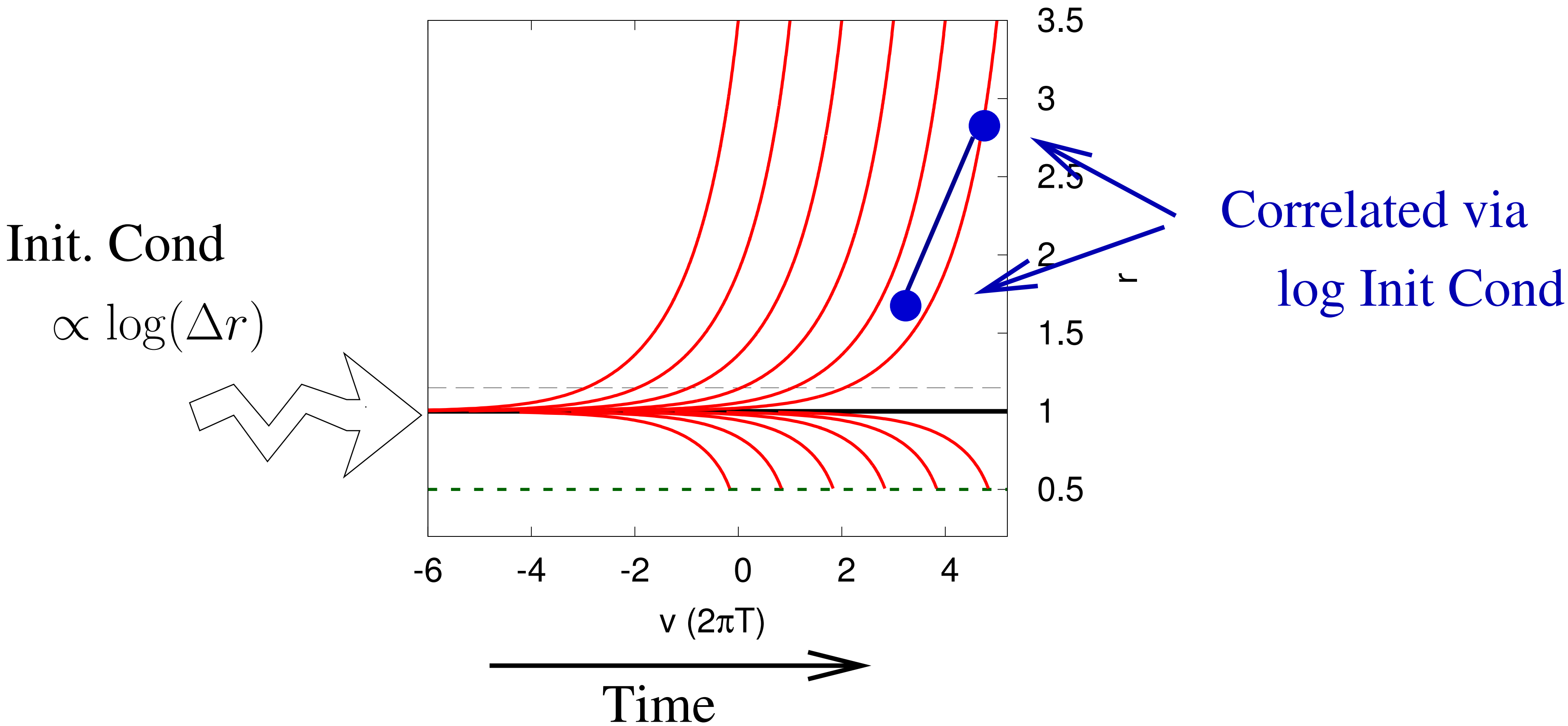
$\eta =$ Drag Coefficient

The near horizon initial condition is

$$G_{rr}(v_1 r_1|v_2 r_2) \to -\frac{1}{4\pi\eta}\log\left|\mu\ \overbrace{2\Delta v\,\Delta r}^{\text{local }\Delta s^2}\right|$$

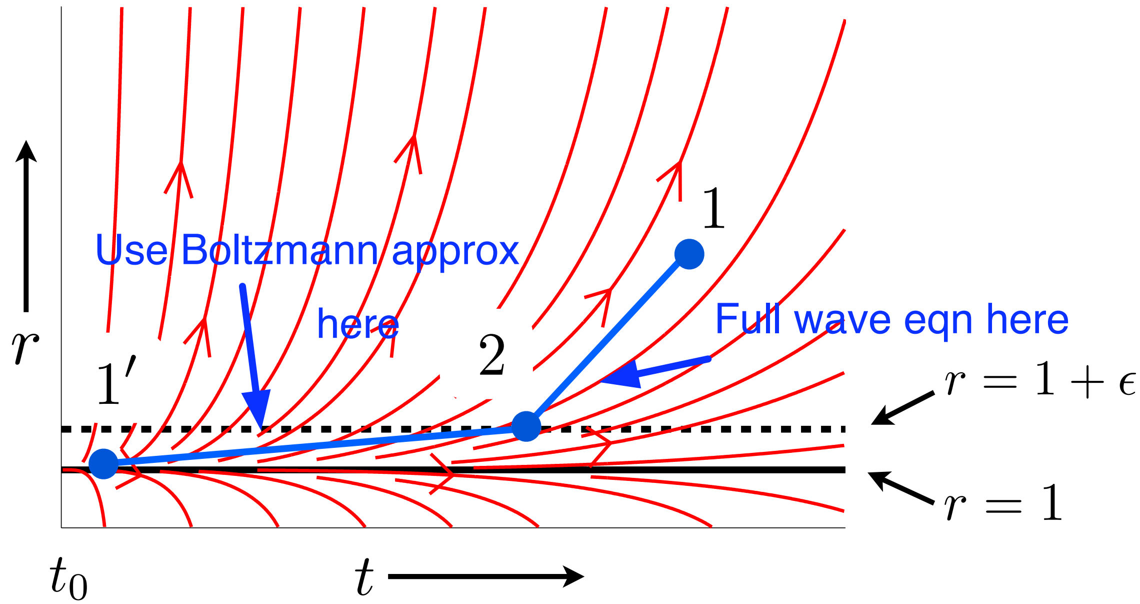
Summary: Specify IC and Solve Equations of Motion

