Strong magnetic fields in lattice gluodynamics

P.V.Buividovich, M.N.Chernodub,T.K. Kalaydzhyan, D.E. Kharzeev, E.V.Luschevskaya, O.V. Teryaev, M.I. Polikarpov

ITEP Lattice

arXiv:1011.3001, arXiv:1011.3795, arXiv:1003.2180, arXiv:0910.4682, arXiv:0909.2350, arXiv:0909.1808, arXiv:0907.0494, arXiv:0906.0488, arXiv:0812.1740

Quarks, Gluons, and Hadronic Matter under Extreme Conditions

15th of March 2011 - 18th of March 2011

Schlosshotel Rheinfels, St. Goar, Germany

Strong magnetic fields in lattice gluodynamics

P.V.Buividovich, M.N.Chernodub,T.K. Kalaydzhyan, D.E. Kharzeev, E.V.Luschevskaya, O.V. Teryaev, M.I. Polikarpov

arXiv:1011.3001, arXiv:1011.3795, arXiv:1003.2180, arXiv:0910.4682, arXiv:0909.2350, arXiv:0909.1808, arXiv:0907.0494, arXiv:0906.0488, arXiv:0812.1740

Quarks, Gluons, and Hadronic Matter under Extreme Conditions

15th of March 2011 - 18th of March 2011

Schlosshotel Rheinfels, St. Goar, Germany

Lattice simulations with magnetic fields, status

1. Chiral Magnetic Effect

1.1 CME on the lattice

1.2 Vacuum conductivity induced by magnetic field

1.3 Quark mass dependence of CME (+ talk of P. Buividovich)

1.4 Dilepton emission rate (+ talk of P. Buividovich)

2. Other effects induced by magnetic field

2.1 Chiral symmetry breaking

2.2 Magnetization of the vacuum

2.3 Electric dipole moment of quark along the direction of the magnetic field

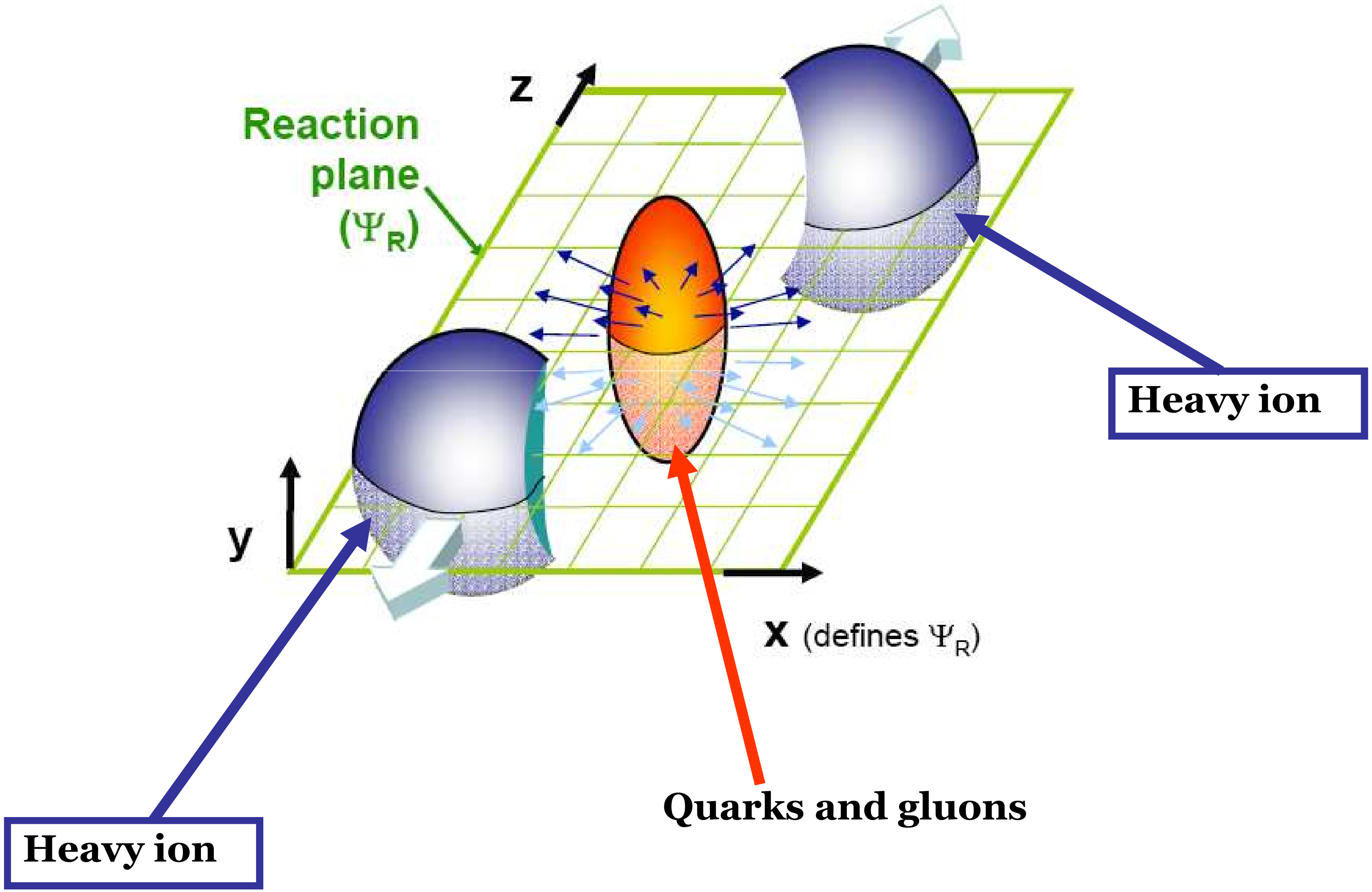
Lattice simulations with magnetic fields, future

1. **Calculations of "OLD" quantities in *SU(3), SU(2)* with dynamical quarks, QCD. Decreasing systematic errors in *SU(2)* calculations**

2. **Calculation of "NEW" physical quantities**

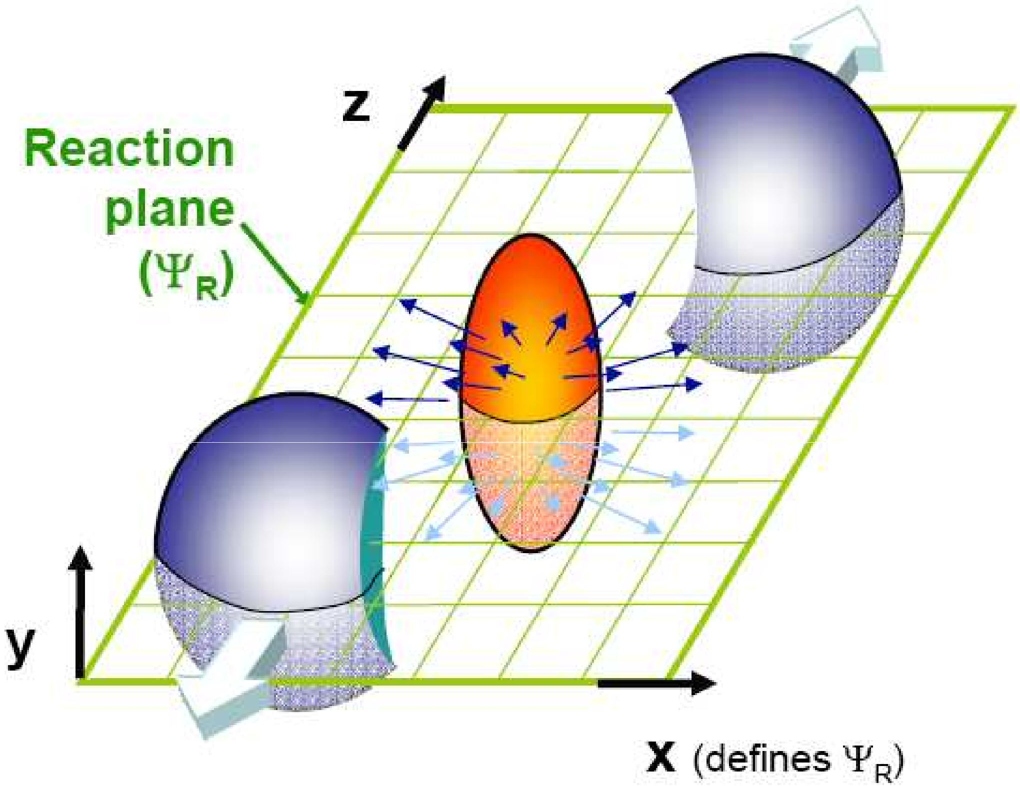
Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran ’07-’08]



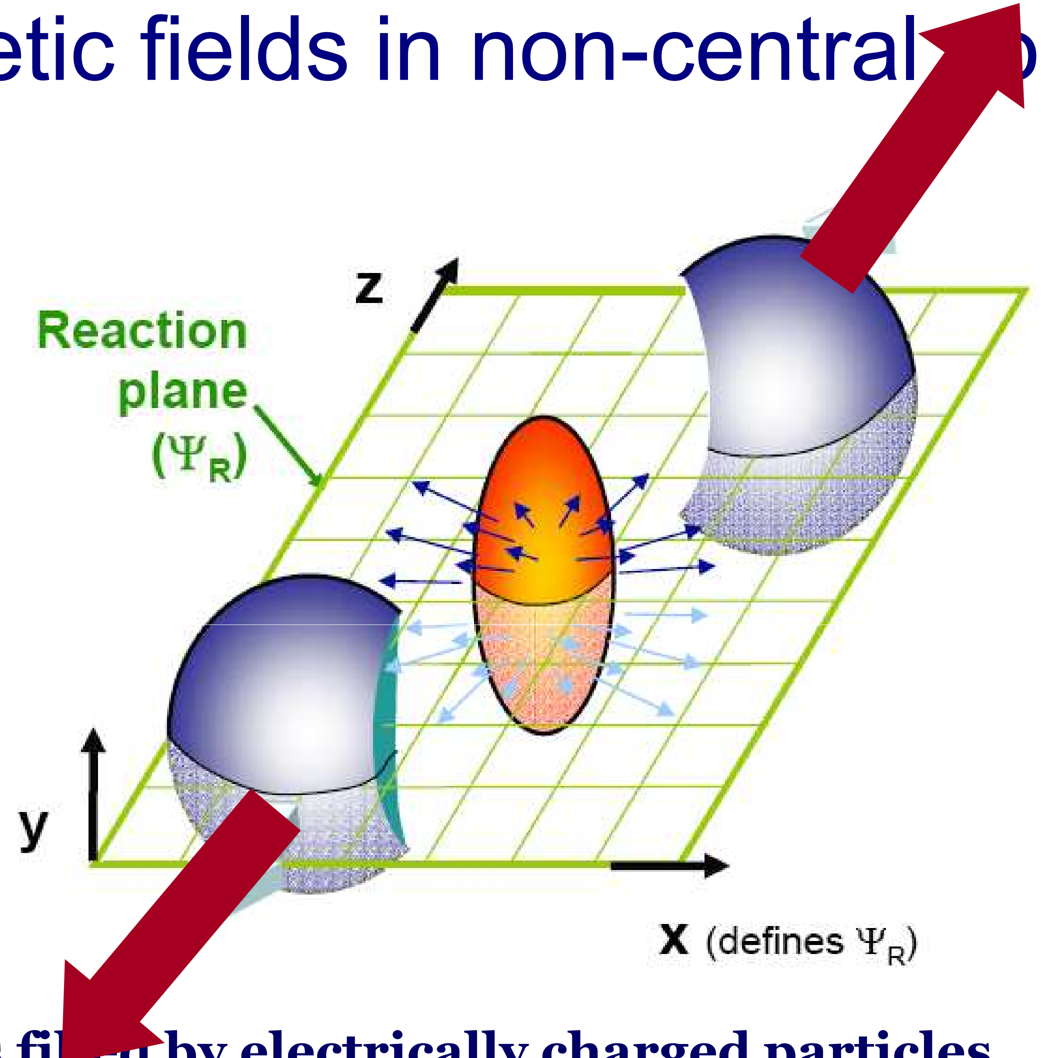
Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran '07-'08]



[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008), URL http://arxiv.org/abs/0808.3382.
[2] D. Kharzeev, R. D. Pisarski, and M. H. G.Tytgat, Phys. Rev. Lett. 81, 512 (1998), URL http://arxiv.org/abs/hep-ph/9804221.
[3] D. Kharzeev, Phys. Lett. B 633, 260 (2006), URL http://arxiv.org/abs/hep-ph/0406125.
[4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008), URL http://arxiv.org/abs/0711.0950.

