Finite-volume effects and the QCD phase diagram

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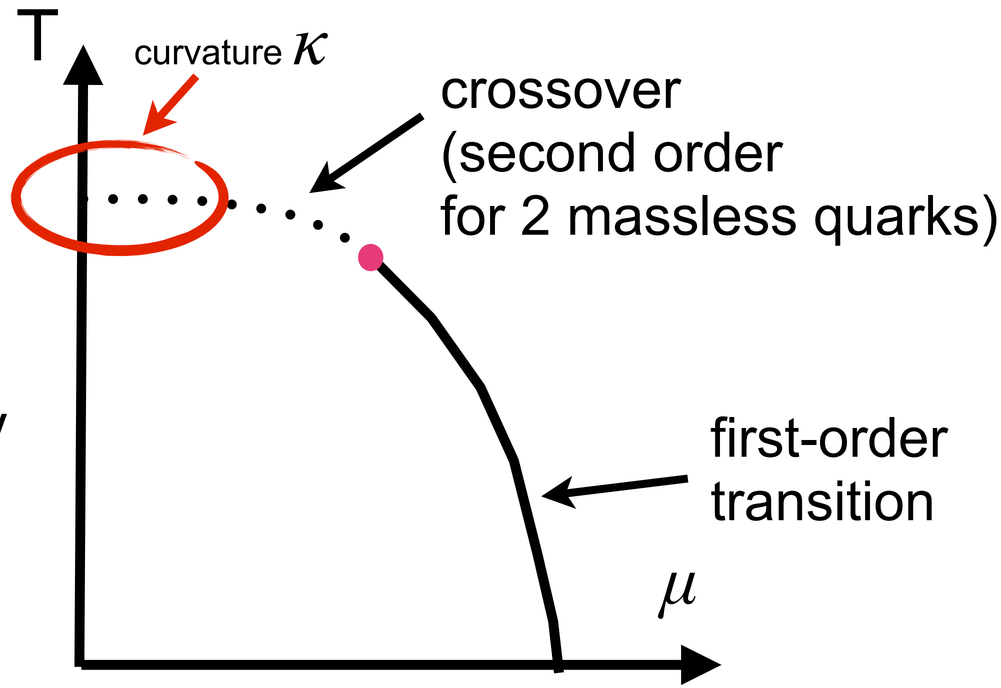
work done in collaboration with

- Jens Braun, Universität Jena
- Bernd-Jochen Schaefer, Universität Graz

[PoS Lattice (2010) 193, arXiv: 1011.1435 [hep-ph]]
[arXiv: 1104. xxxx [hep-ph]]

Transition at finite chemical potential

QCD phase diagram...
... seen very schematically



- second-order phase transition for two flavors in the chiral limit
  [R. D. Pisarski and F. Wilczek, Phys. Rev. D 29 (1984) 338]
- crossover at finite quark masses for finite temperature at $\mu = 0$
- conventional expectation:
  first-order phase transition with critical end point

Curvature of the transition line

- at small baryon chemical potential μ, the phase transition line is characterized by the curvature κ

$$\frac{T_\chi(L, m_\pi^*, \mu)}{T_\chi(L, m_\pi^*, \mu\!=\!0)} = 1 - \kappa \left(\frac{\mu}{(\pi T_\chi(L, m_\pi^*, 0))} \right)^2 + \dots$$

- "sign problem" in lattice QCD:
  simulations are difficult at finite μ
- curvature can be calculated in lattice QCD (imaginary chemical potential, Taylor expansion) [P. de Forcrand and O. Philipsen, Nucl. Phys. B 642 (2002) 290, JHEP 01 (2007) 077; F. Karsch et al., Nucl. Phys. Proc. Suppl. 129, 614 (2004).]

$\Rightarrow$ differences partially due to finite-volume effects?

Some results from lattice QCD and functional RG calculations

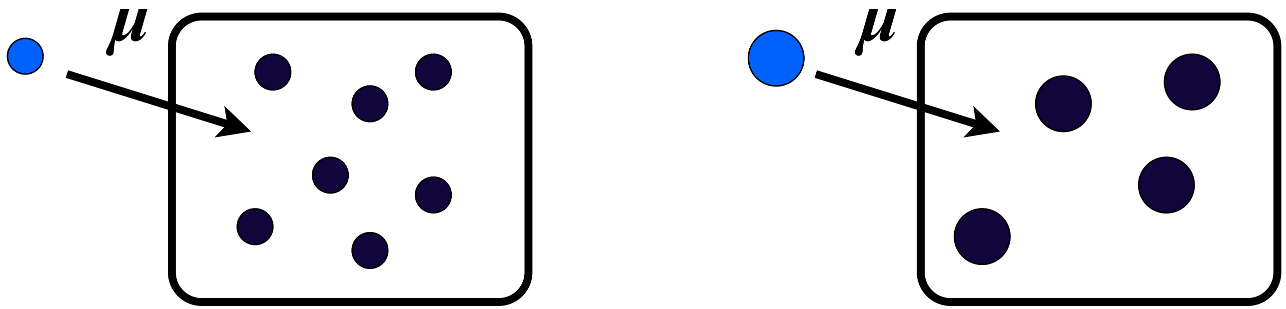
		N_f	am_c	κ
FRG (condensate)	[1]	1	0	1.13(15)
FRG (critical coupling)	[1]	1	0	0.44(4)
lattice, imaginary μ	[2]	2	0.032	0.500(54)
lattice, imaginary μ	[3]	3	0.026	0.667(6)
lattice, Taylor reweighting	[4]	3	0.005	1,13(45)
lattice, Taylor scaling	[5]	2+1		0.58(2)
lattice, Taylor	[6]	2+1		0.088(14)