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G_{rr}(t_1 r_1|t_2 r_2) = 0$$



Logarithmic initial data is inflated by near horizon geometry

From initial data to final correlations in two steps:



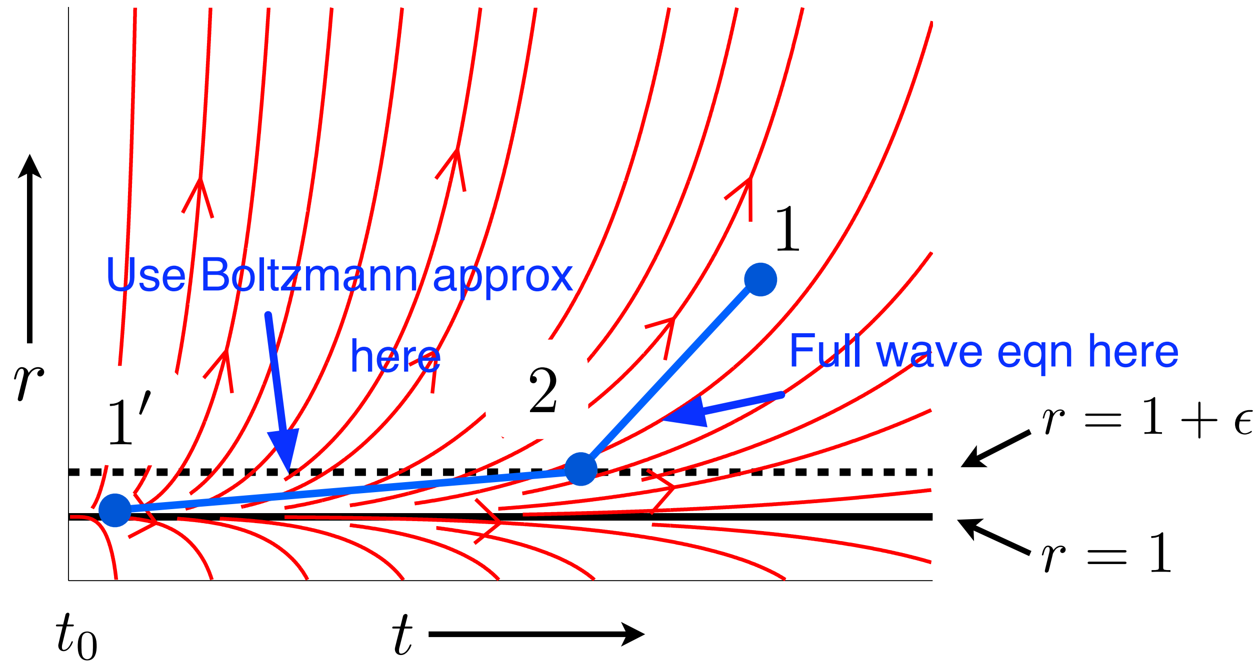
(a) From horizon to stretched horizon – Waves are very short wavelength

– Use collisionless Boltzmann approximation (geodesic/WKB/eikonal approx)

(b) The stretched horizon to boundary – Waves are longer wavelength

– Use full wave equation

Two step evolution:

