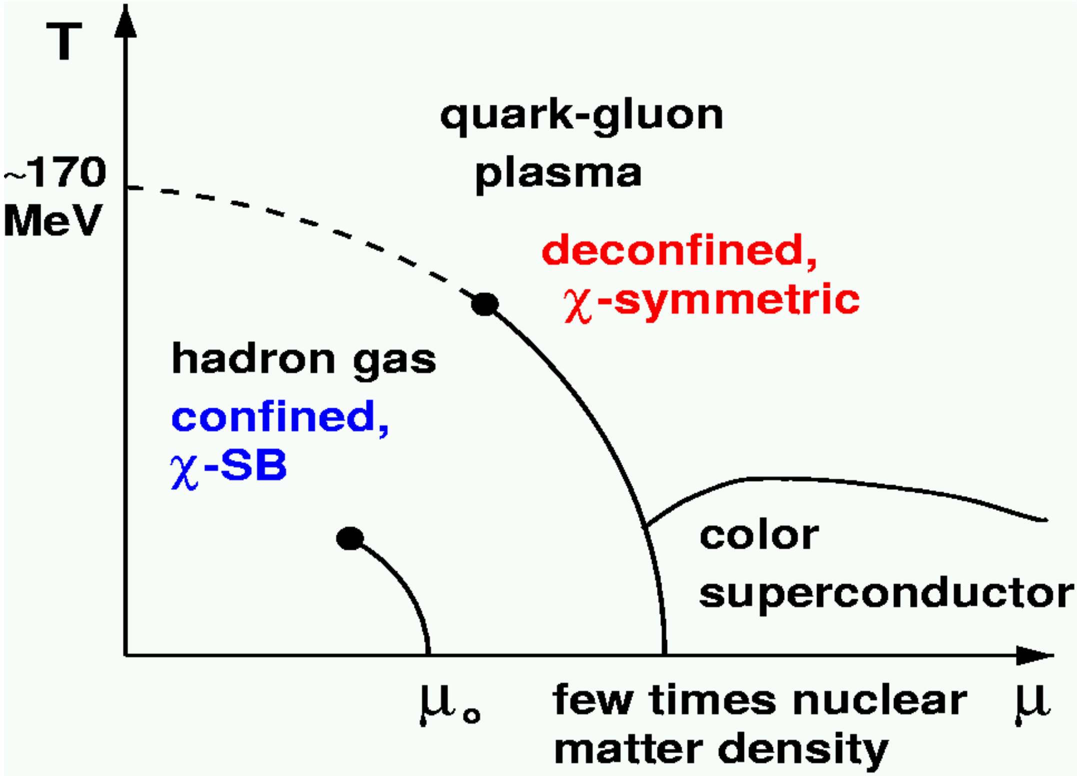
Gluons, color superconductors, and gauge symmetry

Axel Maas

15th of March 2011
Quarks, Gluons, and Hadronic Matter under Extreme Conditions
St. Goar
Germany

Color superconductors



[Karsch, Laermann, 2003]

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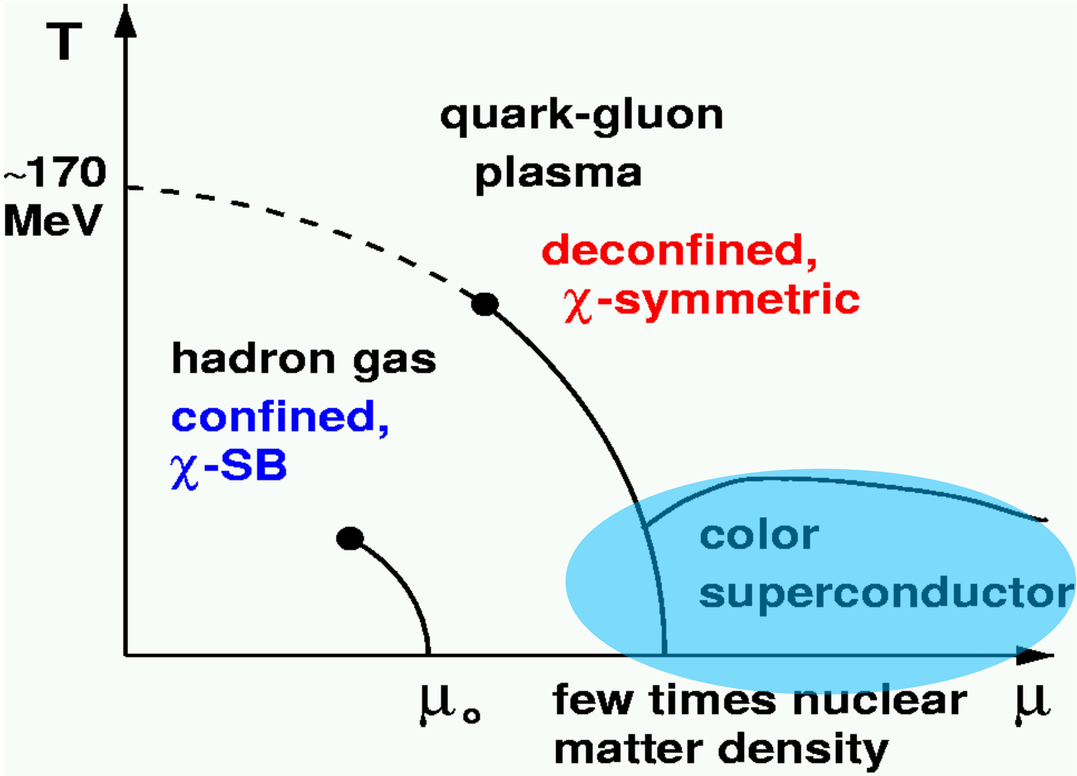

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  - Neutron star density?
  - Various types
    - 2SC, 3SC, CFL,...
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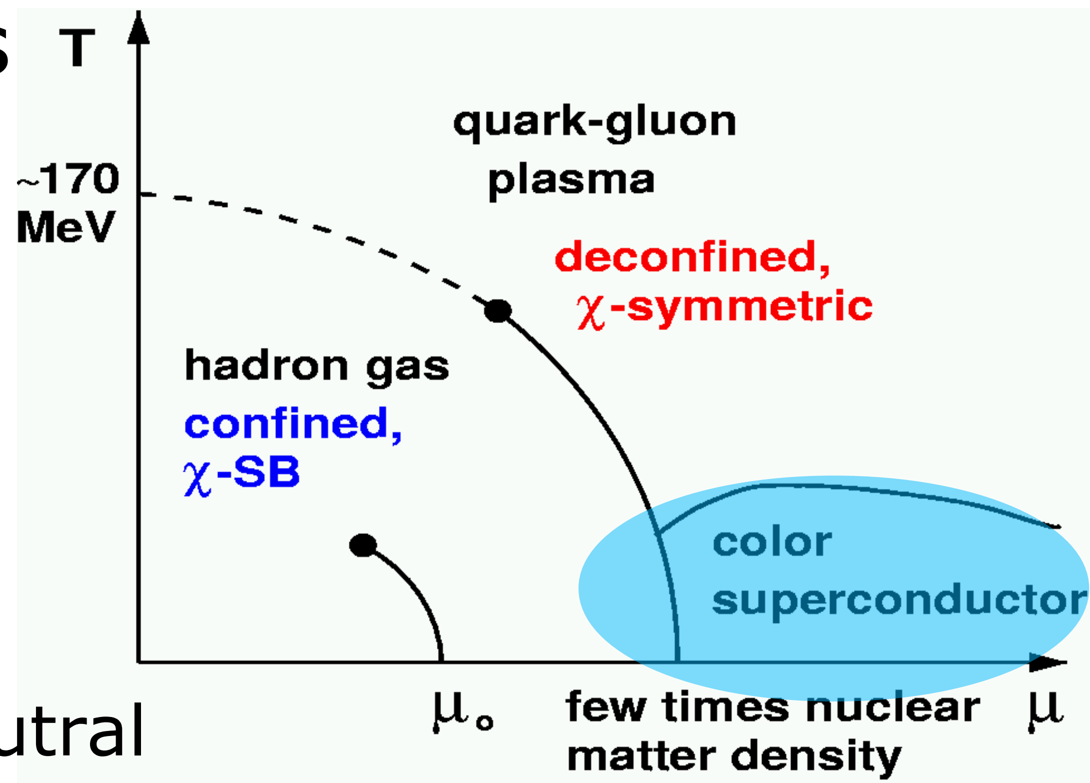

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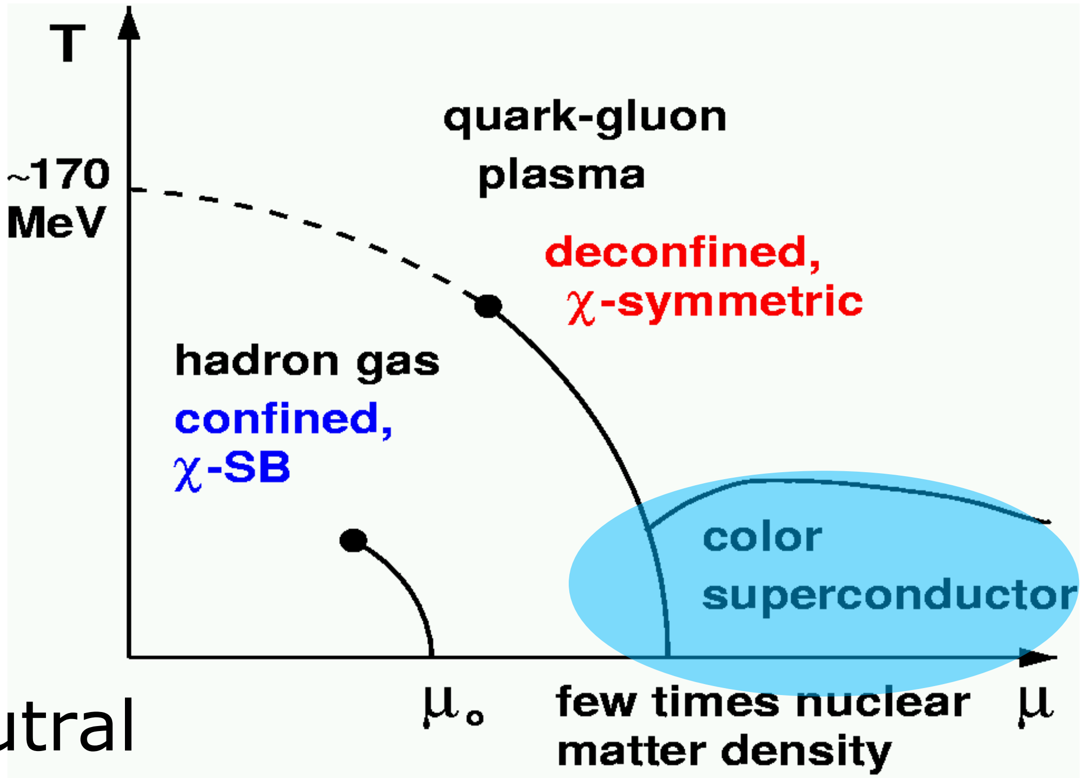

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- Very hard to tackle
  - Model calculations
  - Perturbation theory and HDL
  - Functional calculations