Magnetic fields in non-central collisions



Charge is large
Velosity is high

Thus we have two very big currents

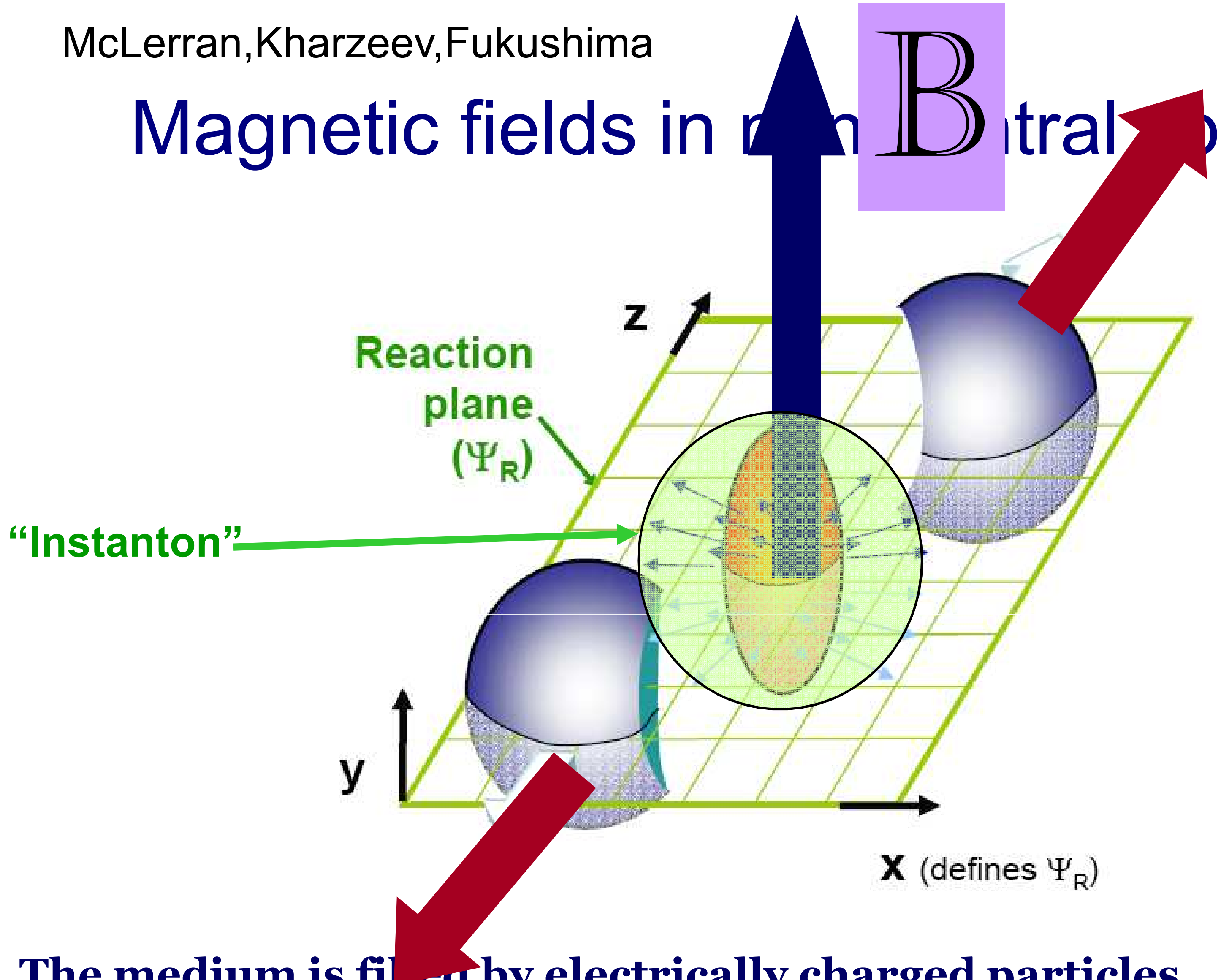
The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

McLerran,Kharzeev,Fukushima

Magnetic fields in non-central collisions



Two very big currents produce a very *big magnetic field*

The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

In heavy ion collisions magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda^2_{QCD}$$

Magnetic forces are of the order of strong interaction forces

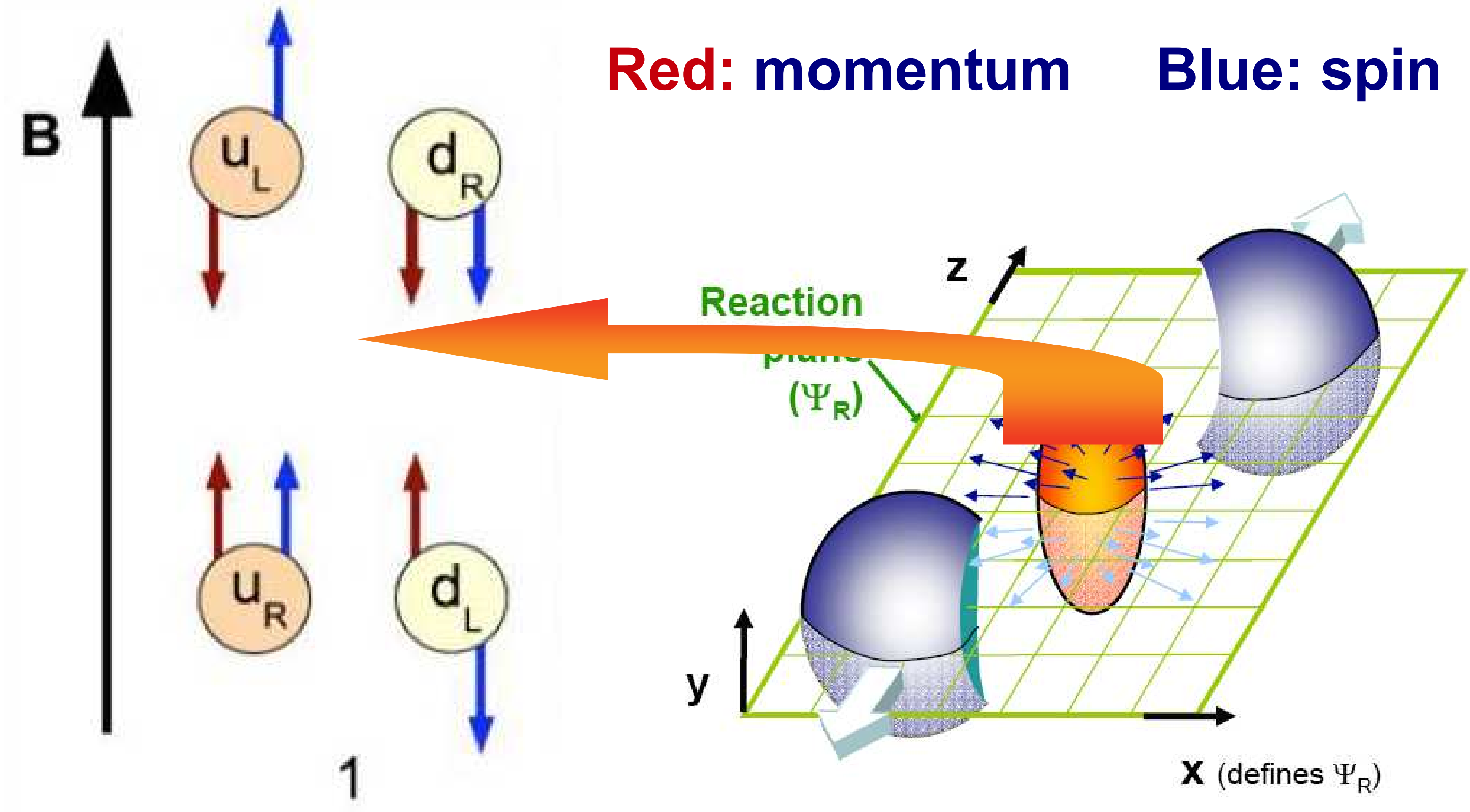
$$eB \approx \Lambda^2_{QCD}$$

We expect the influence of magnetic field on strong interaction physics

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

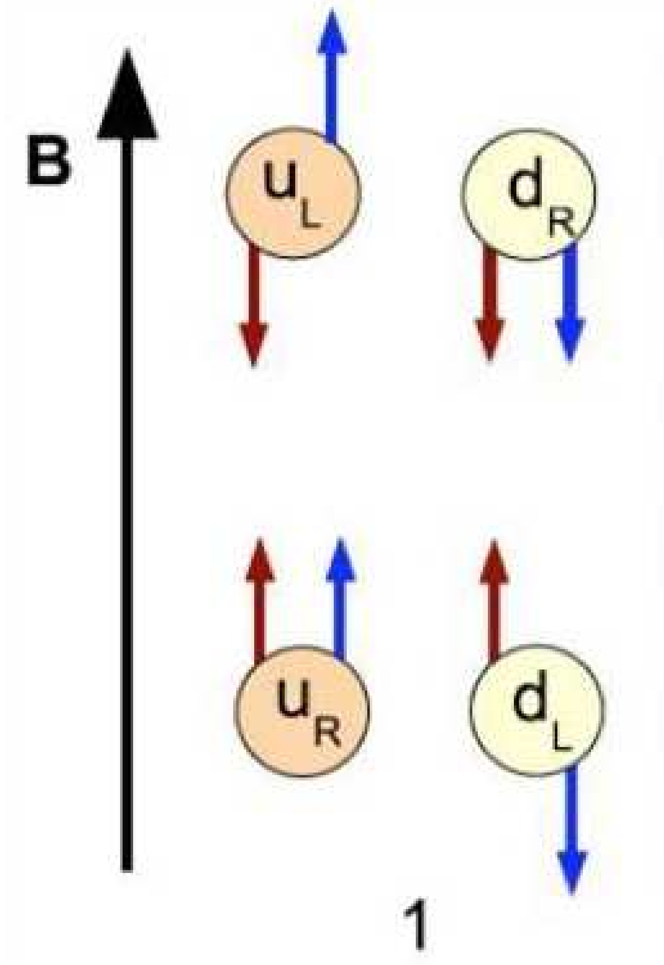
1. Massless quarks in external magnetic field.

Red: momentum **Blue: spin**



Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

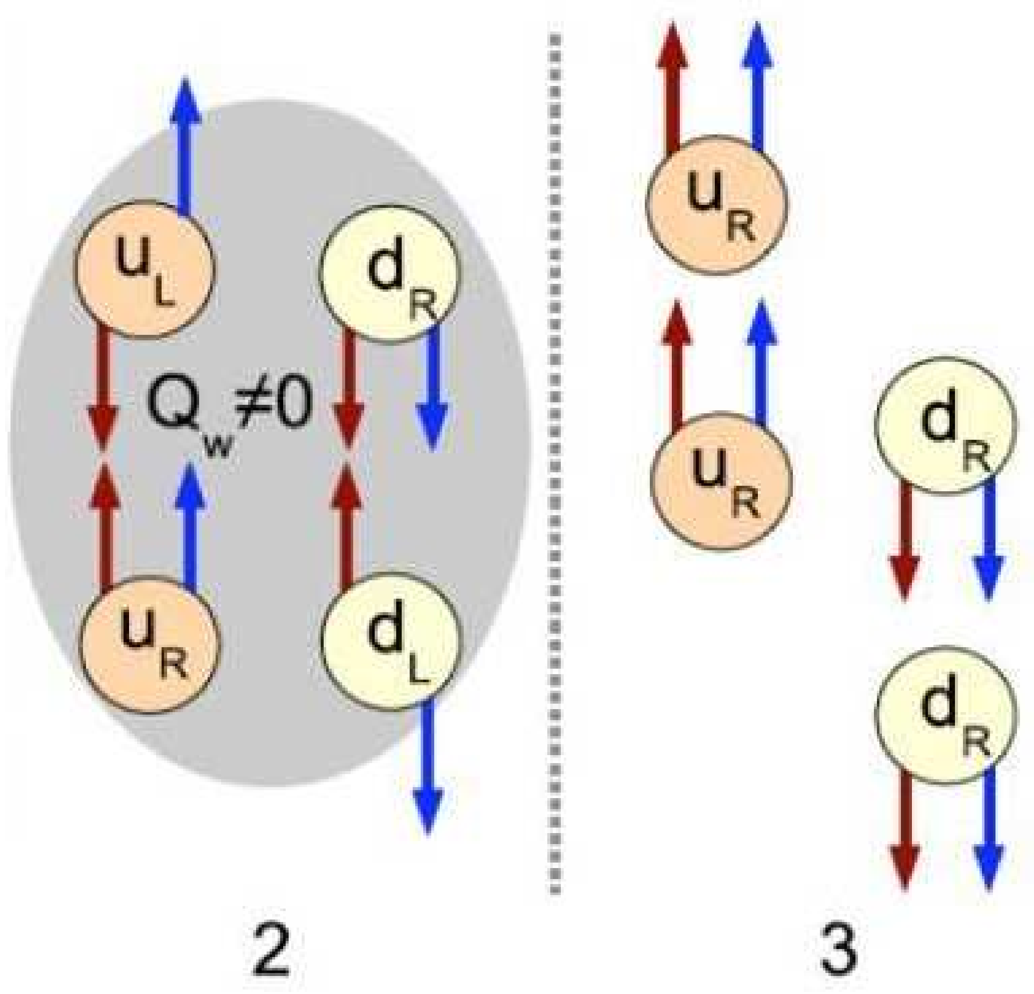
1. Massless quarks in external magnetic field.



