Landau gauge ghost and gluon propagators from NSPT and Monte Carlo

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Outline

1. Introduction
2. Langevin equation and NSPT
3. Propagators in NSPT
   - Ghost propagator
   - Gluon propagator
   - Implementation
4. Redoing the Monte Carlo calculations
   - Gluon field definitions, gauge functionals and all that
   - Comparison between MC results for both definitions
5. Selected results from NSPT
   - Comparing NSPT with the new MC results
   - Higher loops in the $V \to \infty$ and $a \to 0$ limits
6. Summary

Outline

Perturbative results with increasing precision (more loops) are increasingly needed:

- Relate observables measured in lattice QCD to their physical counterpart via renormalisation
- Separate non-perturbative effects from observables assumed to be sensitive to confinement

Common belief: Gauge fixed gluon and ghost propagators encode in their momentum dependence such properties

- extreme IR behavior: dominated by Gribov effects
- intermediate momenta (1 GeV) : vortices, instantons, ...
- large momenta: onset of condensates, first deviations from asymptotic freedom

Lattice perturbation theory (LPT) in diagrammatic form is much more involved than continuum perturbation theory (CPT) of QCD: Thus, only very few higher-loop results from LPT are known !

Alternative

Use stochastic quantization (Parisi and Wu, 1981) realized via Langevin equation

General non-perturbative application:

Langevin simulations of lattice QCD including stochastic gauge fixing instead of standard Monte Carlo (MC) simulations

Zwanziger, Stamatescu, Wolff (1983)

Zwanziger, Seiler, Stamatescu (1984)

Batrouni et al. (1985)

recent application: J. M. Pawlowski, D. Spielmann, I. O. Stamatescu, Lattice Landau gauge with stochastic quantization, Nucl. Phys. B **830** (2010) 291

Perturbative application:

replaces the standard LPT

$\Rightarrow$ Numerical Stochastic Perturbation Theory (NSPT)

(Di Renzo et al.,1994) used for higher order calculations;

for numerical stability stochastic gauge fixing needed

Here: a new application (developed since 2007 Leipzig/Berlin) that requires complete gauge fixing:
Aim: study of higher-loop ghost and gluon propagators in minimal Landau gauge to make predictions (postdictions) for usual LPT and to compare with non-perturbative results
F. Di Renzo, E.-M.I., H. Perlt, A. Schiller, C. Torrero, Nucl. Phys. B **831** (2010) 262

and Nucl. Phys. B **842** (2011) 122

Two other recent, not gauge dependent applications of NSPT :

Very high order lattice perturbation theory for Wilson loops,
R. Horsley, G. Hotzel, E.-M. I., Y. Nakamura, H. Perlt, P.E.L. Rakow, G. Schierholz,

and A. Schiller, arXiv:1010.4674 [hep-lat], Lattice 2010
$\longrightarrow$ extract gluon condensate from Wilson loops (up to 20 loops)

Hunting the static energy renormalon,

C. Bauer and G. Bali, arXiv:1011.1165 [hep-lat], Lattice 2010
$\longrightarrow$ extract the leading renormalon in the perturbative expansion of the static energy (up to 12 loops)

Outline

Langevin equation for lattice QCD

Use Euclidean lattice Langevin equation with "time" t

$$\frac{\partial}{\partial t} U_{x,\mu}(t;\eta) = \mathrm{i}\left(\nabla_{x,\mu} S_G[U] - \eta_{x,\mu}(t)\right) U_{x,\mu}(t;\eta)$$

η Gaussian random noise
S_G lattice action
$\nabla_{x,\mu}$ left Lie derivative

For $t \to \infty$ link gauge fields U are distributed according to equilibrium measure $\exp(-S_G[U])$.

Discretize $t = n\,\epsilon$
Get solution at next time step $n+1$ in the Euler scheme

$$U_{x,\mu}(n+1;\eta) = \exp(\mathrm{i}\,F_{x,\mu}[U,\eta])\,U_{x,\mu}(n;\eta)$$

with force

$$F_{x,\mu}[U,\eta] = \epsilon \nabla_{x,\mu} S_G[U] + \sqrt{\epsilon}\,\eta_{x,\mu}$$

Perturbative Langevin equations

Rescale time step to $\varepsilon = \beta\epsilon$ and use the rescaled equations (discrete time $t = n\varepsilon$) for a perturbative expansion of links:

$$U_{x,\mu}(n;\eta) \rightarrow 1 + \sum_{l>0} \beta^{-l/2} U_{x,\mu}^{(l)}(n;\eta)$$

The Langevin equation at finite ε transforms into a system of simultaneous updates for each order $U_{x,\mu}^{(l)}$, beginning like

$$U^{(1)}(n+1) = U^{(1)}(n) - F^{(1)}(n) \ ,$$

$$U^{(2)}(n+1) = U^{(2)}(n) - F^{(2)}(n) + \frac{1}{2}\left(F^{(1)}(n)\right)^2 - F^{(1)}(n)\,U^{(1)}(n) \ ,$$

etc.
Random noise η enters only in $F^{(1)}$, noise is propagating towards higher orders through lower order fields.
The hierarchy is upward open !