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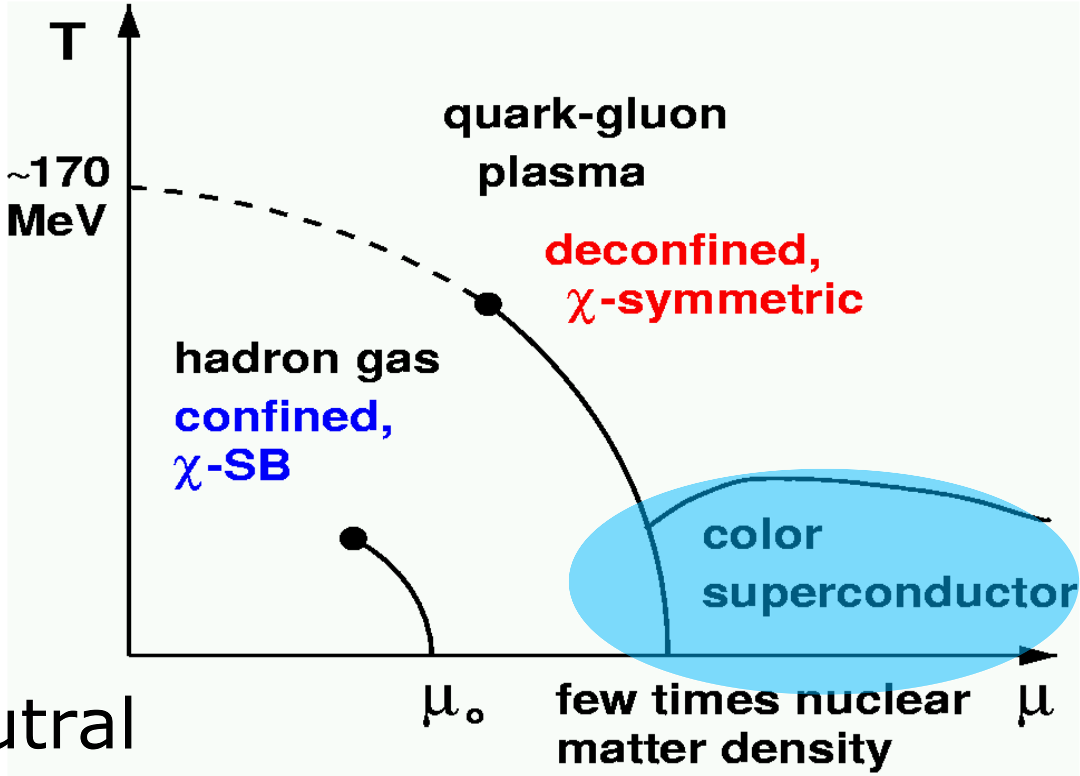

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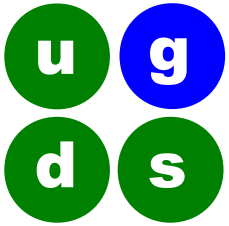
Calculational scheme



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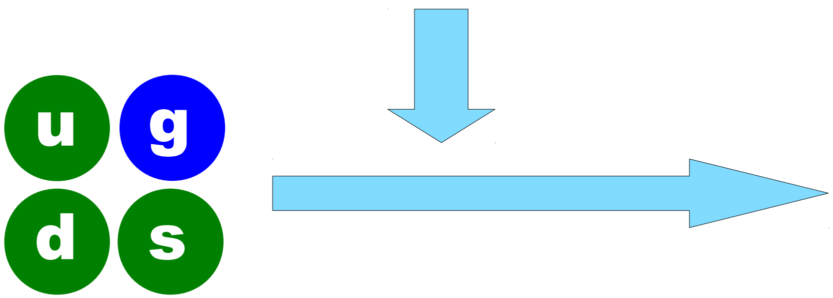
(fixed gauge:
Landau,...)



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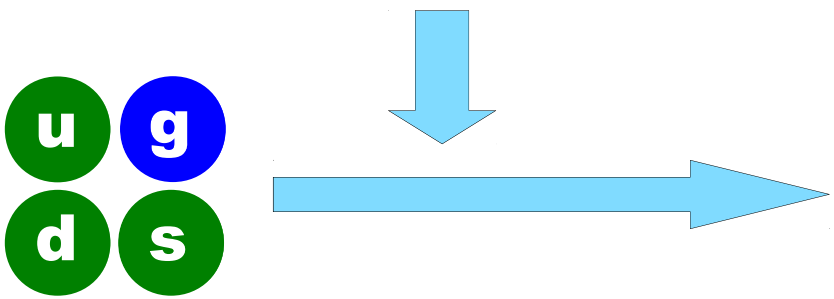

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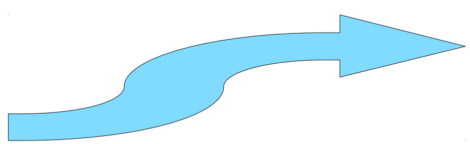

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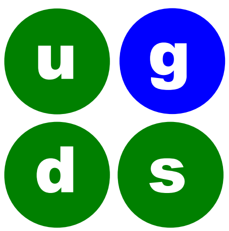

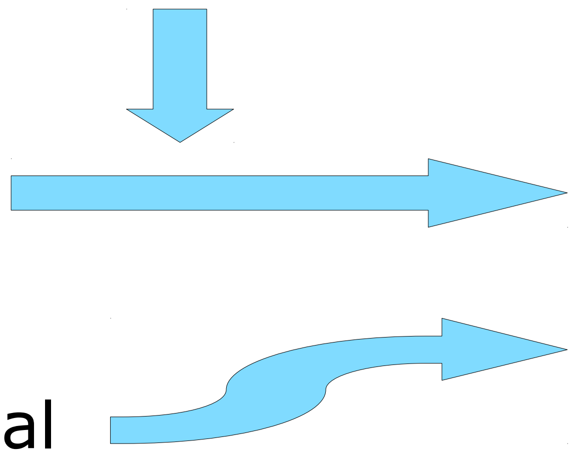

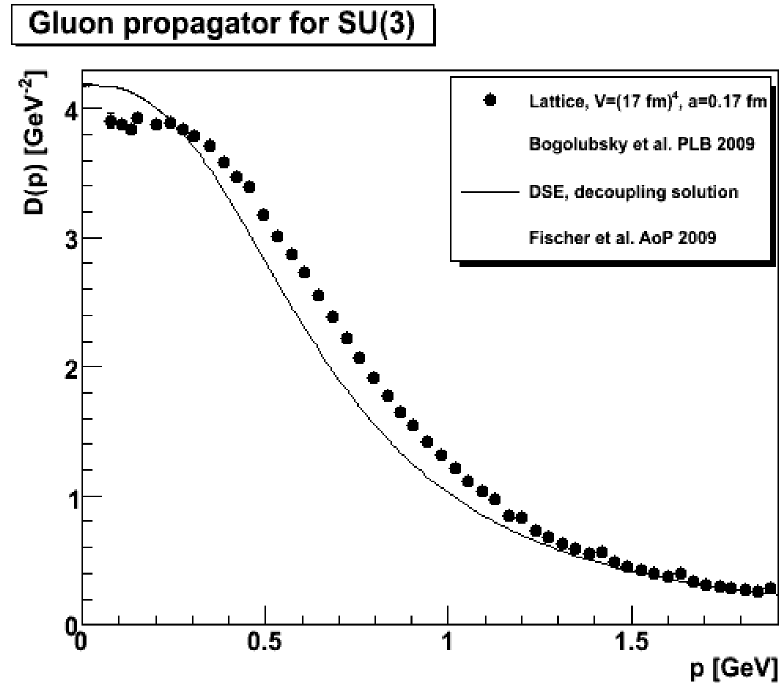

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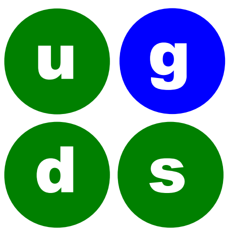

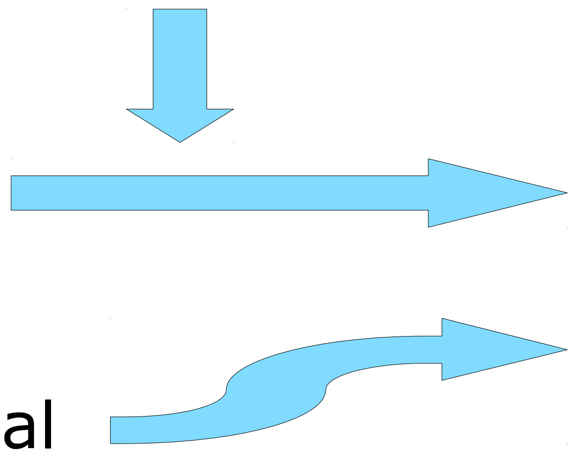

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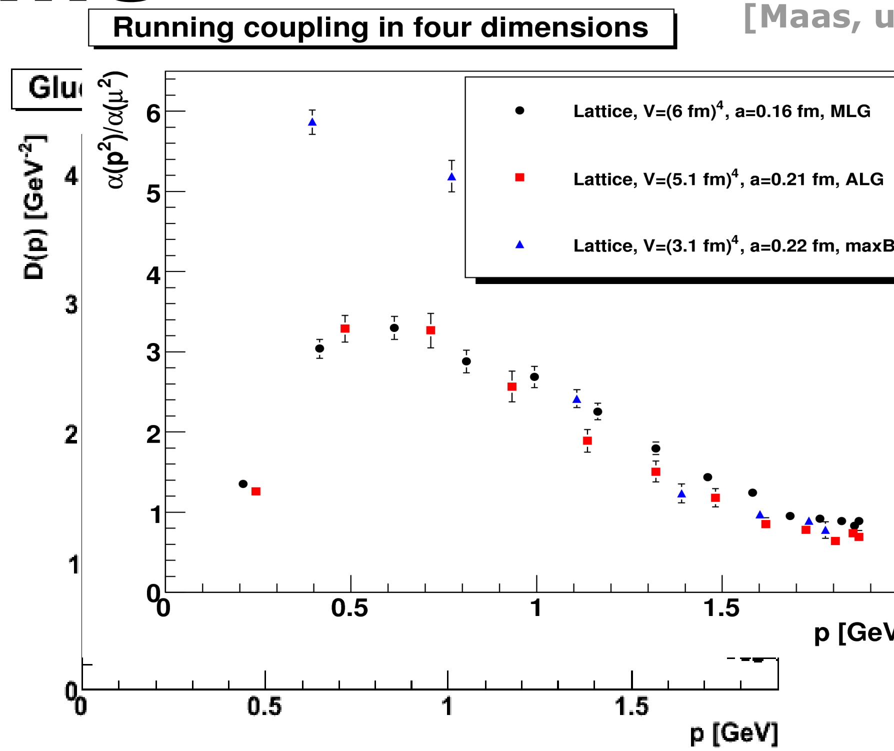

[Maas, unpublished]

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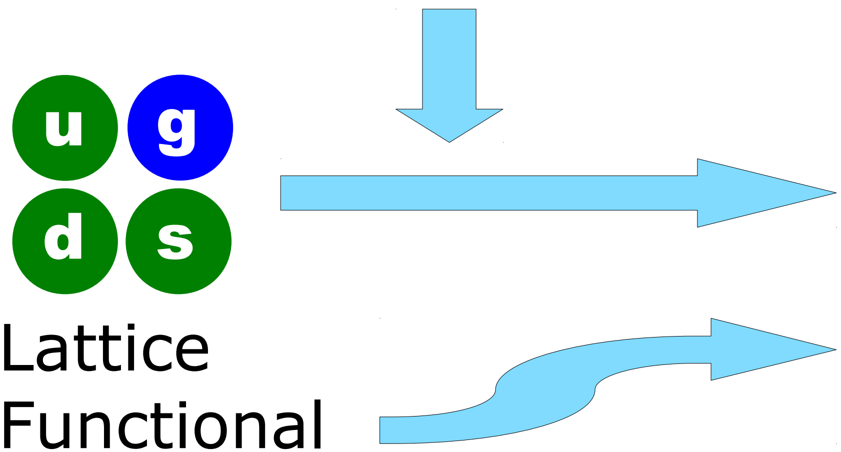