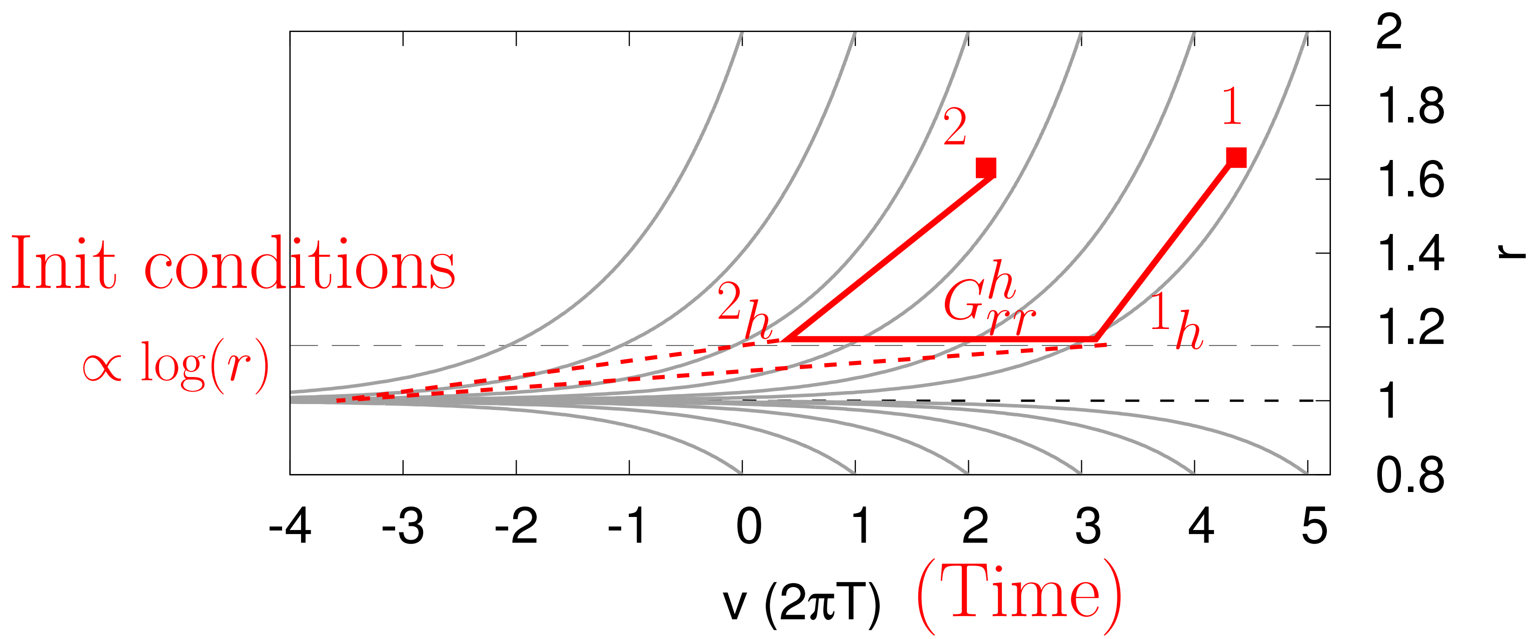


Study the Wronskian find a Green Fcn Composition Rule

$$G_{ra}(1|1') = \int \mathrm{d}t_2 \underbrace{G_{ra}(1|2)}_{\text{response to Force at 2}} \times \underbrace{\left[\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2}\right]_{r_2=r_h} G_{ra}(2|1')}_{\text{Force at 2 from 1'}}$$

Fluctuations from Equations of Motion

$$\underbrace{G_{rr}(1|2)}_{\text{bulk flucts}} = \int dt_{1h} dt_{2h} \underbrace{G_R(1|1_h)\, G_R(2|2_h)}_{\text{outgoing Green fcns}} \underbrace{G^h_{rr}(1_h|2_h)}_{\text{horizon flucts}},$$

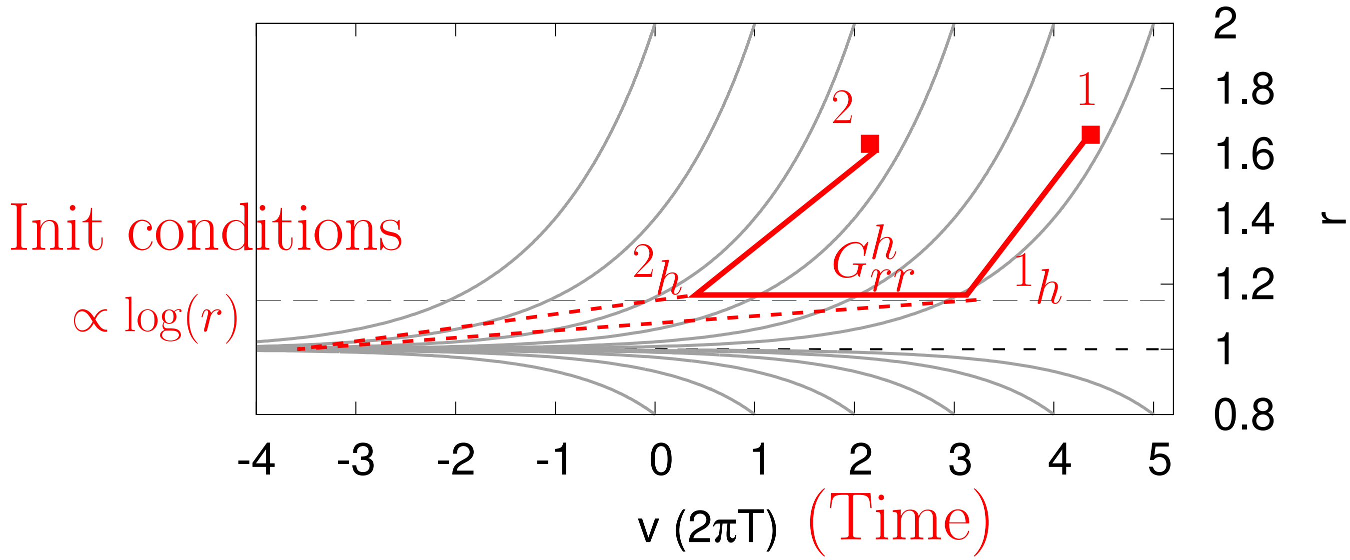


Where the horizon force-force correl. summarizes UV vacuum flucts in past

$$G^h_{rr}(t_1|t_2) = \text{Blow-up of initial data} \propto \log(r)$$

$$= -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2} \log\left|1 - e^{-2\pi T(t_1 - t_2)}\right| .$$

The horizon fluctuations and the Lyapunov exponent



1. Thermal looking:

$$G^h_{rr}(\omega) = \text{Fourier-Trans of } -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2}\log|1-e^{-2\pi T(t_1-t_2)}|$$

$$= \left(\tfrac{1}{2}+n(\omega)\right)2\omega\eta \qquad n(\omega)\equiv\frac{1}{e^{\omega/T}-1}$$