Perturbative Langevin equations

In addition, the gauge field variables (Lie algebra valued, $A = \log U$) are simultaneously stored enforcing antihermitecity to all orders in $1/\sqrt{\beta}$. Similar expansion:

$$A_{x,\mu}(n;\eta) \to \sum_{l>0} \beta^{-l/2} A^{(l)}_{x,\mu}(n;\eta)$$

$$A^{(1)}(n) = U^{(1)}(n)$$

$$A^{(2)}(n) = U^{(2)}(n) - \frac{1}{2}\left(U^{(1)}(n)\right)^2$$

etc.

- To stabilize the Langevin process, stochastic gauge fixing is added to the update.
- Subtraction of zero modes. Possible alternative: twisted boundary conditions (C. Bauer and G. Bali).

Perturbative Langevin equations and observables

Construct observables by expansion, Wilson loops in $U_{x,\mu}^{(l)}$ and propagators in $A_{x,\mu}^{(l)}$!

$$W^{(l)} = \sum_{l_1, l_2, \ldots, l_K, \sum l_i = l} \mathcal{P} \left(\prod_{\mathrm{link}=1}^{K} U_{\mathrm{link}}^{(l_{\mathrm{link}})} \right)$$

For gauge dependent quantities complete gauge fixing is needed !

We need the minimal Landau gauge which is reached by iterative Fourier accelerated gauge trafo's.

One step interchanged with Langevin step $=$ stochastic gauge fixing

Exact Landau gauge fixing

In contrast to the approximate Landau gauge reachable by stochastic gauge fixing (along with the Langevin updates), we can guarantee the minimal Landau gauge (transversality if the gauge field) up to machine precision.

Perform Landau gauge fixing and measure gluon and ghost propagators (after typically 50 Langevin steps).

Condition for perturbative Landau gauge at all orders

$$\sum_{\mu} \partial^L_\mu A^{(l)}_{x,\mu} = 0 \,, \quad \text{with} \quad \partial^L_\mu A^{(l)}_{x,\mu} \equiv A^{(l)}_{x+\hat{\mu}/2,\mu} - A^{(l)}_{x-\hat{\mu}/2,\mu} \,.$$

Landau gauge is reached iteratively, by a perturbative variant of the Fourier accelerated steepest descent algorithm (Davies et al, 1987).

Outline

Momentum space ghost propagator

$$G(p(k)) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{Tr}\, M^{-1}(k) \right\rangle_U$$

color trace of $M^{-1}(k)$, the Fourier transform
(i.e. diagonal element in plane wave basis)
of the inverse of the Faddeev-Popov operator

$$M_{xy}^{ab} = \left[\partial_\mu^L D_\mu\right]_{xy}^{ab}$$

with

$$D_\mu[\varphi] = \left(1 + \frac{i}{2}\Phi_\mu(x) - \frac{1}{12}\left(\Phi_\mu(x)\right)^2 - \frac{1}{720}\left(\Phi_\mu(x)\right)^4 + \ldots\right)\partial_\mu^R + i\,\Phi_\mu(x)$$

where $\varphi_\mu^a = iA_\mu^a$ and $[\Phi_\mu]^{bc} = -if_{abc}\varphi_\mu^a$.

Momentum space ghost propagator

The expansion based on collecting terms of equal power $\beta^{-l/2}$,
$A_{x,\mu}^{(l)} \to M^{(l)} \to \left[M^{-1}\right]^{(l)}$

$$M = M^{(0)} + \sum_{l>0} \beta^{-l/2} M^{(l)}$$

This allows for a recursive evaluation of the inverse without inversions :

$$M^{-1} = \left[M^{-1}\right]^{(0)} + \sum_{l>0} \beta^{-l/2} \left[M^{-1}\right]^{(l)}$$

with $\left[M^{-1}\right]^{(0)} = \left[M^{(0)}\right]^{-1} = \Delta^{-1}$ and

$$\left[M^{-1}\right]^{(l)} = -\left[M^{-1}\right]^{(0)} \sum_{j=0}^{l-1} M^{(l-j)} \left[M^{-1}\right]^{(j)}$$

Momentum space ghost propagator

The n-loop ghost propagator $G^{(n)}$ is obtained from sandwiching $\left[M^{-1}\right]^{(l=2n)}$ between plane wave vectors

$$\xi^a(x) = \delta^{ab} \ \exp\left(2\pi \, i \, k_\mu \, x_\mu / N_\mu\right)$$

for many 4-tuples (k_1, k_2, k_3, k_4) and all colors b (this is expensive !) with lattice momenta

$$\hat{p}_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{N}\right) = \frac{2}{a} \sin\left(\frac{a p_\mu}{2}\right)$$

$$G^{(n)}(\hat{p}(k)) = \left(\xi^\dagger, \left[M^{-1}\right]^{(l=2n)} \xi\right)$$

Ghost dressing function at n loops

$$J^{(n)}(p) = p^2 \, G^{(n)}(p(k)) \qquad \text{and / or} \qquad \hat{J}^{(n)}(\hat{p}) = (\hat{p})^2 \, G^{(n)}(p(k))$$

Momentum space ghost propagator

Warning !

