Quarkyonic Matter

Toru Kojo (RBRC)

Collaborators:

Y. Hidaka, L. McLerran, R.D. Pisarski, A.M. Tsvelik

based on works:

		with
▪ Quarkyonic Chiral Spirals (QCS)	**NPA843: 37 (2010)**	Hidaka, McLerran, Pisarski
▪ Multiple QCSs	**PRD: 074015 (2010)**	Pisarski, Tsvelik
▪ Quarkyonic matter in (1+1) D	1104.xxxx …?	T.K.

Preface

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- for step by step arguments to construct concepts.

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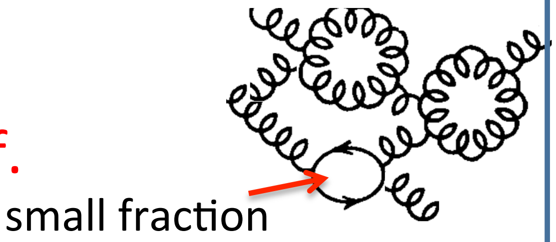
In vacuum: O(Nc) O(Nc^2)

Strength ~ Num. of d.o.f.



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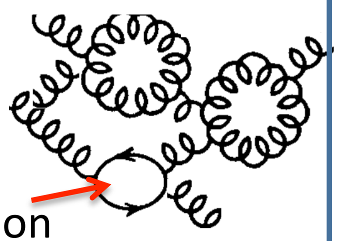
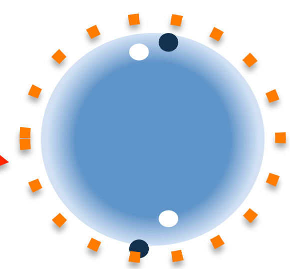


small fraction

In quark matter: O(Nc) $\times(\mu/\Lambda_{QCD})^{d-1}$ O(Nc^2)

(**d**: spatial dimensions)

phase space

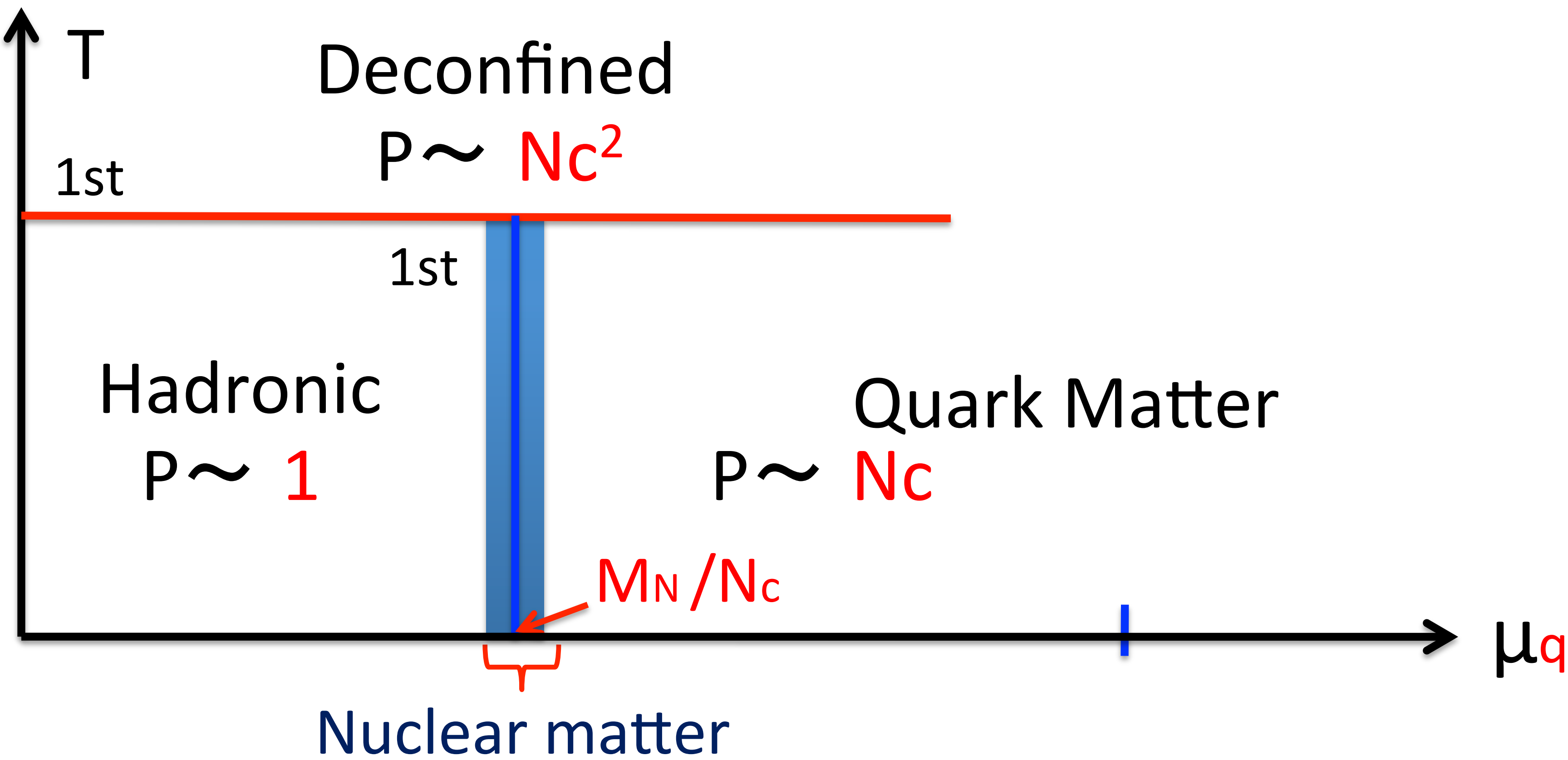


Quark Fermi sea

Large Nc Phase Diagram : McLerran & Pisarski (2007)

(2-flavor)

(ChSB is Not plotted yet!)



$\mu_B \sim N_c\, \mu_q \sim M_N + k_F^2/2M_N + \ldots$

small change in μ_q or μ_B → large change in kF thus $n_B \sim k_F^3$

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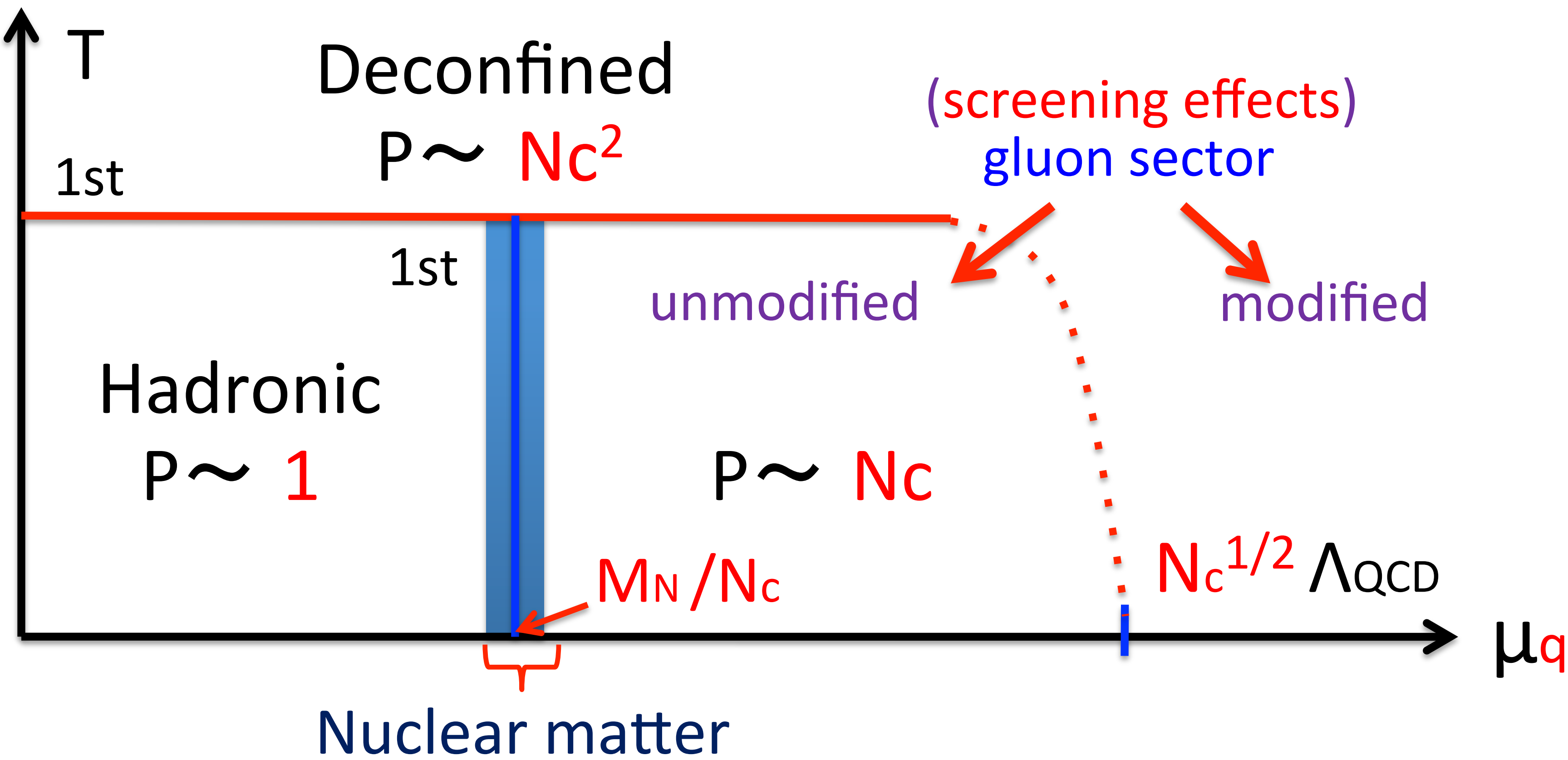

