QCD at Finite Temperature and Density from Dyson-Schwinger Equations

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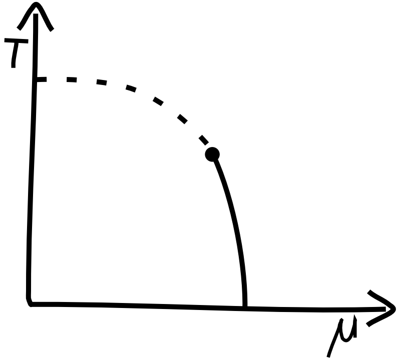
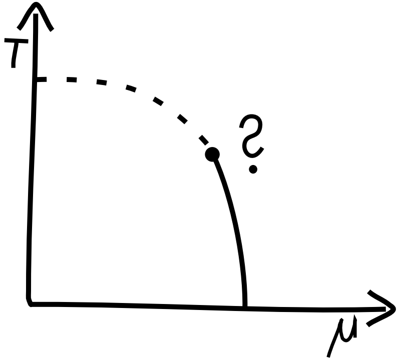
`Christian Fischer, Jan Lücker, Jens Müller in preparation`

Layout

Open questions about the QCD phase diagram

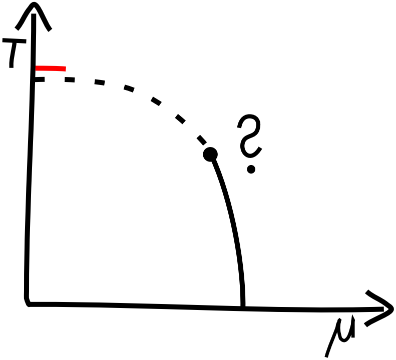


Open questions about the QCD phase diagram



- Is there a critical endpoint, and if so, where?

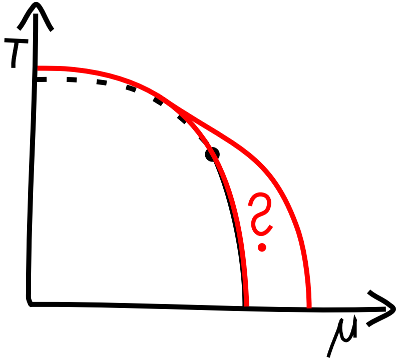
Open questions about the QCD phase diagram



- Is there a critical endpoint, and if so, where?
- At $\mu = 0$ chiral and deconfinement transitions in the same regime[1,2]

[1] Aoki *et al.*, JHEP **0906** (2009)
[2] Bazavov *et al.*, PR**D80** (2009)

Open questions about the QCD phase diagram



- Is there a critical endpoint, and if so, where?
- At $\mu = 0$ chiral and deconfinement transitions in the same regime[1,2]
- Is that still true at all μ, or is there a quarkyonic phase?

[1] Aoki *et al.*, JHEP **0906** (2009)
[2] Bazavov *et al.*, PR**D80** (2009)

Why functional methods?

- Lattice QCD works only for small chemical potential $\rightarrow$ fermion sign problem, light quarks are expensive
- Model calculations ([P]NJL, [P]QM, ...) may miss features of QCD

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Functional methods (Functional Renormalisation Group[1,2], Dyson-Schwinger Equations[3,4])

⊕ Can be applied directly to QCD

⊕ Any quark mass possible

⊕ Finite density no problem

⊖ Truncation is necessary

[1] Gies, hep-ph/0611146, Pawlowski, hep-th/0512261
[2] Braun, Haas, Marhauser, Pawlowski, PRL **106** (2011)
[3] Alkofer, von Smekal, Phys. Rept. **353** (2001)
[4] Roberts, Schmidt, Prog. Part. Nucl. Phys. **45** (2000)

Layout

Motivation

Truncation scheme

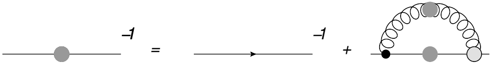
Order parameters

Results

Summary

Truncation scheme I

The quark DSE:



describes the fully dressed quark propagator

$$S^{-1}(\vec{p},\omega_n) = \mathrm{i}\,C(\vec{p}^2,\omega_n)(\omega_n + \mathrm{i}\mu)\gamma_4 + \mathrm{i}A(\vec{p}^2,\omega_n)\vec{p}\vec{\gamma} + B(\vec{p}^2,\omega_n)$$