Red: momentum **Blue: spin**

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

2. Quarks in the instatnton field.



Red: momentum
Blue: spin

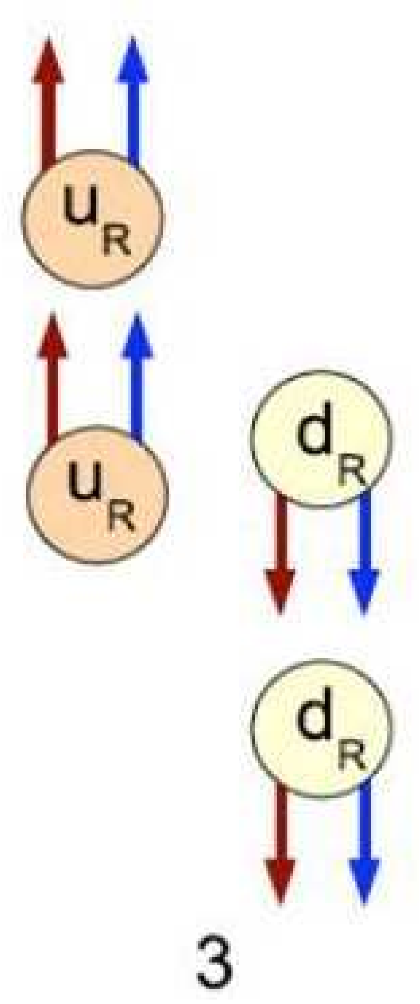
Effect of topology:

$u_L \rightarrow u_R$

$d_L \rightarrow d_R$

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

3. Electric current along magnetic field



Red: momentum
Blue: spin

Effect of topology:

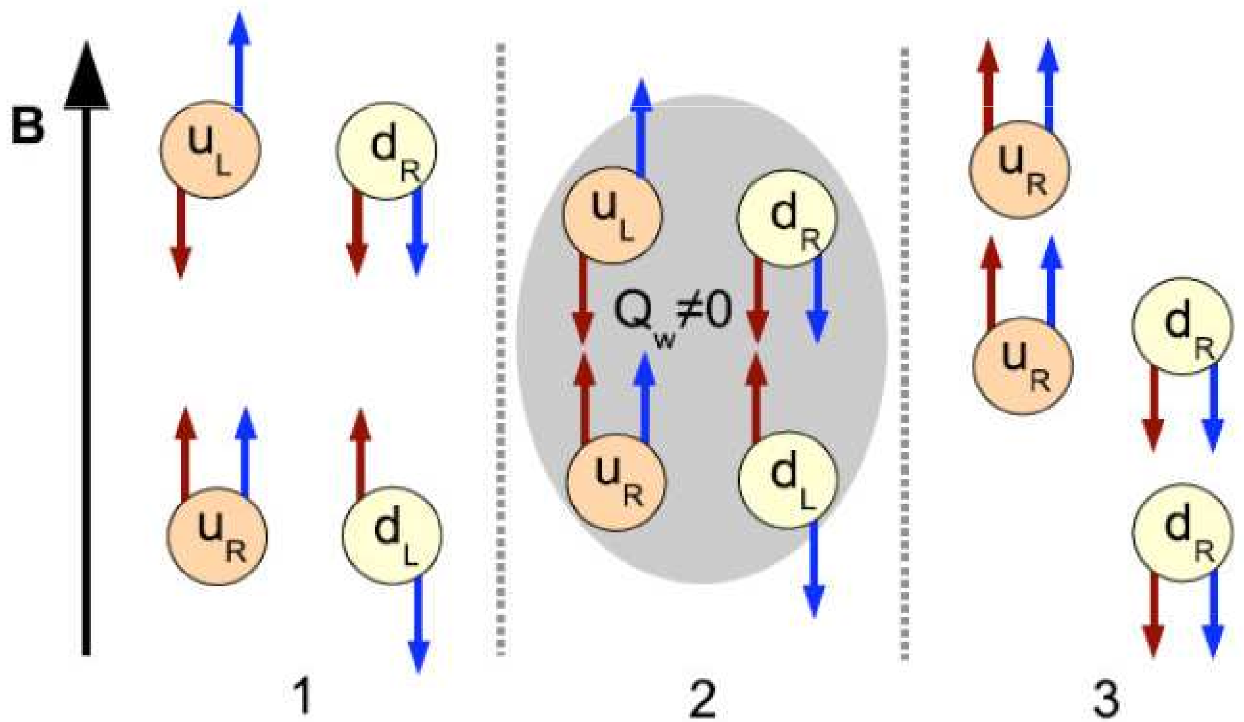
$u_L \rightarrow u_R$

$d_L \rightarrow d_R$

u-quark: q=+2/3
d-quark: q= - 1/3

Chiral Magnetic Effect by **Fukushima, Kharzeev, Warringa, McLerran**

3. Electric current is along magnetic field In the *instanton* field

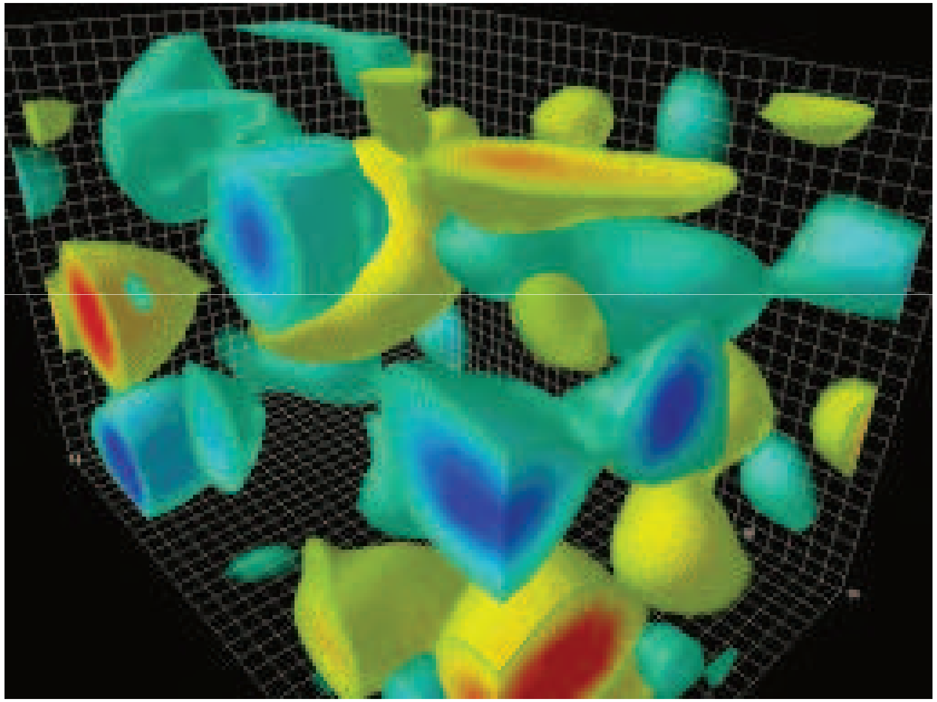


Red: momentum
Blue: spin

Effect of topology:
$u_L \rightarrow u_R$
$d_L \rightarrow d_R$

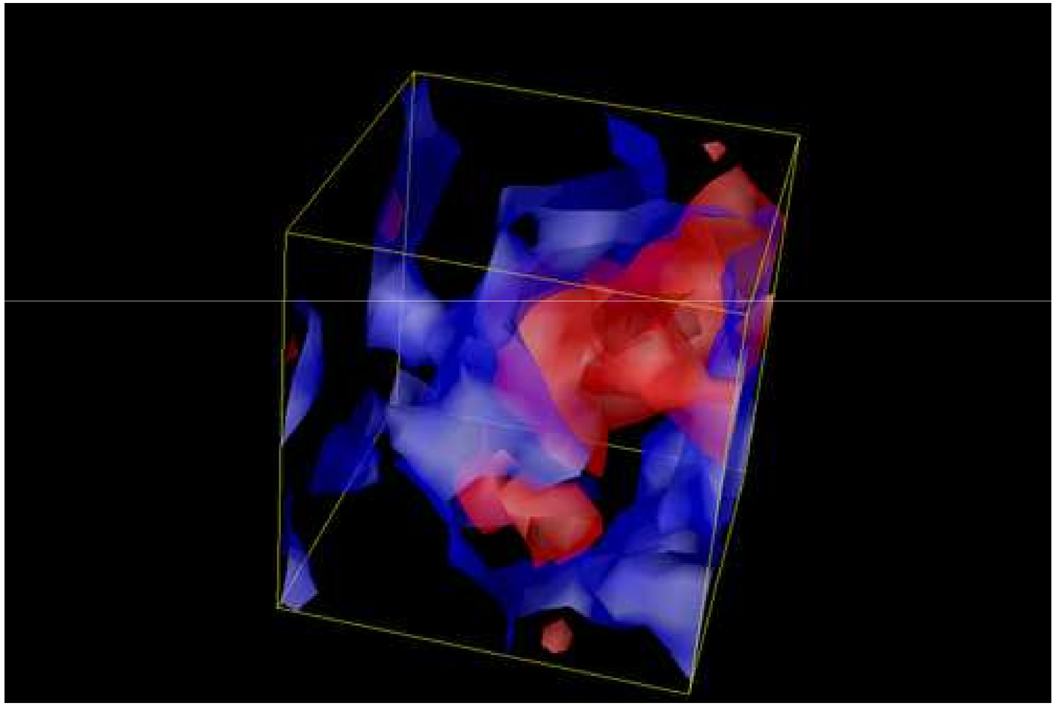
u-quark: q=+2/3
d-quark: q= - 1/3

3D time slices of topological charge density, lattice calculations



D. Leinweber

Topological charge density after vacuum cooling



P.V.Buividovich,
T.K. Kalaydzhyan, M.I. Polikarpov

Fractal topological charge density without vacuum cooling

Magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda^2_{QCD}$$

We expect the influence of magnetic field on strong interaction physics

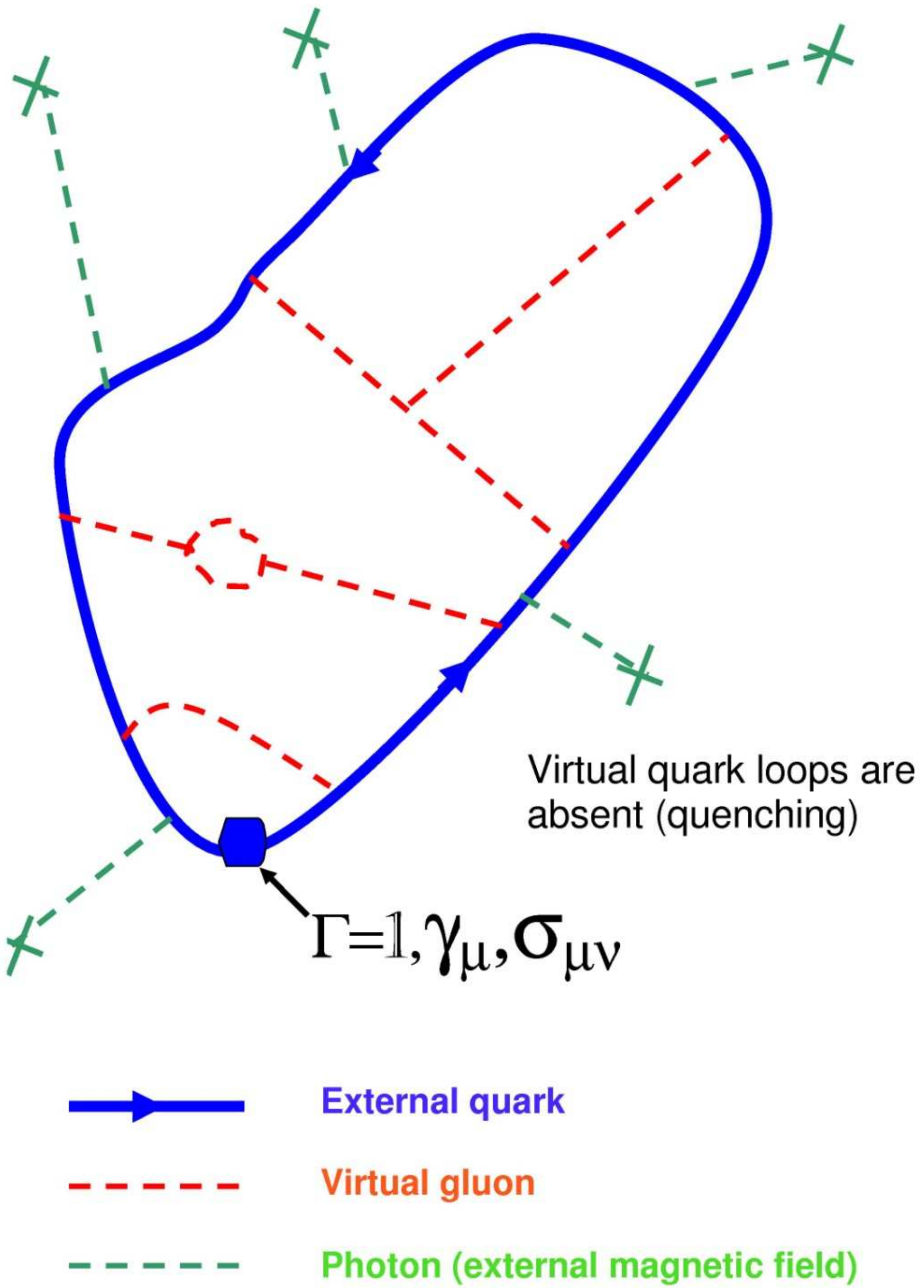
The effects are nonperturbative,

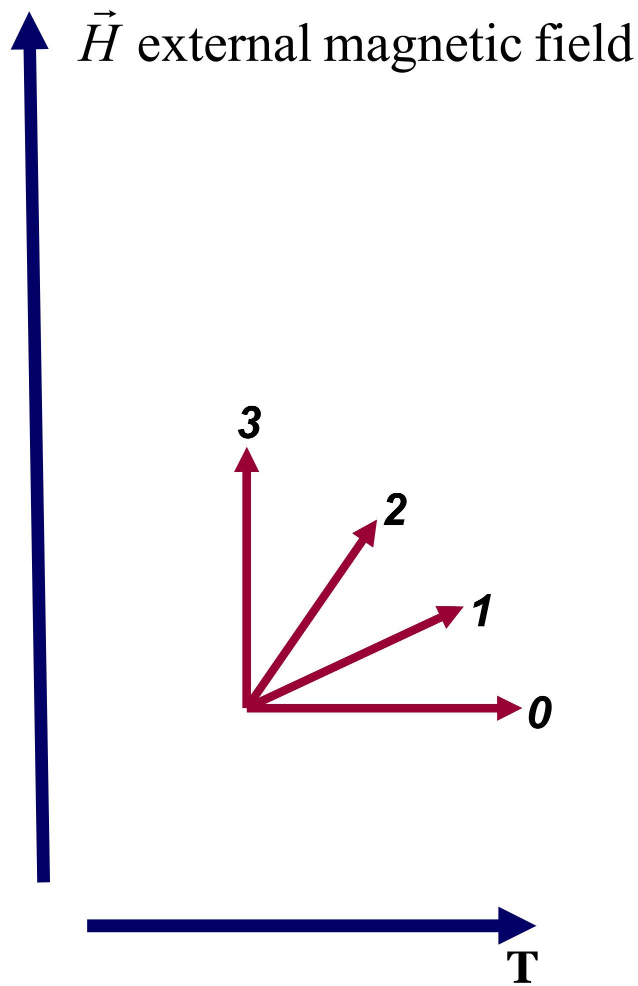
and we use

Lattice Calculations

We calculate $<\overline{\psi}\,\Gamma\,\psi>;\ \Gamma = 1, \gamma_\mu, \sigma_{\mu\nu}$

in the external magnetic field and in the presence of the vacuum gluon fields We consider SU(2) gauge fields and quenched approximation