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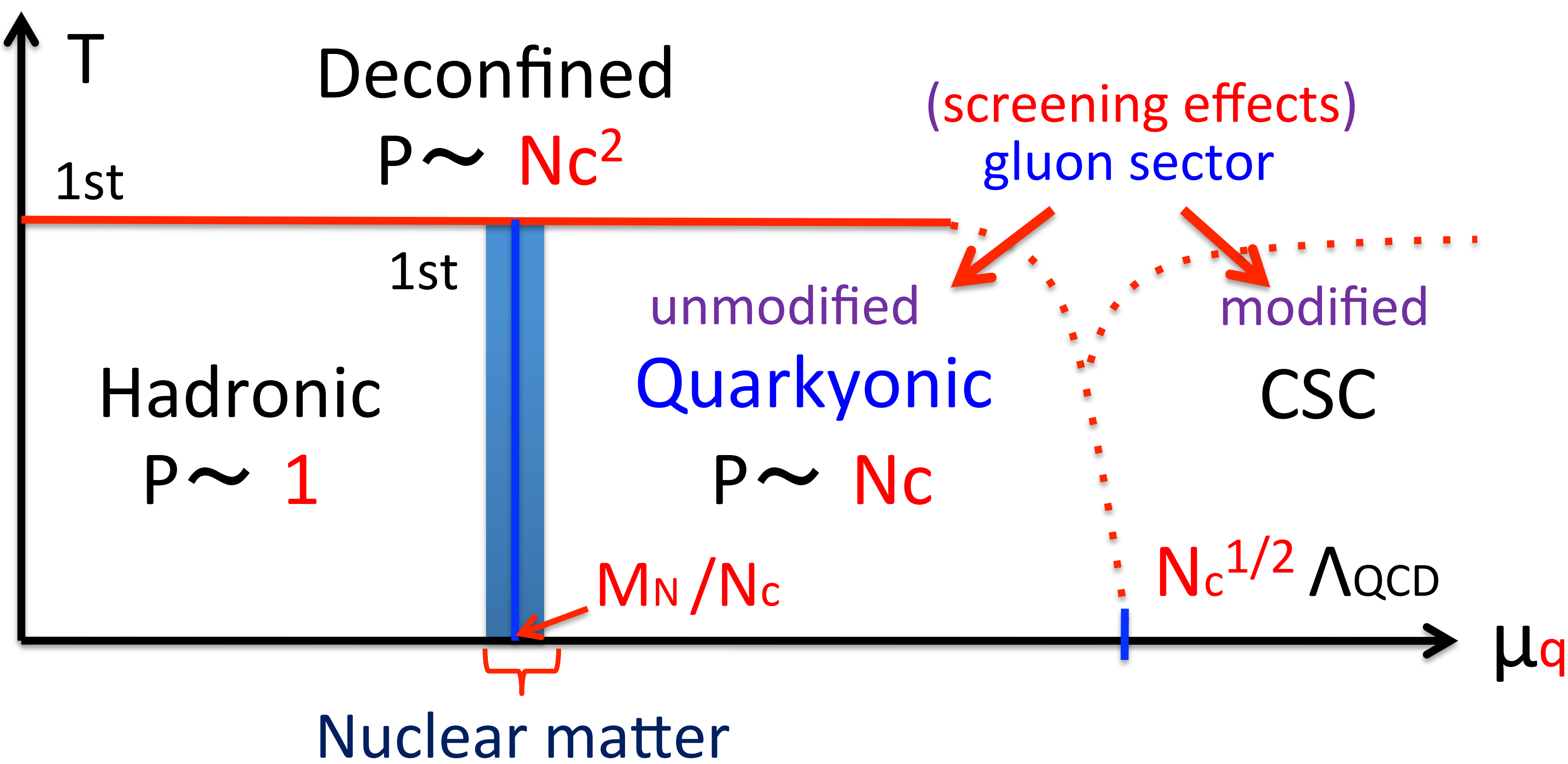

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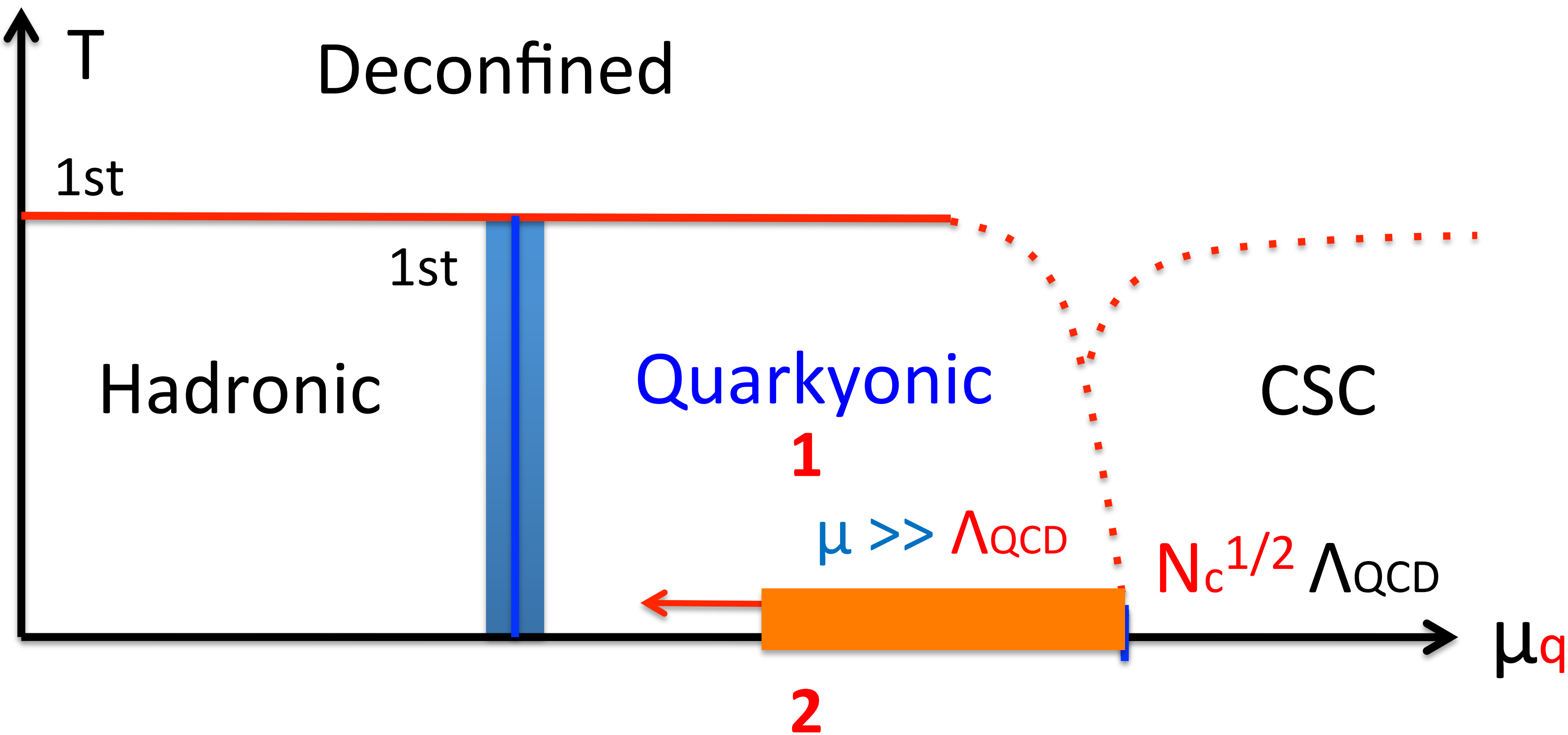
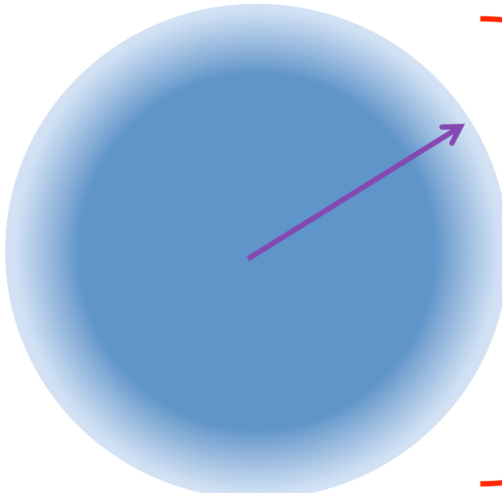