Truncation scheme I

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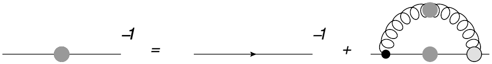

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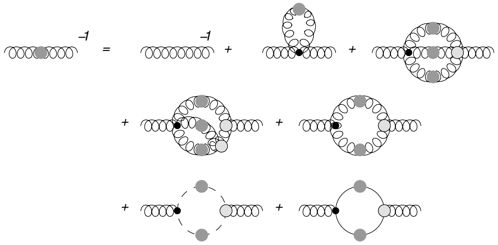
- needs fully dressed gluon
- and fully dressed quark-gluon vertex

Vertex Ansatz:

$$\Gamma_\mu(p,k;q) = \gamma_\mu \cdot \Gamma(q^2) \cdot \left(\delta_{\mu,4} \frac{C(p)+C(q)}{2} + \delta_{\mu,i} \frac{A(p)+A(q)}{2} \right)$$

Truncation scheme II

The gluon DSE:



Truncation scheme II

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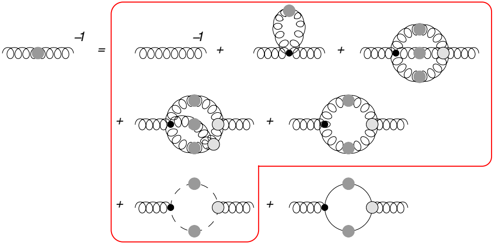
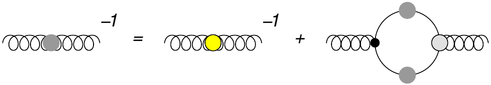


Truncation

⇓

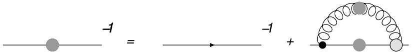


Quenched T-dependent gluon from lattice QCD[1]

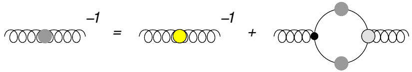
[1] C. S. Fischer, A. Maas and J. A. Mueller, Eur. Phys. J. C **68** (2010) 165

Truncation scheme III

Coupled system of equations



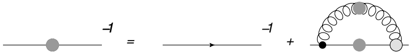
$$S(p)^{-1} = Z_2 S_0^{-1}(p) + Z_{1F} C_F g^2 \int_k \gamma_\mu S(k) \Gamma_\nu D_{\mu\nu}(k-p)$$



$$D_{\mu\nu}^{-1}(p) = D_{\mu\nu,quenched}^{-1}(p) + \frac{N_f Z_{1F} g^2}{2} \int_k \mathrm{Tr}\left[\gamma_\mu S(k) \Gamma_\nu S(k-p)\right]$$

Truncation scheme III

Coupled system of equations



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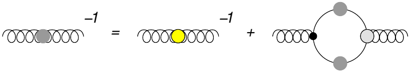

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. . . for now we rely on a Hard Thermal Loop (HTL) approximation of the quark loop, full system work in progress.

Layout

Order parameters 1: chiral symmetry

The chiral condensate is an order parameter for chiral symmetry breaking:

$$\langle\bar{\psi}\psi\rangle = \mathrm{Tr}[S]$$

- **large** for broken chiral symmetry
- **small** for (approximate) restoration of chiral symmetry
- we use physical quark masses $\Rightarrow$ crossover for $\mu = 0$

Order parameters 2: deconfinement I

The dual condensates[1,2,3]:

$$\Sigma_n = \int \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle \bar{\psi}\psi \rangle_\varphi$$

where $\langle \bar{\psi}\psi \rangle_\varphi$ is a condensate for shifted boundary conditions:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0) \quad \varphi \in [0, 2\pi]$$

[1] C. Gattringer, Phys. Rev. Lett. **97** (2006)
[2] F. Synatschke, A. Wipf, C. Wozar, Phys. Rev. **D75** (2007)
[3] E. Bilgici, F. Bruckmann, C. Gattringer, C. Hagen, Phys. Rev. **D77** (2008)