[1] J. Braun, Eur. Phys. J. C64, 459 (2009); arXiv:0810.1727 [hep-ph].
[2] P. de Forcrand and O. Philipsen, Nucl. Phys. B642, 290 (2002), hep-lat/0205016. [8^3 x 4]
[3] P. de Forcrand and O. Philipsen, JHEP 01, 077 (2007), hep-lat/0607017. [8^3 x 4]
[4] F. Karsch et al., Nucl. Phys. Proc. Suppl. 129, 614 (2004), hep-lat/0309116. [12^3 x 4, 16^3 x 4]
[5] O. Kaczmarek et al., arXiv:1011.3130, Phys. Rev. D83 (2011) 014504. [32^3 x 8]
[6] G. Endrődi et al., arXiv:1102.1356. [28^3 x 10]

Why Finite-volume effects?

- curvature depends on the sensitivity of the system on the chemical potential

$$\mu = \left.\frac{\partial F}{\partial N_q}\right|_{T,V}$$



- sensitivity in turn depends on the "constituent quark mass"
- constituent quark mass affected by volume!

Quark-meson model for 2 flavors

- Model for chiral symmetry breaking with 2 quark flavors
- chiral symmetry SU(2) × SU(2) → SU(2) (quark sector) as O(4) → O(3) (meson sector)
- no gauge degrees of freedom

$$\Gamma_\Lambda[\bar{q}, q, \sigma, \vec{\pi}] = \int \mathrm{d}^4x \;\; \bar{q}(i\partial\!\!\!/)q + g\bar{q}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})q$$
$$\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U_\Lambda(\sigma, \sigma^2 + \vec{\pi}^2)$$

- specify effective action for the model at initial scale Λ
- use functional Renormalization Group (Wetterich equation) to obtain effective action, including fluctuations
  [C.Wetterich, Phys. Lett. B 301 (1993) 90.]

Renormalization Group calculation

- RG flow equation with 3*d* optimized cutoff function: change of effective potential with change of RG scale *k*

$$
\begin{aligned}
k\partial_k U_k(\phi^2) \quad = \quad k^5 \Bigg[& \frac{3}{E_\pi}\left(\frac{1}{2}+n_B(E_\pi)\right)\mathcal{B}_{\mathrm{p}}(kL) + \frac{1}{E_\sigma}\left(\frac{1}{2}+n_B(E_\sigma)\right)\mathcal{B}_{\mathrm{p}}(kL) \\
& -\frac{2N_cN_f}{E_q}\Big(1-n_F(E_q,\mu)-n_F(E_q,-\mu)\Big)\mathcal{B}_{\mathrm{p}}(kL)\Bigg]
\end{aligned}
$$

- volume dependence encoded in mode-counting functions

- depends on choice of boundary conditions (for the spatial directions)

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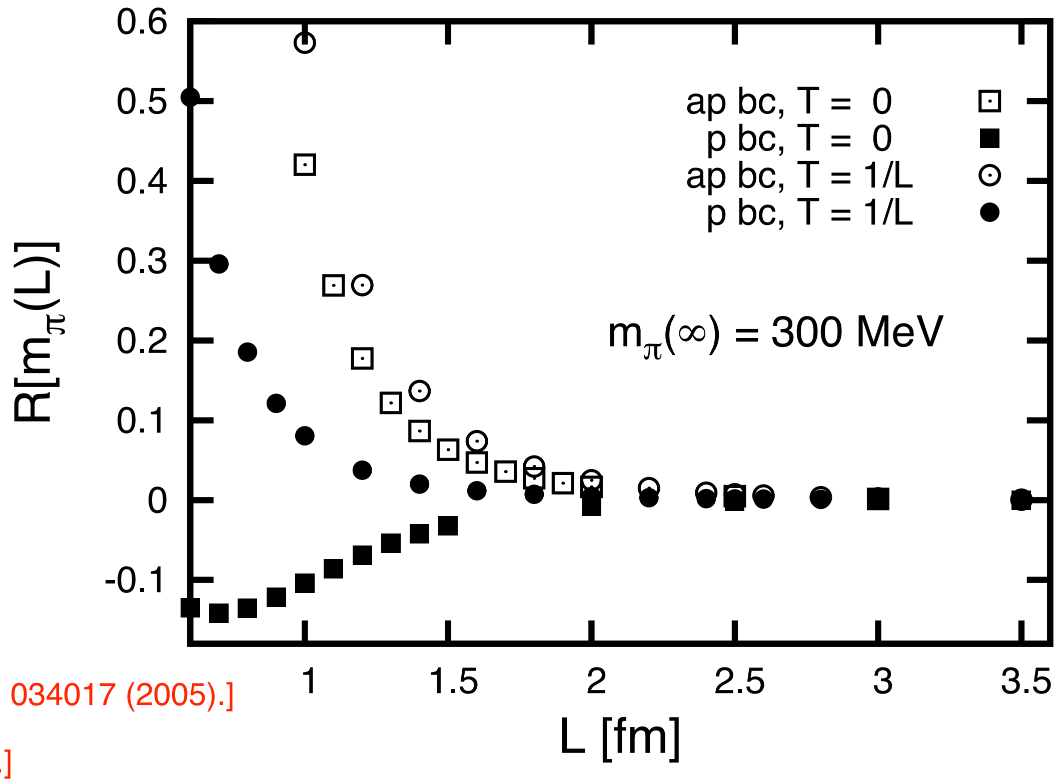
Quark-meson model results in finite volume