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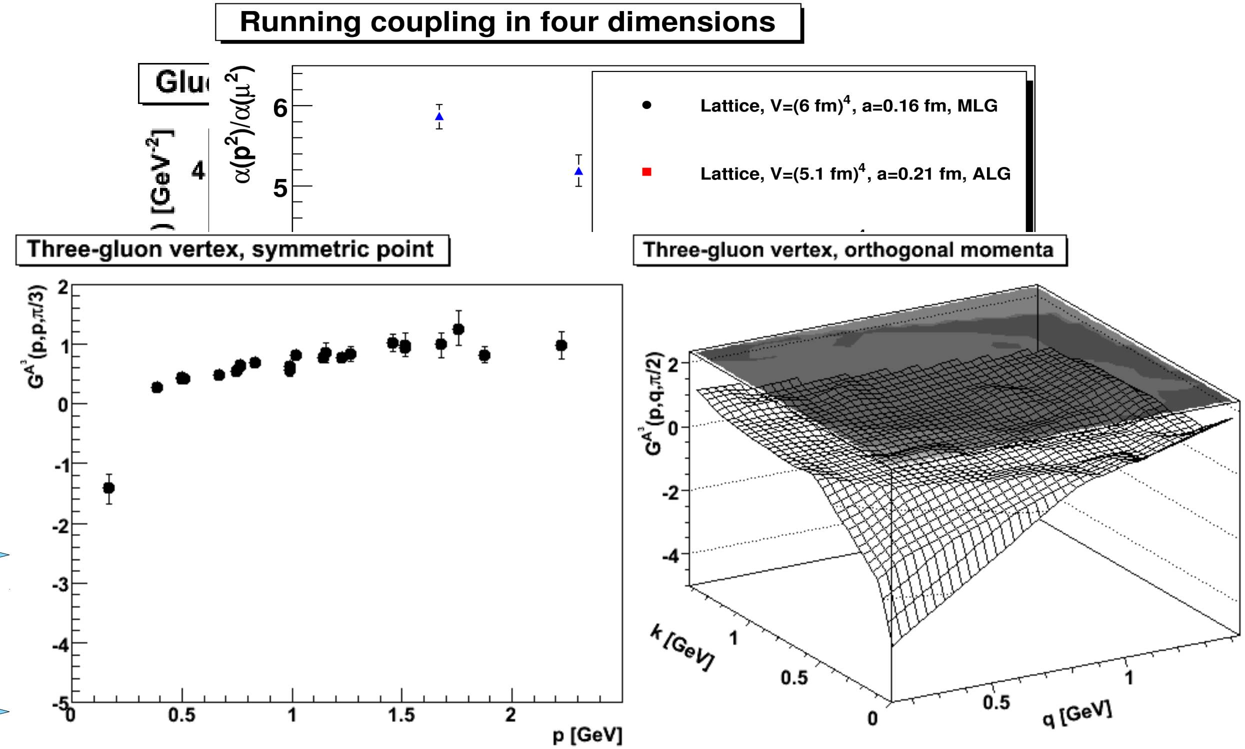

[Cucchieri, Maas, Mendes PRD 2008]

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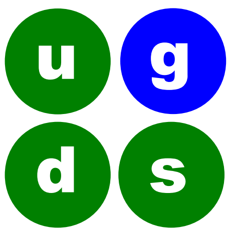

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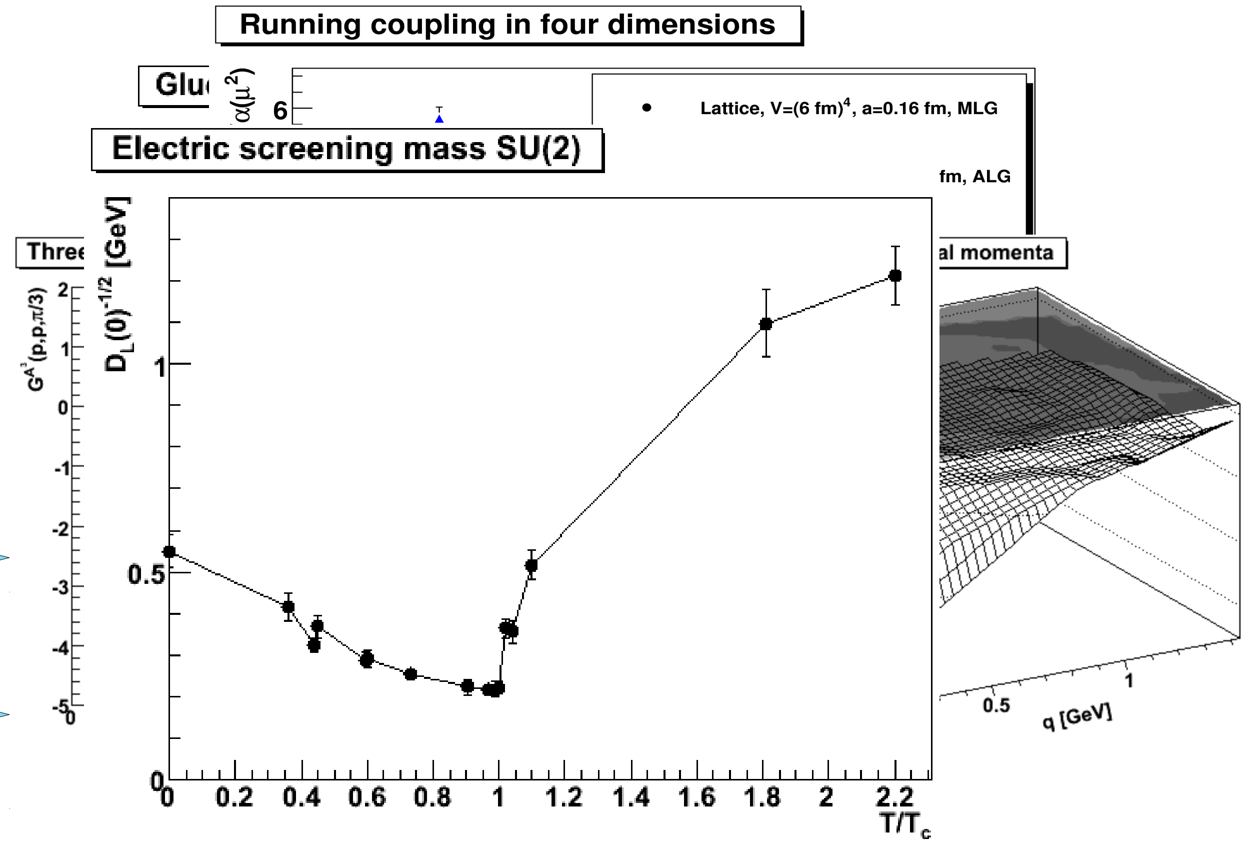

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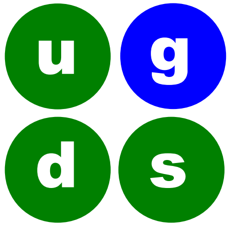

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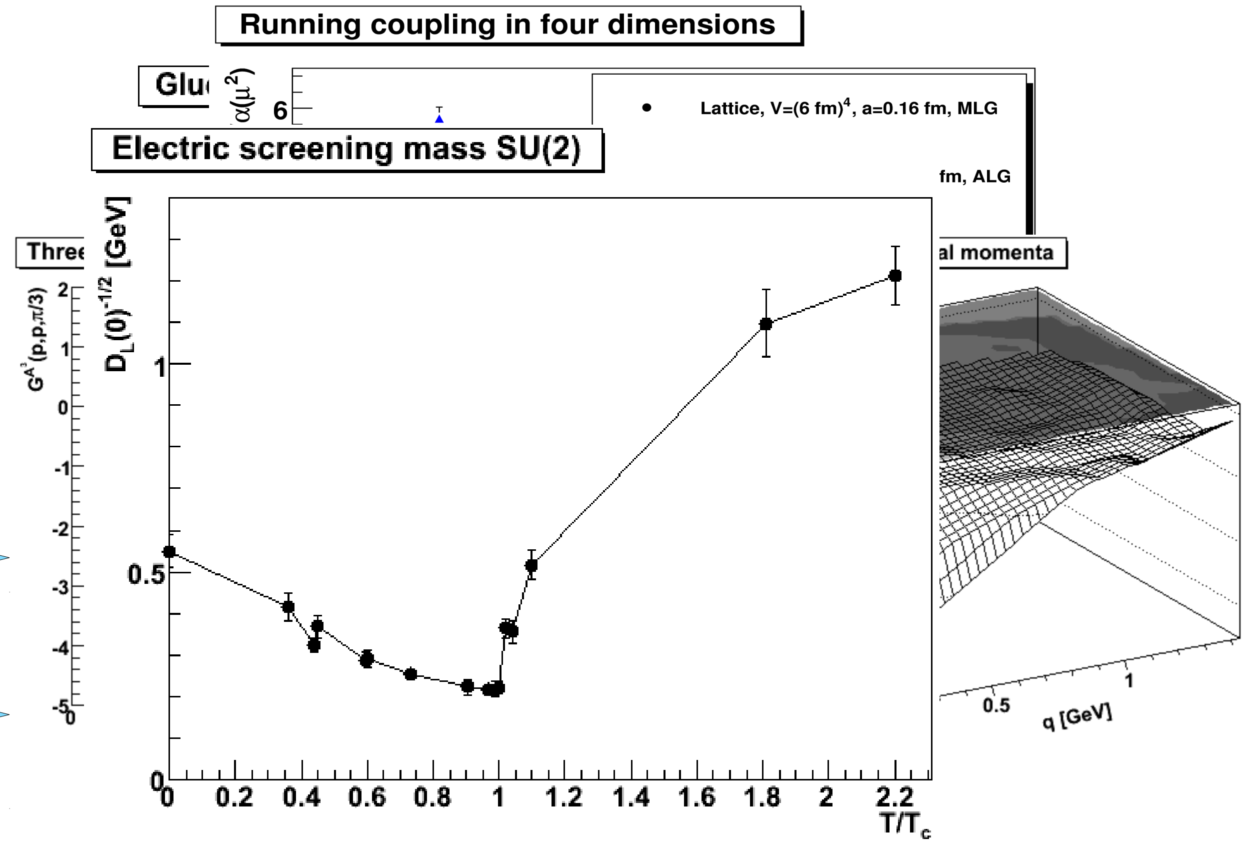

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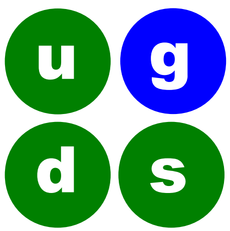