1, Basics of Quarkyonic Matter

2, Chiral symmetry in Quarkyonic Matter

Quarkyonic Matter



Bulk properties:

(EOS, Pressure, etc.)

perturbative quarks

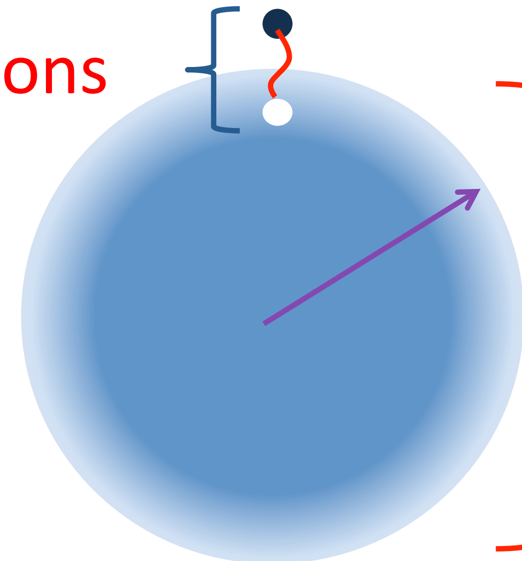
Pauli-blocking.

forming color singlet B.G.

Quarkyonic Matter

hadronic excitations

- Transport properties
- Phase structures (condensation)
- etc.



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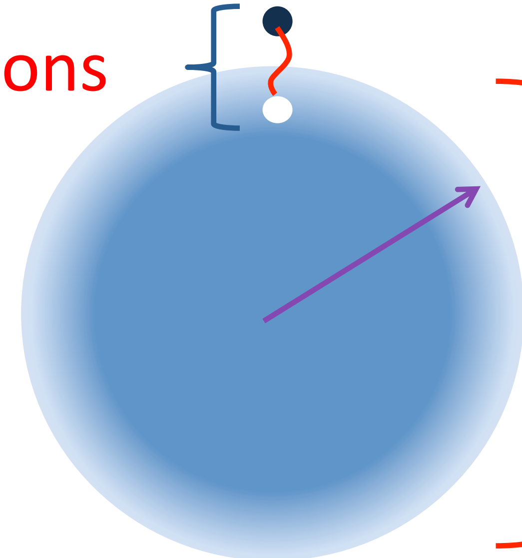

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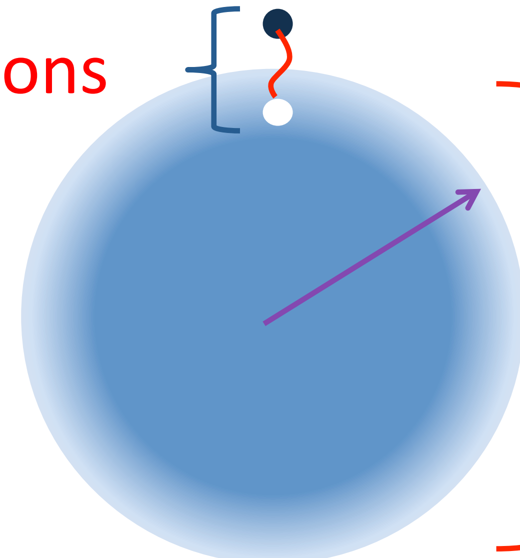

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- Gluon sector unchanged until **quark fluc. ~ gluon fluc.**
- Quarkyonic regime holds until $\mu_q \sim \begin{cases} N_c^{1/2}\,\Lambda_{QCD} & (3+1)\ \mathrm{D} \\ \infty & (1+1)\ \mathrm{D} \end{cases}$

(1+1) QCD, 1: Boring treatments

$$S = \int d^2x \; \bar{\psi}(x)(i\partial\!\!\!/ + \mu\gamma^0)\psi(x) + \int d^2x d^2y \; J^\mu_A(x) \underline{D^{AB}_{\mu\nu}(x-y)} J^\nu_B(y)$$

- Confining propagator: (axial gauge) $\quad D^{AB}_{\mu\nu}(x-y) = \delta^{AB}\delta_{\mu 0}\delta_{\nu 0}|\vec{x}-\vec{y}|$
- Properties of fermions in (1+1) D: $\quad \psi_\pm = \frac{1 \pm \underline{\gamma_0\gamma_z}}{2}\psi = \frac{1 \pm \Gamma_5}{2}\psi$

moving direction → Chirality

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- **Finite density problem can be mapped onto vacuum one:**

Choosing a basis: $\psi = e^{i\Gamma_5 \mu z}\psi' = e^{\pm i\mu z}\psi'_\pm$

$$\bar{\psi}(x)(i\partial\!\!\!/ + \mu\gamma^0)\psi(x) \rightarrow \bar{\psi}'(x) i\partial\!\!\!/ \psi'(x)$$

$$J^\mu_A(x) \rightarrow J'^\mu_A(x)$$ (Color current **unchanged**)

Model at arbitrary high density is also confining

(1+1) QCD, 2 : Charge-Color separation

- Non-Abelian **Bosonization**:

U(1) free bosons & Wess-Zumino-Novikov-Witten action :

"Charge – Flavor – Color Separation"

$$S = \underbrace{S_{U(1)}[\phi]}_{\text{Quark num.}} + \underbrace{S^{flavor}_{k=N_c}[g]}_{\text{flavor}} + \underbrace{S^{color}_{k=N_f}[h]}_{\text{Color}}$$ **+ gauge int.**

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At finite μ

change　　　　unchange

- Free energy = Free Fermi sea + Vacuum energy

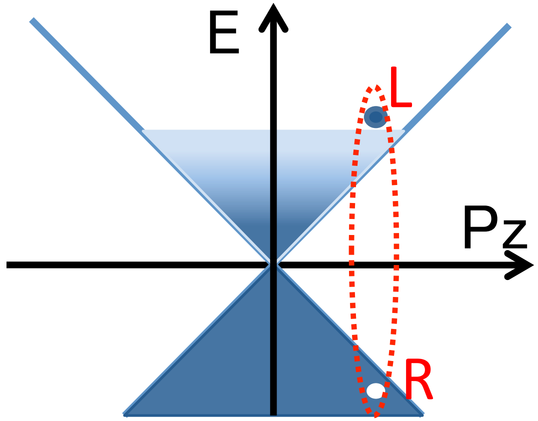
- **For confining aspects:**
  - Whether quarks are packed or not, is **Not a Real** issue.
  - **Phase space** for colored fluc. is relevant quantity.

(in 1+1 D, phase space unchanges at finite μ.)

How is Chiral Sym. realized in (3+1) D?

- Candidates which spontaneously break Chiral Symmetry

Dirac Type



$P_{Tot}=0$ (uniform)

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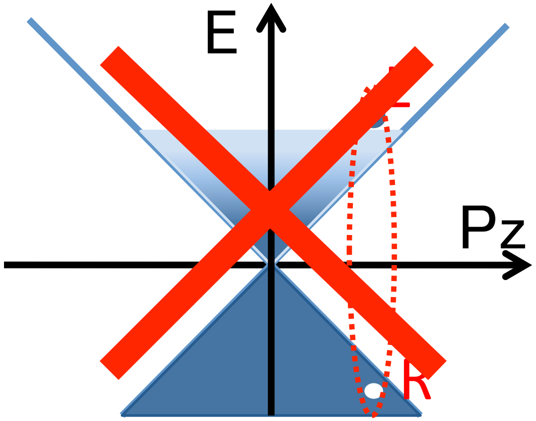

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It costs large energy,
so does not occur spontaneously.