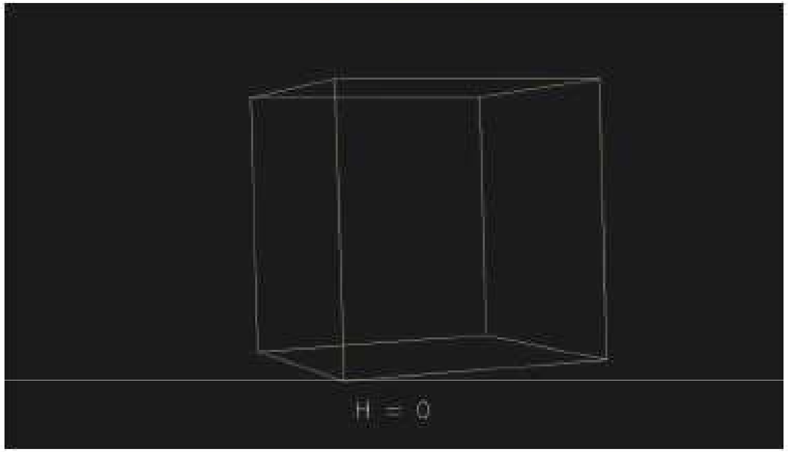


Quenched vacuum, overlap Dirac operator, external magnetic field

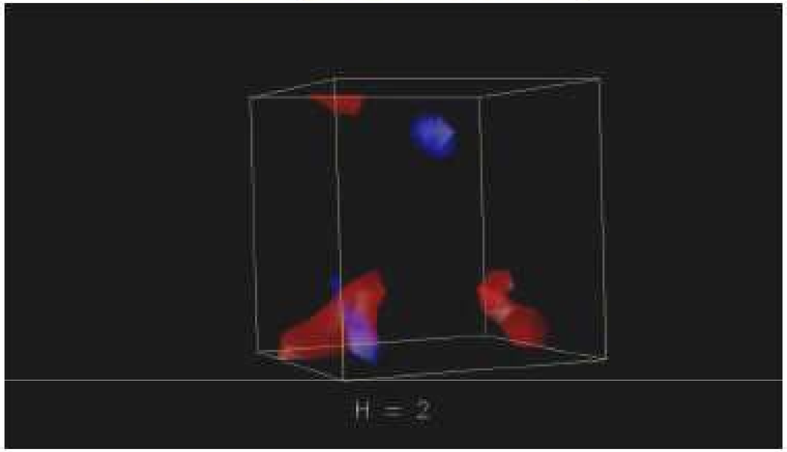
$$eB = \frac{2\pi qk}{L^2};\ eB \geq (250\ Mev)^2$$

Density of the electric charge vs. magnetic field, 3D time slices

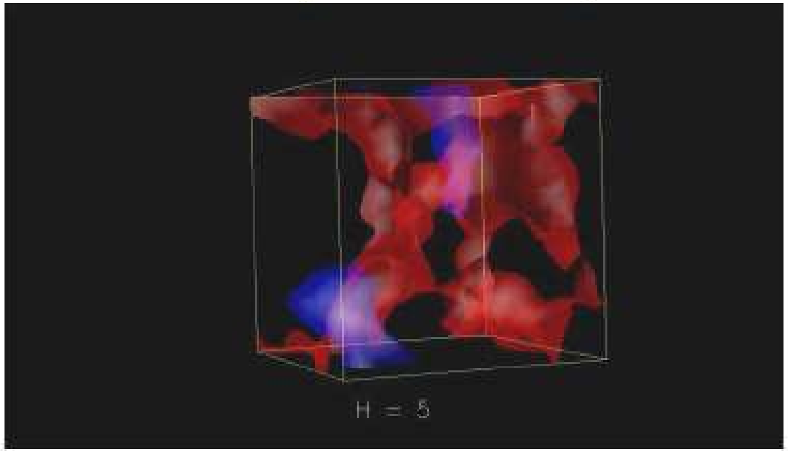
$B = 0$



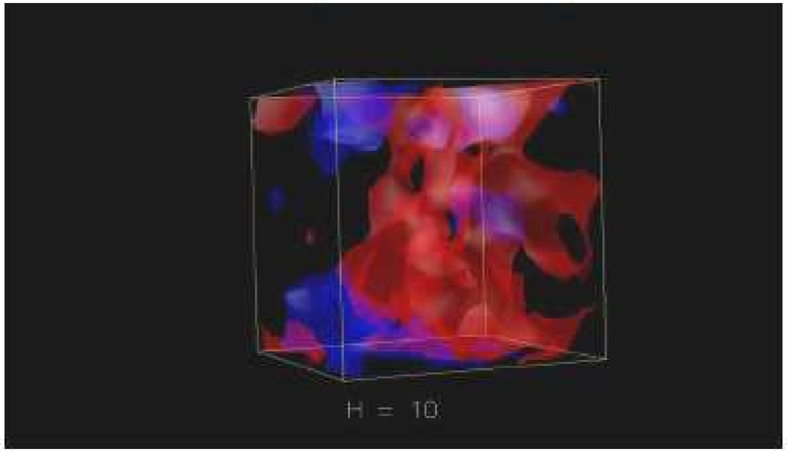
$B = (500\,\mathrm{MeV})^2$



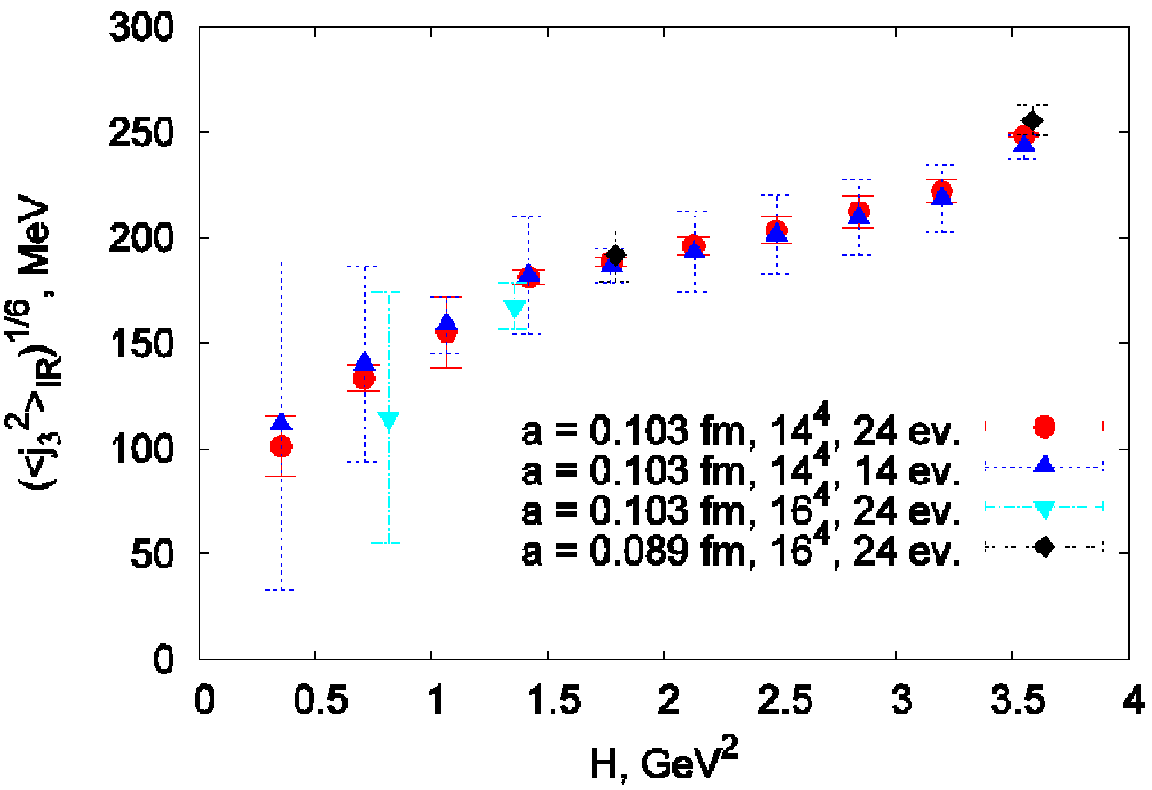
$B = (780\,\mathrm{MeV})^2$



$B = (1.1\,\mathrm{GeV})^2$



1. Chiral Magnetic Effect on the lattice, numerical results T=0



Regularized electric current:

$$< j_3^2 >_{IR} = < j_3^2(H,T) > - < j_3^2(0,0) >, \quad j_3 = \overline{\psi}\gamma_3\psi$$

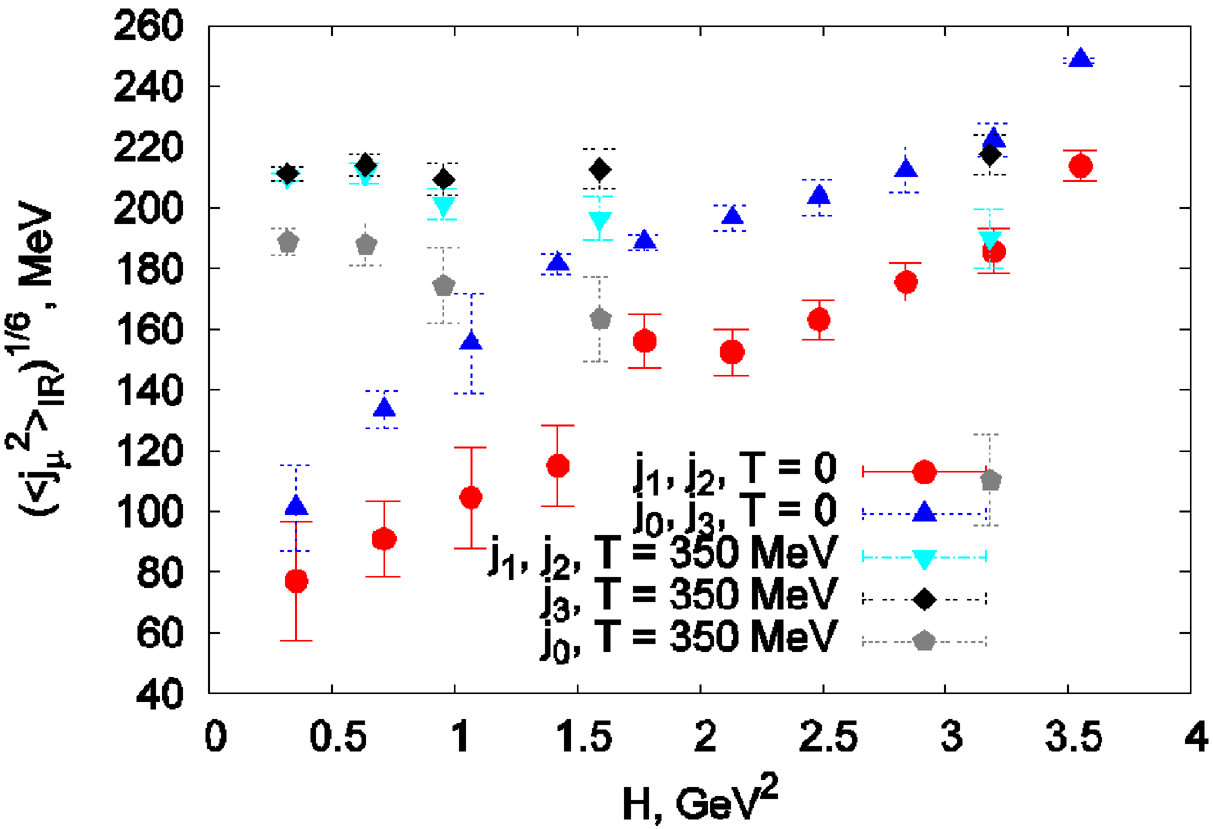
Chiral Magnetic Effect on the lattice, numerical comparison of results nearTc and nearzero

