Gluons, color superconductors, and gauge symmetry

Axel Maas

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Quarks, Gluons, and Hadronic Matter under Extreme Conditions
St. Goar
Germany
Color superconductors

\[ T \approx 170 \text{ MeV} \]

- hadron gas, \( \chi \)-symmetric
- quark-gluon plasma
- color superconductor
- deconfined, \( \chi \)-symmetric

\[ \mu, \mu_0 \]

few times nuclear matter density

[Refs: Karsch, Laermann, 2003]
Color superconductors

![Diagram showing phase transitions in color superconductors](image)

- Introductions
- Framework
- CSC
- Higgs
- Results
- Consequences
- Summary

[Karsch, Laermann, 2003]
Color superconductors

- Color superconductors are expected to form at large densities
  - Neutron star density?
  - Various types
    - 2SC, 3SC, CFL,...
    - Here generic color-neutral

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(fixed gauge: Landau,...)
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Lattice
Functional methods
Calcutational scheme

Quantization (fixed gauge: Landau,...)

Lattice Functional methods

Gluon propagator for SU(3)
Calculational scheme

Quantization (fixed gauge: Landau,...)

Gluon, Color Superconductors, and Gauge Symmetry/Axel Maas

Lattice Functional methods

Running coupling in four dimensions

[Maas, unpublished]
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[Cucchieri, Maas, Mendes PRD 2008]
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[Fischer, Müller, Maas EPJC 2010]
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Mesons (, Baryons)

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Mesons (, Baryons) Phase diagram

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Running coupling in four dimensions

[Electric screening mass SU(2)]

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Phase diagram: Yang-Mills, Quenched QCD, QCD at low and high density (, Critical Phenomena)
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  • What happens in the full quantum gauge theory?
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    • Yields masses for the gauge bosons
  • Perturbative expansion around the mean field
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The gauge-fixing trick

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    - Rewrite the scalar field as $\phi = \rho \exp(i \tau^a \alpha^a)$
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  - (Renormalizable) 't Hooft gauge
    - Rewrite the scalar field as $\phi = \begin{pmatrix} \varphi^1 + i \varphi^2 \\ \eta + i \varphi^3 \end{pmatrix}$
    - Make the choice $\partial^\mu A_\mu^a = \begin{pmatrix} \eta \end{pmatrix} \varphi^a$
What is the trick?

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  • Preferred direction in color space
  • Direction of the Higgs field with non-vanishing expectation value
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• What happens for a different, manifestly symmetric choice?
Symmetric gauge choice

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- Mechanism: No direction of Higgs field preferred – global rotations 'wash out' expectation value
Consequences

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  - Technical: How can they be accessed?
  - Physical: How do both phases differ?
- Caution with the analogy
  - Quantum phase diagram of the Higgs case is non-trivial
  - Confinement and Higgs not completely separated
- Translation to color superconductors?

[Intricated diagram with labels: Higgs phase, Confinement, ~gauge coupling, Higgs mass]
Basic quantities

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- Gluon propagator

\[ D^{ab}_{\mu\nu}(x-y) = <A^a_\mu(x)A^b_\nu(y)> \]

\[ D_{\mu\nu}(p) = (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \frac{Z(p)}{p^2} \]

- Longitudinal part exactly zero
- All directions equivalent (no QED)
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  - **Gluon propagator**
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  - **Higgs propagator**
    \[ D_{S}^{ij}(x-y) = \langle \phi^+_i(x) \phi_j(y) \rangle \]
    - Includes would-be Goldstones
Gluons

4d, Higgs, $24^4 \beta=2.3$, $\kappa=0.32$, $\lambda=1.0$

Introduction – Framework – CSC – Higgs – Results – Consequences – Summary

Gluon propagator

$D(p) \text{[GeV}^2\]}

$\frac{1}{p^2 + (1 \text{ GeV})^2}$

Higgs phase
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- Pole mass yet unclear

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[Maas EPJC 2011]
Gluons

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  - Pole mass yet unclear
- Very similar to confinement – and quenched
  - Changes in the gauge-fixing (ghost) sector
Running coupling

\[ \frac{\alpha(p^2)}{2\alpha(\mu^2)} \]

- Screening leads to suppression of interactions at small momenta

4d, Higgs, 24^4 beta=2.3, kappa=0.32, lambda=1.0

[Maas EPJC 2011]
Running coupling

- Screening leads to suppression of interactions at small momenta
- No qualitative difference seen
Scalar

Scalar propagator

$D_s(p) [1/\text{GeV}^2]$

$p [\text{GeV}]$

$4d, \text{Higgs, } 24^4 \beta=2.3, \kappa=0.32, \lambda=1.0$

Higgs phase

$1/(p^2+(1.5^2))$
Scalar

Scalar propagator

- Close to tree-level with positive mass squared
- Dynamical mass generation

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[Maas EPJC 2011]
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[Maas EPJC 2011]
Summary from the Higgs

- **Higgs effect is manifestly a quantum effect**
  - Can be captured in some gauges using a mean-field+perturbation theory ansatz
  - In other gauges a full dynamical quantum effect
    - Mean field zero
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- What is the consequence for color superconductors?
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    - Calculations can be performed using Landau gauge without explicit condensates
What are the possible consequences?

- In the Higgs system the transition between different phases may be gauge-dependent [Caudy, Greensite PRD 2008]
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  - How is the phase structure in QCD at finite density made gauge-invariant?
  - How proceeds gluon screening?
  - What can be learned further from a comparison to the Higgs system – and where does the analogy ends?
Summary

- Gauge symmetry requires a careful treatment of color condensates
  - Color condensates are in general gauge-dependent
  - Elitzur's theorem implies constraints
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  - Even in Landau gauge