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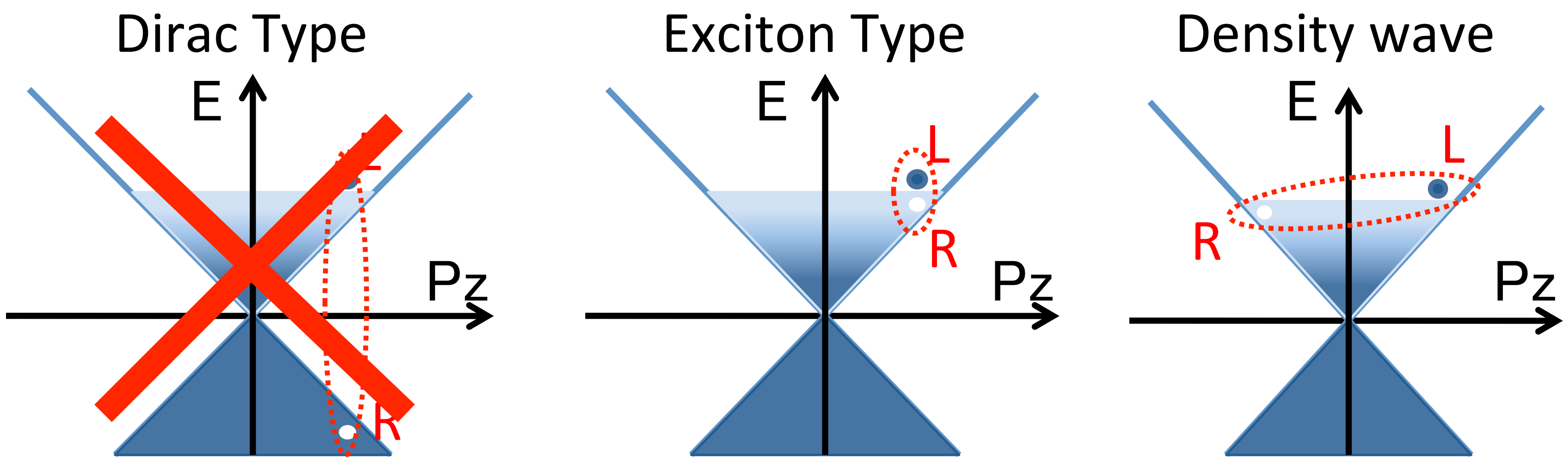

$P_{Tot}=0$ (uniform) $P_{Tot}=0$ (uniform) $P_{Tot}=2\mu$ (nonuniform)

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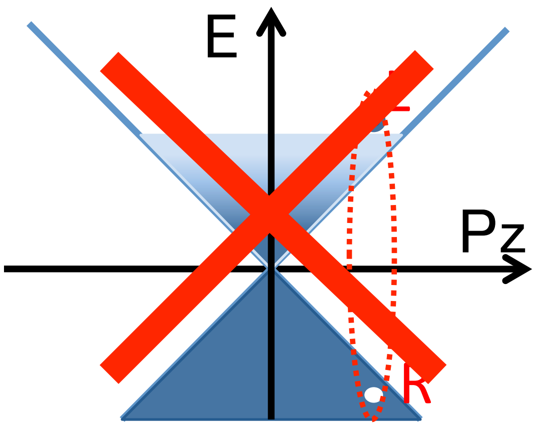

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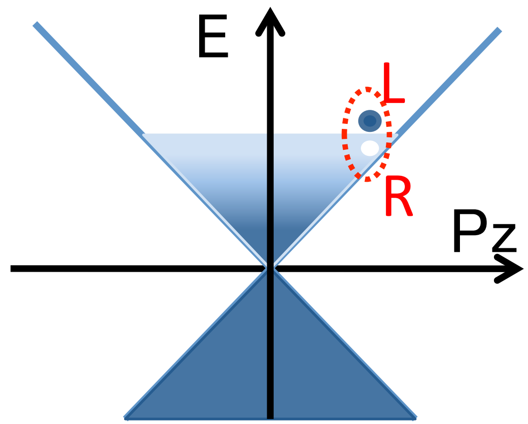
Exciton Type



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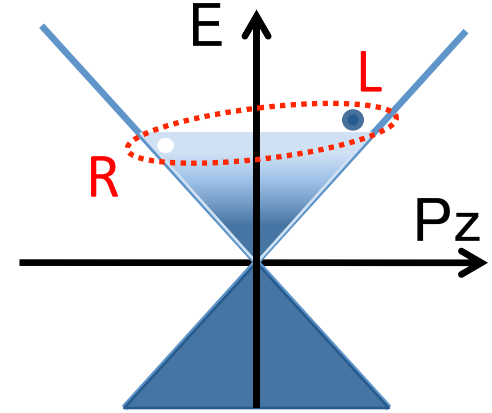
Density wave



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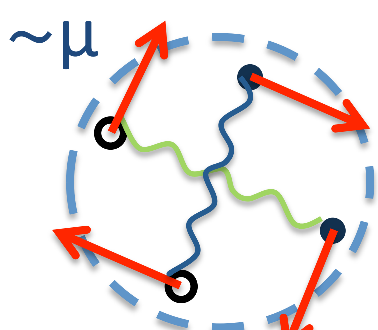
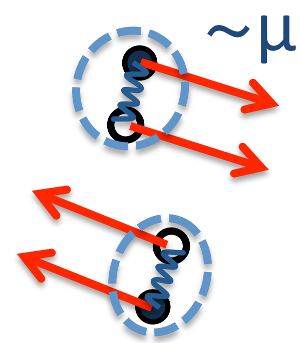
Coordinate space picture





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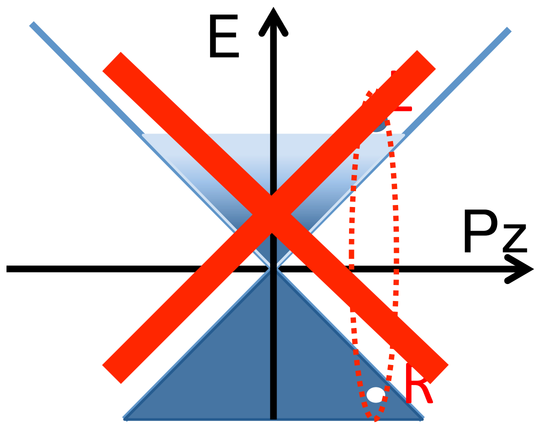

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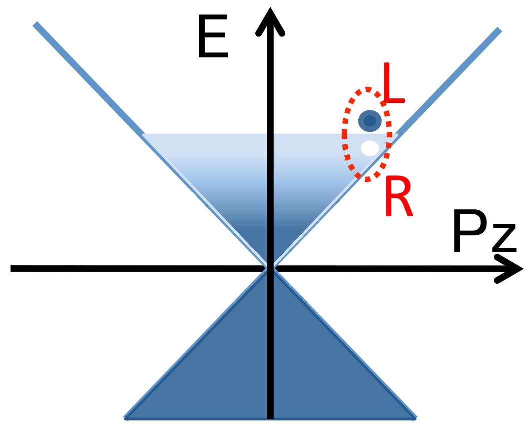

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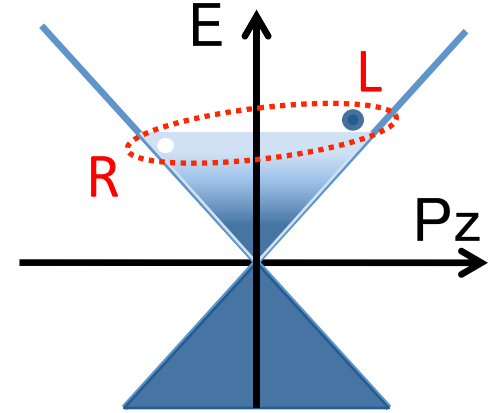

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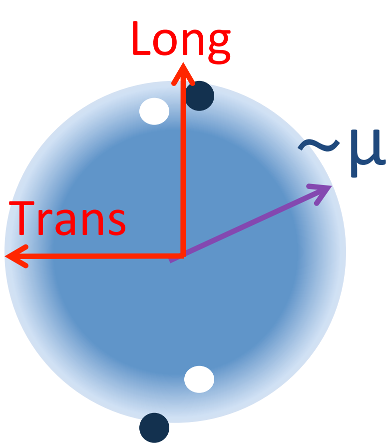

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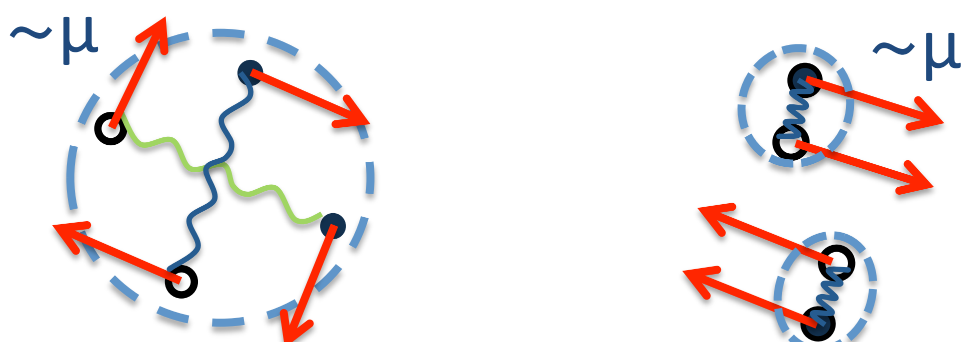

A simple model of linear confinement

- Confining propagator for quark-antiquark (quark-hole):

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$ (linear rising type)

strong IR enhancement

cf) leading part of Coulomb gauge propagator (ref: Gribov, Zwanziger)

Applications to finite μ : Glozman-Wagenbrunn08, Guo-Szczepaniak09

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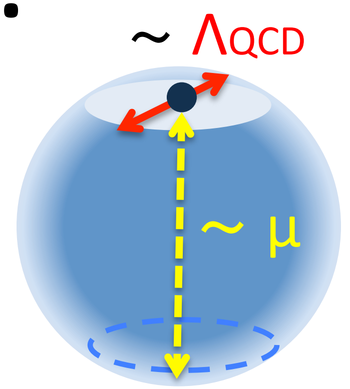
- We will apply nonperturbative treatments:

  Schwinger-Dyson & Bethe-Salpeter equations.