$T=0$

$F_{12}\neq 0$

$<j_1^2>=<j_2^2>$

$<j_3^2>=<j_0^2>$



$T>0$

$F_{12}\neq 0$

$<j_1^2>=<j_2^2>$

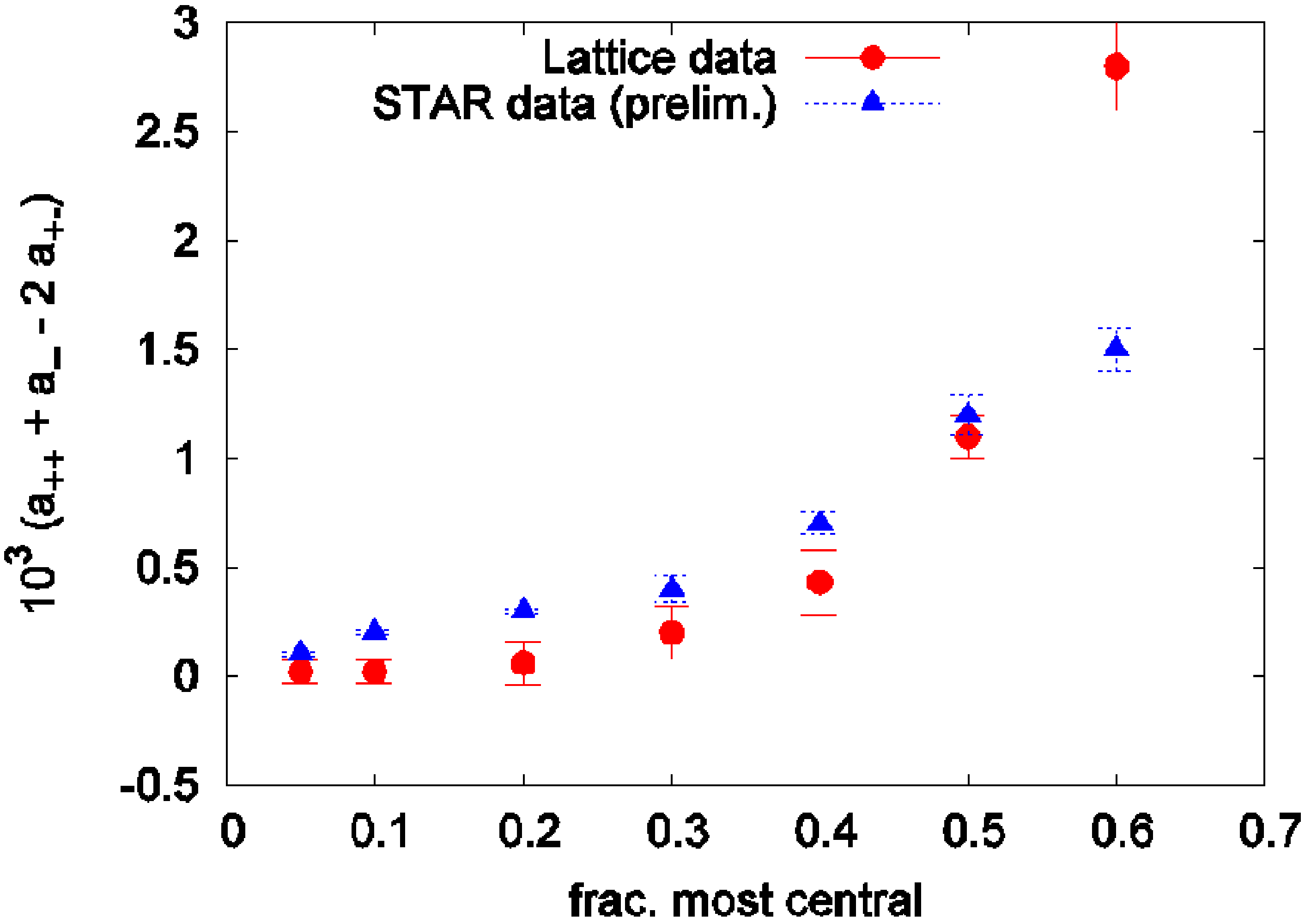
$<j_3^2>\neq<j_0^2>$

Regularized electric current:

$$< j_i^2 >_{IR} = < j_i^2(H,T) > - < j_i^2(0,0) >, \quad j_i = \overline{\psi}\gamma_i\psi$$

Chiral Magnetic Effect,

EXPERIMENT VS LATTICE DATA (Au+Au)



Chiral Magnetic Effect,
EXPERIMENT VS LATTICE DATA

$$a_{ab} = \frac{1}{N_e}\sum_{e=1}^{N_e}\frac{1}{N_a N_b}\sum_{i=1}^{N_a}\sum_{j=1}^{N_b}\cos\left(\phi_{ia}+\phi_{jb}\right)$$

experiment

$$\frac{\langle (\Delta Q)^2 \rangle}{N_q^2} = a_{++} + a_{--} - 2a_{+-}$$

$R \approx 5\ fm$

$\rho \approx 0.2\ fm$

$\tau \approx 1\ fm$

our fit

D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008),

$$= \frac{4\pi\tau^2\rho^2 R^2}{3N_q^2}\left(\langle j_{\parallel}^2 \rangle + 2\langle j_{\perp}^2 \rangle\right)$$

our lattice data at *T=350 Mev*

1.2 Magnetic Field Induced Conductivity of the Vacuum

Qualitative definition of conductivity, σ

$$< j_\mu(x) j_\nu(y) > = \boxed{C} + A \cdot \frac{\exp\{-m|x-y|\}}{r^\alpha}$$

$$j_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$$

$$\boxed{\sigma \propto C}$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

- Conductivity (Kubo formula)

$$G_{ij}(\tau) = \int_0^{+\infty} \frac{dw}{2\pi} K(w,\tau)\, \rho_{ij}(w),$$

Maximal entropy method

$$K(w,\tau) = \frac{w}{2T} \frac{\cosh\left(w\left(\tau - \frac{1}{2T}\right)\right)}{\sinh\left(\frac{w}{2T}\right)},$$

$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i\left(\vec{0},0\right) j_j(\vec{x},\tau) \rangle$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$ **- Conductivity (Kubo formula)**

For weak constant *electric* field

$$< j_i > = \sigma_{ik} E_k$$

Magnetic Field Induced Conductivity of the Vacuum
Calculations in SU(2) gluodynamics

$$\langle \bar{q}(x)\gamma_i q(x)\ \bar{q}(y)\gamma_j q(y)\rangle$$

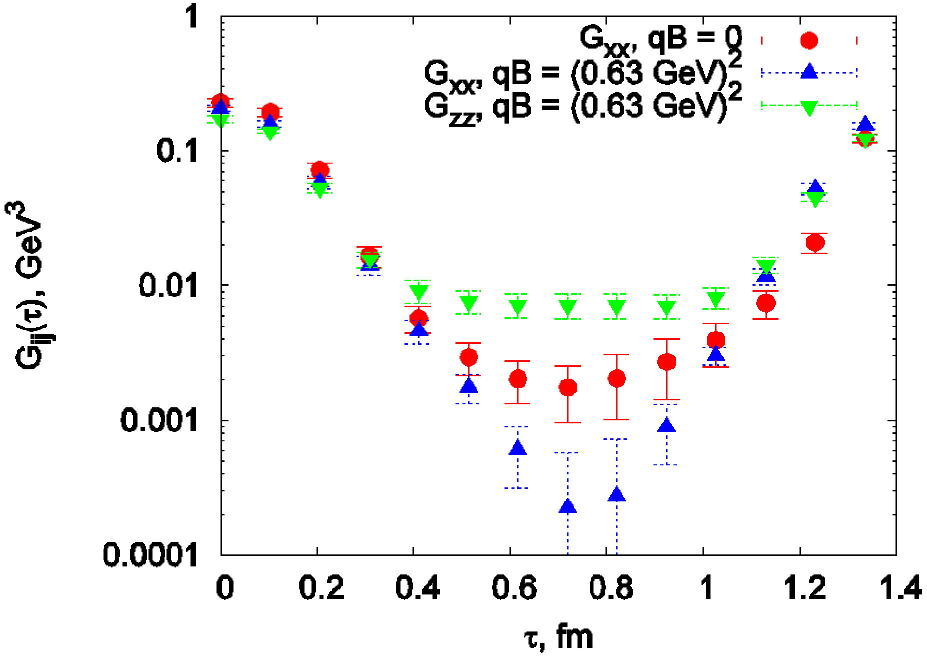
$$= \int \mathcal{D}A_\mu\, e^{-S_{YM}[A_\mu]}\,\mathrm{Tr}\left(\frac{1}{\mathcal{D}+m}\gamma_i\frac{1}{\mathcal{D}+m}\gamma_j\right)$$

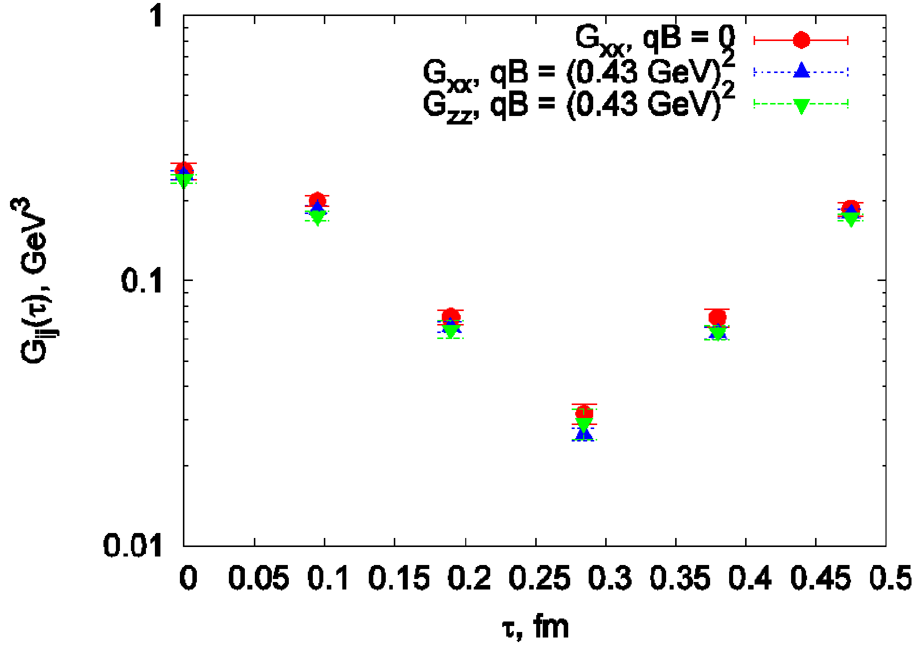
We use overlap operator + Shifted Unitary Minimal Residue Method *(Borici and Allcoci (2006))* to obtain fermion propagator

$$G_{ij}(\tau) = \int d^3\vec{x}\langle j_i\left(\vec{0},0\right) j_j(\vec{x},\tau)\rangle$$

Magnetic Field Induced Conductivity of the Vacuum Calculations in SU(2) gluodynamics

$$G_{ij}(\tau) = \int d^3\vec{x} \langle j_i(\vec{0},0)\, j_j(\vec{x},\tau) \rangle$$