2. Temperature $\propto$ inflation rate

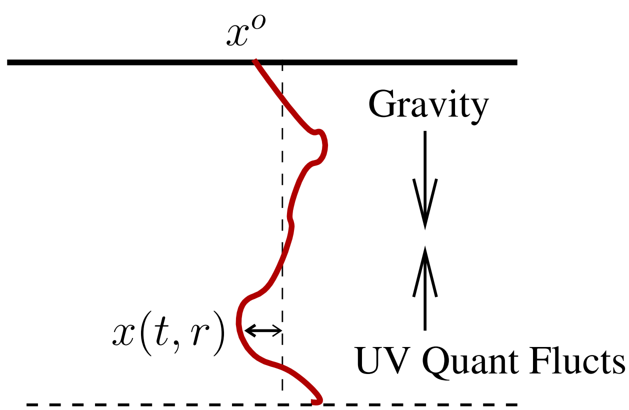
$$2\pi T = \text{Lyapunov exponent of diverging geodesics}$$

Hawking Radiation

- ✓ Fluctuations

1. Dissipation

Dissipation - Spectral Density



$$\rho_{ra-ar} = \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle$$

- The spectral density also obeys the EOM

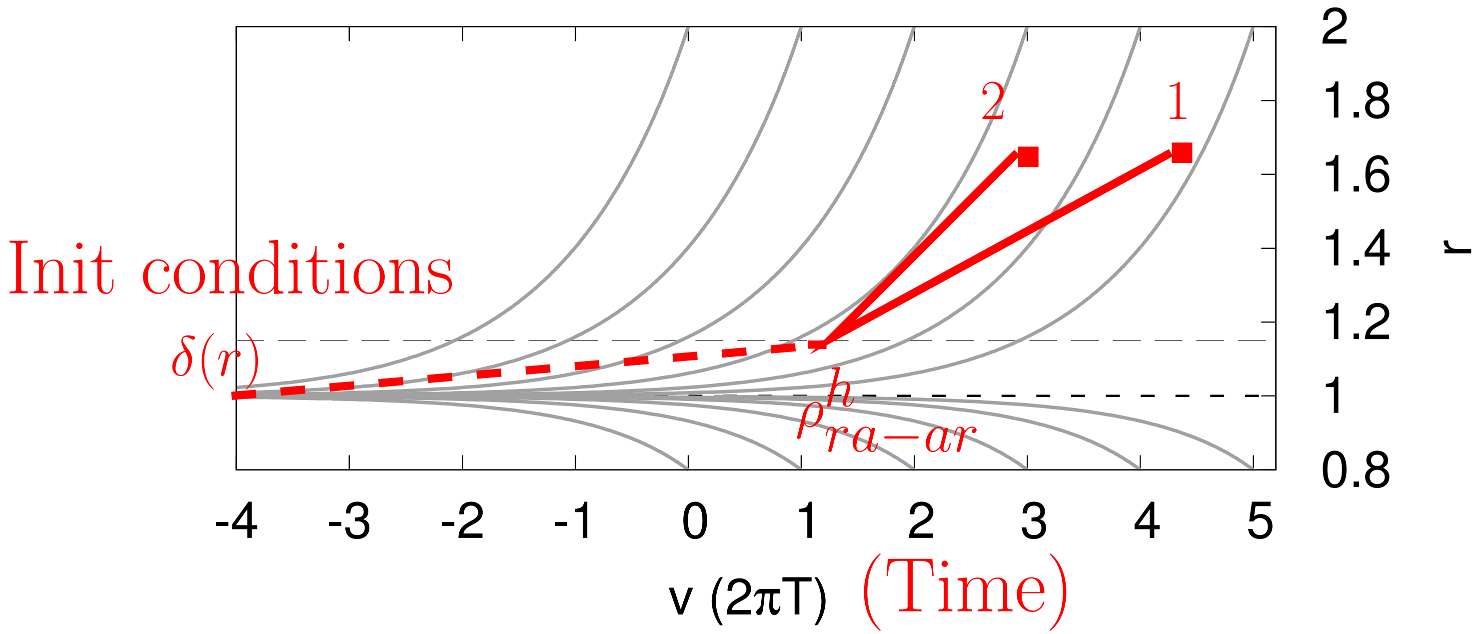
$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] \rho_{ra-ar}(t_1 r_1 | t_2 r_2) = 0$$

- But initial conditions are given by the canonical commutation relations

$$\eta \sqrt{h} h^{tt}(r_1) \lim_{t_2 \to t_1} \partial_{t_1} \rho_{ra-ar}(t_1 r_1 | t_2 r_2) = i\delta(r_1 - r_2)\,.$$

Spectral Density

$$\underbrace{\rho_{ra-ar}(1|2)}_{\text{bulk spectral fcn}} = \int \mathrm{d}t_{1h}\mathrm{d}t_{2h} \underbrace{G_R(1|1_h)\; G_R(2|2_h)}_{\text{outgoing Green fcns}} \underbrace{\rho^h_{ra-ar}(1_h|2_h)}_{\text{horizon spectral fcn}} ,$$



Where the horizon spectral density

$$\rho^h_{ra-ar}(t_1, t_2) = \text{local due to canonical commutation relations}$$