- Pion mass shift in V = $L^3 \times 1/T$ from the model
- periodic vs. anti-periodic quark boundary conditions (b.c.)

$$f_\pi \sim \langle \sigma \rangle$$

$$\langle \bar{\psi}\psi \rangle \sim \langle \sigma \rangle$$

$$m_\pi^2 = m \frac{\langle \bar{\psi}\psi \rangle}{f_\pi^2} \sim \frac{m}{\langle \sigma \rangle}$$

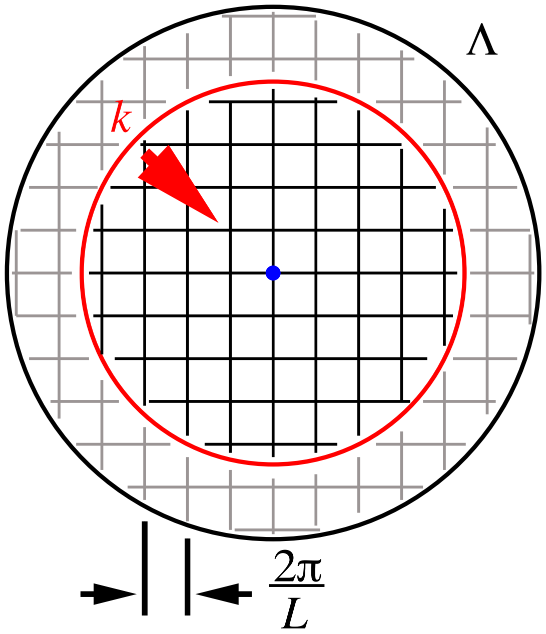


[J. Braun, B. Klein, H.-J. Pirner, Phys. Rev. D72, 034017 (2005).]

[J. Luecker et al., Phys. Rev. D81, 094005 (2010).]

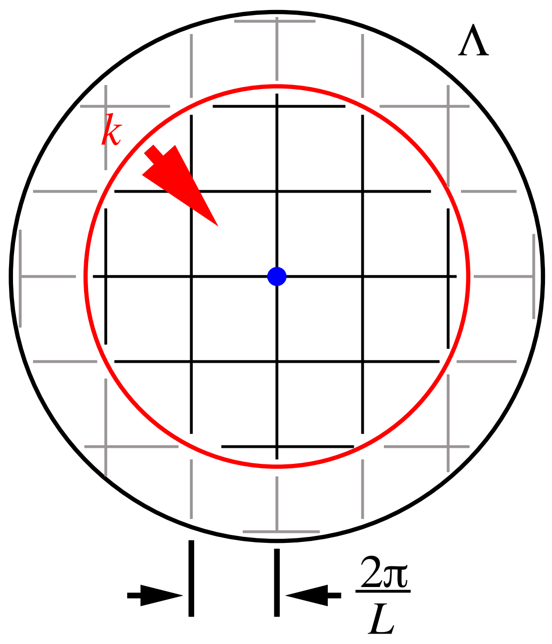
Quark contributions for a finite volume

- quark momentum modes contributing to the condensate (and the constituent quark mass) in a large finite volume
- zero-mode
  for **periodic** b.c.
- no zero mode
  for **anti-periodic** b.c.



Quark contributions for a finite volume

- quark momentum modes contributing to the condensate (and the constituent quark mass) in a small finite volume
- enhancement of the zero-mode contribution ~$1/V$ for **periodic** boundary conditions