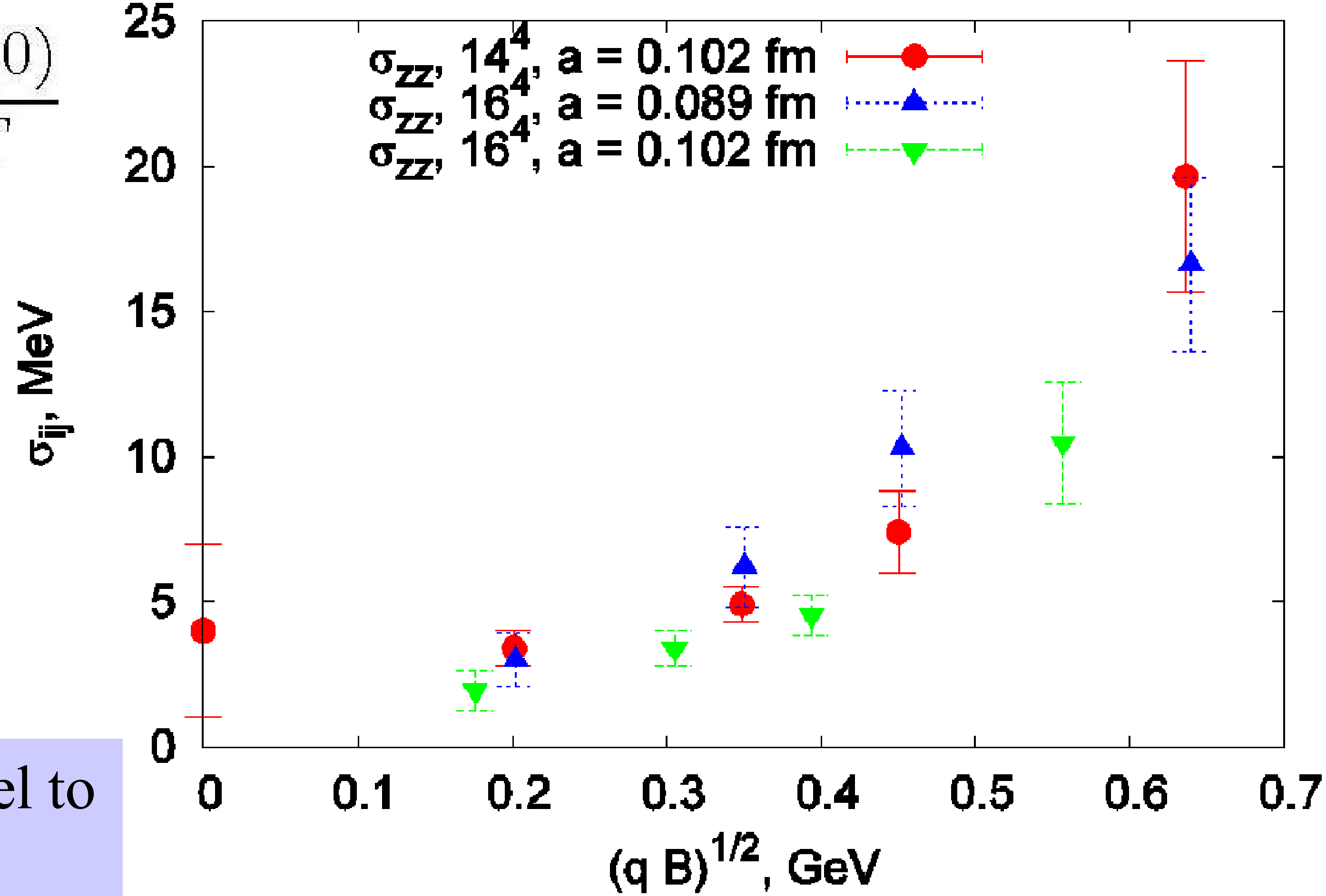


T/Tc = 0.45

T/Tc = 1.12

Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T/Tc=0.45*

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

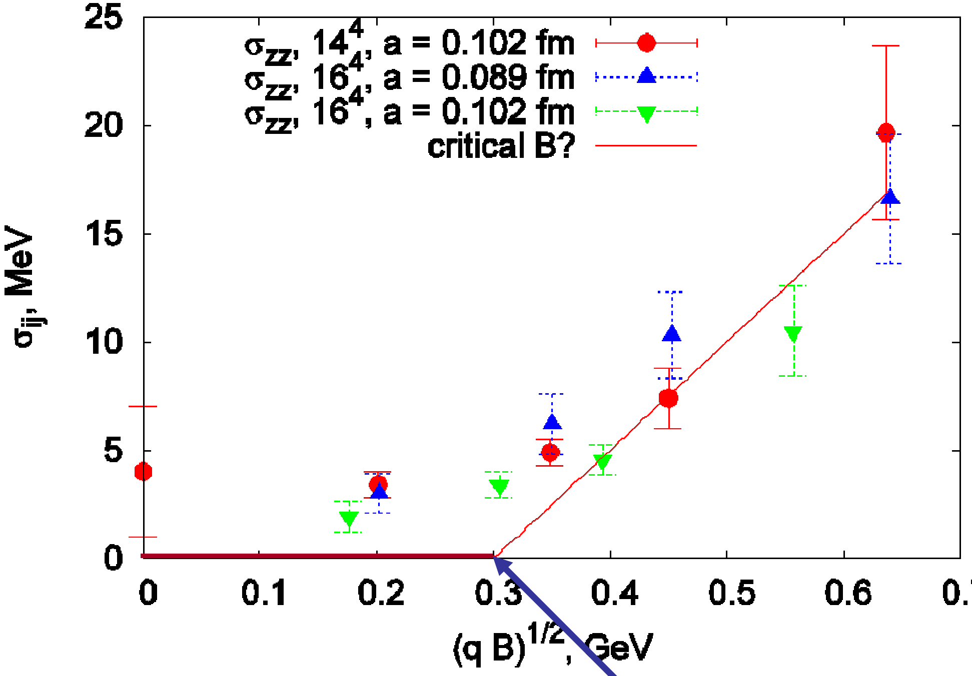


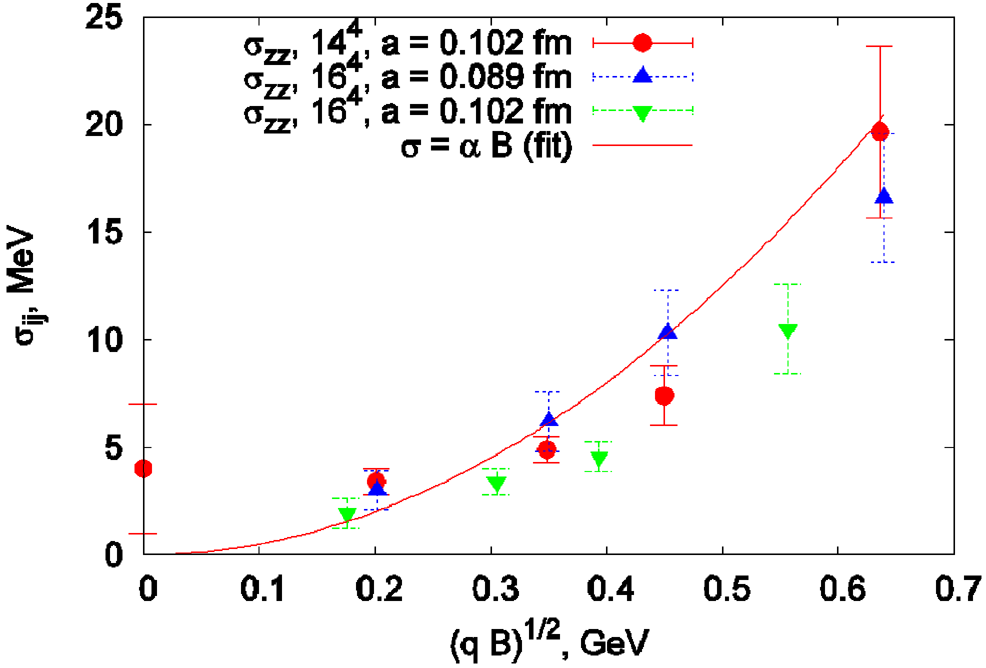
$\vec{B}$ is parallel to $0Z$ axis

Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T/Tc=0.45*

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

At T=0, B=0 vacuum is insulator

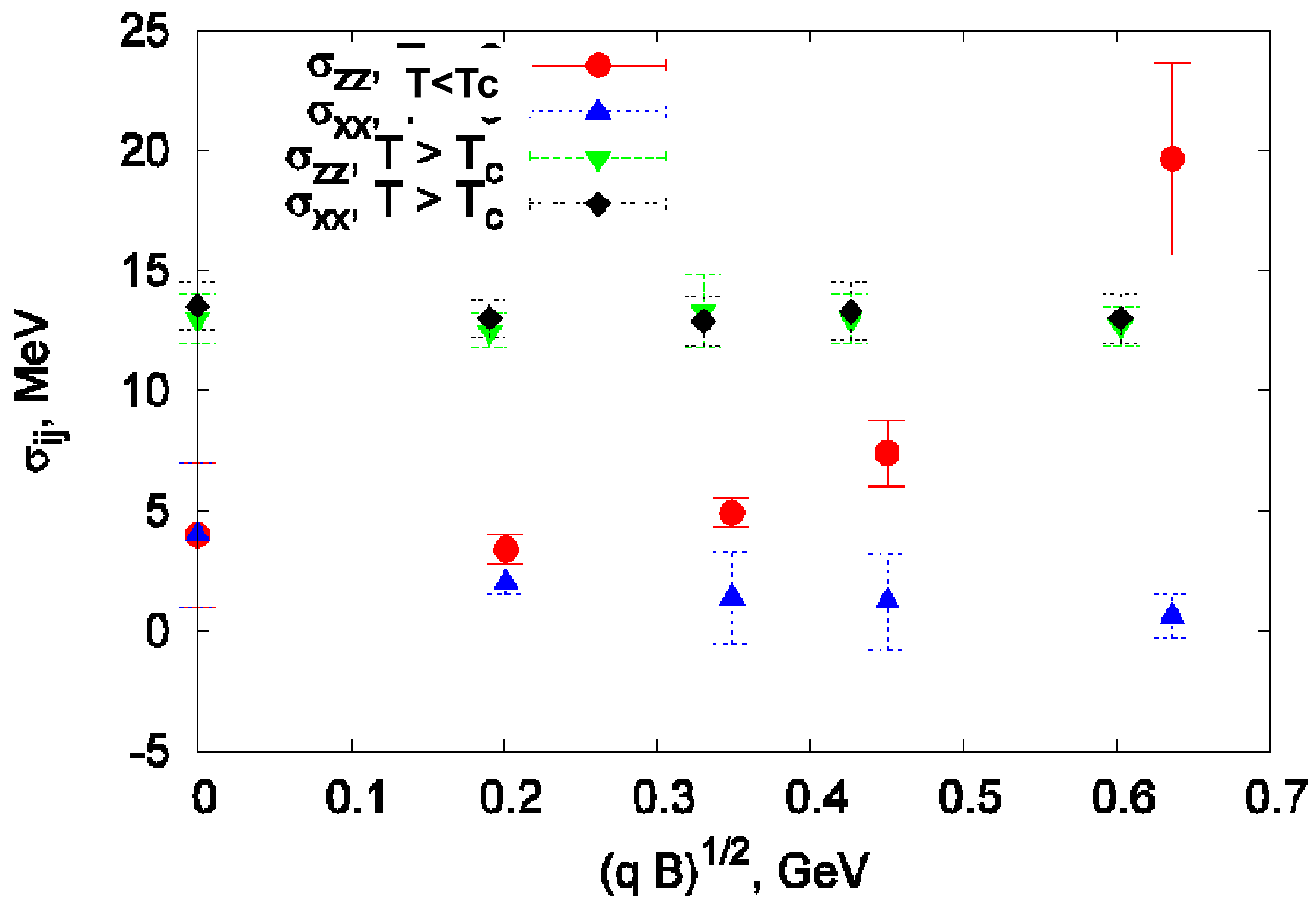




Critical value of magnetic field?

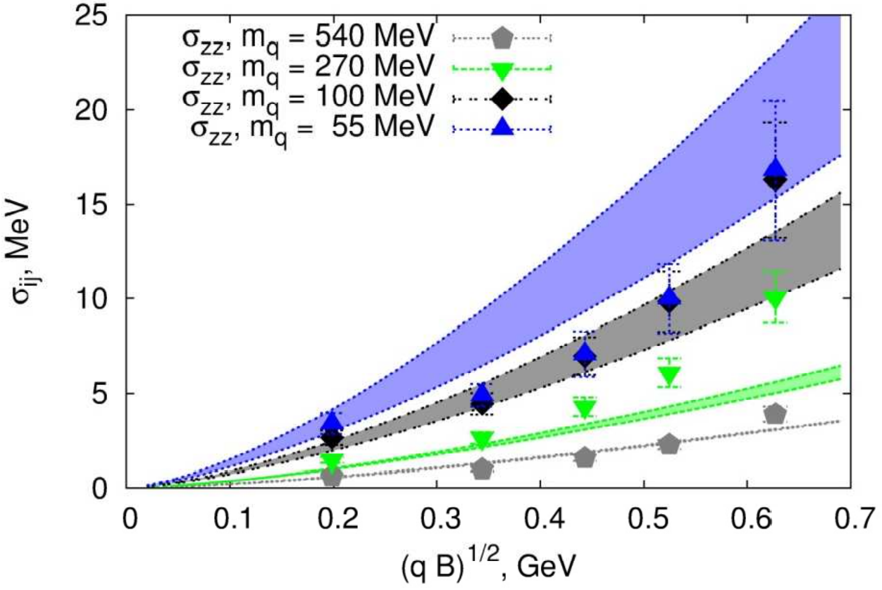
Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T<Tc, T>Tc*

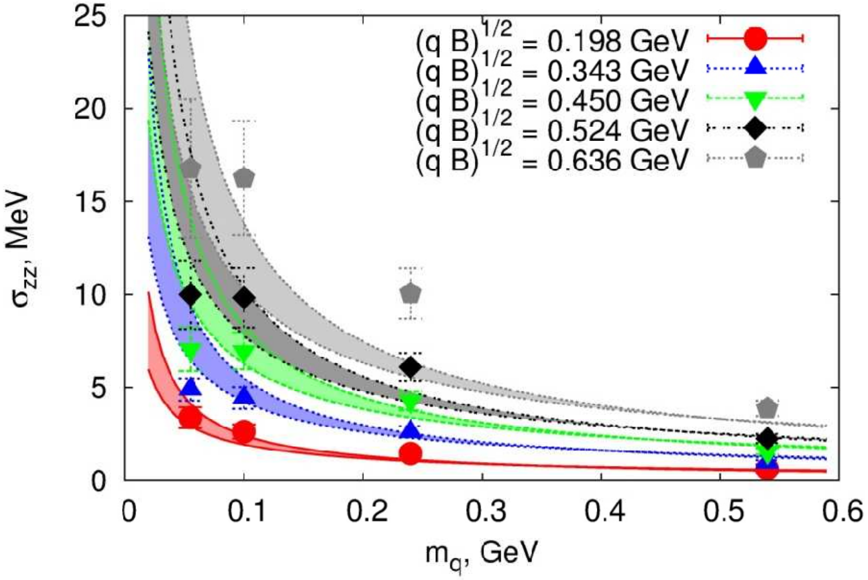
$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$



$\vec{H}$ is parallel to $0Z$ axis

Calculations in SU(2) gluodynamics, conductivity at *T/Tc=0.45*, variation of the quark mass





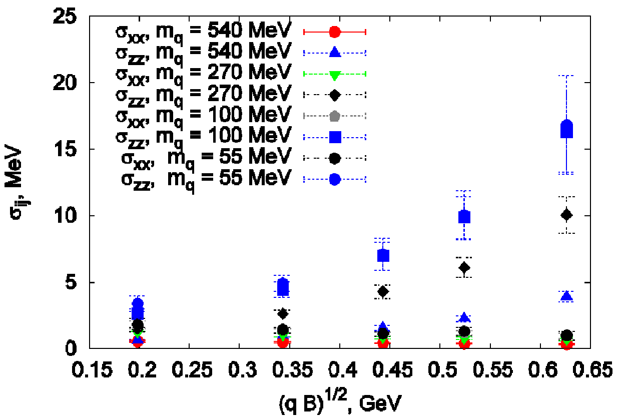
$$\sigma_{zz}(m_q, qB) \sim m_q^{-\alpha} (|qB|)^{\beta}$$

$$\alpha \approx 1$$

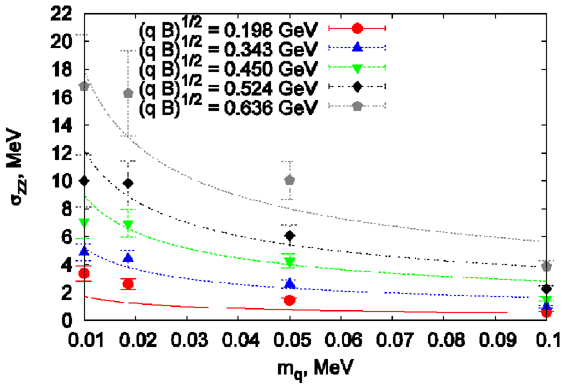
$$\beta \approx 1$$

Calculations in SU(2) gluodynamics, conductivity at *T/Tc=0.45*, variation of the quark mass and magnetic field

$$\sigma_{ij} \propto \frac{B_i B_j}{B m_q} \propto \frac{B_i B_j}{B m_\pi^2}$$



Why?