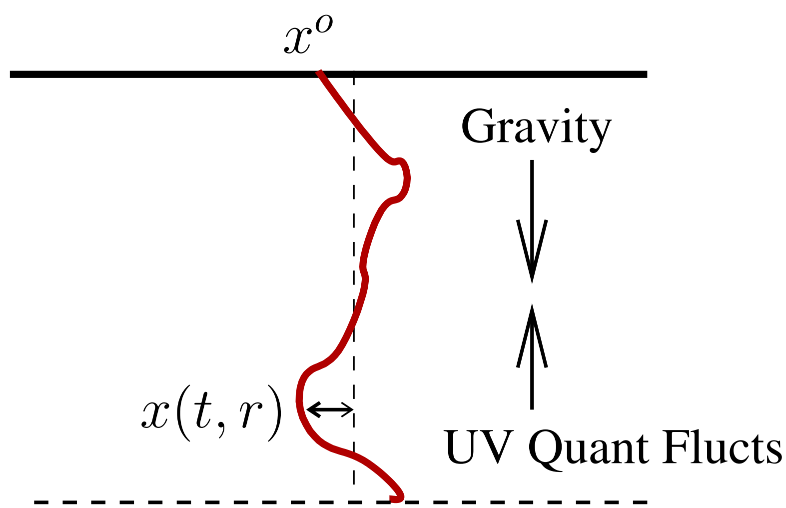
$$= 2\eta \left[-i\delta'(t_1 - t_2) \right] \qquad (2\omega\eta \text{ in Fourier space})$$

Hawking Radiation

- ✓ Fluctuations
- ✓ Dissipation

Conclusion: Detailed Balance

$$G_{rr}(\omega, r_1, r_2) = \left(\tfrac{1}{2} + n(\omega)\right) \rho(\omega, r_1, r_2)$$



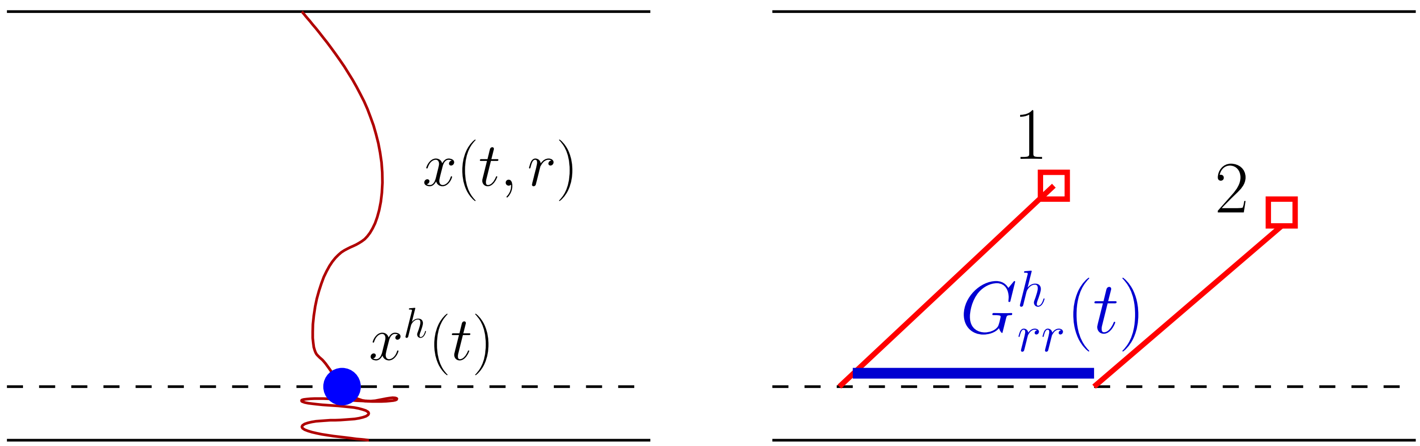
1. Dissipation

$$\underbrace{\rho_{ra-ar}(\omega, r_1, r_2)}_{\text{bulk spec dense}} = \underbrace{G_R(\omega, r_1|r_h)\; G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \quad \underbrace{2\omega\eta}_{\text{Horizon spec dense}}$$

2. Fluctuations

$$\underbrace{G_{rr}(\omega, r_1, r_2)}_{\text{bulk flucts}} = \underbrace{G_R(\omega, r_1|r_h)\; G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \underbrace{\left(\tfrac{1}{2} + n(\omega)\right) 2\omega\eta}_{\text{Horizon-flucts}}$$

Horizon Effective Action



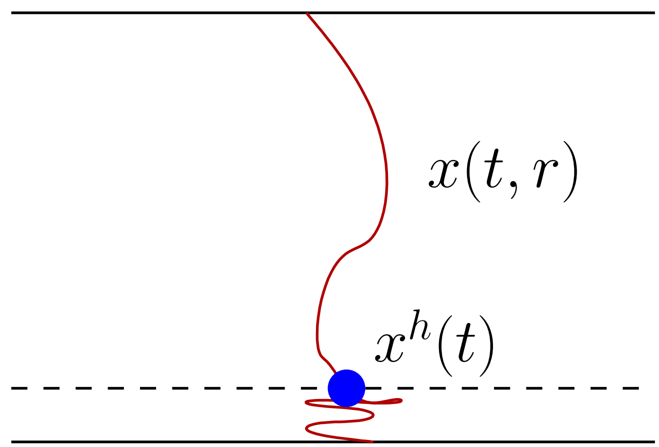
1. G^h_{rr} and G^h_{ra} summarize the dynamics for r below r_h on $x^h(t)$

$$G^h_{rr}(t) = \left\langle x^h_r(t) x^h_r(0) \right\rangle$$

2. The complete set of these correlators determine a Horizon Effective Action

3. The full action is

$$S = \underbrace{S_{\rm out}}_{\text{String action for } r > r_h} + \underbrace{S^h_{\rm eff}}_{\text{Horizon effect action}}$$



$$S^h_{\rm eff} = -\int_\omega x^h_a \Big[\underbrace{-i\omega\eta}_{\text{Horizon dissipation}} \Big] x^h_r + \frac{i}{2}\int_\omega x^h_a \Big[\underbrace{(1+2n)\omega\eta}_{\text{Horizon flcts}} \Big] x^h_a$$