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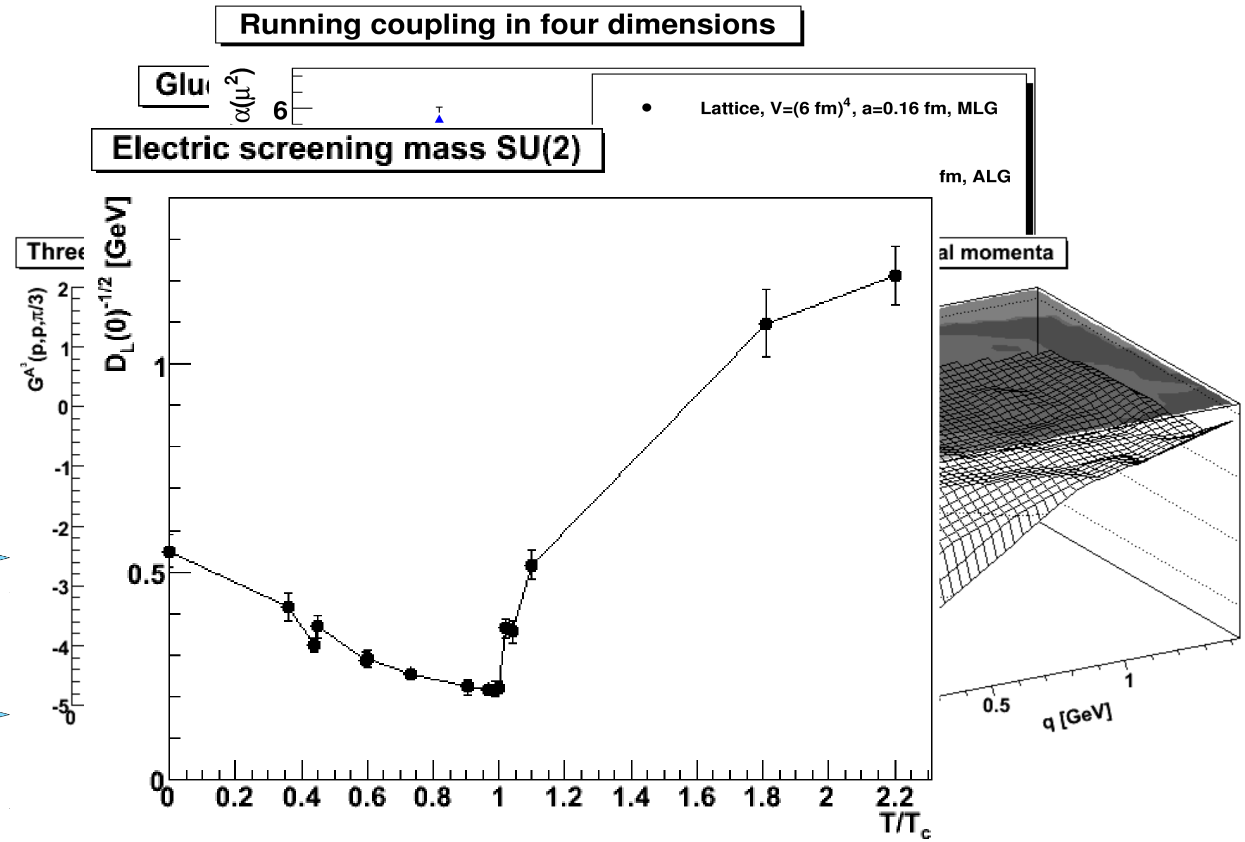

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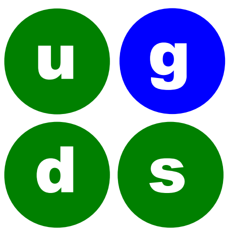

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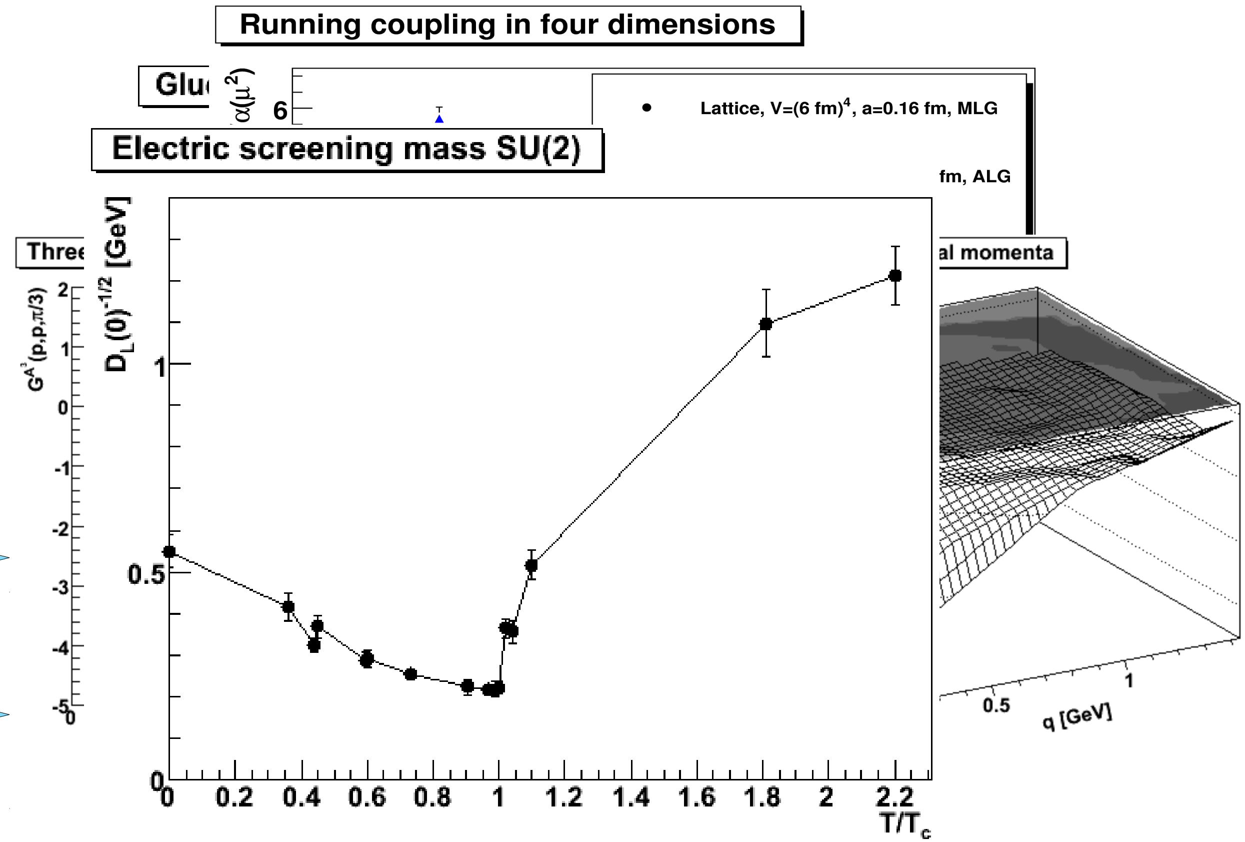

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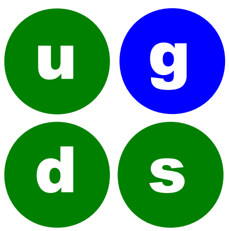

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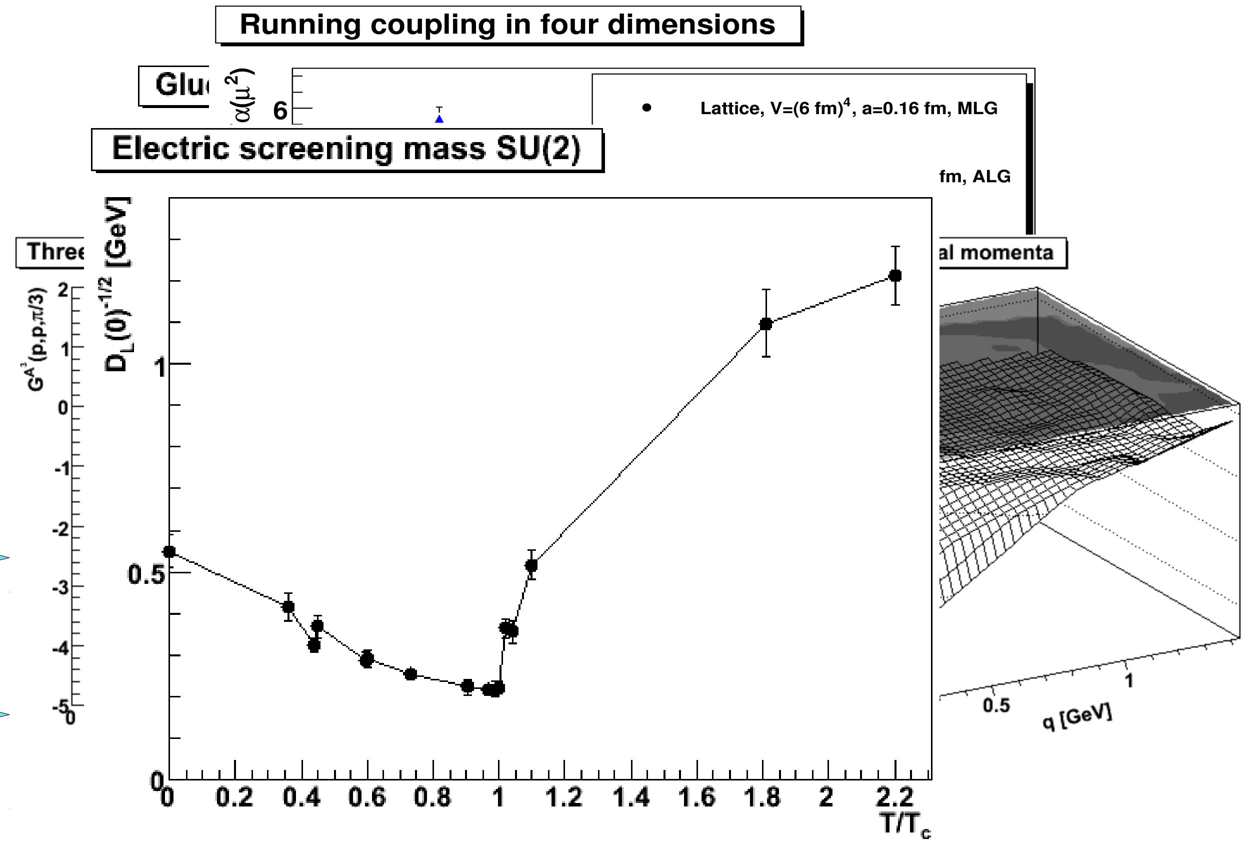

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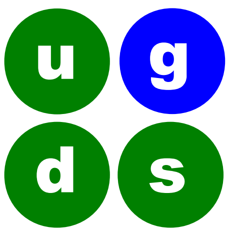