1.3 Dilepton emission rate

L. D. McLerran and T. Toimela, Phys. Rev. D 31, 545 (1985),
E. L. Bratkovskaya, O. V. Teryaev, and V. D.Toneev, Phys. Lett. B 348, 283 (1995)

$$\frac{R}{V} = -4e^4 \int \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} L^{\mu\nu}(p_1, p_2) \frac{\rho_{\mu\nu}(q)}{q^4}, \quad (7)$$

where p_1 and p_2 are the momenta of the leptons, $q = p_1 + p_2$, m is their mass and $L^{\mu\nu} = \left(\left(p_1 \cdot p_2 + m^2\right)\eta^{\mu\nu} - p_1^\mu p_2^\nu - p_2^\mu p_1^\nu\right)$ is the dilepton tensor. If the electric conductivity is nonzero in the direction of the magnetic field, for sufficiently small p_1, p_2 one has $\rho_{ij}(q) \approx \rho_{ij}(0) \sim \sigma_{ij} \sim B_i B_j / |B|$, and hence

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

$$\frac{R}{V} \sim \int \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} \frac{(p_1 \cdot B)(p_2 \cdot B)}{|B|}. \quad (8)$$

- There should be more soft dileptons in the direction ***perpendicular*** to magnetic field

$$\frac{d\sigma}{dp_1 dp_2} \propto \frac{|B|}{m_\pi} \sin^2\theta$$

θ is the angle between the spatial momentum of the leptons and the magnetic field, in the center of mass of dilepton pair

2. Other effects induced by magnetic field

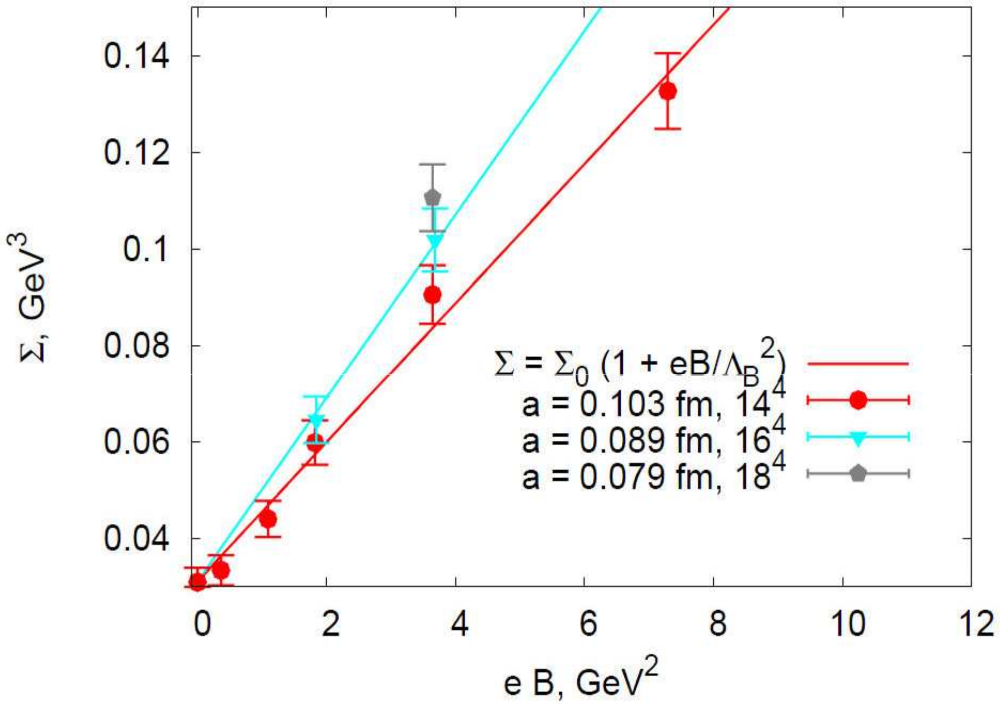
2.1 Chiral symmetry breaking
2.2 Magnetization of the vacuum
2.3 Electric dipole moment of quark along the direction of the magnetic field

3. Chiral condensate in QCD

$$\Sigma = - < \overline{\psi} \psi >$$

$$m_\pi^2 f_\pi^2 = m_q < \overline{\psi} \psi >$$

Chiral condensate vs. field strength, SU(2) gluodynamics



$$\Sigma = \Sigma_0 \left(1 + \frac{eB}{\Lambda_B^2}\right)$$

- Our value for Λ_B:

$$\Lambda_B^{\rm fit} = (1.41 \pm 0.14 \pm 0.20)\,\mathrm{GeV}$$

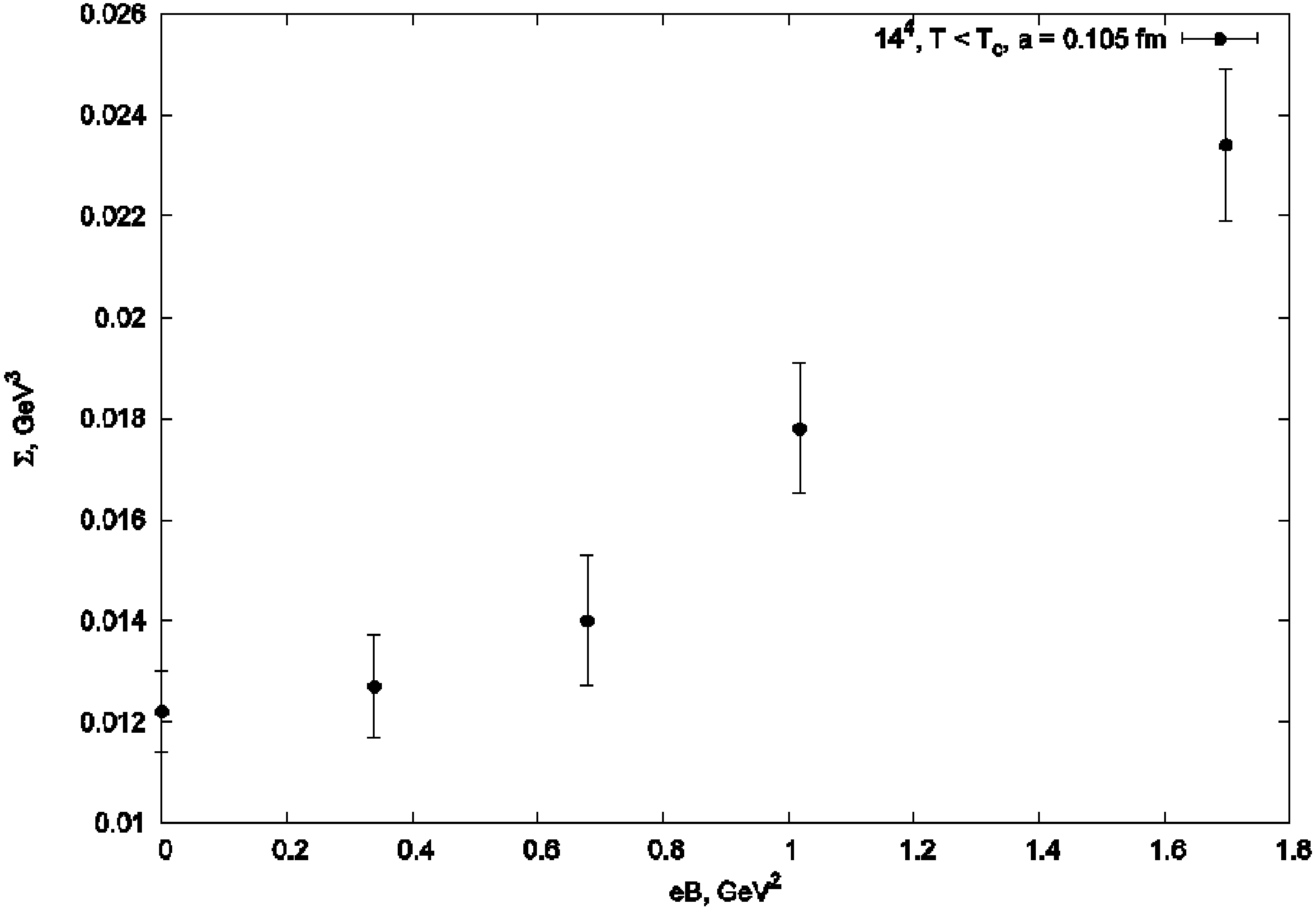
- χPT result:

$$\Lambda_B^{\chi PT} = 1.96\,GeV \quad (F_\pi = 130\,MeV - real\ world)$$
$$\Lambda_B^{\chi PT} = 1.36\,GeV \quad (F_\pi = 90\,MeV - quenched)$$

- Chiral condensate at $B = 0$: $\Sigma_0^{\rm fit} = [(310 \pm 6)\ \mathrm{MeV}]^3$

We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!

Chiral condensate vs. field strength, SU(3) gluodynamics (+ talk of Kalaydzhyan)

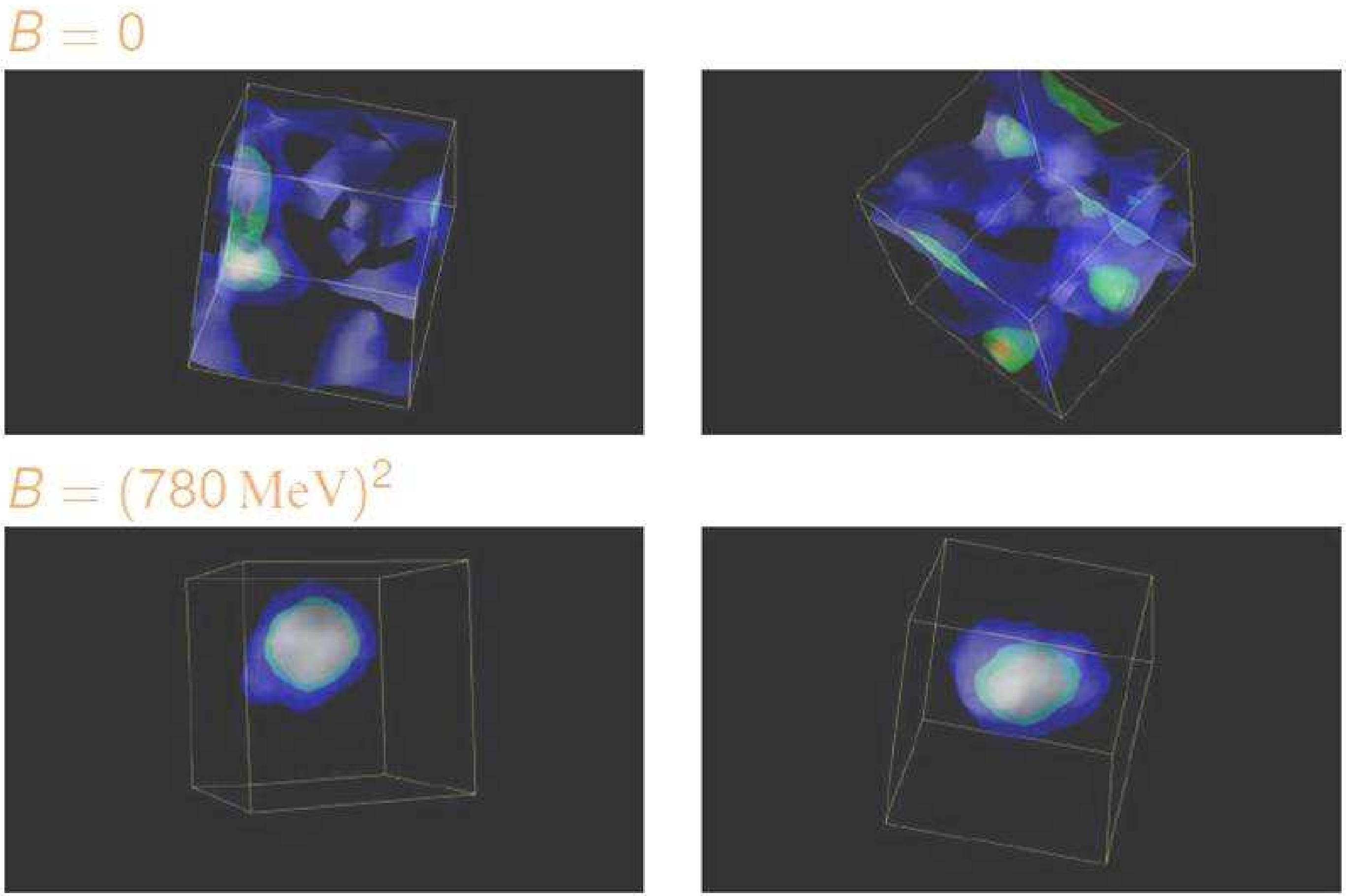


Localization of Dirac Eigenmodes

Typical densities of the nearzero eigenmodes vs. the strength of the external magnetic field