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- Σ_n corresponds to loops that wind n-times around the time direction
- Spatial fluctuations are included but $1/m$ suppressed

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Order parameters 2: deconfinement II

- $\Rightarrow \Sigma_{\pm 1}$ is the Polyakov loop for $m \to \infty$
- $\Sigma_{+1} \to$ dressed Polyakov loop
- $\Sigma_{-1} \to$ conjugated dressed Polyakov loop

Order parameters 2: deconfinement II

- $\Rightarrow \Sigma_{\pm 1}$ is the Polyakov loop for $m \to \infty$
- $\Sigma_{+1} \to$ dressed Polyakov loop
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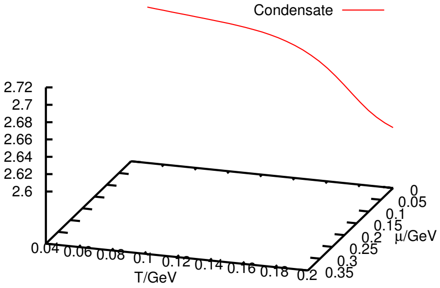
$\Rightarrow$ order parameters for confinement, accessible by functional methods

- **small** in the confined phase
- **large** in the quark-gluon plasma
- crossover, since finite quark masses are used

Layout

The chiral transition



- $N_f = 2$
- Crossover at small μ

C. Fischer, J. Müller, JL in preparation

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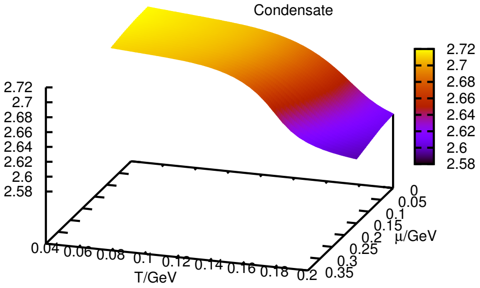

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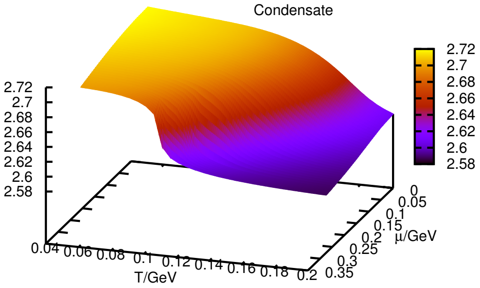

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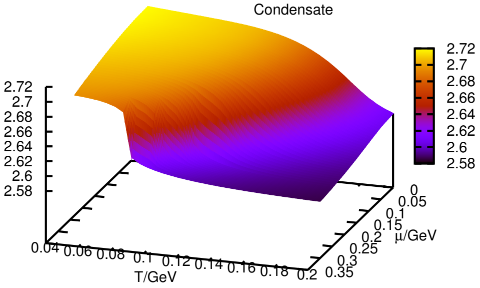

- $N_f = 2$
- Crossover at small μ
- 1st order at large μ

C. Fischer, J. Müller, JL in preparation

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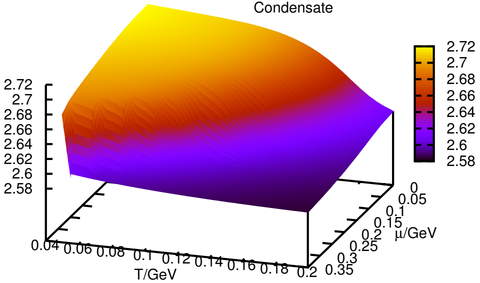

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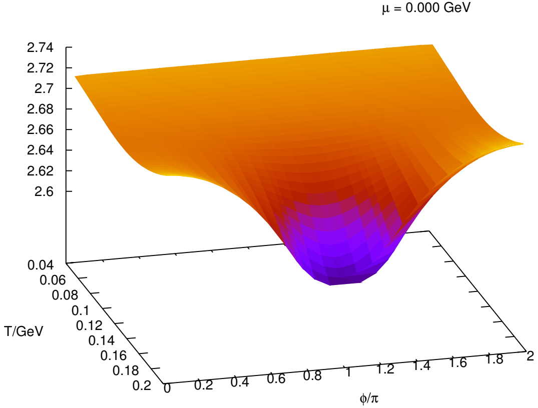
C. Fischer, J. Müller, JL in preparation