The effective action provides a horizon boundary condition

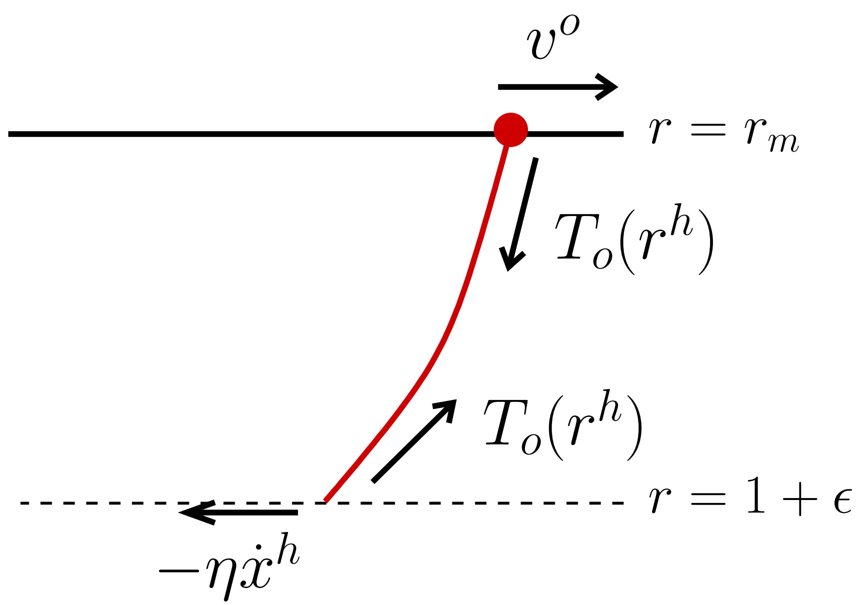
Classical Boundary Conditions(No fluctuations)

Herzog et al; DT, Casalderrey-Solana; Gubser

- Classical Boundary condition

$$\overbrace{T_o(r_h)\partial_r x(t,r_h)}^{\text{Tension}} = \overbrace{\eta\,\dot{x}^h(t)}^{\text{Drag}} \qquad \underbrace{T_o(r_h)}_{\text{Tension}} \equiv \frac{\sqrt{\lambda}}{2\pi} g_{xx}(r_h)\sqrt{h}h^{rr}$$

- Tension is opposed by drag and horizon motion is over-damped



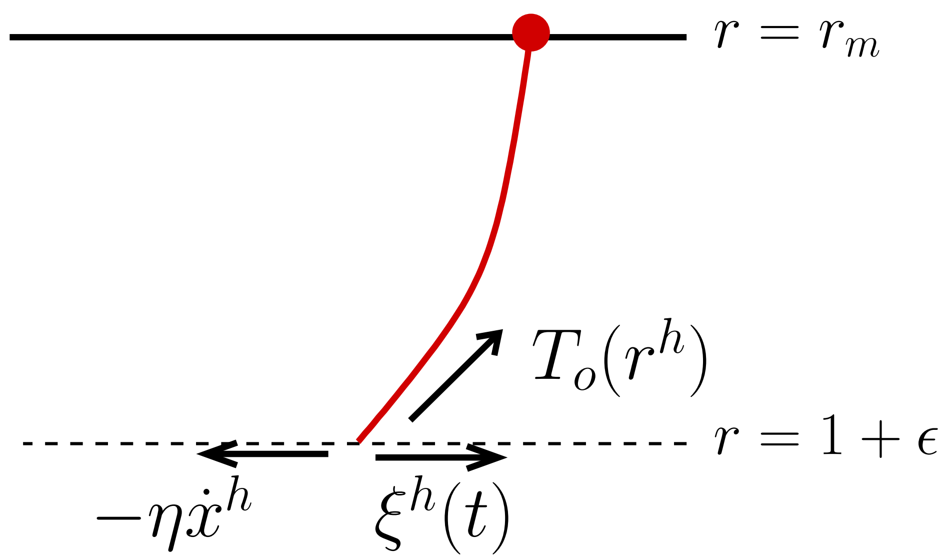
$$F_{\text{quark}} = -\eta v^o$$

Boundary conditions with fluctuations

- Still overdamped horizon motion with a random horizon force

$$\overbrace{T_o(r_h)\partial_r x(t,r_h)}^{\text{Tension}} + \overbrace{\xi^h(t)}^{\text{Random force}} = \overbrace{\eta\,\dot{x}^h(t)}^{\text{Drag}}$$