- We dimensionally reduce these from (3+1)D to (1+1)D.

  (Pert. regime; Deryagin-Grigoriev-Rubakov '92, Shuster-Son 99, etc)

Dim. reduction of integral eqs.



1, Virtual flucts. are limited within small mom. domain.

2, Quark energies are insensitive to small ΔkT.

(due to flatness of Fermi surface in trans. direction)

e.g.) Schwinger-Dyson eq.

insensitive to kT

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(k) \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

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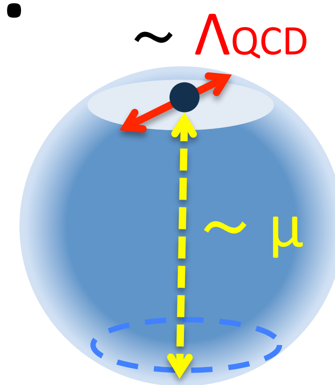

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factorization

$$\longrightarrow \int \frac{dk_4 dk_z}{(2\pi)^2} \gamma_4 S(k_4, k_z, \underline{\vec{0}_T}) \gamma_4 \otimes \underline{\int \frac{d\vec{k}_T}{(2\pi)^2} \frac{\sigma}{|\vec{p}-\vec{k}|^4}}$$

smeared gluon propagator

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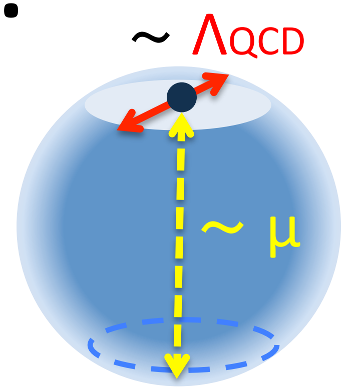

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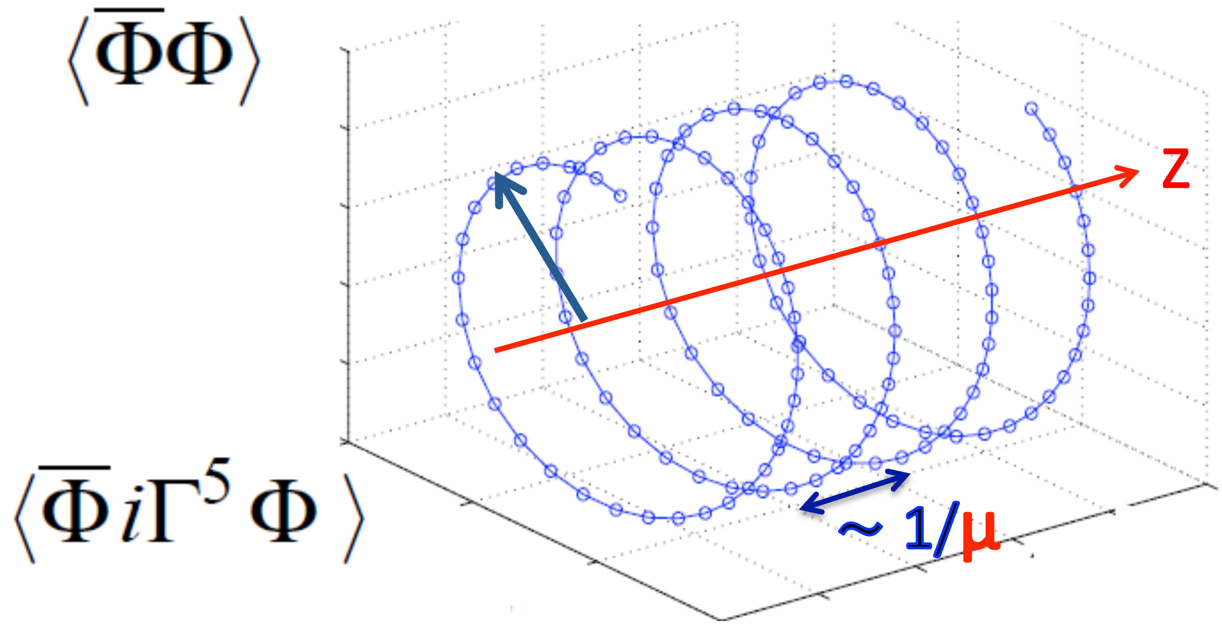
- At leading order:

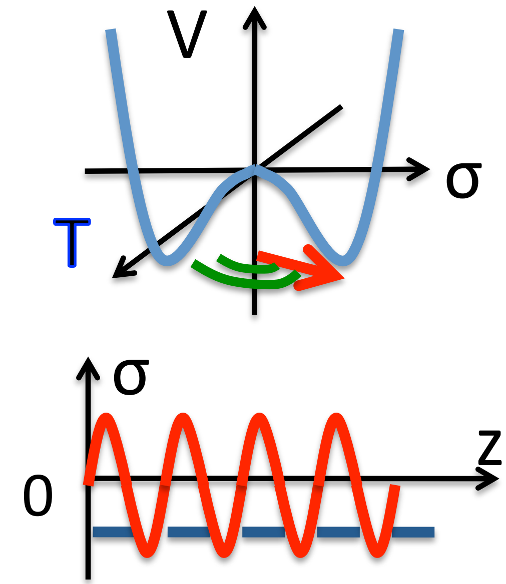
Dimensional reduction of Non-pert. self-consistent eqs :

4D “QCD” in Coulomb gauge $\longleftrightarrow$ 2D QCD in A_1=0 gauge

(confining model)

Solutions: Chiral Spirals in (1+1)D





cf) ▪ Chiral Gross Neveu model

Schon & Thies, hep-ph/0003195; 0008175; Thies, 06010243,....

Basar & Dunne, 0806.2659; Basar, Dunne & Thies, 0903.1868,...

▪ \`tHooft model, massive quark (1-flavor)

B. Bringoltz, 0901.4035,....

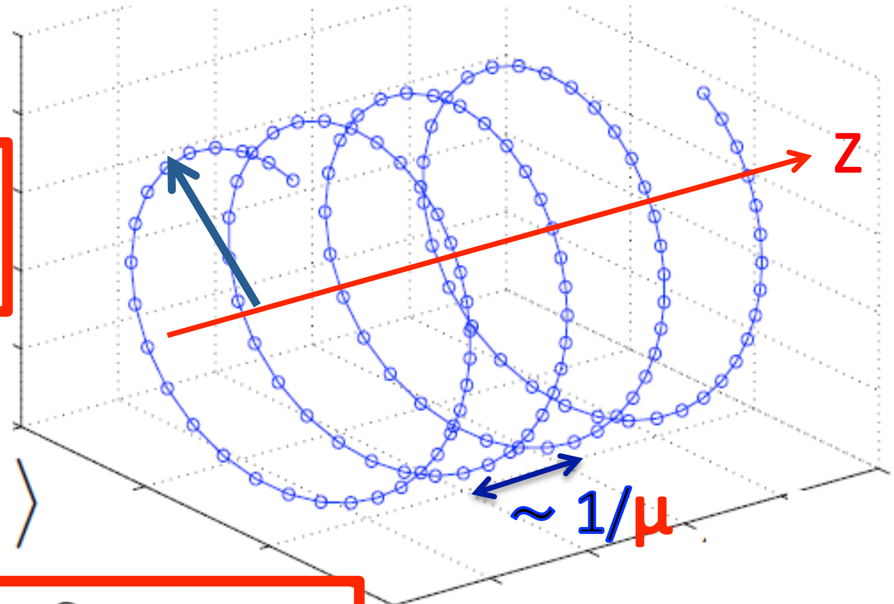
Quarkyonic Chiral Spirals in (3+1)D

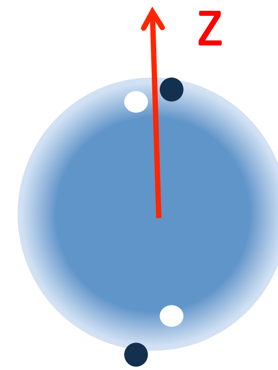
$\langle \bar{\Phi}\Phi \rangle$

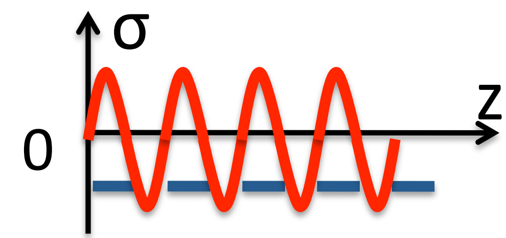
$\boxed{\langle \bar{\psi}\psi \rangle}$

$\langle \bar{\Phi} i\Gamma^5 \Phi \rangle$

$\boxed{\langle \bar{\psi}\gamma^0\gamma^z\psi \rangle}$







(characterized by moving direction)

- At leading order:
  - Baryon number is spatially constant.
  - No other condensates.