Curvature in infinite volume

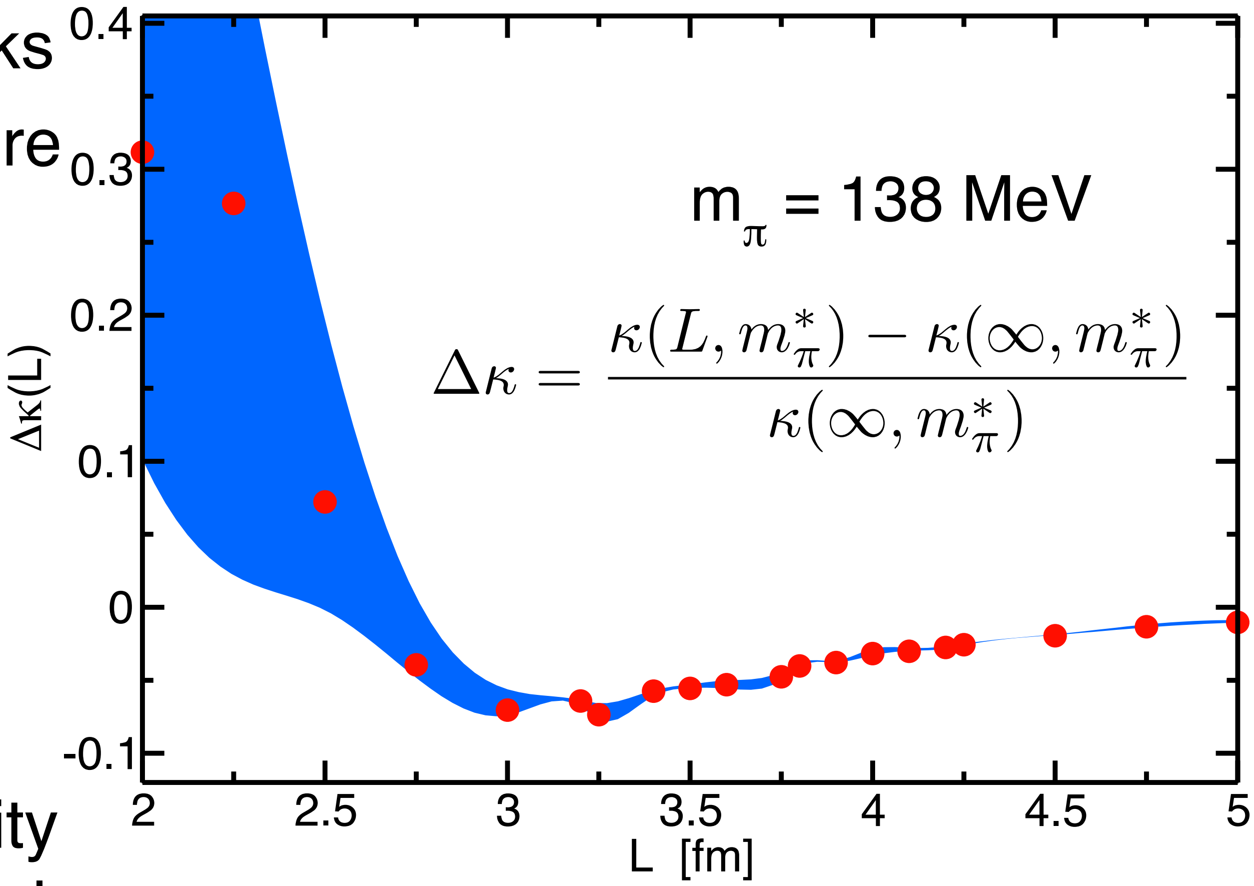
m_π [MeV]	100	150	200
$\kappa(L \to \infty)$	1.391	1.392	1.440
$T_\chi(L \to \infty)$	178 MeV		

- model: curvature increases slightly with pion mass (transition temperature sensitive!)

General observation: NJL-type models for chiral symmetry breaking tend to be more sensitive to changes in the pion mass than QCD