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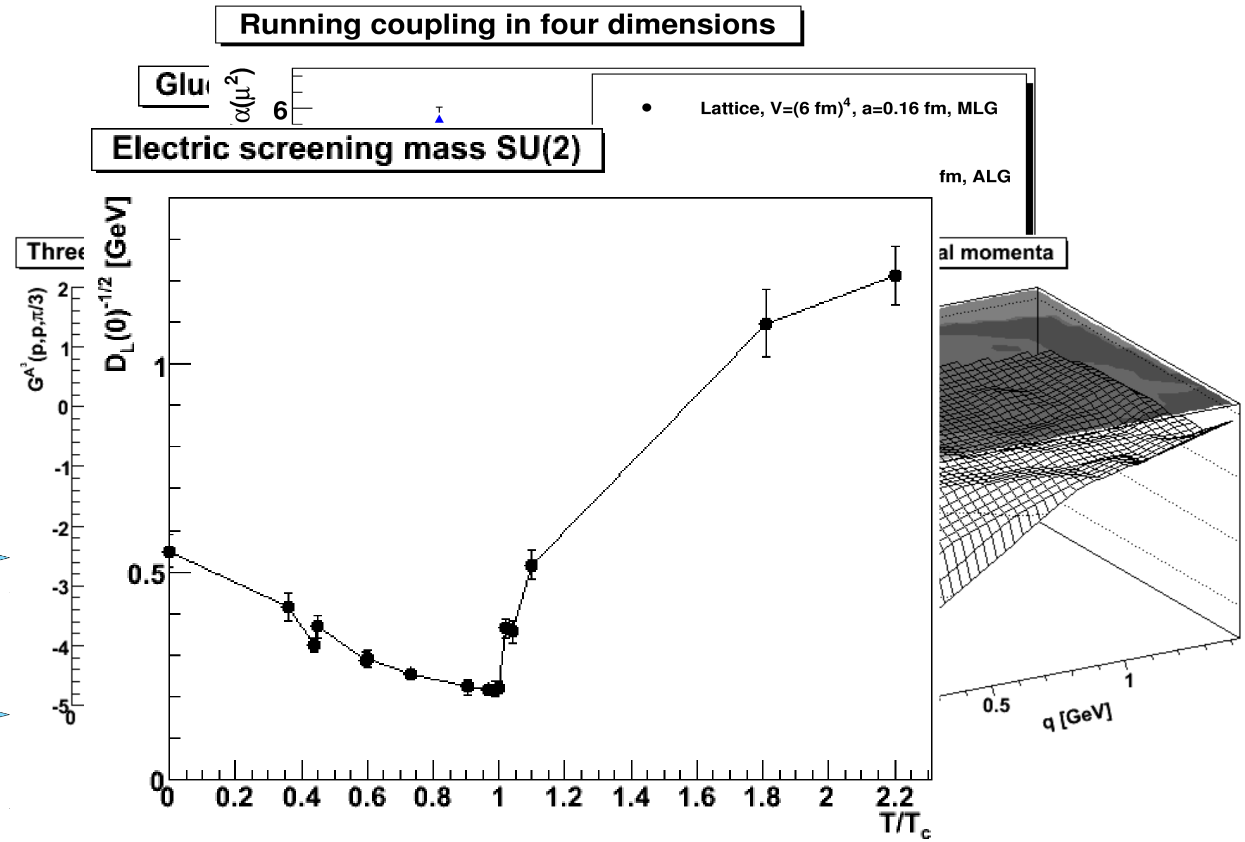

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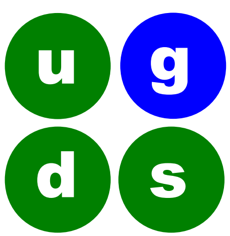

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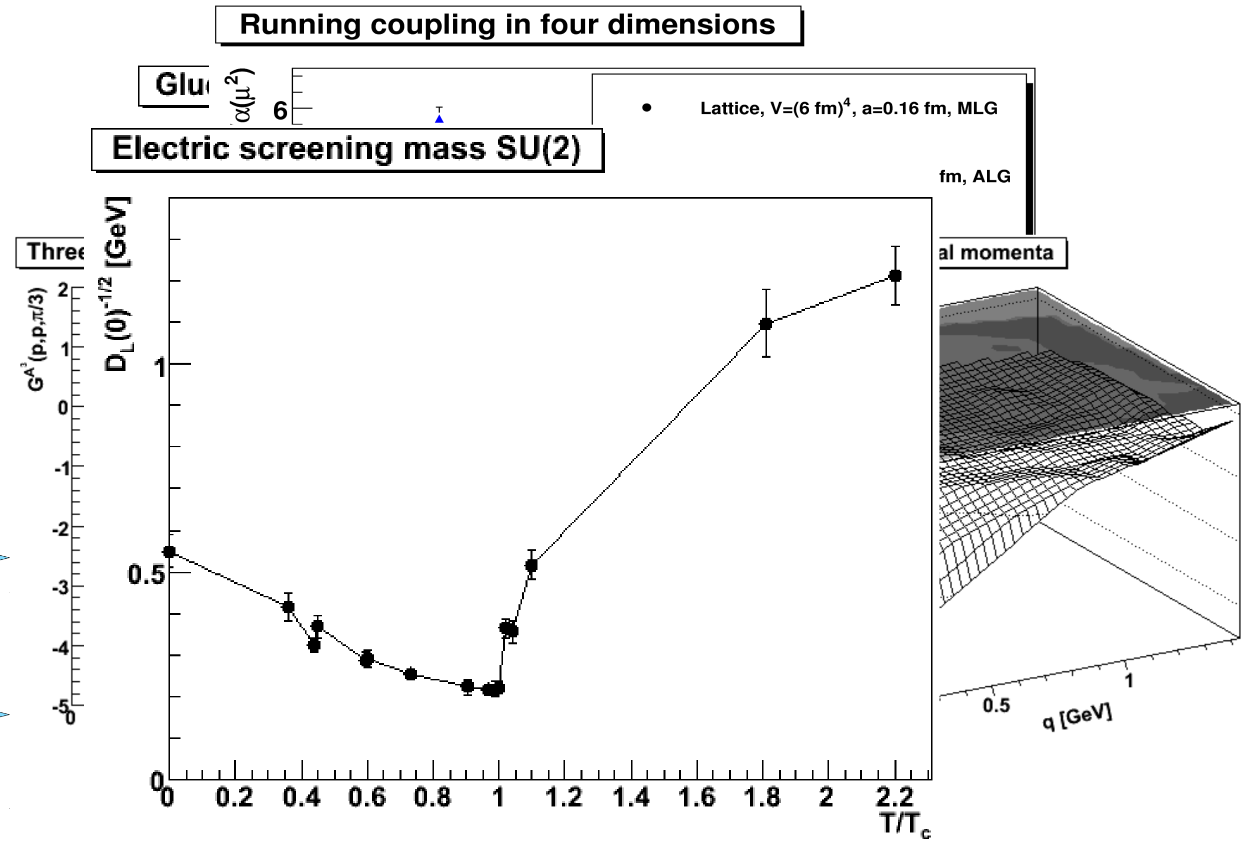

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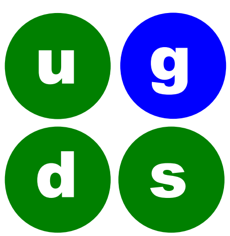

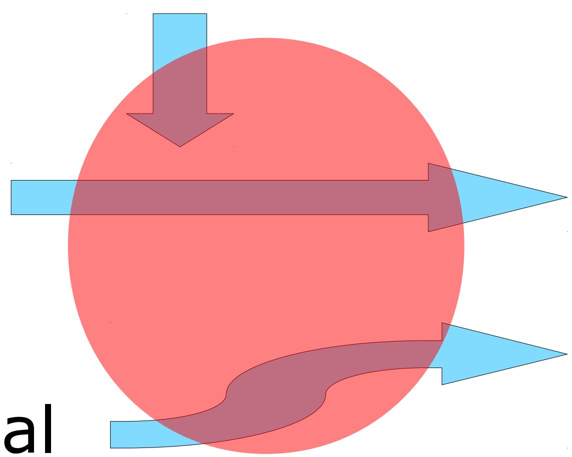

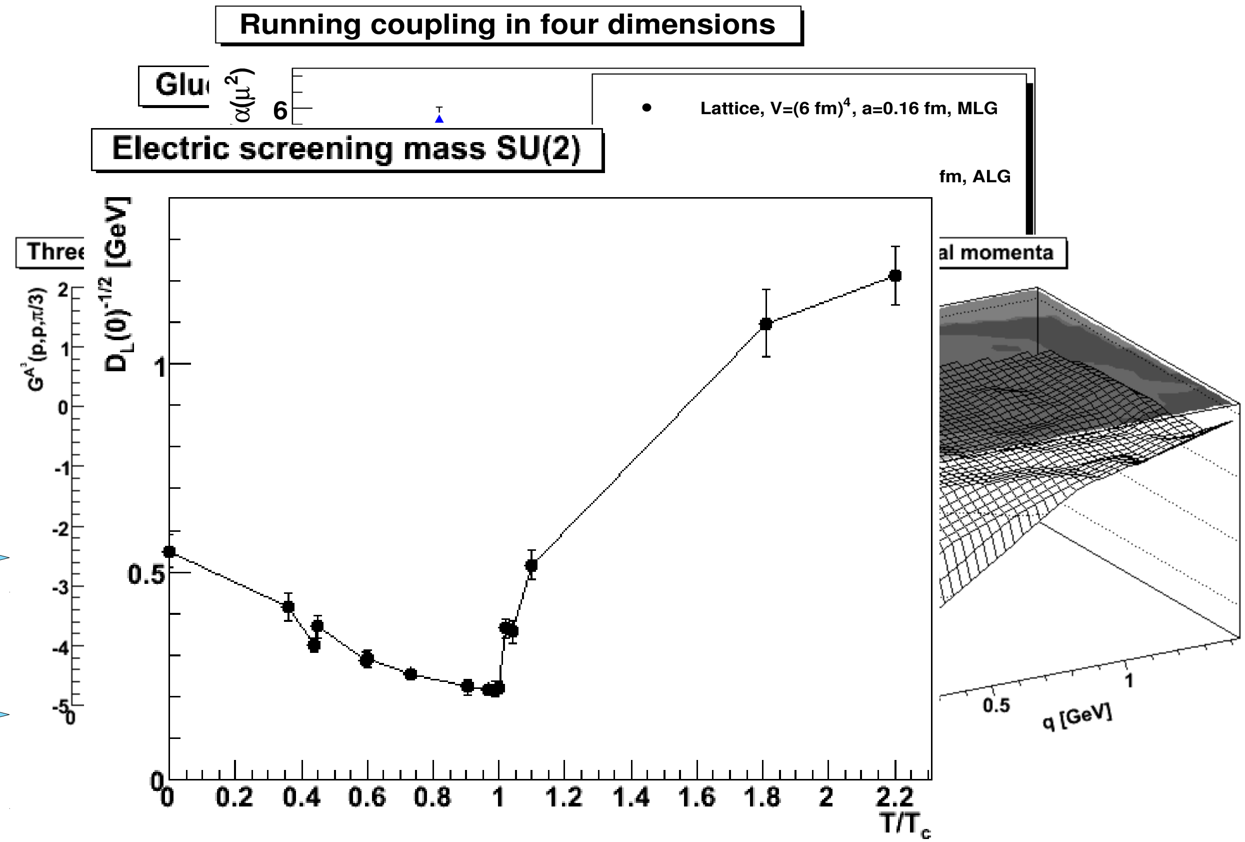

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- Mechanism: No direction of Higgs field preferred – global rotations 'wash out' expectation value

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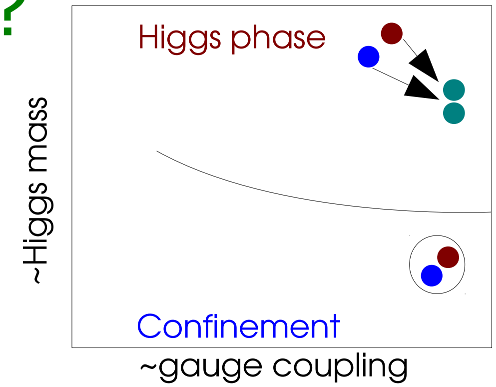
  - Technical: How can they be accessed?
  - Physical: How do both phases differ?
- Caution with the analogy
  - Quantum phase diagram of the Higgs case is non-trivial
  - Confinement and Higgs not completely separated
  - Translation to color superconductors?