It is known that (1+1) QCD at finite μ leads chiral spirals, but **WHY**?

- Key observation: Moving direction = (1+1) D Chirality

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Opposite phase

$$\boxed{\langle \bar{\varphi} \Gamma_5 \varphi \rangle = \langle \bar{\varphi}_- \varphi_+ \rangle - \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta i \sin 2\mu z \neq 0}$$

Density wave of $\bar{\Psi}\Psi$ naturally accompanies $\bar{\Psi}\Gamma^5\Psi$

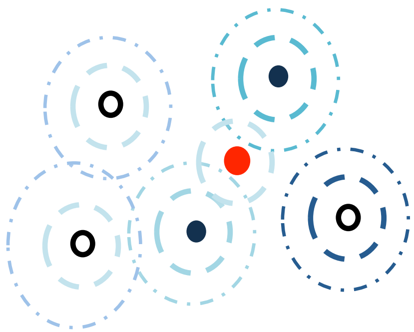
Toward multiple patch construction

One patch results may be good starting point.

(see also: Rapp, Shuryak, Zahed: hep-ph/0008207)

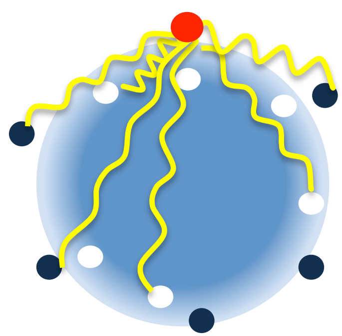
Perturbative gluons

- r - space)



Influence by all other quarks must be treated simultaneously.

- p - space)



Gap ⟶ strongly density dependent.

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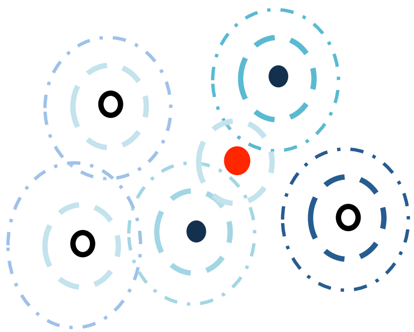

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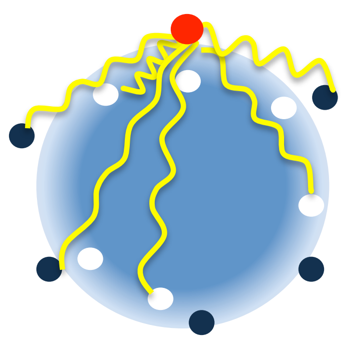
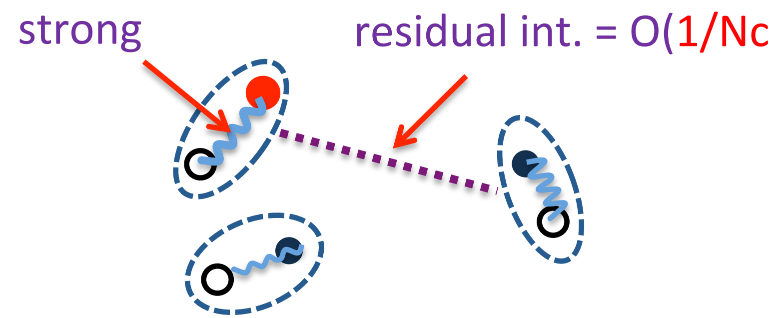
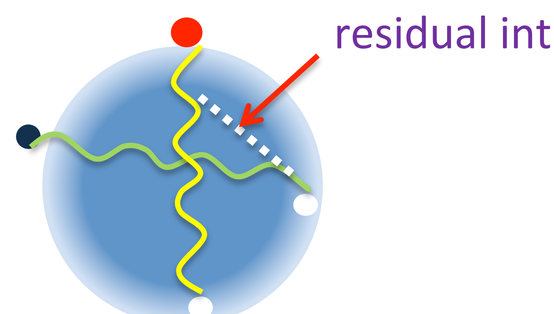


Gap → strongly density dependent.

Confining gluons

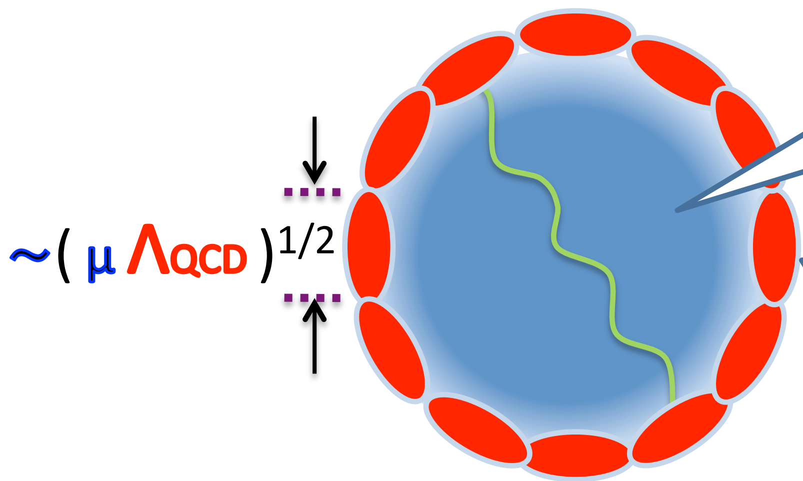


After mesonic objects are formed, residual interactions enter.



Gap → weakly density dependent.
(confinement - origin)

Summary about Chiral Symmetry



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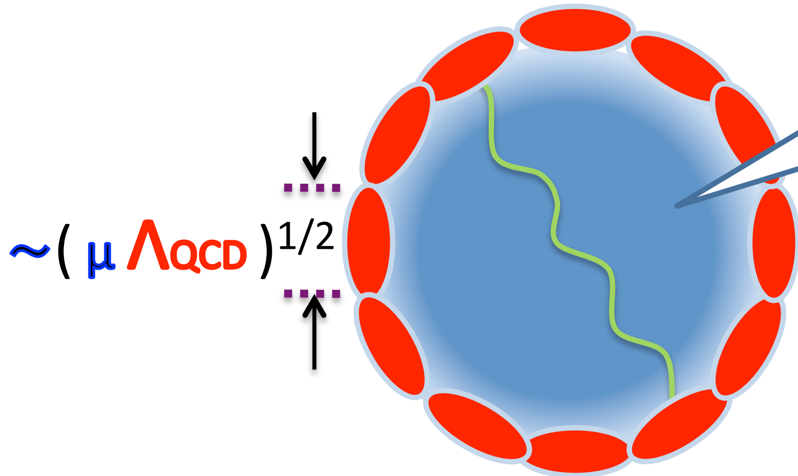

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  total surface area (μ^2), 1-patch area ($\mu\,\Lambda_{QCD}$)

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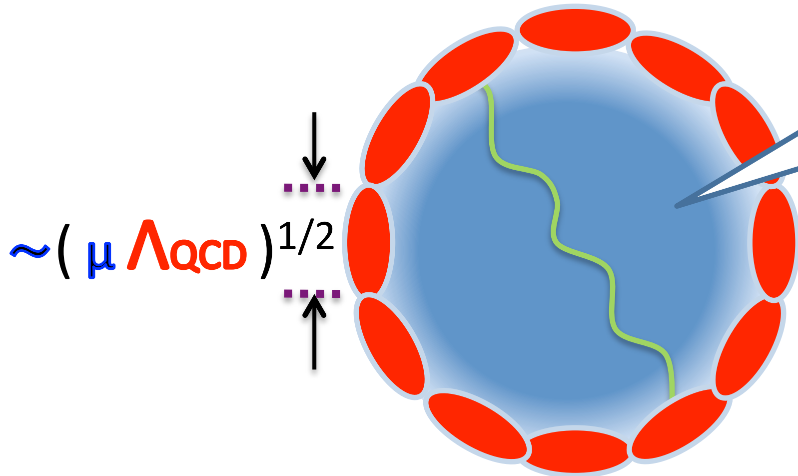