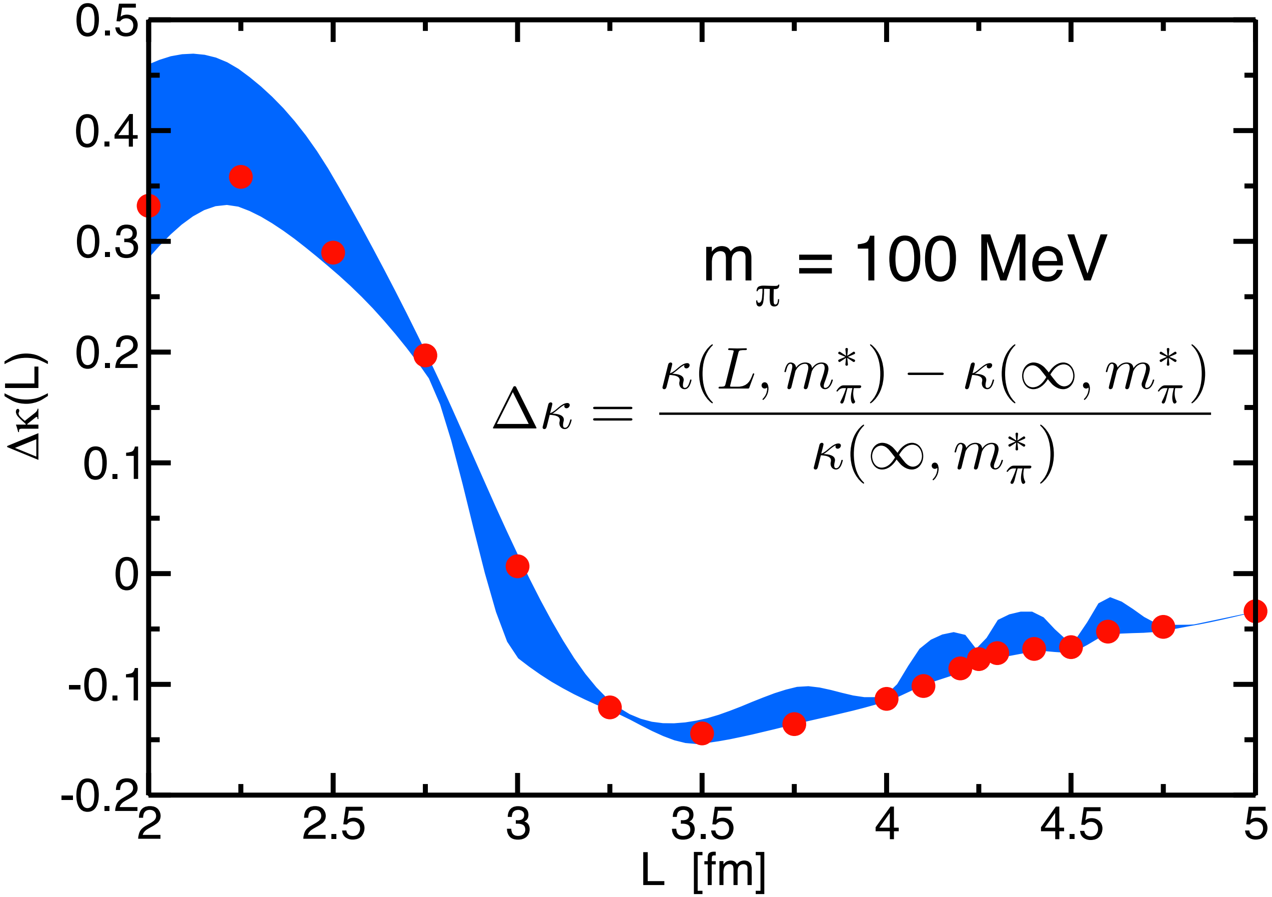
Change of curvature in finite volume

- **periodic** boundary conditions for quarks
- decreasing curvature in intermediate volume
- corresponds to decreasing pion mass/increasing constituent quark mass
- decreased sensitivity to chemical potential



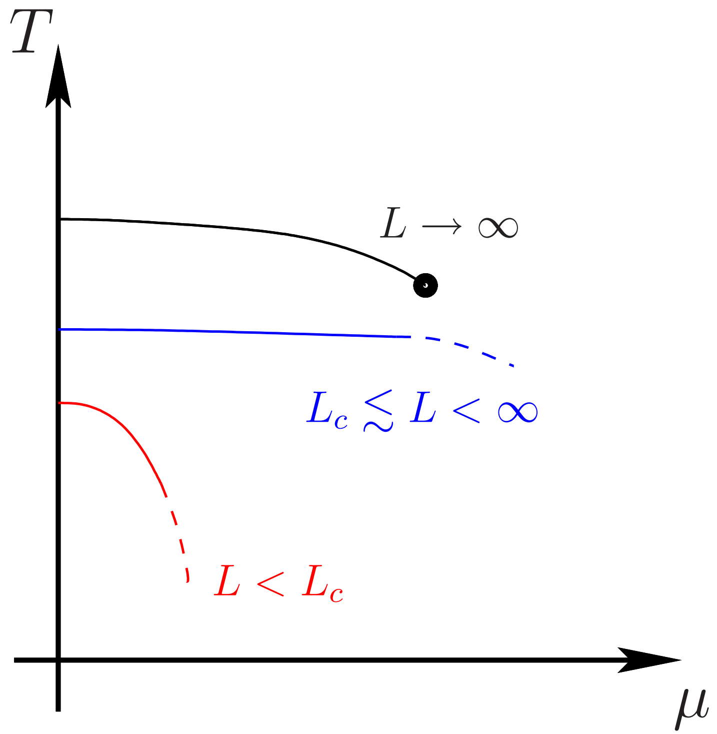
Change of curvature in finite volume

- effects stronger for smaller quark masses
- sensitivity decreases with increasing pion mass
- in agreement with expectations: constituent quark mass rises with pion mass!



Phase diagram for QCD models in finite volume - qualitative results

- qualitatively clear effects of finite volume on curvature
- phase transition line tends to *flatten* in an intermediate volume range
- curvature increases dramatically for very small volumes