[Fradkin & Shenker, PRD 1979]

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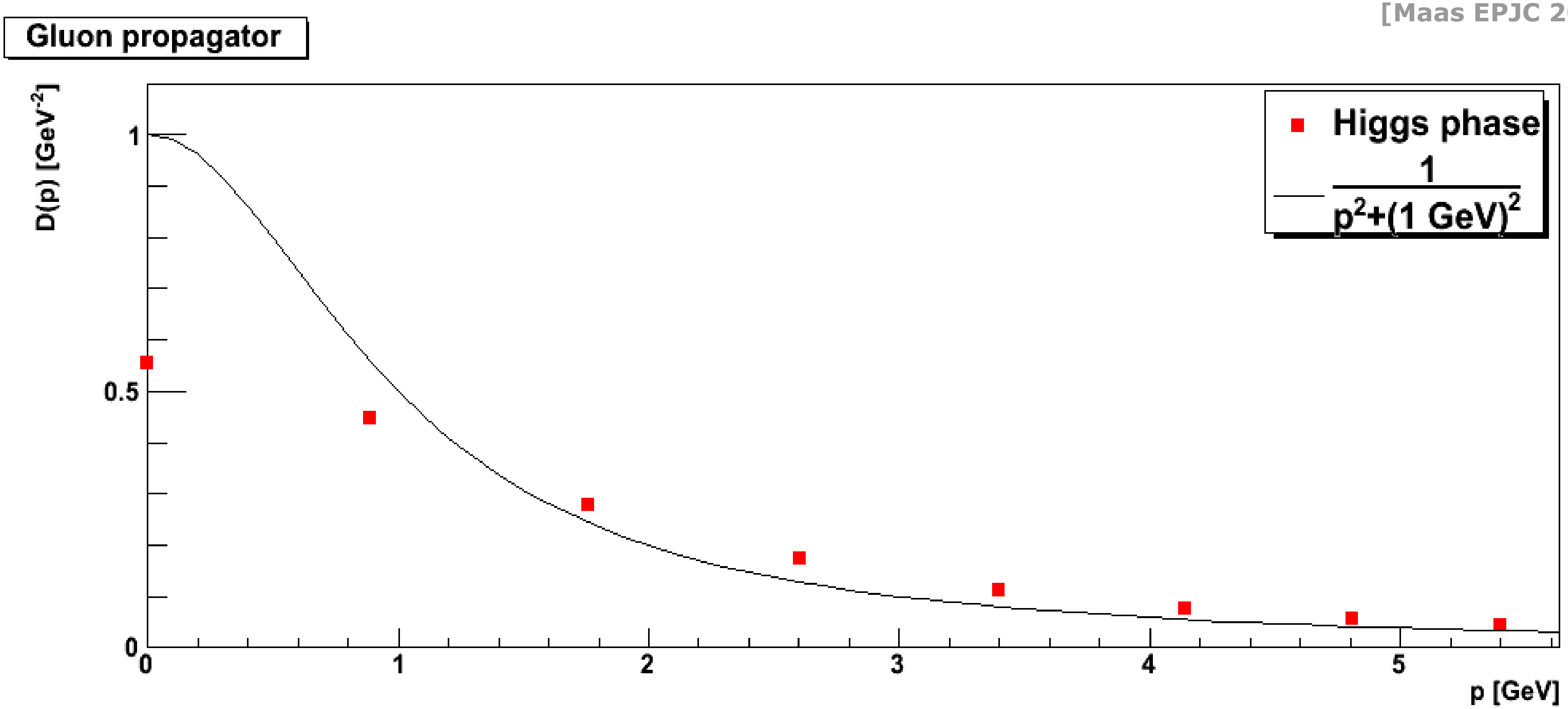
$$D^{ij}_S(x-y) = \ < \phi^+_i (x) \phi_j(y) >$$

  - Includes would-be Goldstones

Gluons

4d, Higgs, 24^4 beta=2.3, kappa=0.32, lambda=1.0

[Maas EPJC 2011]



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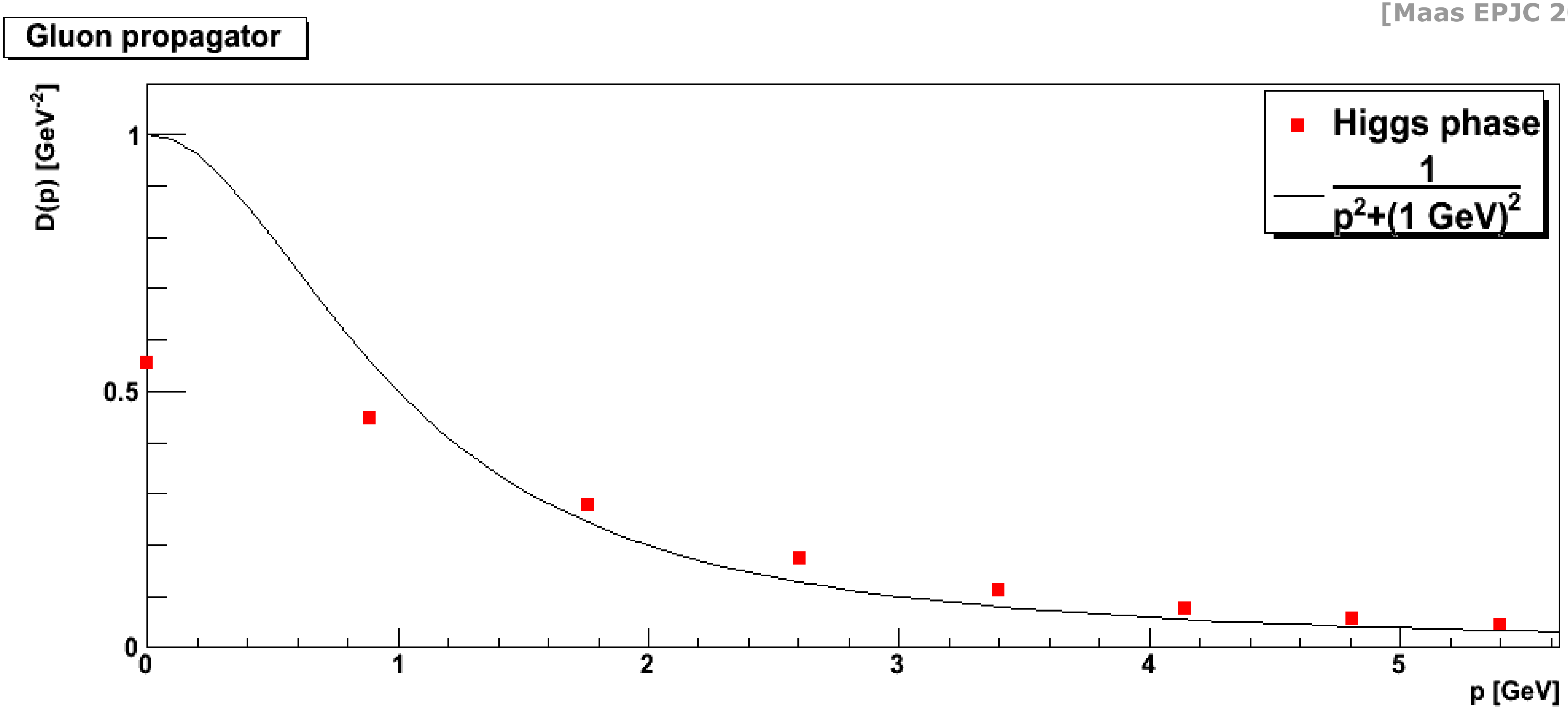

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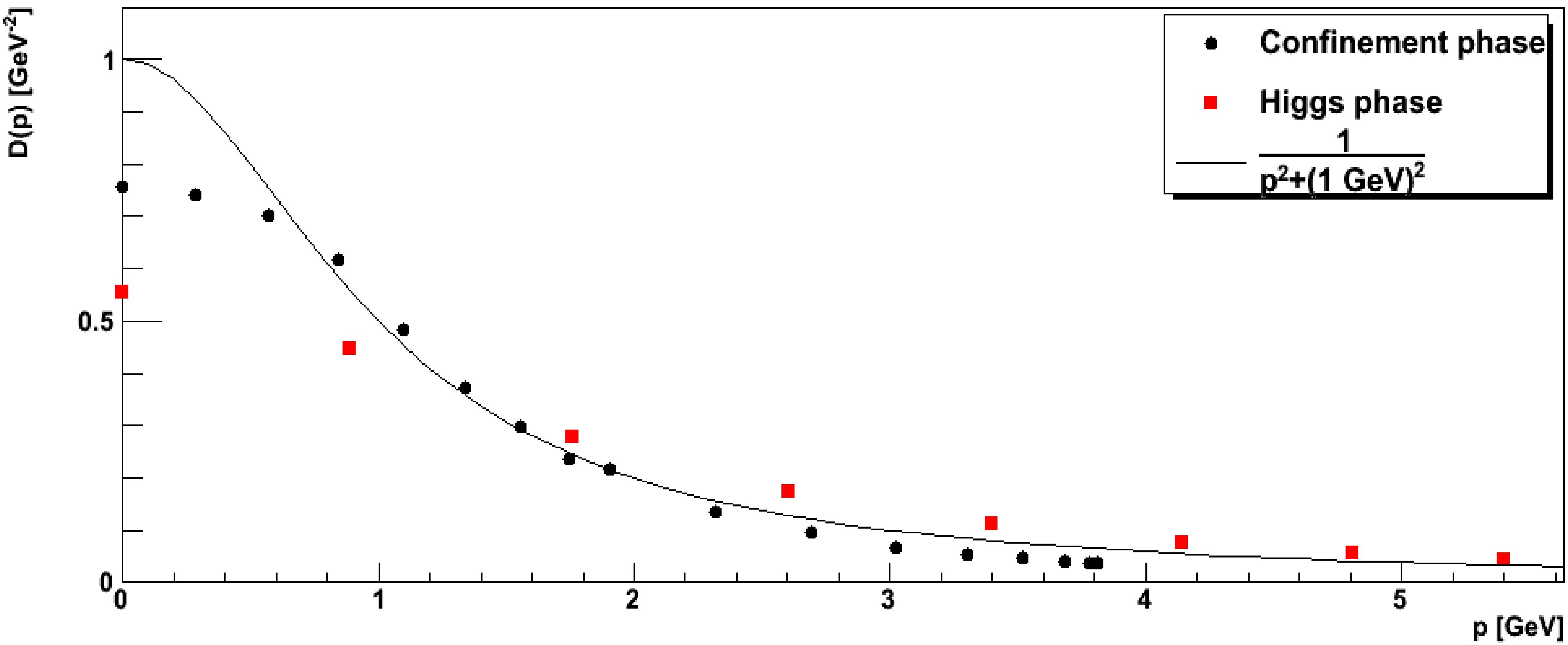
  - Pole mass yet unclear

Gluons

4d, unquenched: 24^4 beta=2.0, kappa=0.25, lambda=0.5
4d, Higgs, 24^4 beta=2.3, kappa=0.32, lambda=1.0

[Maas EPJC 2011]

Gluon propagator



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- Very similar to confinement