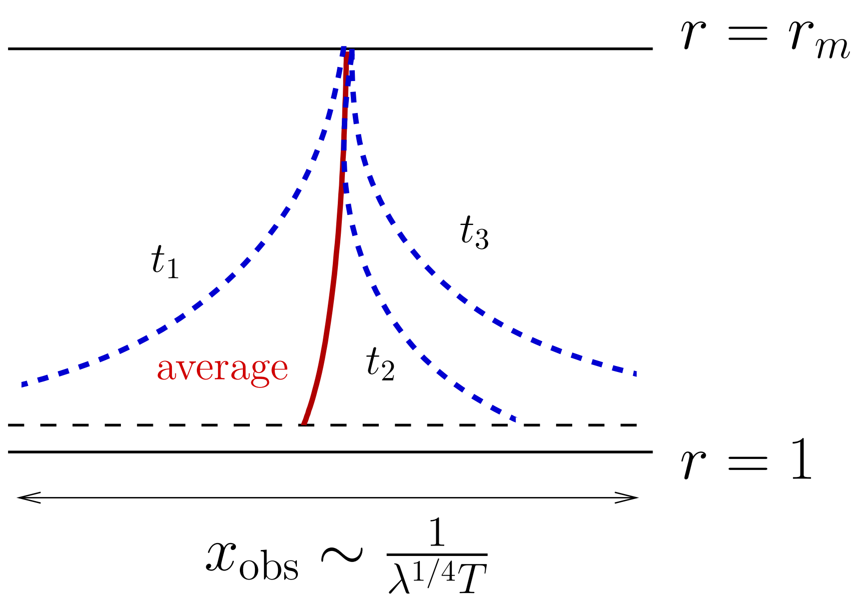
- Picture



- Random horizon force satisfies a horizon fluctuation dissipation theorem

$$\left\langle \xi^h(t)\xi^h(t') \right\rangle = 2T\eta\delta(t-t')$$

What happens when noise is included:



1. Every step t_1, t_2, t_3 fluctuates to a new trailing string – $\longrightarrow$ random force
2. The *average* of the trailing strings gives the drag – average string $\rightarrow$ drag
3. Boundary endpoint satisfies Langevin equation of motion

$$M \frac{d^2 \mathbf{x}^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}} \, \dot{\mathbf{x}}^o + \underbrace{\xi}_{\text{Noise}}$$

Non-equilibrium

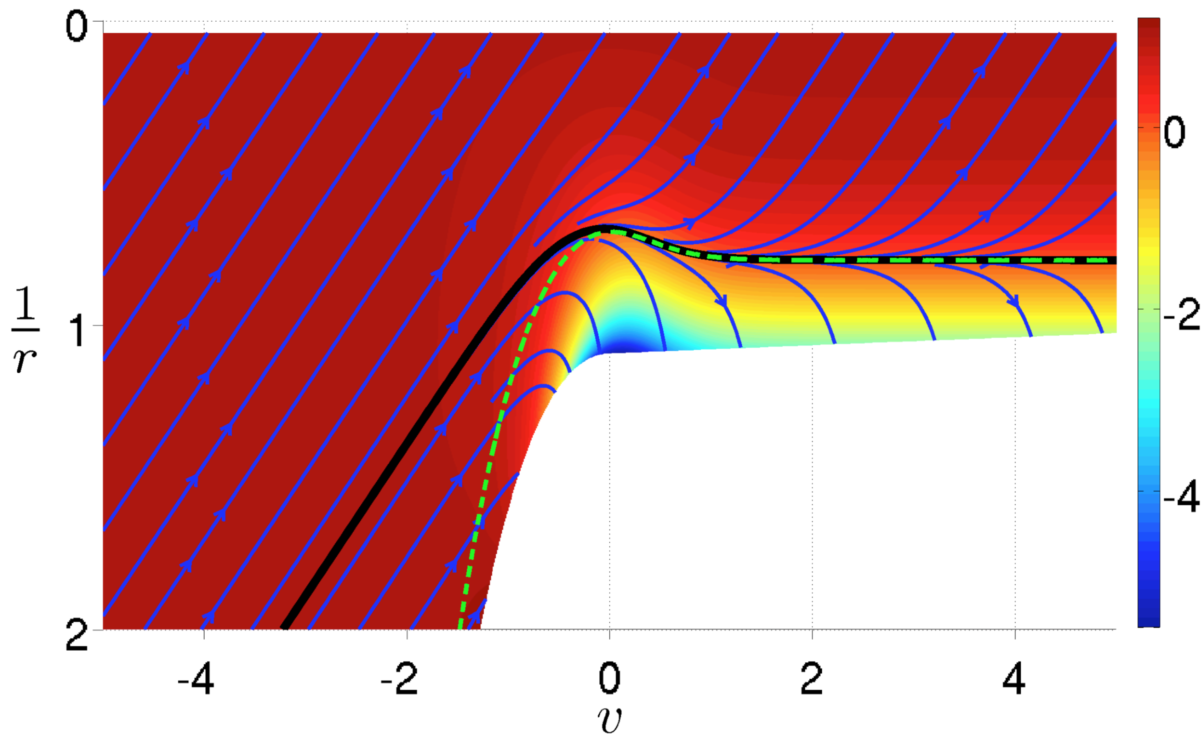
Non-equilibrium: (Chesler-Yaffe)

- God&Devil turn on a strong gravitational pulse in "Our" world

$$ds^2 = -dt^2 + e^{B_o(t)} d\mathbf{x}_\perp^2 + e^{-2B_o(t)} dx_\parallel^2$$

Non-equilibrium

Chesler Yaffe



- Surface Properties

$$\underbrace{2\pi T_{\rm eff}(v)}_{\text{Lyapunov exponent}} \equiv \overbrace{\frac{1}{2}\frac{\partial A(r,v)}{\partial r}}^{\rm Metric-coeff}\Bigg|_{r=r_h(v)} \qquad \underbrace{\eta(v) \equiv \frac{\sqrt{\lambda}}{2\pi} g_{xx}(r_h(v),v)}_{\text{coupling to horizon metric}}\ .$$