Conclusions

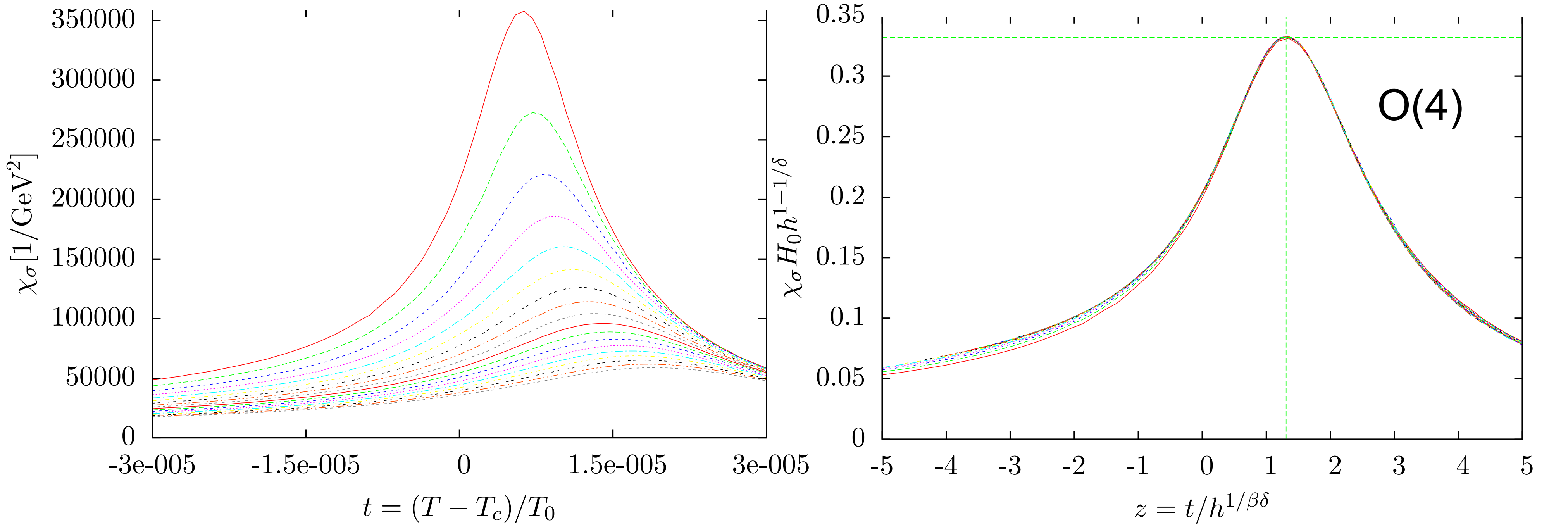
- Curvature of finite chemical-potential temperature phase transition line calculated from an NJL-type model *including fermionic and mesonic fluctuations*
- Curvature much larger than in gauge theories
- Finite volume: phase transition line *flattens* in intermediate volume range ➔ curvature smaller!
- possible effects in QCD lattice simulations: expect curvature in small volumes to be smaller

Outlook

- critical end point of first-order line: study by means of a full functional solution (potential on a mesh grid), with Bernd-Jochen Schaefer and Arno Tripolt, Graz
- investigate change in a finite volume with anti-periodic boundary conditions for the quark fields
- investigate confinement effects:
  include Polyakov-loop potential

Susceptibility χ_σ from the model: O(4)

- susceptibility χ_σ for small values of $m_\pi < 0.9$ MeV

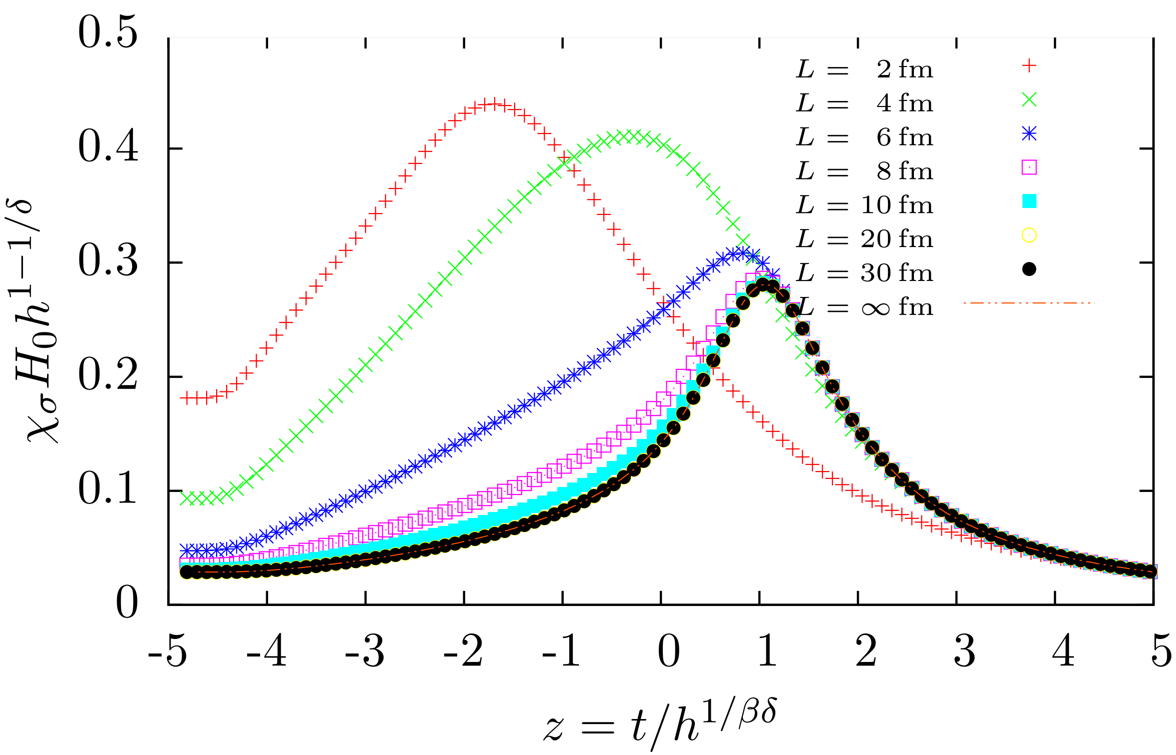


[P. Piasecki, J. Braun, and B. Klein (2010)]

- rescaled susceptibility $\chi_\sigma H_0 h^{1-1/\delta}$

Infinite volume scaling in finite volume?