- $M = M(A)$ (constructed via logarithmic definition of A in terms of U) differs from the Faddeev-Popov definition (the Hessian of the linear gauge fixing functional) which is adopted in almost all MC calculations !
- New Monte Carlo simulations needed compatible with NSPT !
- Will all previous lattice MC results be obsolete ? We checked that they are not !

Momentum space gluon propagator

Construct tree-level ($n = 0$) and different loop orders $n \neq 0$ from Fourier transformed gauge fields $\widetilde{A}_\mu^{a,(l)}(k)$

$$\delta^{ab} D_{\mu\nu}^{(n)}(p(k)) = \left\langle \sum_{l=1}^{2n+1} \left[\widetilde{A}_\mu^{a,(l)}(k)\, \widetilde{A}_\nu^{b,(2n+2-l)}(-k) \right] \right\rangle$$

Remarks:

- Only even orders $l = 2n$ in $1/\sqrt{\beta}$ are nonvanishing.
- Odd orders (half-integer n) are vanishing within errors !
- The tree level propagator $D_{\mu\nu}^{(0)}$ arises from quadratic fluctuations of the gauge field $A^{(l)}$ with $l = 1$.

In Landau gauge we consider

$$\sum_{\mu=1}^{4} D_{\mu\mu}^{(n)} \equiv 3D^{(n)}$$

Gluon dressing function

Dressing function of gluon propagator

$$Z^{(n)}(p) = p^2\, D^{(n)}(p(k))$$

and/or

$$\hat{Z}^{(n)}(\hat{p}) = (\hat{p})^2\, D^{(n)}(p(k))$$

Remarks:

- $Z^{(n)}$ is calculated simultaneously (FFT) for all momenta (cheap !)
- Gauge fixing must correspond to the $A = \log U$ definition !

Implementation of NSPT: three limits to be taken

- Choose maximally addressable loop order, take $l_{\max} = 2n_{\max}$! (only restrictions: computer time, memory and machine precision)
- Solve coupled system of equation for $U^{(l)}$'s ($l = 1, \ldots, l_{\max}$) at several ε and lattice volumes !
- Get time series of gauge fields $A^{(l)}$ to all chosen orders !
- Perform minimal Landau gauge fixing to machine precision !
- Evaluate the perturbative ghost and gluon propagators !

• Limit $\varepsilon \to 0$

This allows for comparison of results at finite volume with Monte Carlo measurements.

• Limits $V \to \infty$ and $pa \to 0$

A strategy is worked out to handle finite a and finite volume effects. Compare with analytic results of standard LPT (as far as available). Predict new precise numerical results for higher loops: these limits are obviously implied in standard LPT and CPT !

Raw data: some examples

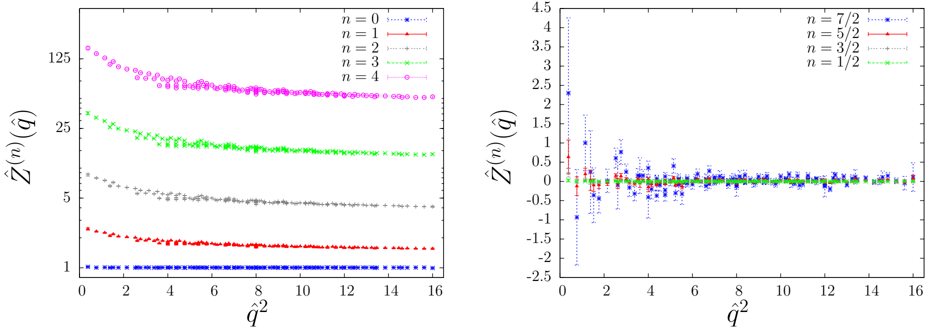


Figure: $\hat{Z}^{(n)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 10$ and $\varepsilon = 0.01$.
Left: Separate loop contributions. Right: Vanishing contributions.

Limit $\varepsilon \to 0$

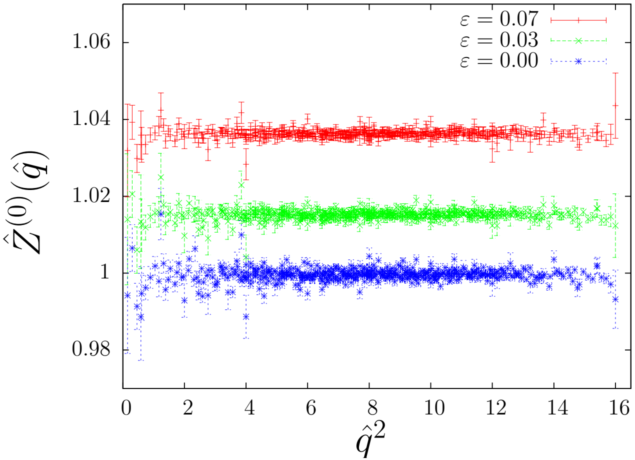


Figure: Tree level dressing function $\hat{Z}^{(0)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 16$.