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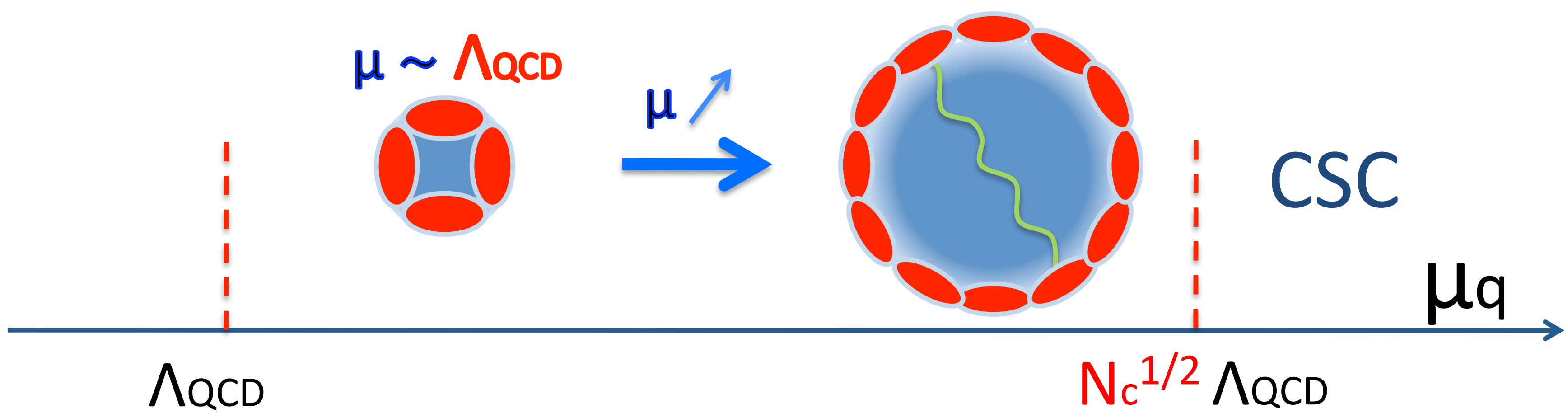

Outlook: Next Step ??

- **Conceptual part:**
  - Better understandings of multiple chiral spirals:
    Can it be smoothly connected to high density nuclear matter??
  - Convert **QCD scale** condensates → EW sectors, observables?

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- **Quantitative part:** (My personal To Do List)
  - Estimations of **width of Quarkyonic region**, quite tough, because:
  - 1, Confining force **at spatial level** is especially relevant.

    To take into account baryonic d.o.f.
    To avoid overcounting of quark phase space, entropy, flucts., etc.
  - 2, Quark energy **gaps from Fermi surface** are also relevant as well.
    (Quark dispersion under chiral spirals B.G.)

Appendix

Quarkyonic Chiral Spirals vs .....

1, Perturbative gluon propagator : Deryagin, Grigoriev, & Rubakov '92

- Scalar CDW (not spirals) was studied in large Nc, high density regime.
- Gaps are small, and reach $\sim \Lambda_{QCD}$ when $\mu \sim$ **100 GeV.**

2, + Screening effects : Shuster & Son 99 Park-Rho-Wirzba-Zahed 99

- Spirals (same structure as QCS) are found in large Nc.
- Screening mass develops faster than pert. gap, so no spirals in Nc=3.

3, Effective models : Nakano-Tatsumi 04, Nickel08, Carignano-Nickel-Buballa10 Ralf-Shuryak-Zahed01

- Relatively low density regime.
- CDW or CS or solitons in σ-π (not σ-Tensor) channels are studied.

4, Non-Perturbative gluon propagator :This work

- Spirals are studied in large Nc, relatively high density regime.
- gap is confinement origin $\sim \Lambda_{QCD}$ (>> perp. gap) ,
  it may be possible to have QCS before screening mass fully develops.

Projection of moving direction & Flavor doubling

- Proj. of moving direction: $\psi_{R\pm} = \dfrac{1 \pm \gamma^0\gamma^z}{2}\psi_R \quad \psi_{L\pm} = \dfrac{1 \pm \gamma^0\gamma^z}{2}\psi_L$

$$\mathscr{L}_{\rm kin}^{\rm lightcone} = i[\psi_{R+}^{\dagger}(\partial_0 + \partial_z)\psi_{R+} + \psi_{R-}^{\dagger}(\partial_0 - \partial_z)\psi_{R-}]$$ + (Left-handed)

(At leading order: no mixing terms of moving direction & chirality)

- Spin doublet ⟶ Flavor doublet in (1+1)D

$$\varphi_{\uparrow} = \begin{bmatrix} \varphi_{\uparrow +} \\ \varphi_{\uparrow -} \end{bmatrix} = \begin{bmatrix} \psi_{R+} \\ \psi_{L-} \end{bmatrix} \quad \varphi_{\downarrow} = \begin{bmatrix} \varphi_{\downarrow +} \\ \varphi_{\downarrow -} \end{bmatrix} = \begin{bmatrix} \psi_{L+} \\ \psi_{R-} \end{bmatrix}$$

moving direction
↕
(1+1) D Chirality

- Without spin mixing ⟶ Flavor singlet op. in (1+1)D

(Only 4-candidates)

$$\bar{\psi}\psi \to \bar{\Phi}\Phi, \quad \bar{\psi}\gamma^0\psi \to \bar{\Phi}\Gamma^0\Phi, \quad \bar{\psi}\gamma^z\psi \to \bar{\Phi}\Gamma^z\Phi, \quad \underline{\bar{\psi}\gamma^0\gamma^z\psi \to \bar{\Phi}\Gamma^5\Phi}$$

- With spin mixing ⟶ Flavor non-singlet op.

e.g.) $\bar{\psi}\gamma^5\psi \to \bar{\Phi}\tau_3\Gamma^5\Phi, \quad i\bar{\psi}\gamma^1\psi \to \Phi\tau_2\Gamma^5\Phi,$

Pert. of transverse curvature

- 1-D chains with transverse coupling:

$$H = \sum_{\boldsymbol{r}} H_{1D}(\boldsymbol{r}) + T_{tunn}$$

$\propto 1/\mu$

continuum limit:

$$\frac{1}{2p_F}(\nabla_\perp R^\dagger \nabla_\perp R + \nabla_\perp L^\dagger \nabla_\perp L)$$

- H_{1D} → Bosonization → "Charge-Flavor-Color separation"

U(1) free bosons (g, h: matrix field for flavored & colored bosons)

$$S = \underbrace{S_{U(1)}[\phi] + S^{flavor}_{k=N_c}[g]}_{\text{conformal}} + \underbrace{S^{color}_{k=N_f}[h] \ \text{+ gauge int.}}_{\text{dimensionful}}$$

- Trans. terms → expanded perturbatively and then resumed:

Massive sector (colored) → integrated out.

→ leaving only color singlet sectors.

Collective modes (near the center of patches)

・U(1): $$\mathcal{L}^{U(1)}_{k=N_c N_f'} = \frac{N_f' N_c p_F M}{8}\left[\, (\partial_L \Phi)^2 + \frac{\eta M}{p_F}(\partial_\perp \Phi)^2 \,\right]$$

・SU(2Nf): $$\mathcal{L}^{SU(N_f')}_{k=N_c} = \frac{N_c p_F M}{4}\left[\, \mathcal{L}_{WZW}[g] + \frac{\eta' M}{p_F}\mathrm{tr}[\partial_\perp g \partial_\perp g^\dagger] \,\right]$$

(momentum measured from Fermi surface) $M \sim \Lambda_{\mathrm{QCD}} \quad \eta,\ \eta' \sim 1$

・A number of Goldstone modes: $(4N_f^2 - 1) + \underline{1}$

(anomaly is not included yet)

・Decay constant ~ $(N_c\, \underline{\mu\, \Lambda_{QCD}})^{1/2}$

degeneracy in trans. direction

Interactions between Goldstone modes → O(1/Nc)

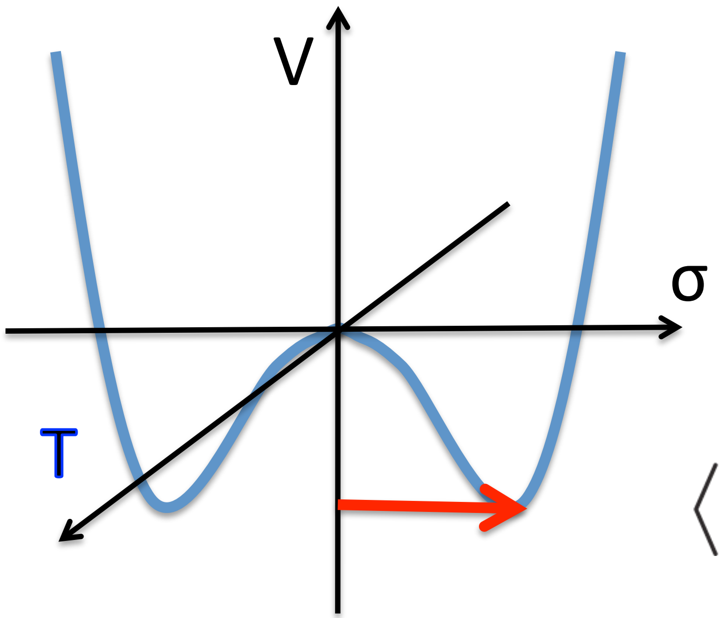
・Transverse dispersion is suppressed by Λ_{QCD} / μ .