- rescaled susceptibility $\chi_\sigma H_0 h^{1\text{-}1/\delta}$ in finite volume



[P. Piasecki, J. Braun, and B. Klein (2010), arXiv:1008.2155]

- m_π = 75 MeV
- deviations from infinite-volume scaling for L < 6 fm
- effects probably weaker in lattice QCD

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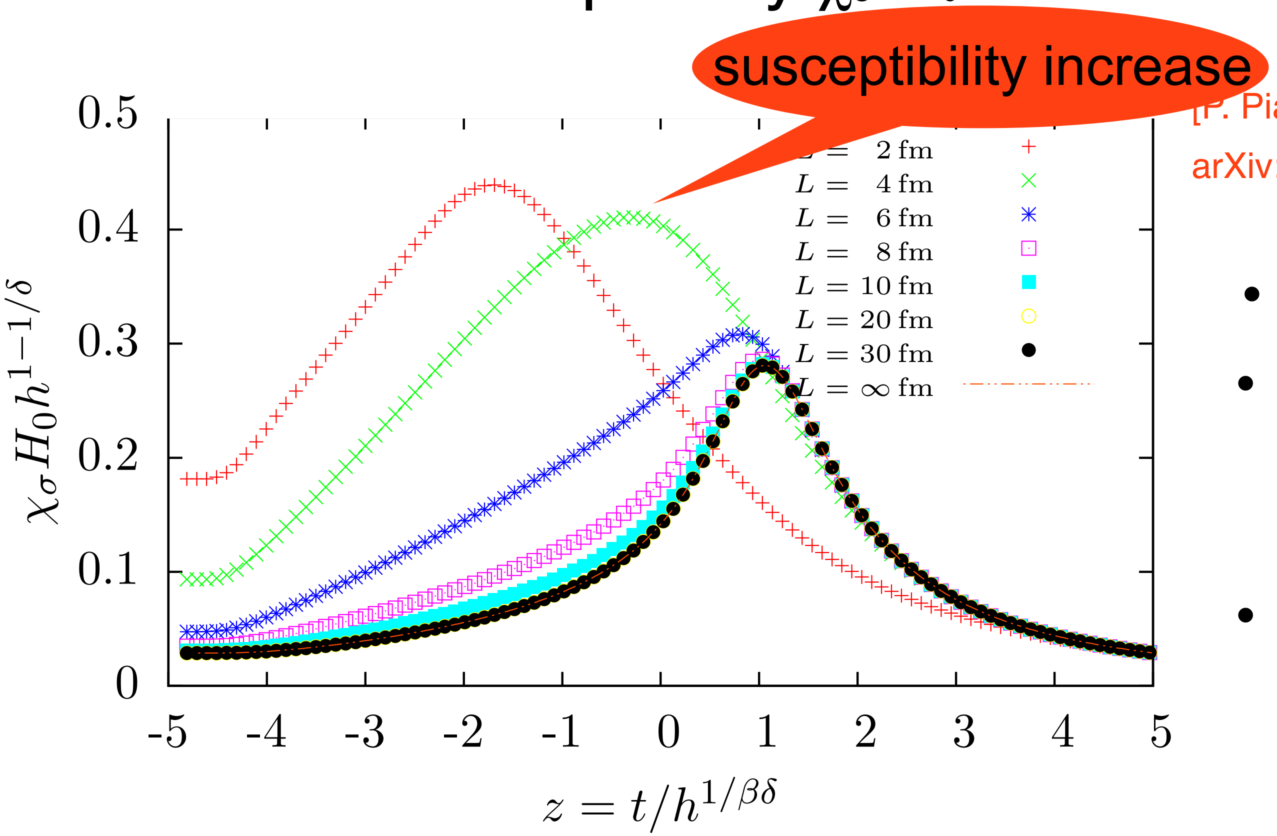