Broken rotational symmetry

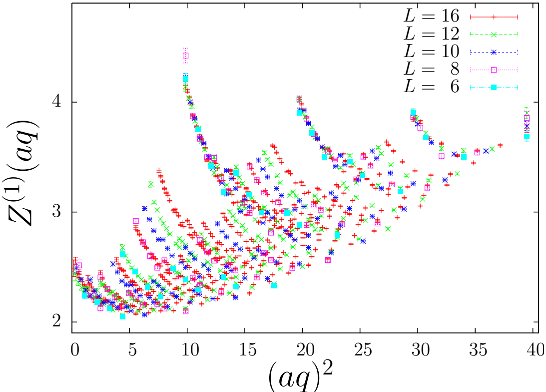


Figure: One-loop dressing function $Z^{(1)}(aq)$ vs. $(aq)^2$ at all volumes shown for *all* inequivalent 4-tuples.

An ad hoc remedy are momentum cuts (like cone cut, etc.).
Fitting with $H(4)$ invariants uses all this information $\rightarrow$ continuum limit.

Momentum cuts ameliorate lower rotational symmetry

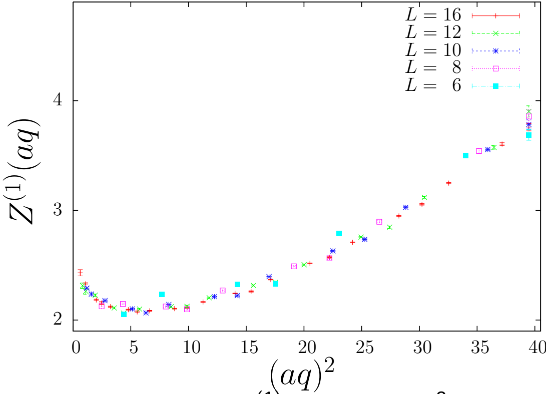


Figure: One-loop dressing function $Z^{(1)}(aq)$ vs. $(aq)^2$ at all volumes for near-diagonal 4-tuples $(k, k, k, k), (k \pm 1, k, k, k), k > 0$.

A smooth $(aq)^2$ dependence emerges for near-diagonal momenta. This happens similarly for *all* loop contributions.

Outline

Gluon field definition and resp. gauge functional

Landau gauge (transversality) defined as

$$\left(\sum_\mu \partial_\mu A_\mu\right)(x) \equiv \sum_\mu \left(A_{x+\frac{\hat\mu}{2},\mu} - A_{x-\frac{\hat\mu}{2},\mu}\right) = 0$$

Suppose the gluon field definition is replaced by

$$A^{(\mathrm{lin})}_{x+\frac{\hat\mu}{2},\mu} = \frac{1}{2iag_0}\left(U_{x,\mu} - U^\dagger_{x,\mu}\right)\Big|_{\mathrm{traceless}} \rightarrow A^{(\mathrm{log})}_{x+\frac{\hat\mu}{2},\mu} = \frac{1}{iag_0}\log\left(U_{x,\mu}\right) .$$

Then the Landau gauge is fulfilled if

$$F^{(\mathrm{lin})}_U[g] = \frac{1}{4V}\sum_{x,\mu}\left(1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\,{}^gU_{x,\mu}\right) \rightarrow \mathrm{Min}$$

is replaced by

$$F^{(\mathrm{log})}_U[g] = \frac{1}{4VN_c}\sum_{x,\mu}\mathrm{Tr}\left[{}^gA^{(\mathrm{log})}_{x+\frac{\hat\mu}{2},\mu}\,{}^gA^{(\mathrm{log})}_{x+\frac{\hat\mu}{2},\mu}\right] \rightarrow \mathrm{Min}$$

E.-M. I, Ch. Menz, M. Müller-Preussker, A. Schiller, A. Sternbeck, Phys. Rev. D in print

Iterative gauge fixing for the logarithmic definition

$$ {}^{g}U_{x,\mu} \rightarrow {}^{(rg)}U_{x,\mu} = r_x \, {}^{g}U_{x,\mu} \, r^{\dagger}_{x+\mu} $$

local gauge fixing

$$ r_x = \exp\left(-i\alpha \, (\sum_{\mu} \partial_{\mu} {}^{g}A^{(\log)}_{\mu})(x) \right) $$

Fourier accelerated gauge fixing

$$ r_x = \exp\left(-i\alpha \hat{F}^{-1} \left[\frac{q^2_{\max}}{q^2} \hat{F} \left[(\sum_{\mu} \partial_{\mu} {}^{g}A^{(\log)}_{\mu})(x) \right] \right] \right) $$

Multigrid accelerated gauge fixing

$$ r_x = \exp\left(-i\alpha \, q^2_{\max} \, \Delta^{-1} (\sum_{\mu} \partial_{\mu} {}^{g}A^{(\log)}_{\mu})(x) \right) $$

Gluon and ghost propagators: decoupling solution

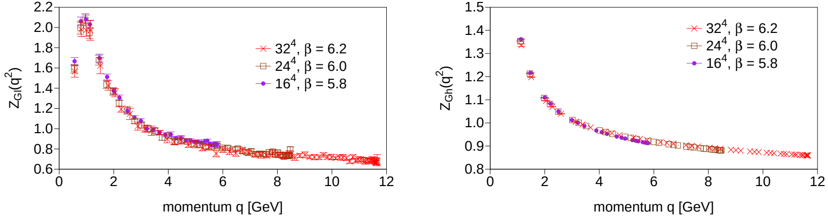
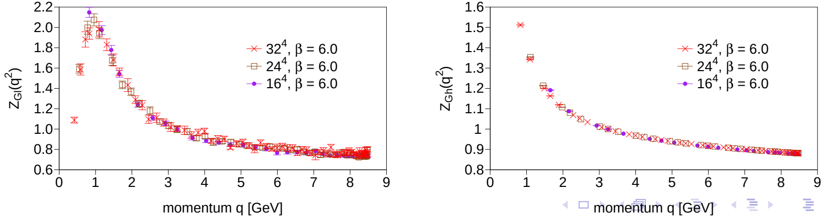


Figure: Top: Renormalized gluon (left) and ghost dressing function (right) for the logarithmic definition and various $a = a(\beta)$. The physical volume is fixed to $V = (2.2\ \mathrm{fm})^4$. Bottom: The same for fixed $\beta = 6.0$ for different volumes V. Data has been renormalized at $q = \mu \approx 3.2\ \mathrm{GeV}$.



Bare propagators are multiplicatively “renormalized”

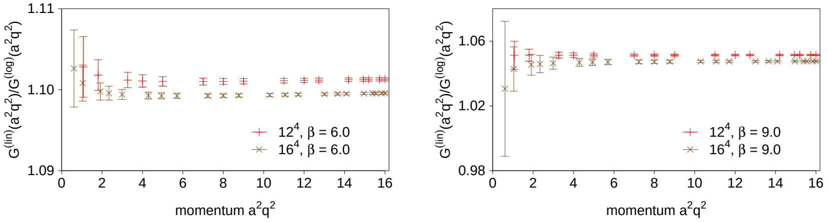
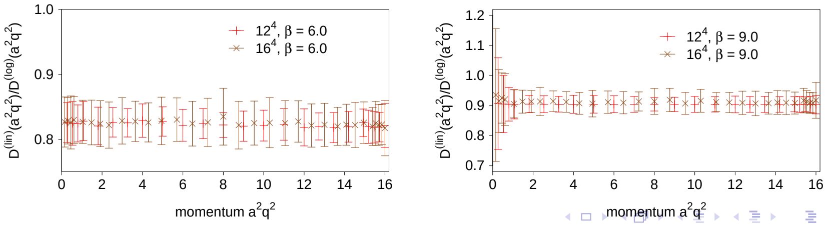


Figure: Top: Ratio of ghost propagators relating the two definitions for $\beta = 6.0$ (left)and $\beta = 9.0$ (right). Bottom: The same for the gluon propagators.



The running coupling is reproduced

A. Sternbeck et al., PoS (LAT2009) 210

$$\alpha_s^{\mathrm{MM}}(q^2) = \frac{g_0^2}{4\pi} Z_{\mathrm{Gl}}(a^2, q^2) Z_{\mathrm{Gh}}^2(a^2, q^2)$$

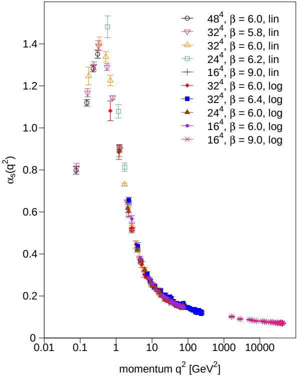
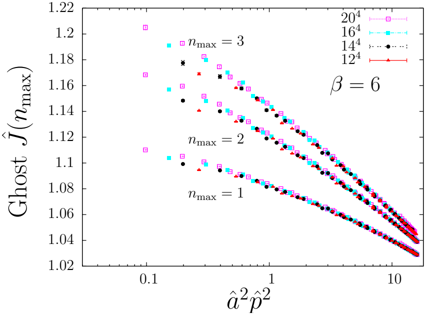


Figure: The running coupling obtained from Monte Carlo gluon and ghost dressing functions.

Outline

Summed NSPT propagators: β restored

$$\hat{J}(n_{\max}) = \sum_{n=0}^{n_{\max}} \frac{1}{\beta^n} \hat{J}^{(n)}, \quad \beta = 6/g^2, \quad \left(\hat{J}^{(n)} = (\hat{p}^2/p^2) J^{(n)}\right)$$



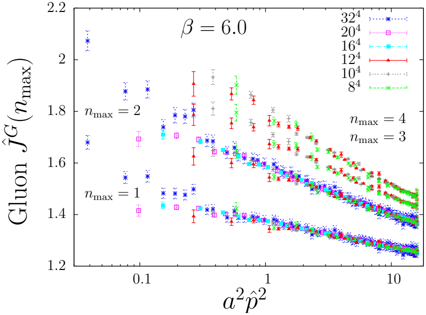


Figure: The cumulatively summed perturbative ghost (left) and gluon (right) dressing functions

Propagators, MC vs. NSPT (naive and boosted)

Bare coupling in LPT bad expansion parameter (Lepage, Mackenzie,1993)
Use a variant of boosted LPT with boosted coupling

$$g_b^2 = g^2/P_{\mathrm{pert}}(g^2) > g^2$$

Reorder into a series in g_b^2 with smaller expansion coefficients

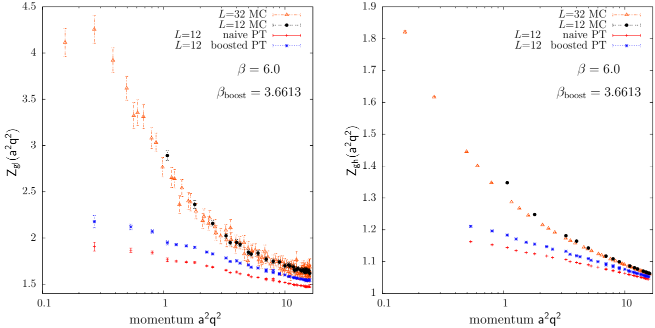


Figure: Comparison of naive and boosted LPT with MC data at $L = 12$ at $\beta = 6.0$. (left): bare gluon dressing function (up to 4-loop); (right): bare ghost dressing function (up to 3-loop)

Running coupling, MC vs. NSPT (naive and boosted)

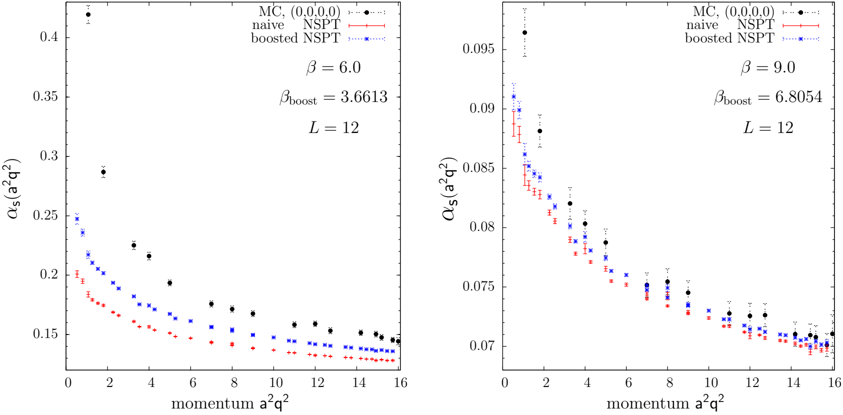


Figure: Comparison of naive and boosted LPT with MC data for the running coupling $\alpha_s(q^2)$ for a $L = 12$ lattice, with the gluon (ghost) dressing function up to 4-loop (3-loop) accuracy. (left): $\beta = 6.0$; (right): $\beta = 9.0$.

Running coupling, MC vs. NSPT (zoomed into high momenta)

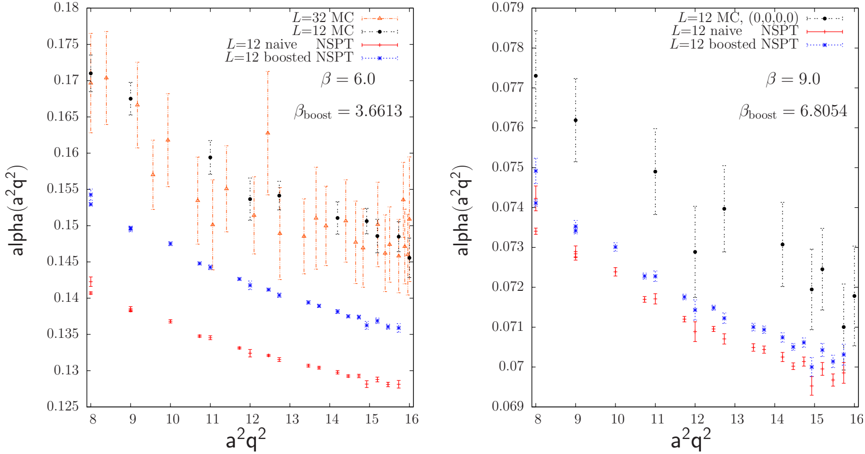


Figure: Comparison of naive and boosted LPT with MC data for the running coupling $\alpha_s(q^2)$ for a $L = 12$ lattice, with the gluon (ghost) dressing function up to 4-loop (3-loop) accuracy. (left):

Relation to standard LPT, handling of finite a effects

Example: extract non-log constant $J_{1,0}$ in one-loop measurement

At infinite volume and in continuum limit

$$J^{(1)}(pa) = J_{1,1} \log(pa)^2 + J_{1,0}$$

logarithmic behavior $J_{1,1}$ is assumed to be known

Anticipating lattice artifacts (non-zero a, infinite volume) rewrite

$$J^{(1)}(pa) = J_{1,1} \log(pa)^2 + J_{1,0}(pa)$$

Use hypercubic-invariant Taylor expansion $[(pa)^n = \sum_\mu (ap_\mu)^n]$

$$\begin{aligned} J_{1,0}(pa) &= J_{1,0} + c_{1,1}\,(pa)^2 + c_{1,2}\,\frac{(pa)^4}{(pa)^2} + c_{1,3}\,(pa)^4 + c_{1,4}\,\left((pa)^2\right)^2 \\ &\quad + c_{1,5}\,\frac{(pa)^6}{(pa)^2} + \cdots \end{aligned}$$

Relation to standard LPT, handling of finite volume

Take into account finite size ($L = aN$)

$$\begin{aligned} J^{(1)}(pa, pL) &= J_{1,1} \log(pa)^2 + J_{1,0;L}(pa, pL) \\ &= J_{1,1} \log(pa)^2 + J_{1,0}(pa) + [J_{1,0;L}(pa, pL) - J_{1,0}(pa)] \\ &= J_{1,1} \log(pa)^2 + J_{1,0}(pa) + \delta J_{1,0}(pa, pL) \end{aligned}$$

Neglect corrections on corrections

$$\delta J_{1,0}(pa, pL) \to \delta J_{1,0}(0, pL)$$

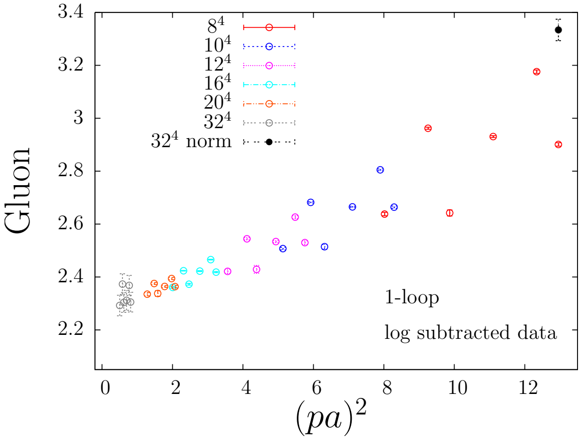
This allows for a non-linear fit of the non-log part:
for a given 4-tuple $k_\mu = (k_1, k_2, k_3, k_4)$ measurements at different lattice sizes N are affected by same pL effect due to the trivial identity

$$p_\mu L = p_\mu aN = 2\pi k_\mu$$

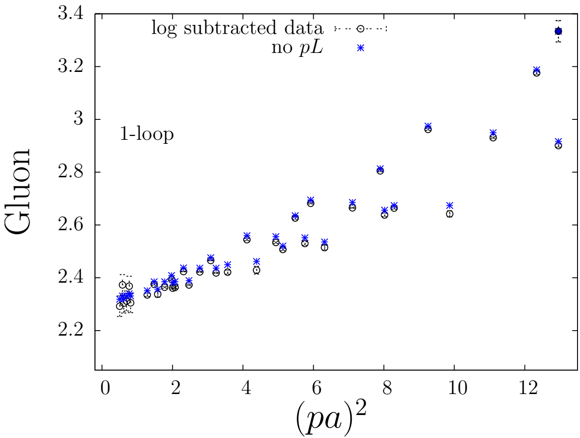
Note: no need to guess a functional form of the finite size effect
need of “renormalisation” data point for infinite volume

Fitting strategy

- select interval $[(pa)^2_{\rm min}, (pa)^2_{\rm max}]$ where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed
- choose data in that interval from a sufficiently large amount of 4-tuples common to all chosen lattice sizes
- subtract all logarithmic pieces (for higher loops use fit results from lower loops to get coefficients of non-leading log's)
- take an additional data point at $(ap)^2 \approx (ap)^2_{\rm max}$ ($\Rightarrow$ extra 4-tuple $k_{\rm max}$) from the largest lattice as reference point for infinite volume: putting $\delta J_{i,0}(0, pL(k_{\rm max})) = 0$
- perform a non-linear fit using all data points from different lattice sizes L^4 plus the reference data point (no functional form guessed) and assuming a specific functional behavior for the $H(4)$ dependence
- vary the momentum squared window and find an optimal χ^2 region for “best” values $J_{i,0}$

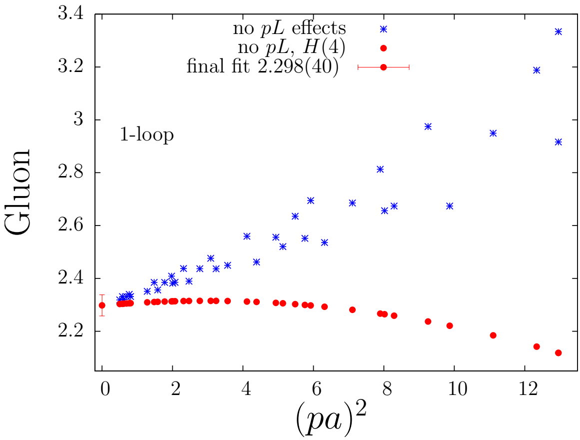


Choose log-subtracted data in interval $[(pa)^2_{\min}, (pa)^2_{\max}]$ from 4-tuples **common** to all lattice sizes (here we have 6 L's) !
Add data point at $\approx (ap)^2_{\max}$ ($\Rightarrow$ extra 4-tuple $k_{\max}$) from largest lattice as reference point for infinite volume: $\delta J_{1,0}(0, pL(k_{\max})) = 0$!



○: original-log subtracted data from all lattice sizes

⋆: data after correcting for finite-volume effects pL



Corrected data at infinite volume in the non-linear fit following only

$$J_{1,0}(pa) \quad = \quad J_{1,0} + c_{1,1}\,(pa)^2 + c_{1,4}\,\left((pa)^2\right)^2$$

Extrapolation examples

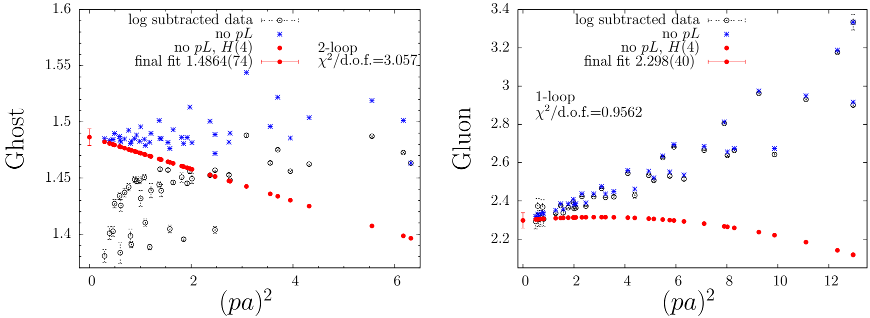


Figure: Fitting of 2-loop ghost $J_{2,0}$ (left) and 1-loop gluon $J^G_{1,0}$ (right) non-log constants following the outlined procedure

○ : raw data from different lattice sizes L^4 (logarithms subtracted)

⋆ : data after correcting for finite-volume effects pLa

● : data after correcting pLa and (some) hypercubic effects

Example for final non-logarithmic constants

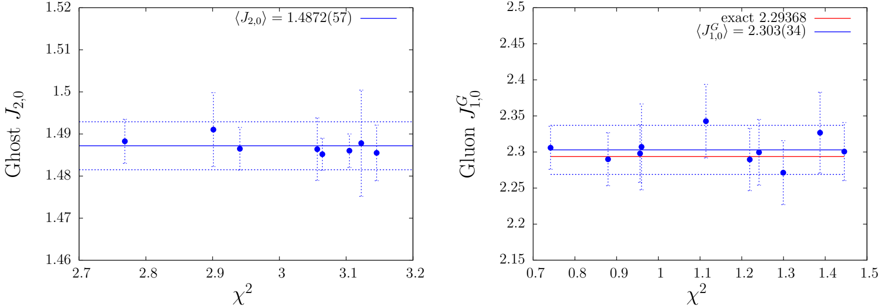


Figure: 2-loop ghost (left) and 1-loop gluon (right) non-log constants for “best” fits

Final results for LPT ($\beta = 6/g^2$, $Ln \equiv \log(pa)^2$)

one-loop non-logarithmic constant known since 30 years

H. Kawai, R. Nakayama, K. Seo, Nucl. Phy. B189 (1981) 40

Ghost dressing function [$\langle J_{1,0} \rangle = 0.52523(95)$]

$$J^{3-\mathrm{loop}}(a,p,\beta) = 1 + \frac{1}{\beta}\left[-0.0854897\,Ln + 0.525314\right] + \\ + \frac{1}{\beta^2}\left[0.0215195\,Ln^2 - 0.358423\,Ln + 1.4872(57)\right] + \\ + \frac{1}{\beta^3}\left[-0.0066027 Ln^3 + 0.175434\,Ln^2 - 1.6731(1)\,Ln + 4.94(27)\right]$$

Gluon dressing function [$\langle Z_{1,0} \rangle = 2.303(34)$]

$$Z^{3-\mathrm{loop}}(a,p,\beta) = 1 + \frac{1}{\beta}\left[-0.24697\,Ln + 2.29368\right] + \\ + \frac{1}{\beta^2}\left[0.08211 Ln^2 - 1.48445\,Ln + 7.93(12)\right] + \\ + \frac{1}{\beta^3}\left[-0.02964 Ln^3 + 0.81689\,Ln^2 - 8.13(3)\,Ln + 31.7(5)\right]$$

Relations to standard *RI'MOM* or $\overline{MS}$ schemes known

Outline

Summary

- We have applied NSPT to calculate in Landau gauge the gluon propagator up to four loops and the ghost propagator up to three loops.
- The summed dressing functions are compared to recent MC results obtained in Berlin using the same definition of gauge fields, the corresponding Landau gauge fixing and Faddeev-Popov matrix as in LPT.
- To improve the comparison, we have used boosted LPT to achieve faster convergence.
- One key goal of the lattice study of propagators is to learn about their genuinely non-perturbative content. The knowledge of higher loop perturbative results is therefore desirable.
- Commonly the large-momentum tail is fitted by continuum-like formulae. Further ambiguities are possible, since irrelevant discretization artefacts might substantially contribute to the perturbative tail.
- At large lattice momenta our calculations indicate that the perturbative dressing functions from NSPT with more than four loops will match the MC measurements, enabling a fair accounting of the perturbative tail.

Summary continued

- The strong difference left over in the intermediate and – moreover – in the infrared momentum region not considered here should then be attributed to non-perturbative effects (power corrections and contributions from non-perturbative localized excitations).

Relation to standard LPT in limits $V \to \infty$ and $pa \to 0$

- Our fitting strategy of lattice artifacts and finite-size corrections seems to be sufficiently accurate.
- Good agreement is found with one-loop results of diagrammatic LPT which are known since many years.
- We have communicated original two- and three-loop results for the propagators.