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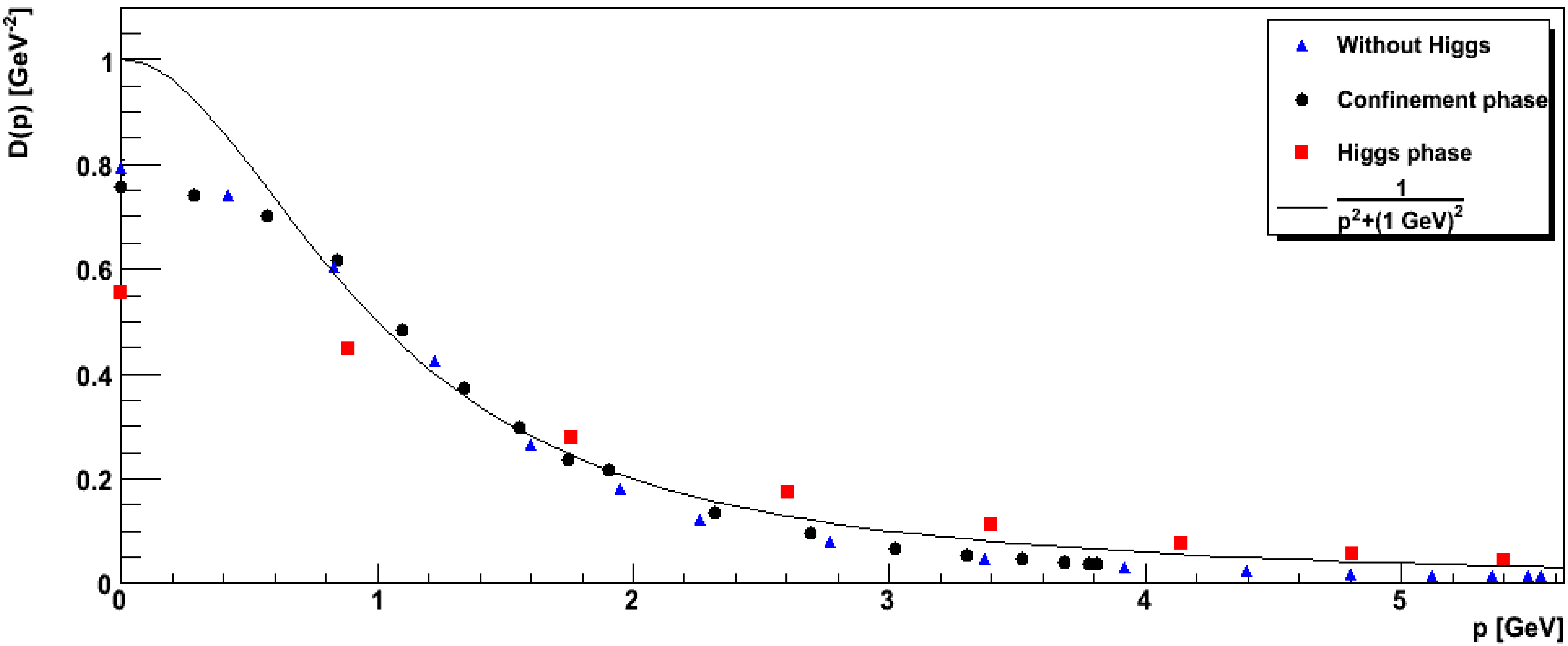
[Maas EPJC 2011]

Gauge boson propagator



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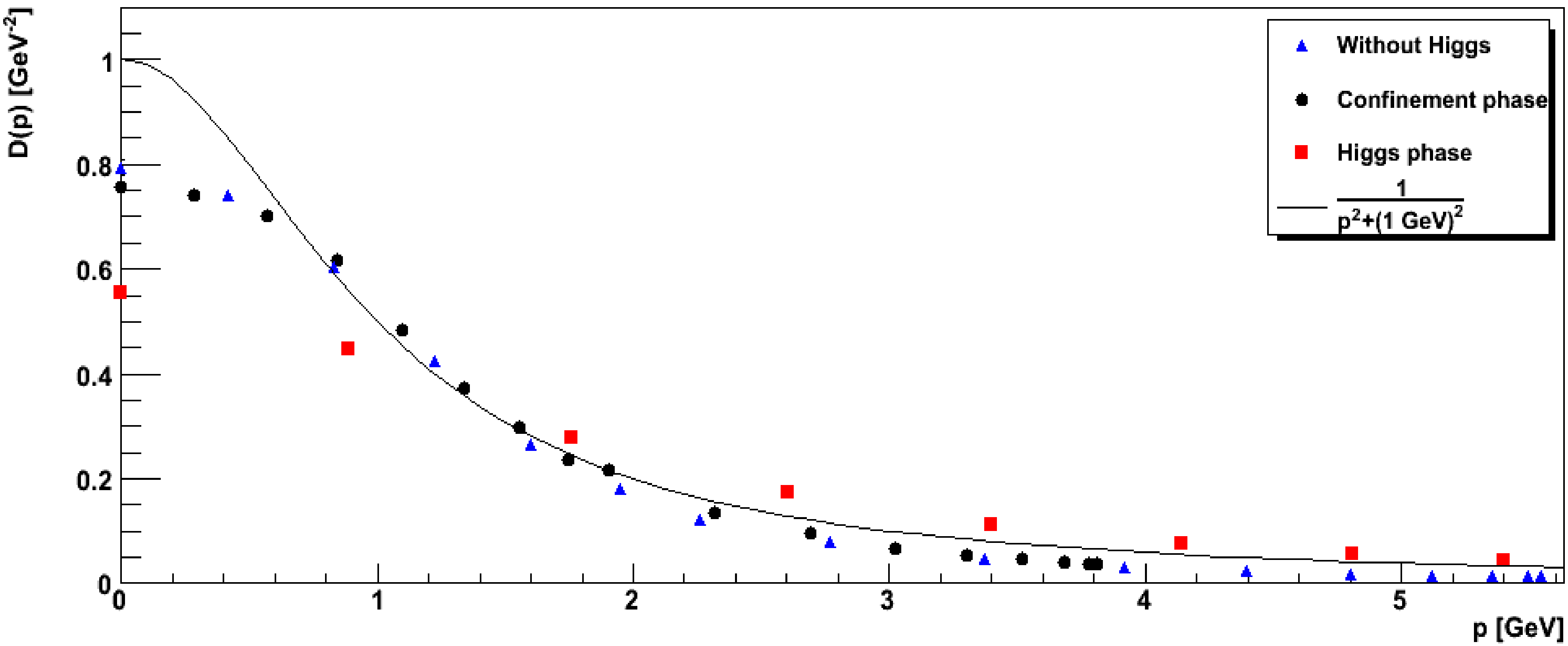

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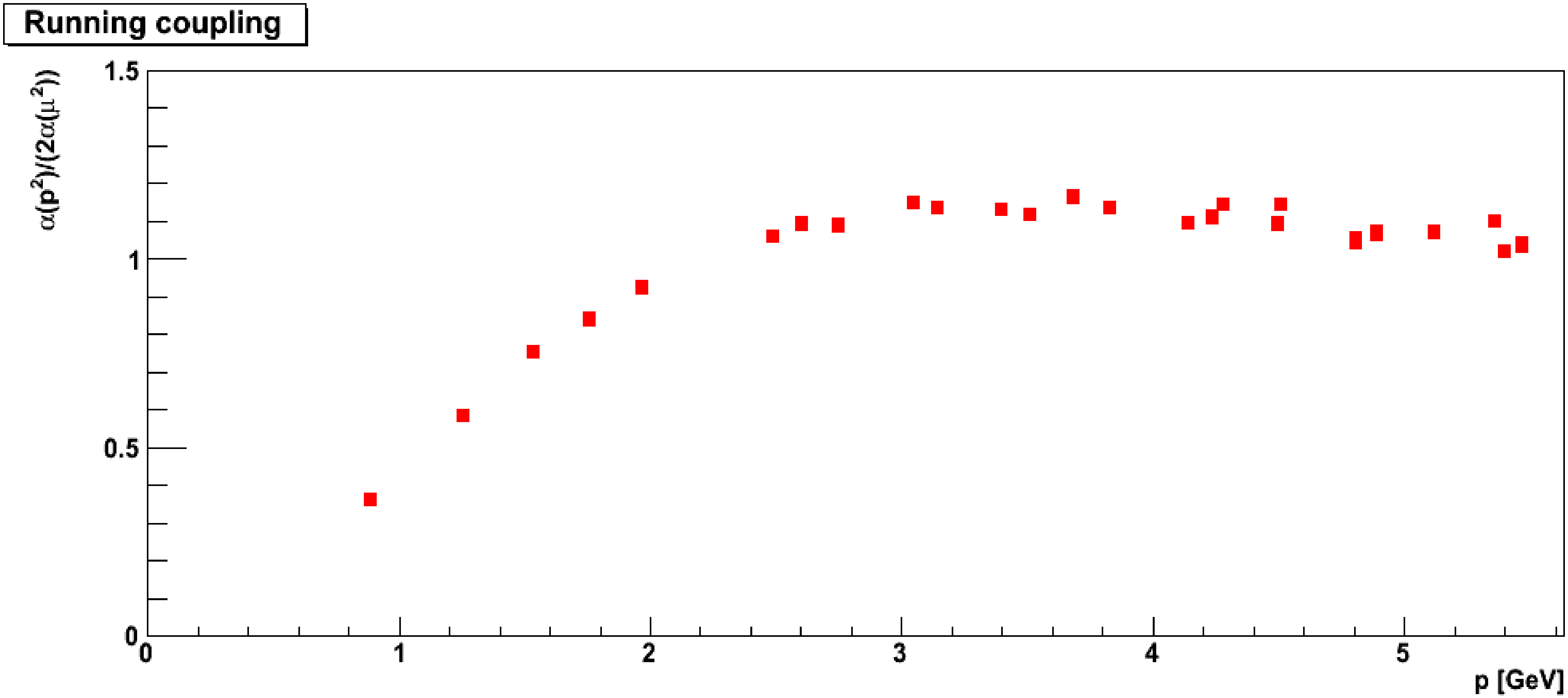

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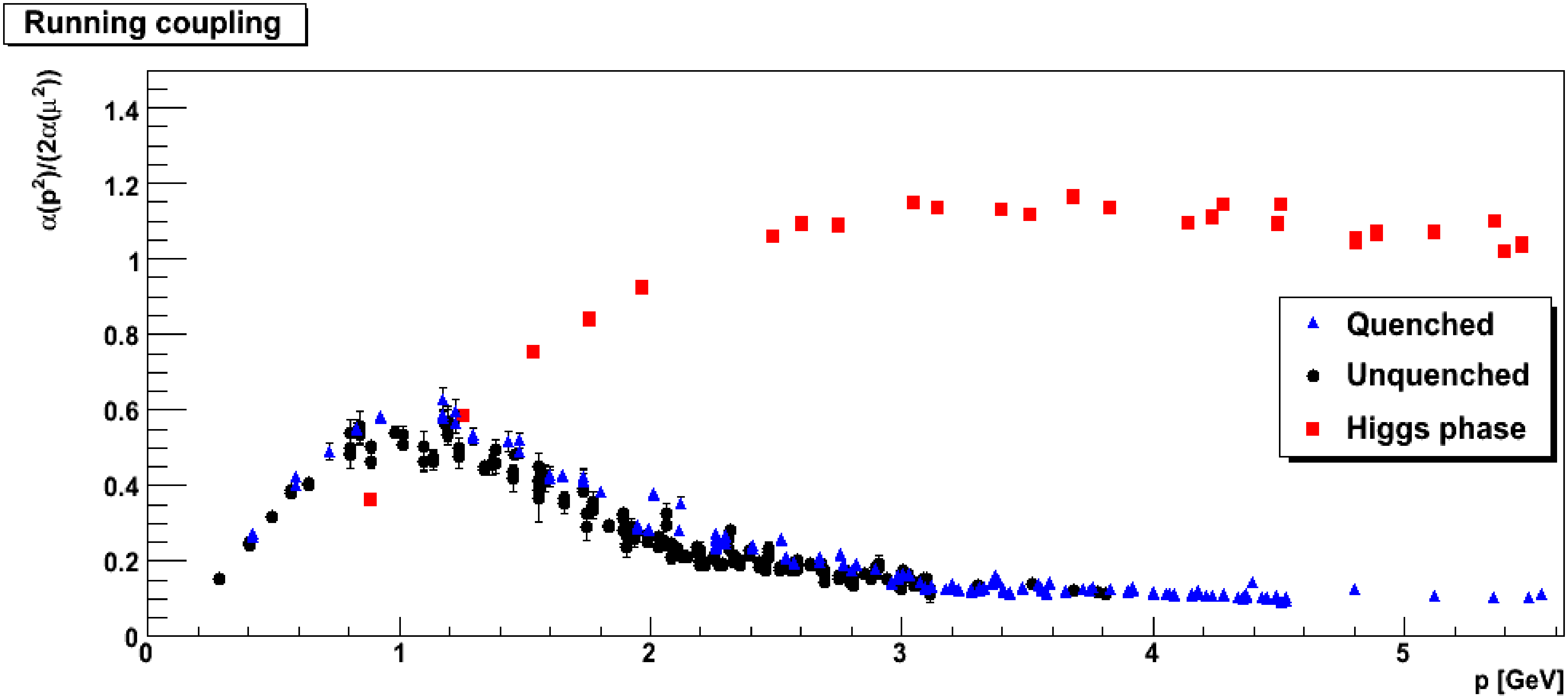