Deconfinement



C. Fischer, J. Müller, JL in preparation

- physical quark at $\varphi = \pi$
- nearly independent on φ for small T
- valley develops for growing T

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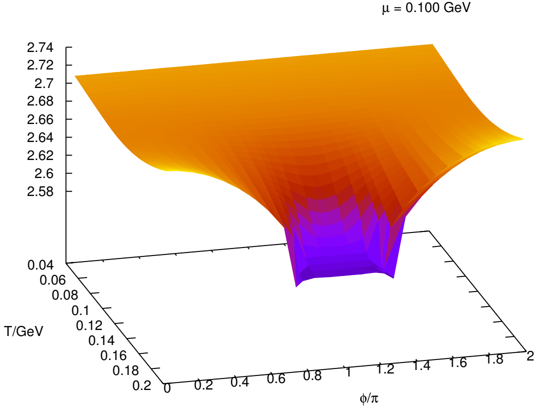

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- valley becomes steeper for growing μ

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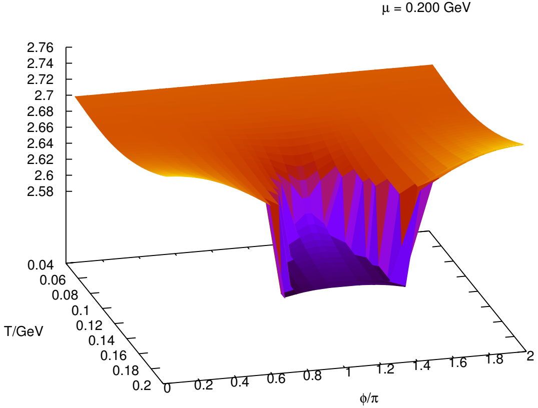


C. Fischer, J. Müller, JL in preparation

- physical quark at $\varphi = \pi$
- nearly independent on φ for small T
- valley develops for growing T
- valley becomes steeper for growing μ
- for large μ 1st order phase transition in φ

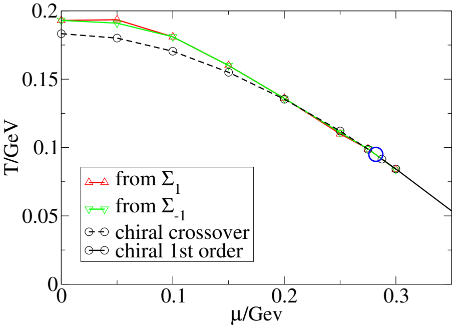
The phase diagram I

To calculate the resulting phase diagram take

- $\max\left[\frac{d\langle\bar{\psi}\psi\rangle}{dm}\right] \rightarrow$ chiral transition

- $\max\left[\frac{d\Sigma_1}{dm}\right] \rightarrow$ deconfinement transition (quarks)
- $\max\left[\frac{d\Sigma_{-1}}{dm}\right] \rightarrow$ deconfinement transition (antiquarks)

The phase diagram II



C. Fischer, J. Müller, JL in preparation

- Chiral and deconfinement transitions closely related
- $T_c(\mu = 0) \approx 183$ MeV
- CEP at $\mu \approx 280$ MeV $\Rightarrow \ \mu_c / T_c > 1$
- Consistent with PQM results[1]

[1]Herbst, Pawlowski, Schaefer PLB 696

Layout

Summary & Outlook

- Truncation scheme with explicit unquenching in the gluon propagator
- Critical end point at relatively large μ
- Chiral and deconfinement transition temperatures nearby, equal near CEP
  $\Rightarrow$ no sign of a quarkyonic phase so far

What next?

- Full quark loop
- Positivity violations and relation to deconfinement
- The area beyond the CEP $\Rightarrow$ need to evaluate the pressure
- Truncation: a better vertex construction, pion (baryon?) back couplings

Thank you for your attention!