Results

1. Anti-commutator

$$G_{rr}(1|2) = \int \mathrm{d}v_{1h}\mathrm{d}v_{2h}\, G_R(1|1_h)\; G_R(2|2_h)\; G^h_{rr}(1_h|2_h)\,.$$

where

$$G^h_{rr}(v_1|v_2) = -\frac{\sqrt{\eta(v_1)\eta(v_2)}}{\pi}\partial_{v_1}\partial_{v_2}\log\left|1-e^{-\int_{v_1}^{v_2} 2\pi T_{\mathrm{eff}}(v')dv'}\right|.$$

2. Commutator – initial conditions from canonical commutation relations

$$\rho(1|2) = \int \mathrm{d}v_{1h}\mathrm{d}v_{2h}\, G_R(1|1_h)\, G_R(2|2_h) \underbrace{\rho^h(v_{1h}|v_{2h})}_{\text{from commutation rel.}}\,,$$

where

$$\rho^h(v_{1h}|v_{2h}) = 2\sqrt{\eta(v_{1h})\eta(v_{2h})}\, i\delta'(v_1 - v_2)\,.$$

Equilibration

- Take Wigner transforms of horizon correlator. Dissipation is local

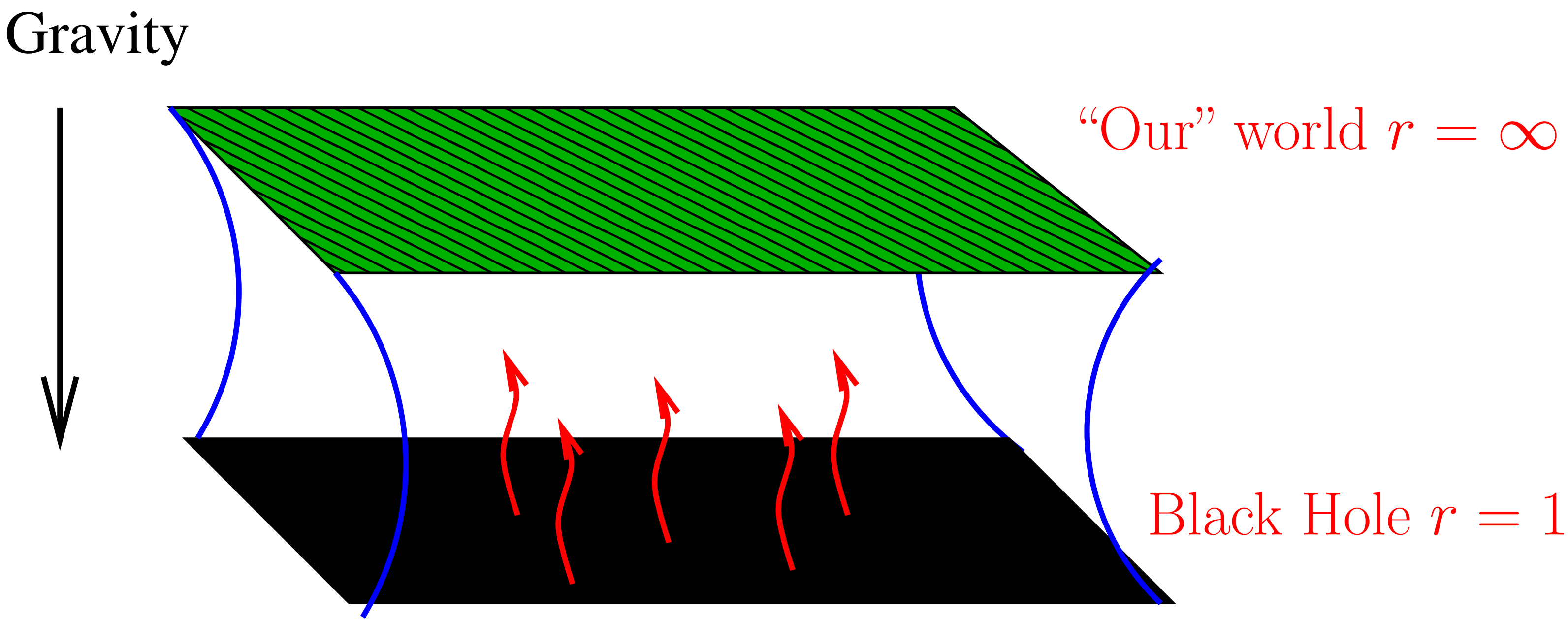
$$\rho^h(\bar{v},\omega) = \int_{-\infty}^{\infty} \mathrm{d}(v_1 - v_2) e^{+i\omega(v_1-v_2)} \rho^h(v_1,v_2)\,,$$
$$= 2\eta(\bar{v})\,\omega\,.$$

- For a non equilibrium timescale τ we have

$$G^h_{rr}(\bar{v},\omega) \simeq \left(\frac{1}{2} + n(\omega)\right) \rho^h(\bar{v},\omega) \, + \, O\left(\frac{1}{\tau^2\omega^2}\right)\,.$$

High frequencies are born into equilibrium on the event horizon

Not conclusions, but answer:



Gravity pulls down, but quantum fields fluctuate, reaching equilibrium