- Screening leads to suppression of interactions at small momenta
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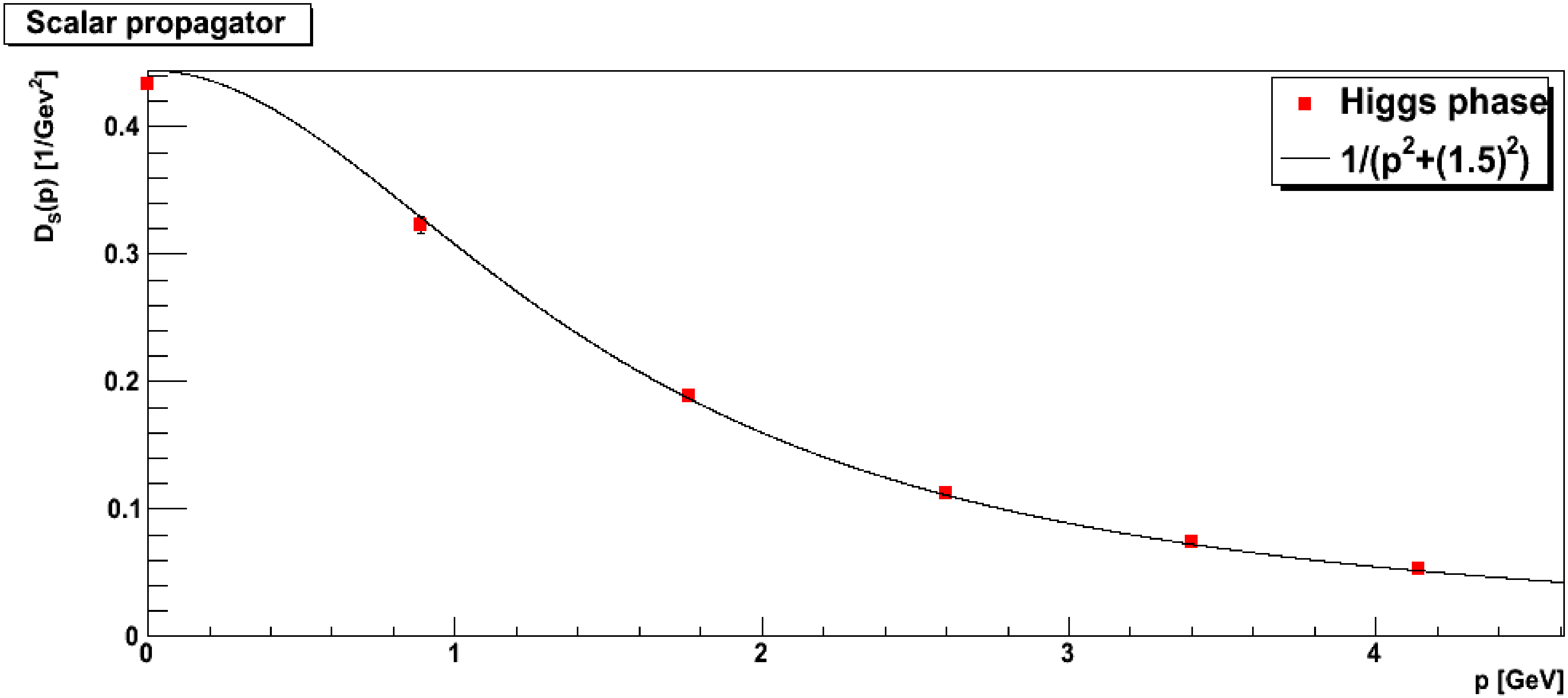

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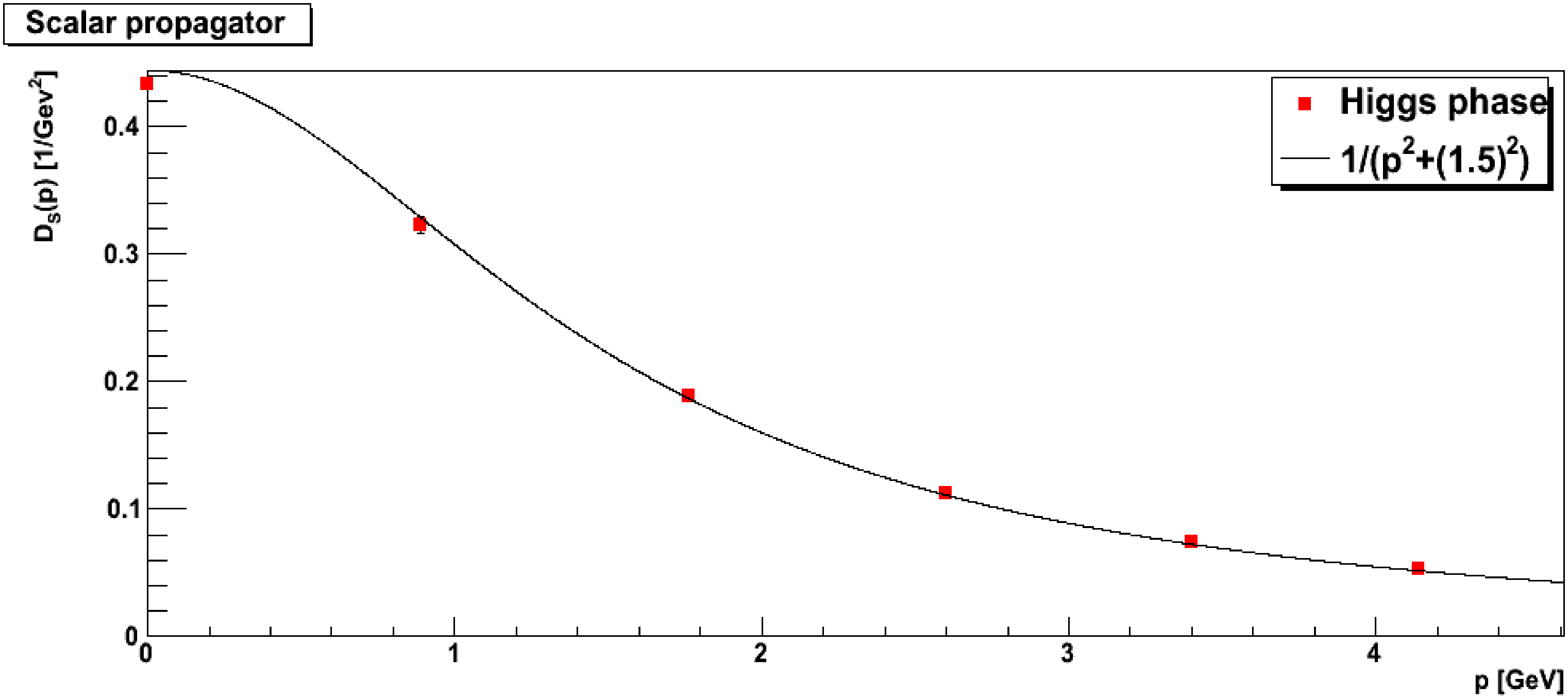

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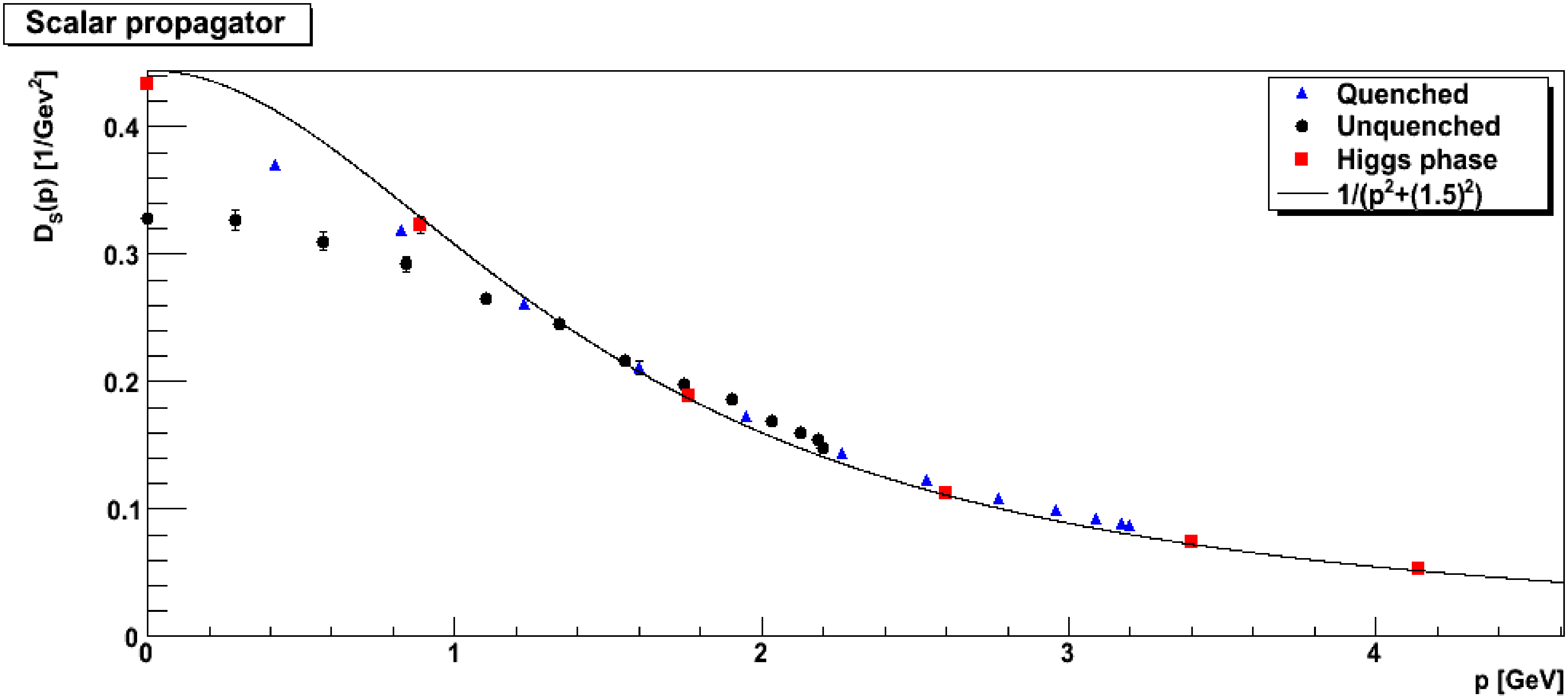

- Close to tree-level with positive mass squared
- Dynamical mass generation
- Little difference between the 'phases'

Summary from the Higgs

- Higgs effect is manifestly a quantum effect
  - Can be captured in some gauges using a mean-field+perturbation theory ansatz
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  - The choice is arbitrary, but something non-perturbative is always involved!

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- What is the consequence for color superconductors?

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  - What can be learned further from a comparison to the Higgs system – and where does the analogy ends?

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