System gets closer to quasi-long range order as μ increases.

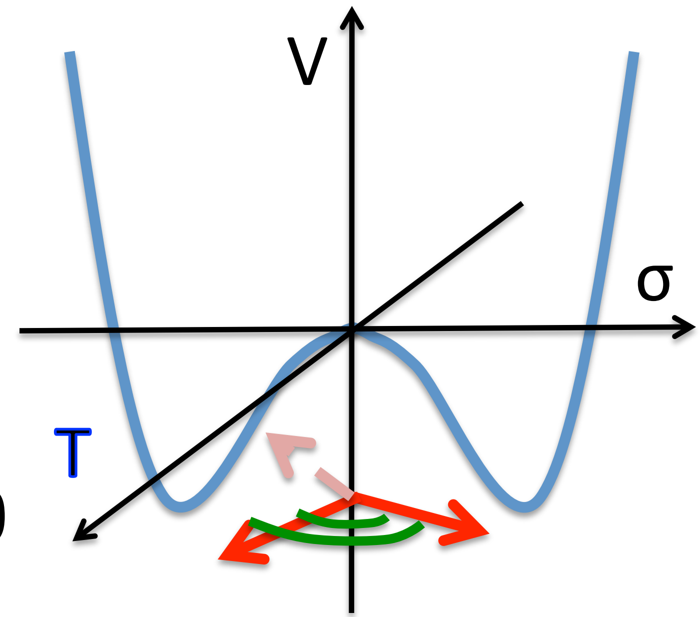
(fluctuations become more and more important in higher density)

Coleman's theorem ?

- Coleman's theorem: No Spontaneous sym. breaking in 2D



$\langle \rho \rangle \neq 0$



$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

Excitations (physical pion spectra)	ground state properties (No pion spectra)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes　　gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f}\phi} \rangle \otimes \langle \mathrm{tr} g \rangle \otimes \langle \mathrm{tr} h \rangle$$

due to IR divergent phase dynamics → 0　　0　　finite

But this does not mean the system is in the usual symmetric phase!

- Non-Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim \begin{cases} \cancel{e^{-m|x|}} & \text{: symmetric phase} \\ \langle \bar{\Psi}_+ \Psi_- \rangle^2 & \text{: long range order} \\ |x|^{\underline{-C/N_c}} & \text{: quasi-long range order} \end{cases}$$

(including disconnected pieces)

large Nc limit (Witten `78)

(power law)

On the IR prescription

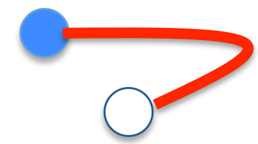
$$\frac{\sigma}{(\vec{k}^2)^2} \xrightarrow{\text{IR cut}} \frac{\sigma}{(\vec{k}^2+\Lambda_{\mathrm{IR}}^2)^2} \xrightarrow{\text{F.T.}} \underline{-\frac{\sigma}{\Lambda_{\mathrm{IR}}}} + \sigma r + O(\Lambda_{\mathrm{IR}} r^2)$$

linear potential

- Probe colored objects:

IR div.: const. from naïve IR cutoff

- Color singlet sector:



IR const. → irrelevant.
(Linear conf. without IR const.)

- As far as color-singlet sector is concerned,
  we can get the same results even if we drop off div. const.
  (principal value IR regulation; e.g., Coleman, Aspects of Symmetry)

- S-D eqs. → just sub-diagrams in B-S eqs.

- Div. of poles will be used as color selection rules at best.
  (Actually div. of poles may not be necessary condition: Callan-Coote-Gross76)

Toward multiple patch construction. 2

- e.g.) Quark propagator in the presence of many QCSs

Sum over all Chiral spirals

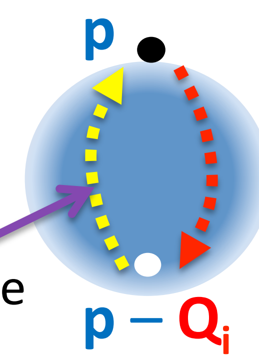
$$\sum_{i=1}^{N_p} \int \frac{d^4p}{(2\pi)^4} \bar{\psi}(p - \boldsymbol{Q}_i) M(\underline{p;} \, \boldsymbol{Q}_i) \psi(p)$$

Space-dependent mass self-energy

- Hypothesis: quarks with high virtuality feel small Chiral Sym. breaking

For both of p^2 and $(p - Q_i)^2$ to be close to Minkovski region:

Angle between **p** and **Q_i** $\longrightarrow$ $|\theta| < \Lambda_{QCD}/p_F$



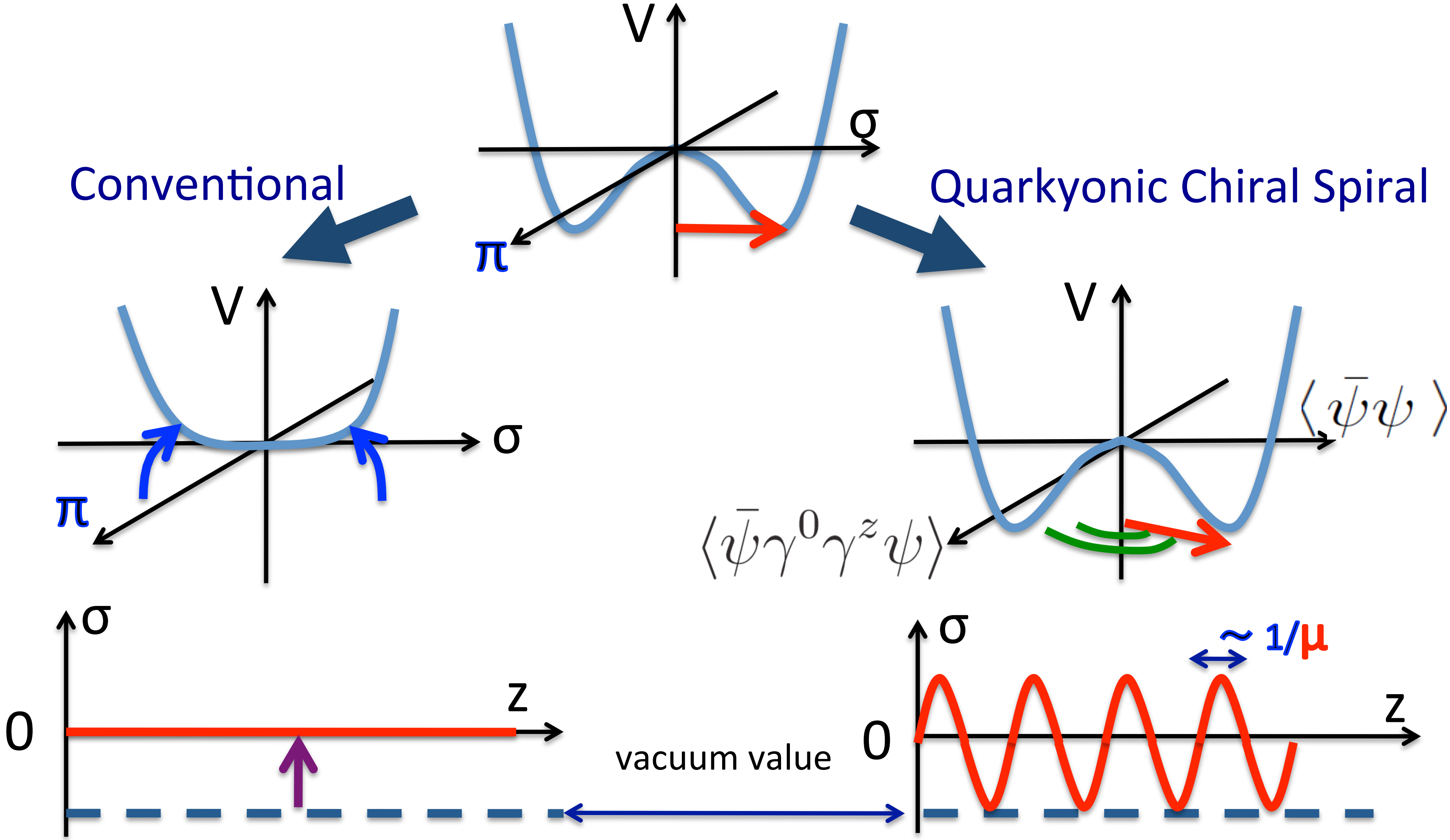
e.g.) θ ~ 0 case

If angles between quark moving direction and QCS are large:

$\longrightarrow$ Chirality changing scatterings are suppressed.

$\longrightarrow$ Each QCS behaves incoherently (except matching point of patches)

Chiral sym. breaking/restoration in (3+1)D



zero everywhere.

nonzero locally, but globally zero!

(cf: chiral sym. restoration in Skyrme model)