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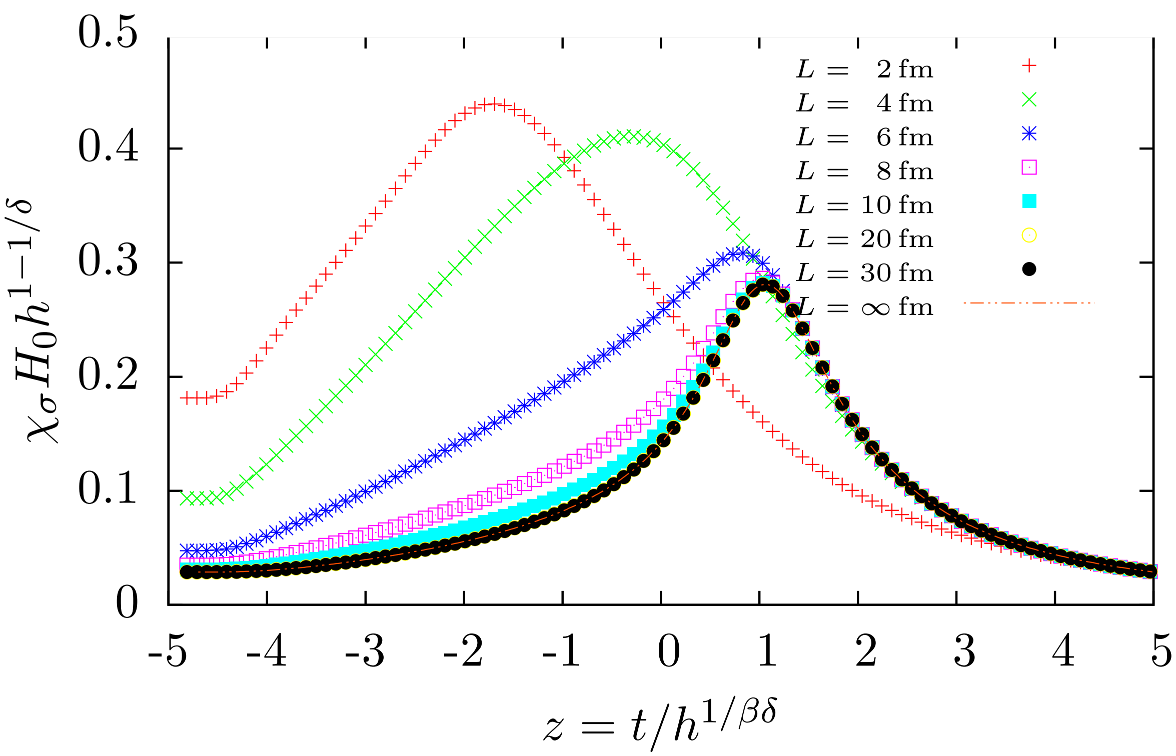

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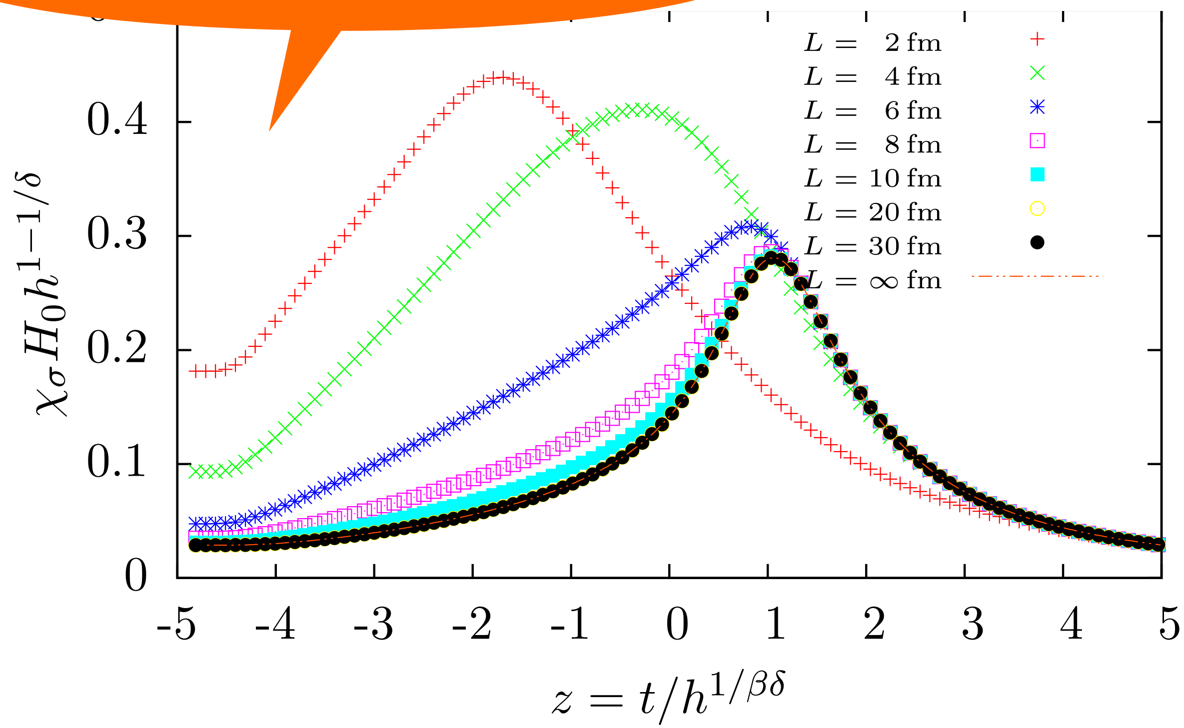
susceptibility decrease $\chi_\sigma \sim L^2$



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