B=0

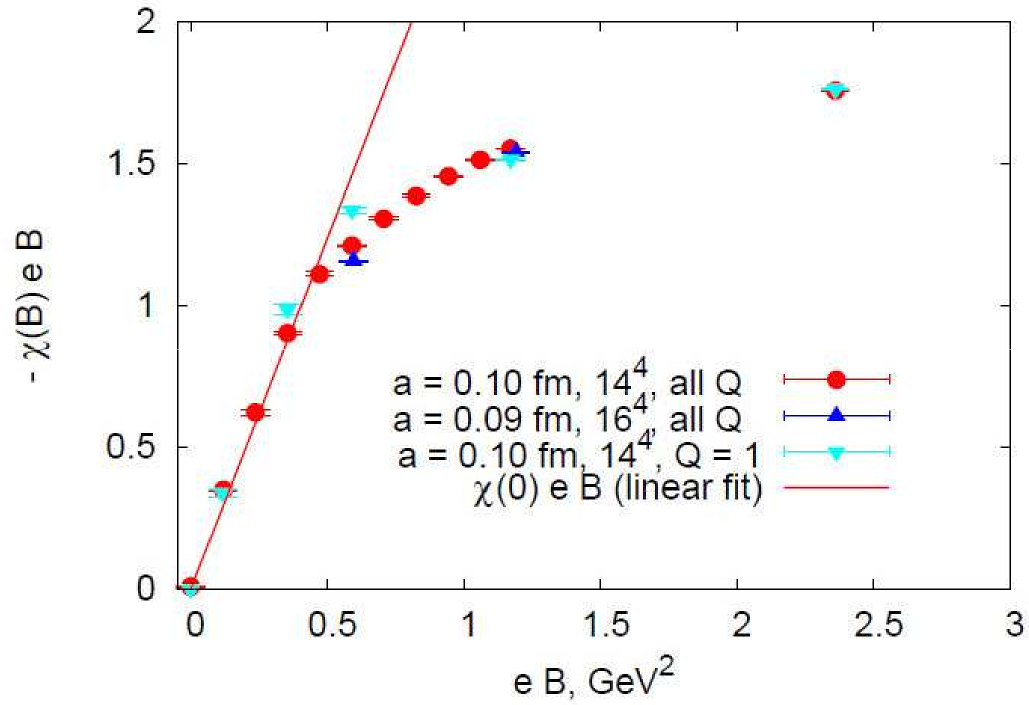
$B=(780Mev)^2$



Chiral condensate Simulations with dynamical quarks

- Massimo D'Elia, Francesco Negro, Swagato Mukherjee, Francesco Sanfilippo, **arXiv:1103.2080, PoS LATTICE2010:179,2010**.
- Michael Abramczyk, Tom Blum, and Gregory Petropoulos, R. Zhou, **arXiv:0911.1348**

4. Magnetization of the vacuum as a function of the magnetic field



Spins of virtual quarks turn parallel to the magnetic field

$$< \bar{\psi}\sigma_{\alpha\beta}\psi > = \chi < \bar{\psi}\psi > F_{\alpha\beta}$$

$$\sigma_{\alpha\beta} = \frac{1}{2i}[\gamma_\alpha, \gamma_\beta]$$

$< \bar{\psi}\psi > \chi = -46(3)\,Mev \leftrightarrow$ our result

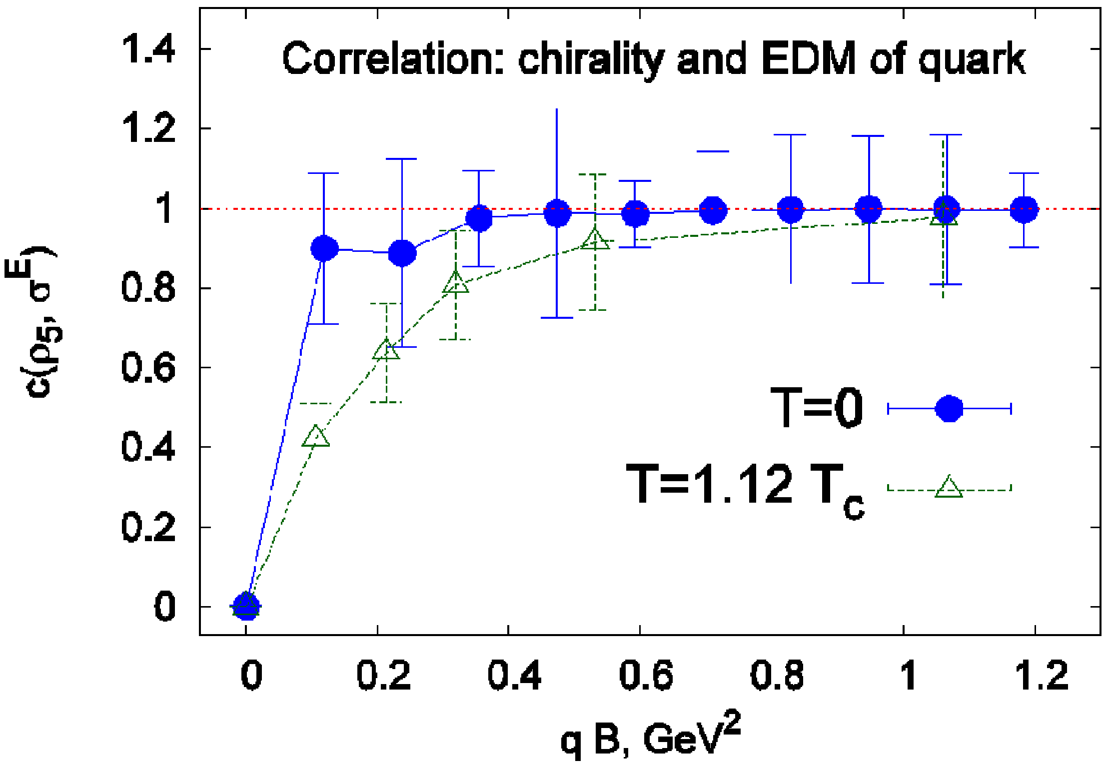
$< \bar{\psi}\psi > \chi \approx -50\,Mev \leftrightarrow$ QCD sum rules

(I. I. Balitsky, 1985, P. Ball, 2003.)

5. Generation of the anomalous quark electric dipole moment along the axis of magnetic field

- **Large correlation between square of the electric dipole moment**

$\sigma_{0i} = i\,\overline{\psi}[\gamma_0, \gamma_i]\psi$ **and chirality** $\rho_5 = \overline{\psi}\gamma_5\psi$



Summary

- CME

a) Visualization

b) Large fluctuations of the electric current in the direction of the magnetic field $< j_3^2 >_{IR}$

c) Conductivity of the vacuum in the direction of the magnetic field $< j_\mu(x) j_\nu(y) >$

- Chiral condensate $< \bar{\psi}\psi >$

- Magnetization of the vacuum $< \bar{\psi}\sigma_{\alpha\beta}\psi >$

- Quark local electric dipole moment $< i\bar{\psi}[\gamma_0, \gamma_i]\psi \cdot \bar{\psi}\gamma_5\psi >$

SU(2), variation of ***T,H,Mq,a, SU(3)*** for $< \bar{\psi}\psi >$

Systematic errors

- SU(2) gluodynamics instead of QCD
- Moderate lattice volumes
- Not large number of gauge field configurations
- In some cases we calculate the overlap propagator using summation over eigenfunctions:

$$\langle \bar{\Psi} \Sigma_{\alpha\beta} \Psi \rangle = 2m \langle \sum_{\lambda_k > 0} \frac{\psi_k^\dagger(x)\, \Sigma_{\alpha\beta}\, \psi_k(x)}{\lambda_k^2 + m^2} \rangle$$

It is interesting to study

SU(3) gluodynamics
SU(2) with dynamical quarks
Lattice QCD
variation of $\boldsymbol{T, H, m_q, a}$

$$<\bar{\psi}\sigma_{\alpha\beta}\psi> \quad <\bar{\psi}\psi> \quad <i\bar{\psi}[\gamma_0,\gamma_i]\psi\cdot\bar{\psi}\gamma_5\psi> \quad <j_3^2>_{IR}$$

$$<j_\mu(x)j_\nu(y)>$$

- CME, vacuum conductivity
- Chiral condensate
- Magnetization of the vacuum
- Quark local electric dipole moment
- Dilepton pair angular distribution
- Shift of the phase transition

New proposals for calculations

- D. Kharzeev

$$\mu \leftrightarrow \mu_5 \qquad L_\mu = i\mu\bar{\psi}\gamma_0\psi + \mu_5\bar{\psi}\gamma_5\psi$$

$$\sigma \leftrightarrow \sigma_5 \qquad \sigma \to < j_\mu(x) j_\nu(y) > \qquad \sigma_5 \to < j_{5\mu}(x) j_{5\nu}(y) >$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks

Chiral Magnetic Effect (CME) + Chiral Separation Effect (CSE) =
= Chiral Magnetic Wave

Dmitri E. Kharzeev, Ho-Ung Yee, arXiv:1012.6026

New proposals for calculations

- D. Kharzeev

$$\mu = 0,\ \mu_5 \neq 0 \qquad L_\mu = \mu_5 \bar{\psi} \gamma_5 \gamma_0 \psi$$

$$\sigma_{5ij}(\mu_5, B, T)$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD (imaginary unity is absent)

<u>*New proposals for calculations*</u>

- D.T. Son, N.Yamamoto **Holography and Anomaly Matching for Resonances.** e-Print: **arXiv:1010.0718**

$$< j_\mu(-q) j^5{}_\nu(q) >= -\frac{q^2}{4\pi^2} P_\mu^{\alpha\perp} [P_\nu^{\beta\perp} \omega_T(q^2) + P_\nu^{\beta\parallel} \omega_L(q^2)] \tilde{F}_{\alpha\beta}$$

$$\omega_L(q^2) = \frac{2N_c}{q^2} \quad \Leftarrow \quad \text{no quantum corrections}$$

$$\omega_T(q^2) = \frac{N_c}{q^2} \quad \Leftarrow \quad \text{there are nonperturbative corrections}$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

New proposals for calculations

- D.T. Son, N.Yamamoto **Holography and Anomaly Matching for Resonances.** e-Print: **arXiv:1010.0718**

$$\omega_L(q^2) = \frac{2N_c}{q^2} \quad \Leftarrow \quad \text{no quantum corrections}$$

$$\omega_T(q^2) = \frac{N_c}{q^2} \quad \Leftarrow \quad \text{there are nonperturbative corrections}$$

$$\omega_T(q^2) = \frac{N_c}{q^2} - \frac{N_c}{f_\pi^2}[< j_\mu^5 j_\mu^5 > - < j_\mu j_\mu >]$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

New proposals for calculations

- D.T. Son, N.Yamamoto **Holography and Anomaly Matching for Resonances.** e-Print: **arXiv:1010.0718**

$$< j_\mu(-q) j^5{}_\nu(q) >= -\frac{q^2}{4\pi^2} P^{\alpha\perp}_\mu [P^{\beta\perp}_\nu (\frac{N_c}{q^2} - \frac{N_c}{f_\pi^2}[< j^5_\mu j^5_\mu > - < j_\mu j_\mu >])$$

$$+P^{\beta\parallel}_\nu \frac{2N_c}{q^2}] \tilde{F}_{\alpha\beta}$$

$$P^{\alpha\perp}_\mu = \eta^\alpha_\mu - \frac{q_\mu q^\alpha}{q^2}, \quad P^{\alpha\parallel}_\mu = \frac{q_\mu q^\alpha}{q^2}$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

New proposals for calculations

- **D.T. Son, N.Yamamoto** $\Longrightarrow$ $$< \bar{\psi}\gamma_\mu\psi(x)\,\bar{\psi}\gamma_\mu\gamma_5\psi(y) >$$

- **D. Kharzeev** $\Longrightarrow$ $$< \bar{\psi}\psi(x)\,\bar{\psi}\gamma_5\psi(y) >$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks,

QCD

New proposals for calculations

- L. McLerran "Nonsymmetric condensate"

Calculate $<\bar{\psi}\psi(x)\,\bar{\psi}\psi(y)>$ parallel and perpendicular to the field. Chiral condensate may depend on the direction!

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD