The Vacuum State, Dimensional Reduction at T=0, and a Torelon Probe at High T

Jeff Greensite
talk at "Quarks etc. under Extreme Conditions"

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co-starring:

AT LOW TEMPERATURES

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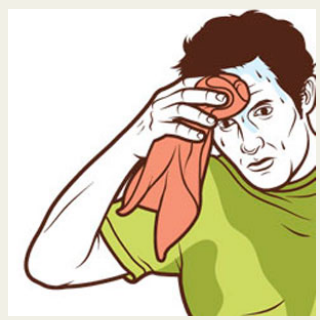
AT HIGH TEMPERATURES



Stefan Olejnik
(in progress)

an elementary formula....

$$\langle O \rangle_{thermal} = \frac{\sum_n \langle n|O|n\rangle e^{-E_n/kT}}{\sum_n e^{-E_n/kT}}$$

Two questions, in pure Yang-Mills theory:

- What is $|0\rangle$? i.e., in Schrodinger representation, what is

$$\Psi_0[A] = \langle A|0\rangle$$

- What is

$$|n\rangle = Q_n|0\rangle \quad ?$$

At low T, the relevant Q_n are just glueball creation operators. But what are the relevant Q_n at high T, past the deconfinement transition?

Confinement and Casimir Scaling from Dimensional Reduction

Confinement and chiral symmetry breaking are properties of the QCD vacuum. Maybe we could learn more about them, if we knew the explicit form of the QCD vacuum wavefunctional, satisfying

$$H\Psi_0[A] = E_0\Psi_0[A]$$

An old proposal: Perhaps, at large scales, the vacuum looks like

$$\Psi_0^{eff}[A] = \mathcal{N}\exp\left[-\frac{1}{2}\mu\int d^3x\ \mathrm{Tr}[F_{ij}^2]\right]$$

JG (1979)

This state has the property of ***dimension reduction:*** large Wilson loops in 3+1 dimensions can be computed in 3 Euclidean dimensions.

Dimensional reduction from D = 4 → 3 → 2
for large spacelike Wilson loops

$$
\begin{aligned}
W(C) &= \langle \mathrm{Tr}[U(C)] \rangle^{D=4} = \langle \Psi_0^{(3)} | \mathrm{Tr}[U(C)] | \Psi_0^{(3)} \rangle \\
&\sim \langle \mathrm{Tr}[U(C)] \rangle^{D=3} = \langle \Psi_0^{(2)} | \mathrm{Tr}[U(C)] | \Psi_0^{(2)} \rangle \\
&\sim \langle \mathrm{Tr}[U(C)] \rangle^{D=2}
\end{aligned}
$$

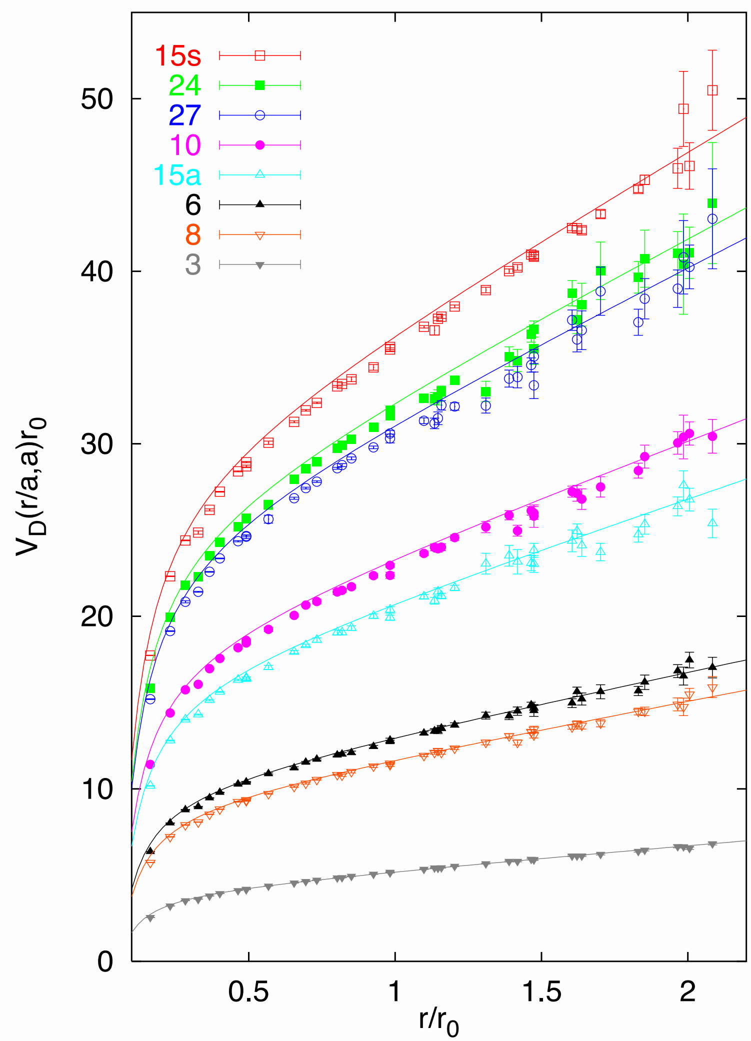
In D=2 dimensions the Wilson loop can be calculated analytically, and we know there is an area-law falloff, and ***Casimir-scaling*** string tensions.

- Is this the right explanation for Casimir scaling?
- Can we test it?

Casimir scaling in SU(3) lattice gauge theory Bali (2000)

string tensions at intermediate distances, prior to color screening, are proportional to the quadratic Casimir of the color representation of the sources.

$$\sigma_r = \frac{C_r}{C_F}\sigma_F$$



On the other hand, dimensional reduction can't be exactly right:

- no color screening (in D=2+1), wrong N-ality properties
- wrong high momentum behavior

At strong couplings, it can be shown that small corrections to the dimensional reduction wavefunction are responsible for color screening.

So what does the vacuum state look like at all scales, not just large scales?

There are now a number of proposals on the table. I will concentrate on D=2+1 dimensions…

GO Wavefunctional (Olejnik & JG)

$$\Psi_{GO}[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y\ F_{12}^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}}\right)_{xy}^{ab} F_{12}^b(y)\right]$$

successes: mass gap, Coulomb ghost propagator

KKN Wavefunctional (Karabali, Kim& Nair)

$$\Psi_{KKN}[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y\ F_{12}^a(x)\left(\frac{1}{\sqrt{-\nabla^2+m^2}+m}\right)_{xy}^{ab} F_{12}^b(y)\right]$$

successes: string tension

problem: not gauge-invariant

Hybrid Wavefunctional

$$\Psi_{hybrid}[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y\ F_{12}^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}+m}\right)_{xy}^{ab} F_{12}^b(y)\right]$$

D^2 is the covariant Laplacian, λ_0 is its lowest eigenvalue, m is a parameter

Coloumb-Gauge Wavefunctional (Reinhardt et al, Szczepaniak et al)

$$\Psi_{CG}[A] = \Big(\det[-\nabla \cdot D]\Big)^{-1/2} \exp\left[-\frac{1}{2}\int d^3k \; \omega(k) A_i^a(k) A_i^a(-k)\right]$$
$$\approx \exp\left[-\frac{1}{2}\int d^3k \; (\omega(k) - \chi(k)) A_i^a(k) A_i^a(-k)\right]$$

where ω(k), χ(k) are determined by a set of integral equations.

successes: enhancement of the Coulomb ghost propagator, confining Coulomb potential.

problem: no area law for spacelike Wilson loops

GO Wavefunctional

The YM ground state is soluble in three different limits:

1. free field,

$$\Psi_0[A] = \exp\left[-\int d^2x d^2y \; B^a(x) \left(\frac{1}{\sqrt{-\nabla^2}}\right)^{ab}_{xy} B^b(y)\right]$$

2. lattice strong-coupling (which has the dimensional reduction form)

$$\Psi_0[U] = \exp\left[\frac{N}{g^4(N-1)}\sum_P \mathrm{Tr}U(P) + \mathrm{c.c.}\right] \sim e^{-\mu \int \mathrm{Tr}[B^2]}$$

3. zero mode: $A_k(x)$ independent of x , V is the volume of space

$$\Psi_0 = \exp\left[-\frac{V}{2g^2}\frac{(A_1 \times A_2)\cdot(A_1 \times A_2)}{\sqrt{|A_1|^2 + |A_2|^2}}\right]$$

The Ψ_{GO} proposal is obtained by looking for the simplest, gauge-invariant wavefunctional which obtains dimensional reduction at large scales, and agrees with the two other analytic solutions in the appropriate limits. In 2+1 dimensions, the proposal is

$$\Psi_{GO}[A] = \exp\left[-\frac{1}{2g^2}\int d^2x d^2y \; B^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

where

$B^a = F^a_{12}$	is the color magnetic field strength
$D^2 = D_k D_k$	is the covariant Laplacian
λ_0	is the lowest eigenvalue of ***$-D^2$***
m	is a parameter with dimensions of mass

We have a finite string tension (from dimensional reduction) if m>0

$$\sigma = \frac{3}{16} m g^2$$

KKN Wavefunctional

Change variables from A_μ^a to gauge-invariant J^a, the tradeoff is local gauge invariance for local holomorphic invariance under

$$J \to hJh^{-1} + \frac{C_A}{\pi}\partial h h^{-1}$$

KKN wavefunction in new (J-field) and old (A-field) variables:

$$\Psi_{KKN} = \exp\left[-\frac{2\pi^2}{g^2 C_A^2}\int d^2x d^2y\, \bar{\partial}J^a(x)\left(\frac{1}{\sqrt{-\nabla^2+m^2}+m}\right)_{xy}\bar{\partial}J^a(y)\right]$$
$$= \exp\left[-\frac{1}{2g^2}\int d^2x d^2y\, B^a(x)\left(\frac{1}{\sqrt{-\nabla^2+m^2}+m}\right)_{xy}B^a(y)\right]$$

where $m = g^2 C_A/2\pi$. If we just throw away $-\nabla^2$ to get to the dimensional reduction form, then the predicted string tension is

$$\sigma = \frac{g^4}{8\pi}(N^2-1)$$

in excellent agreement with numerical simulations extrapolated to large N.

Hybrid Wavefunctional

The problem with Ψ_{KKN} is that in new variables the bilinear is not holomorphic invariant, and in old variables it is not gauge-invariant.

The wavefunctional is therefore ***not a physical state*** as it stands, and the use of dimensional reduction to compute the string tensions is questionable.
Some gauge-invariant completion is required.

So we look for a wavefunctional with the following properties:

1. Gauge-invariant
2. has the dimensional reduction property
3. reduces to Ψ_{KKN} on abelian configurations

$$\Psi_{hybrid} = \exp\left[-\frac{1}{2g^2}\int d^2x d^2y \; B^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}+m}\right)^{ab}_{xy} B^b(y)\right]$$

Coulomb Gauge Wavefunctional

The approach was initiated by Szczepaniak and collaborators (Indiana), and refined/corrected by Reinhardt and collaborators (Tübingen). Here we follow Feuchter and Reinhardt. The proposal to be tested is

$$\Psi_{CG}[A] = \exp\left[-\frac{1}{2}\int \frac{d^2k}{(2\pi)^2} A(-k)\overline{\omega}(k)\, A(k)\right]$$

where $\overline{\omega}(k)$ is to be determined by minimizing the vacuum energy to one loop. The result is a complicated set of integral equations which are solved numerically.

Here is the set of equations determining $\overline{\omega}(k)$.

c_1, c_2 are renormalization constants. Given those constants, the set is solved numerically.

$$\overline{\omega}(k) \equiv \omega(k) - \chi(k),$$

$$\omega^2(k) = k^2 + \chi^2(k) + c_2 + \Delta I^{(2)}(k) + 2\chi(k)\left[\Delta I^{(1)}(k) + c_1\right]$$

$$\Delta I^{(n)}(k) = I^{(n)}(k) - I^{(n)}(0)\,,$$

$$I^{(n)}(k) = \frac{N_C}{2}\int \frac{d^2q}{(2\pi)^2}(\hat{k}\cdot\hat{q})^2\, V(q-k)\,\frac{\overline{\omega}^n(q) - \overline{\omega}^n(k)}{\omega(q)}$$

$$d^{-1}(k) = d^{-1}(\mu) - (I_d(k) - I_d(\mu)),$$

$$I_d(k) \equiv \frac{N_C}{2}\int \frac{d^2q}{(2\pi)^2}\,\left[1 - (\hat{\kappa}\cdot\hat{q})^2\right]\,\frac{d(q-k)}{\omega(q)\,(q-k)^2}$$

$$V(k) \equiv \int d^Dx\; e^{ikx}\langle\Psi|F^{ab}(x,0,[A])|\Psi\rangle = \delta^{ab}\frac{f(k)d^2(k)}{k^2}$$

$$f(k) = \frac{\int d^Dx e^{ikx}\langle\Psi|\left[\frac{\nabla^2}{(-D\cdot\nabla)}\right]^2|\Psi\rangle_{x,a;0,b}}{\left[\int d^Dx e^{ikx}\langle\Psi|\frac{\nabla^2}{(-D\cdot\nabla)}|\Psi\rangle_{x,a;0,b}\right]^2}$$

$$\chi(k) = \frac{N_C}{2}\int \frac{d^2q}{(2\pi)^2}\,\left[1 - (\hat{k}\cdot\hat{q})^2\right]\,\frac{d(q)\,d(q-k)}{(q-k)^2}$$

In this approach, the color Coulomb potential

$$V_c(R) = -\left\langle \left(\frac{1}{\nabla \cdot D} (-\nabla^2) \frac{1}{\nabla \cdot D} \right)^{aa}_{xy} \right\rangle_{|x-y|=R}$$

rises almost (but not quite) linearly in 2+1 dimensions.

Spacelike Wilson loops, however,

$$W(C) = \langle \Psi_{CG} | \mathrm{Tr}[U(C)] | \Psi_{CG} \rangle$$

do not seem to obtain the required area law falloff; this would require some refinement going beyond the Gaussian ansatz.

Testing Vacuum Proposals by Measurement of the Vacuum

A modified Monte Carlo: on time-slice *t=0*, restrict the available configurations to some subset

$$\mathcal{U} \equiv \{U_k^{(m)}(\mathbf{x}), m = 1, 2, ..., M\}$$

and select them, at t=0, via the Metropolis algorithm.

Let N_m be the number of times the m-th configuration is generated. In the limit that $N_{m,n} \rightarrow \infty$

$$\left| \frac{\Psi_0[U^{(m)}]}{\Psi_0[U^{(n)}]} \right|^2 = \frac{N_m}{N_n}$$

So, given

$$\Psi_0^2[U] = \mathcal{N} e^{-R[U]}$$
$$= e^{-R[U]-R_0}$$

with *anybody's* proposal for **R[U]** , all we have to do is to plot

$$-\log\left[\frac{N_n}{N_{tot}}\right] \quad \text{vs.} \quad R[U^{(n)}]$$

If the proposal is right, then the data should fall on a straight line, with **slope = 1**.

Reason: Start with the identity

$$\Psi_0^2[U_i^{(n)}(\mathbf{x})] = \frac{1}{Z}\int DU \ \left\{\prod_{\mathbf{x}}\prod_{k=1}^{2}\delta[U_k(\mathbf{x},0) - U_k^{(n)}(\mathbf{x})]\right\} e^{-S}$$

where

$$\widetilde{\Psi}_0^2[U_i^{(n)}(\mathbf{x})] = \frac{\Psi_0^2[U_i^{(n)}(\mathbf{x})]}{\sum_{m=1}^{M}\Psi^2[U_i^{(m)}(\mathbf{x})]}$$

$$= \frac{\int DU \ \left\{\prod_{\mathbf{x}} \prod_{k=1}^{2}\delta[U_k(\mathbf{x},0) - U_k^{(n)}(\mathbf{x})]\right\} e^{-S}}{\sum_{m=1}^{M}\int DU \ \left\{\prod_{\mathbf{x}} \prod_{k=1}^{2}\delta[U_k(\mathbf{x},0) - U_k^{(m)}(\mathbf{x})]\right\} e^{-S}}$$

This is a statistical system with the t=0 timeslice restricted to a set of M lattices. The system can be simulated numerically, and

$$\widetilde{\Psi}_0^2[U^{(n)}] = \lim_{N_{tot}\to\infty}\frac{N_n}{N_{tot}}$$

Since $\widetilde{\Psi}_0[U]$ is just a rescaling of $\Psi_0[U]$, it follows that

$$\frac{\Psi_0^2[U^{(n)}]}{\Psi_0^2[U^{(m)}]} = \lim_{N_{tot}\rightarrow\infty} \frac{N_n}{N_m}$$

Abelian Plane Waves

Abelian lattice configuration: $[U_1,U_2]=0$, variable amplitude.
Oscillation in the y-direction with wavelength equal to the lattice length L:

$$U_1^{(m)}(n_1,n_2) = \sqrt{1-(a^{(m)}(n_2))^2}\,\mathbb{1}_2 + i a^{(m)}(n_2)\sigma_3$$

$$U_2^{(m)}(n_1,n_2) = \mathbb{1}_2$$

$$a^{(m)}(n_2) = \frac{1}{L}\sqrt{\alpha+\gamma m}\cos\left(\frac{2\pi n_2}{L}\right)$$

squared lattice momentum $\widetilde{k}^2 = 2(1-\cos(2\pi/L))$

For physical units, set string tension $\sigma = (440\ \mathrm{MeV})^2$

lattice spacing $a = \sqrt{\frac{\sigma_{lat}}{\sigma}}$ as usual.

Predictions:

$$-\log\left(\frac{N_j}{N_{tot}}\right) = 2(\alpha + \gamma j)\omega(\widetilde{k^2}) + r_0$$

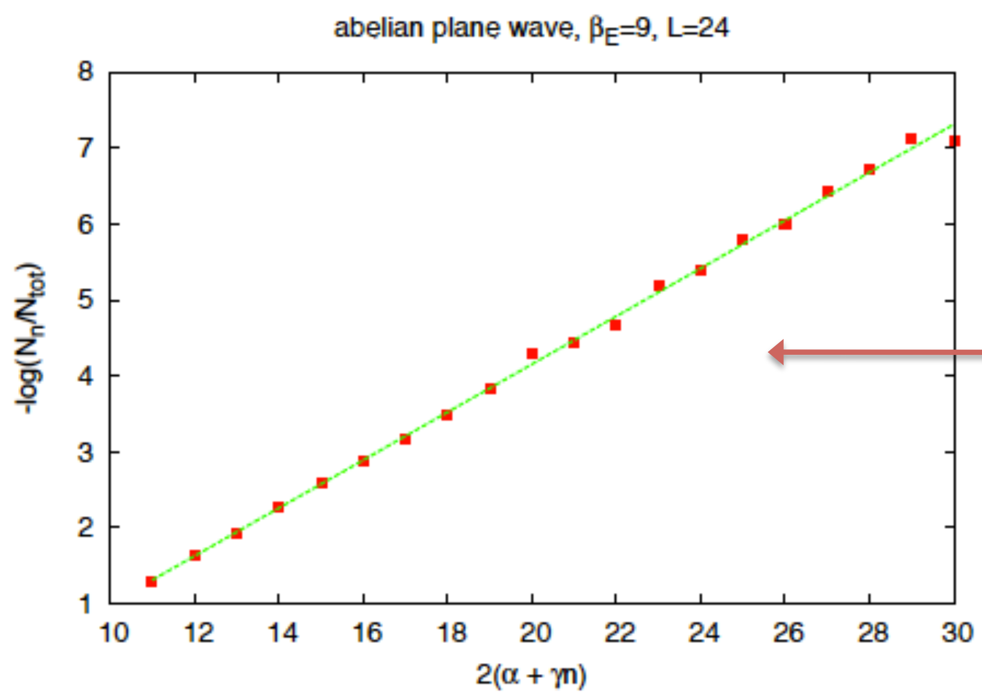
where

$$\omega(k^2) = \begin{cases} \frac{1}{g^2}\frac{k^2}{\sqrt{k^2+m^2}} & \text{GO} \\ \\ \frac{1}{g^2}\frac{k^2}{\sqrt{k^2+m^2}+m} & \text{KKN} \end{cases}$$

For CG, a numerical solution is required. However, $\omega_{CG}(0) = 0$ if $c_1=0$, otherwise $\omega_{CG}(0) > 0$.

Fit the Monte Carlo data to

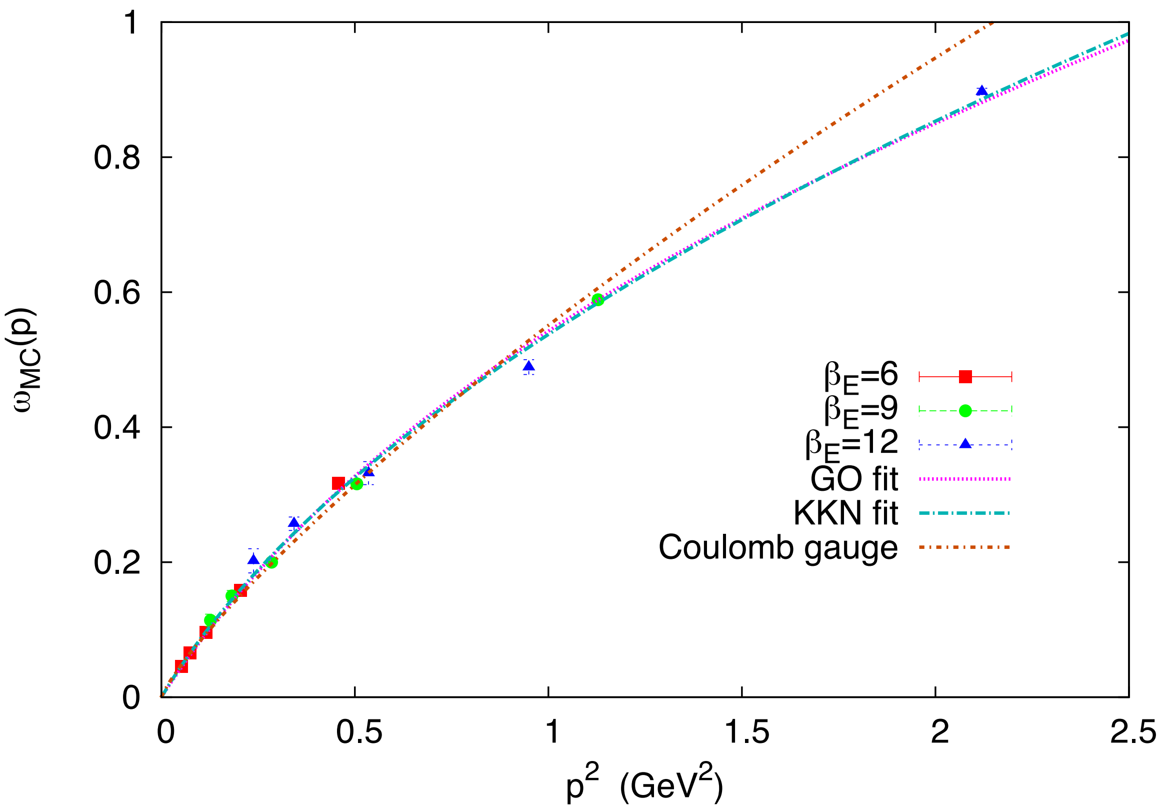
$$-\log\left(\frac{N_j}{N_{tot}}\right) = 2(\alpha + \gamma j)\omega_{MC}(\widetilde{k}^2) + r_0$$



slope is $\omega_{MC}(k^2)$

Compare to the predicted values for $\omega(k^2)$.

Results:



g^2/m is the fitting parameter

GO, KKN both work well, and are indistinguishable in this range of momenta. CG also works well, for the choice (shown here) of c_1=0.

Non-Abelian Constant Configurations

For abelian plane waves in the low-momentum limit, for GO, KKN, *and* CG,

$$R[A] \propto \omega(k^2)(\vec{A}_1 \cdot \vec{A}_1 + \vec{A}_2 \cdot \vec{A}_2)$$

and it is just a question of how much the $\omega(k^2)$ agree in the given range studied.

In contrast for weak-field, non-abelian constant configurations

$$R[A] \propto \begin{cases} (\vec{A}_1 \times \vec{A}_2) \cdot (\vec{A}_1 \times \vec{A}_2) & \text{GO, hybrid} \\ \omega(0)(\vec{A}_1 \cdot \vec{A}_1 + \vec{A}_2 \cdot \vec{A}_2) & \text{CG} \end{cases}$$

In this case, GO & hybrid are *qualitatively different* from CG.

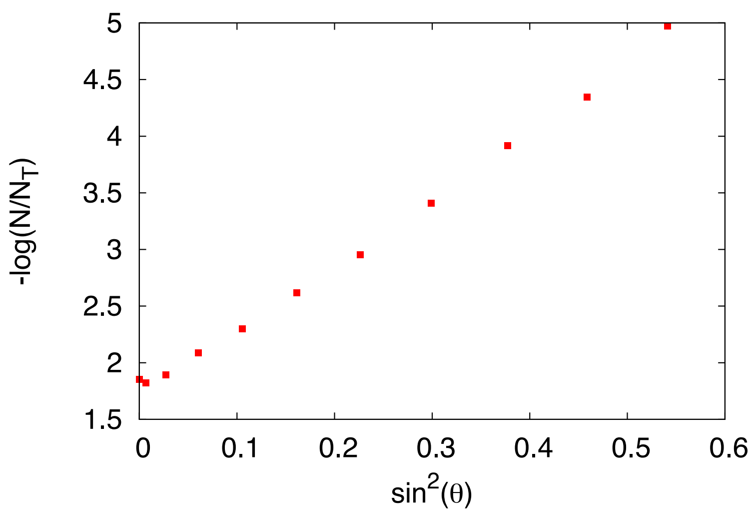
fixed amplitude, variable “non-abelianicity” controlled by angle θ

$$U_1^{(n)} = \sqrt{1-\alpha^2}\mathbb{1}_2 + i\alpha\sigma_1$$
$$U_2^{(n)} = \sqrt{1-\alpha^2}\mathbb{1}_2 + i\alpha(\cos(\theta_n)\sigma_1 + \sin(\theta_n)\sigma_2)$$

for small α ,

$$R_{GO,hybrid}[U^{(n)}] \propto \sin^2(\theta_n)$$
$$R_{CG}[U^{(n)}] \propto \overline{\omega}(0)$$

and this is easy to check, since we should have $-\log[N_n/N_{tot}] = R[U^{(n)}] + r_0$



GO and hybrid are fine,

CG doesn’t work.

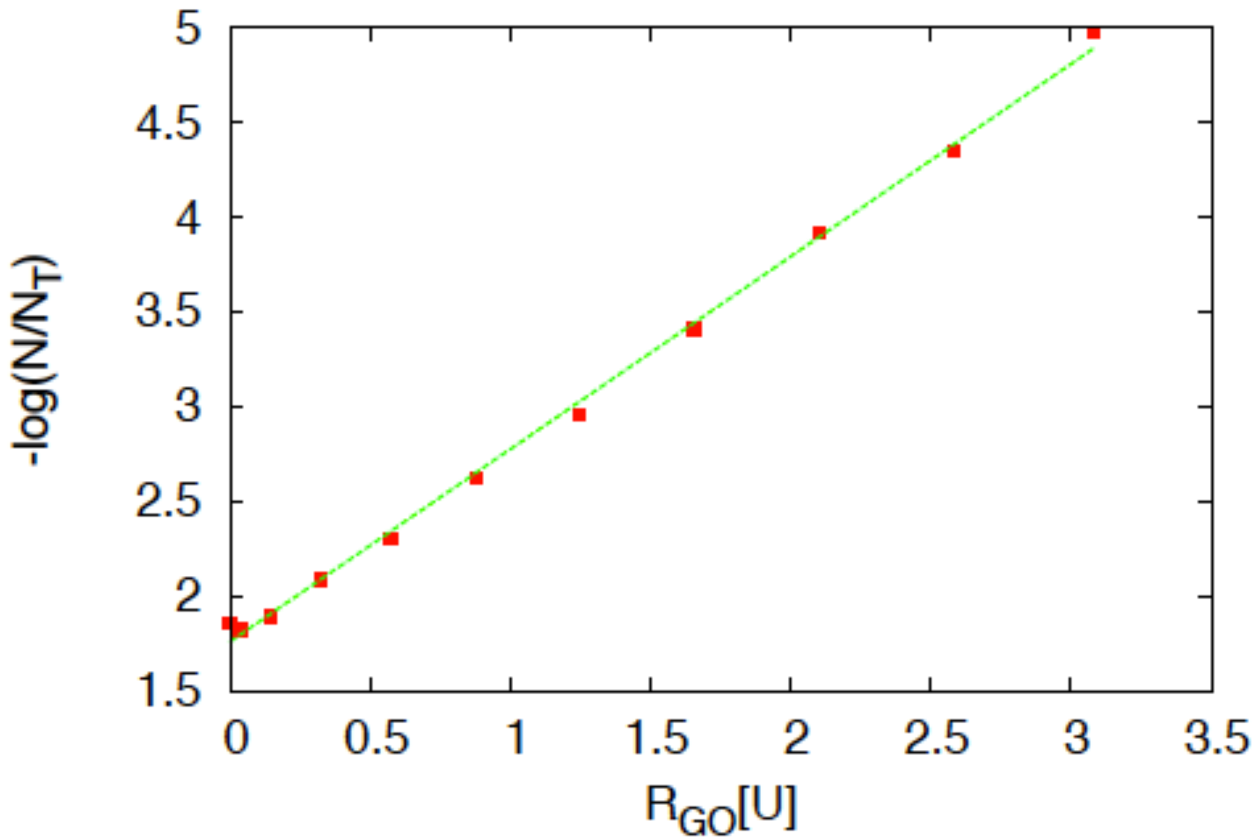


FIG. 6. Plot of $-\log(N_n/N_T)$ vs. R_{GO} for the non-abelian constant configurations with variable non-abelianicity. The straight line fit has slope = 1.02.

variable amplitude, maximal "non-abelianicity" $\theta=\pi/2$

$$U_1^{(m)}(n_1,n_2) = \sqrt{1-(a^{(m)})^2}\,\mathbb{1}_2 + ia^{(m)}\sigma_1$$
$$U_2^{(m)}(n_1,n_2) = \sqrt{1-(a^{(m)})^2}\,\mathbb{1}_2 + ia^{(m)}\sigma_2$$
$$a^{(m)} = \left[\frac{\alpha+\gamma m}{20L^2}\right]^{1/4}$$

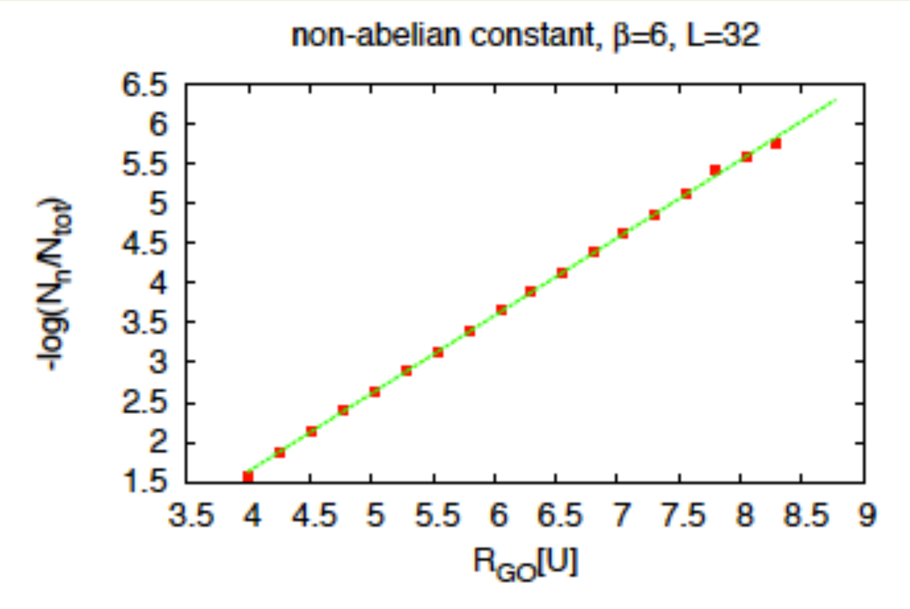
We plot

$$-\log(N_n/N_{tot}) \quad \text{vs.} \quad R[U^{(n)}]$$

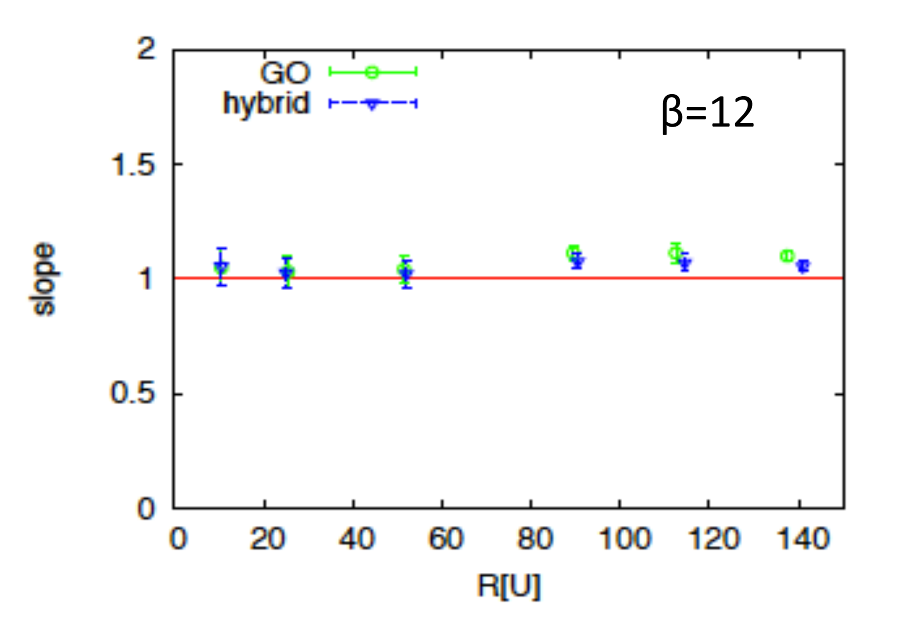
The proposal $\Psi[U] = \exp[-R[U]]$ works if

1. the data in this plot falls on a straight line
2. with slope = 1

Both GO and hybrid work well:



slope = 0.98 in this case



slope is close to unity over a wide range of R[U]

Numerical Simulation of the GO/hybrid Wavefunctionals

It is also possible to simulate the probability distribution $\Psi_0^2[U]$ for the GO and hybrid proposals. Olejnik & JG PRD77 (2008)

Using this method, one can compute the

1. Coulomb gauge ghost propagator $$G(R) = -\left\langle \frac{1}{\nabla \cdot D} \right\rangle^{aa}_{|x-y|=R}$$

2. color Coulomb potential $$V_C(R) = -\left\langle \frac{1}{\nabla \cdot D}(-\nabla^2)\frac{1}{\nabla \cdot D} \right\rangle^{aa}_{|x-y|=R}$$

3. mass gap, extracted from $$\left\langle \mathrm{Tr}[F_{12}^2(x)]\mathrm{Tr}[F_{12}^2(y)] \right\rangle$$

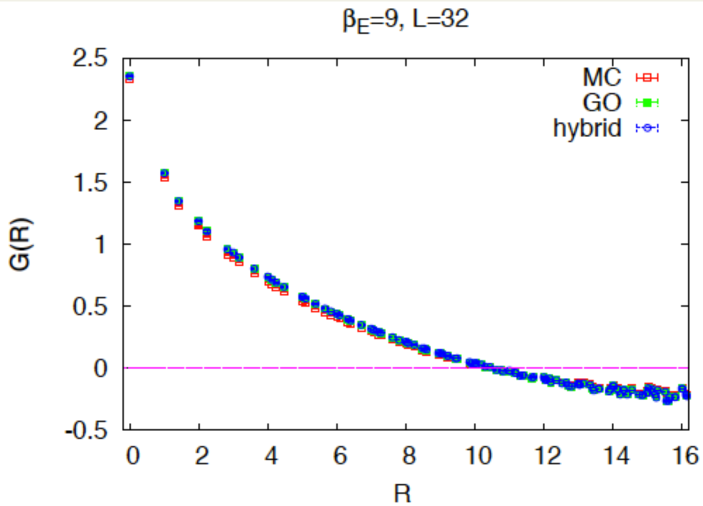
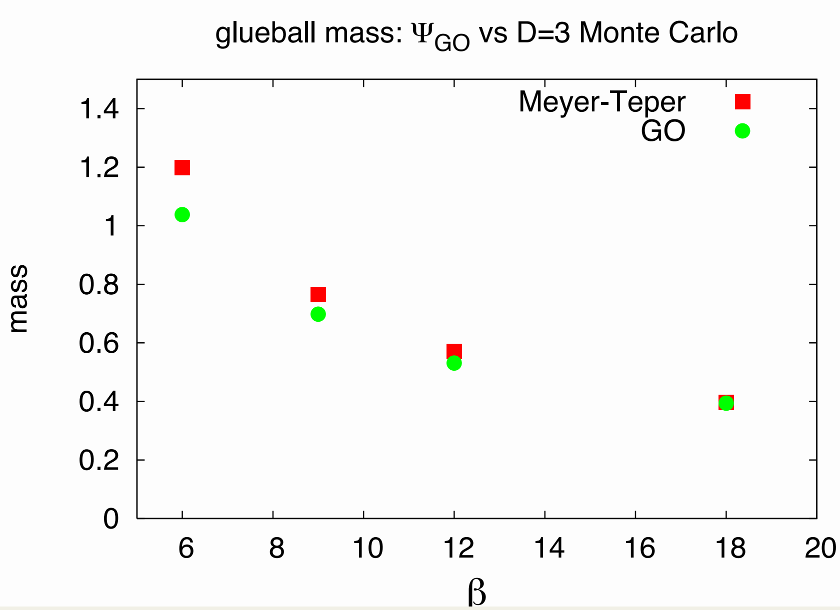


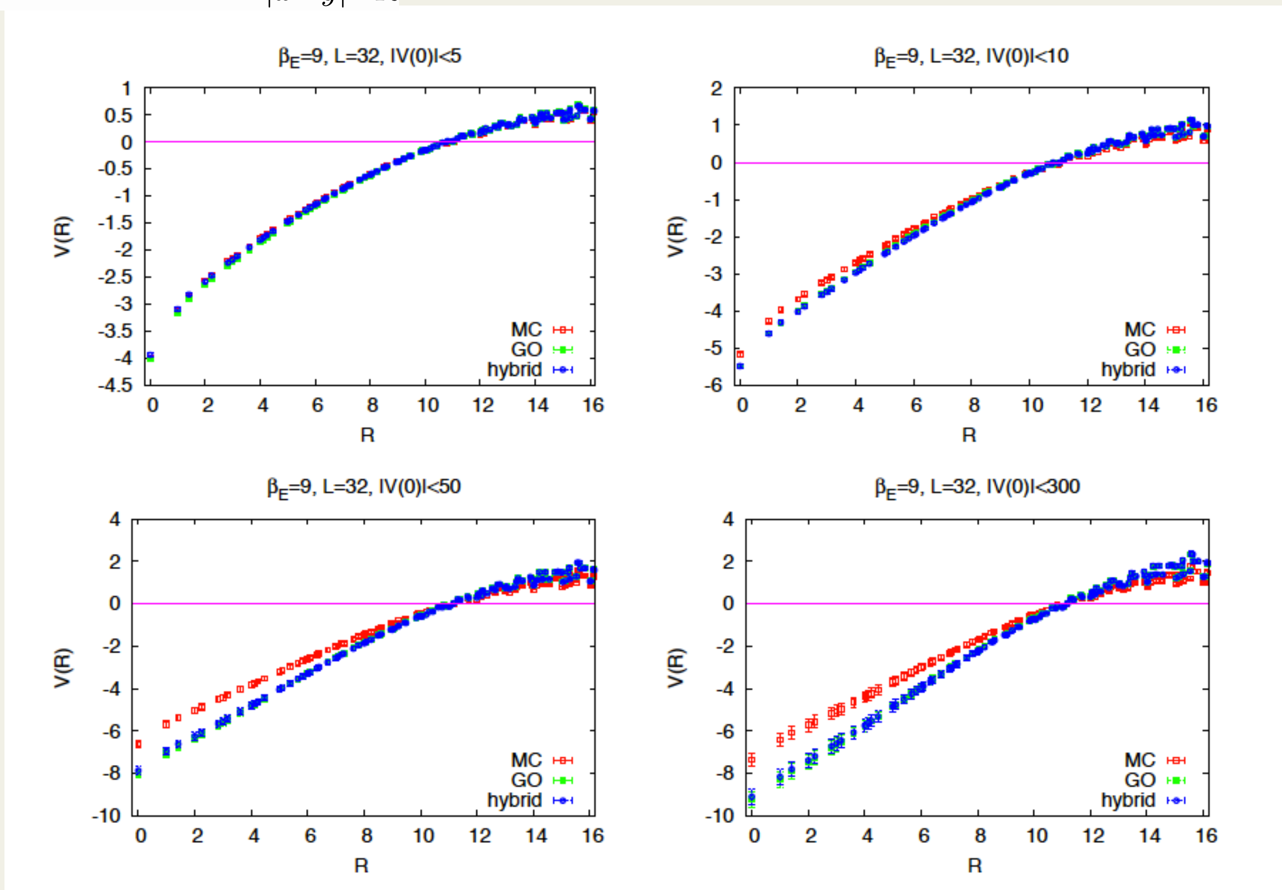
FIG. 10. The ghost propagator derived from standard Monte Carlo (MC) simulation at $\beta_E = 9$, and the same quantity calculated by simulation of the GO and hybrid wavefunctionals, by the technique described in ref. [8].

ghost propagator



mass gap

$$V_C(R) = -\left\langle \frac{1}{\nabla\cdot D}(-\nabla^2)\frac{1}{\nabla\cdot D}\right\rangle^{aa}_{|x-y|=R}$$



agreement with the Coulomb potential is not as good, but the disagreement stems from rare “exceptional” configurations, for which the Fadeev-Popov operator $-\nabla\cdot D$ has an unusually small eigenvalue. The Coulomb potential is more sensitive to these than the ghost propagator. When exceptional configurations are excluded, agreement is restored.

Conclusions at T=0

- Both GO and Hybrid (=KKN for abelian) fit the data almost perfectly, for both abelian plane wave and non-abelian constant lattice configurations.
- The Coulomb gauge proposal also works for abelian plane waves, if we choose $c_1=0$. It fails on non-abelian constant configurations.
- So far, we cannot distinguish the GO from the Hybrid proposal. Will need to go to shorter wavelengths.
- Physics conclusion: There is some truth to ***Dimensional Reduction in the Infrared.*** Long wavelength vacuum fluctuations in 2+1 at fixed time resemble fluctuations in a two-dimensional Euclidean theory. (Explains Casimir scaling.)
- Next step: D=3+1 dimensions.

What are the important high-T states?

We assume that at any temperature, energy eigenstates have the general form

$$|n\rangle = Q_n|0\rangle$$

where Q_n is some gauge-invariant operator. The possibilities are infinite. We can always build gauge-invariant operators from, e.g.

1. eigenstates and eigenvalues of covariant differential operators

$$F[\phi_n^a(x)\phi_m^a(x);\lambda_n] \quad \text{where} \quad -D^2\phi_n = \lambda_n\phi_n$$

2. transformations g[x;A] which take the gauge field A to a physical (e.g. Coulomb) gauge, i.e.

$$G\Big[g[x;A]\circ A_k(x)\Big]$$

There are also

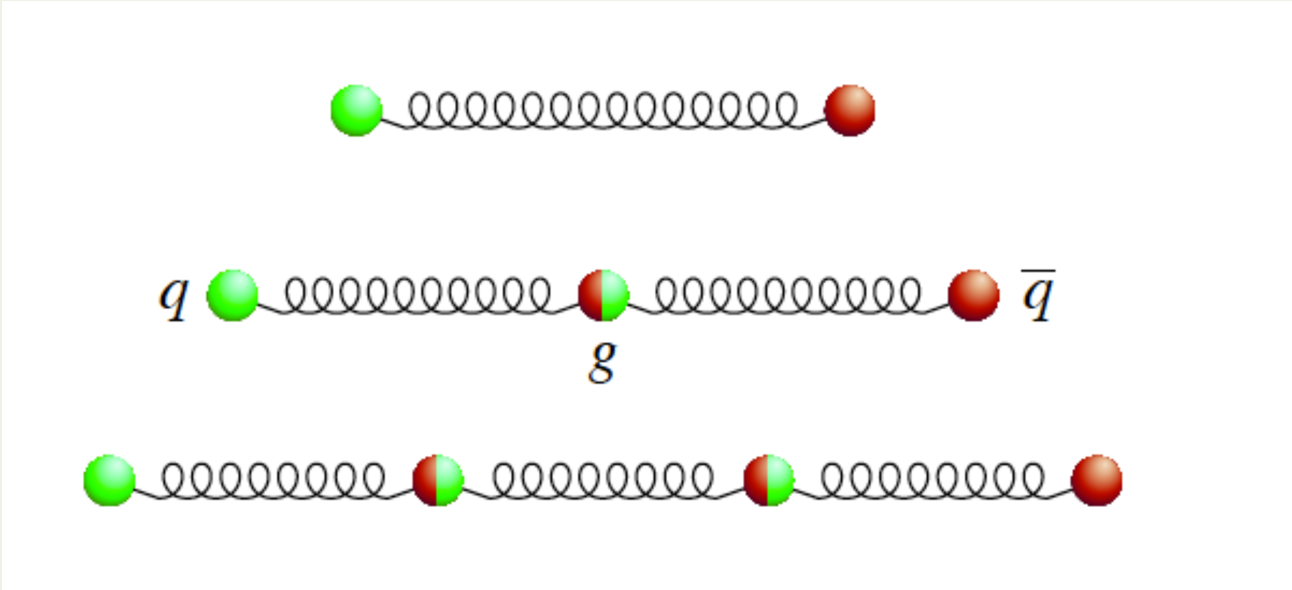
- Wilson loop operators $W(C)$ - create electric flux tubes;
- 't Hooft loop operators $B(C)$ - create magnetic flux tubes.

These are "local" in the sense that their construction does not require solving differential equations in the full volume of space.

Also, in fixed gauges, electric and magnetic flux tubes are seen (in numerical simulations) to have a constituent structure

Gluon-Chain Model (Thorn & JG, 1980's) - in Coulomb gauge, electric flux tubes appear as a chain of constituent gluons, bound by confining Coulombic forces.

Recent numerical investigation: Olejnik & JG PRD 79 (2009)



A similar picture was advocated at *high* temperatures, $T>T_C$, by Shuryak and Liao (2006).

Center vortices (magnetic flux tubes) in maximal center gauge:
a monopole-antimonopole chain.

There is quite a lot of numerical evidence that these objects play a role in generating a confining force which respects N-ality. (review: hep-lat/0301023)

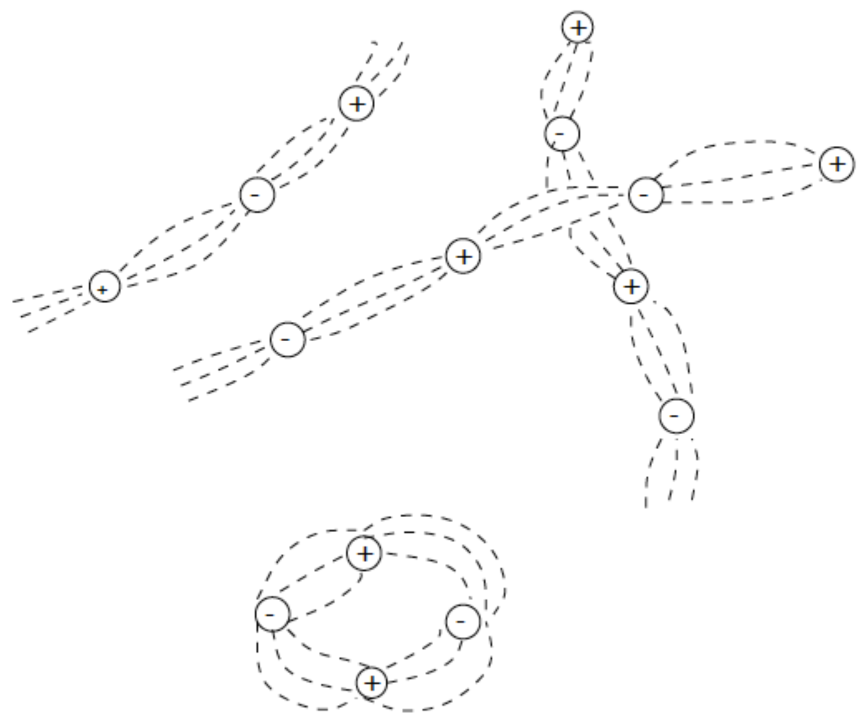


Fig. 8.6 Hypothetical collimation of monopole/antimonopole flux into center vortex tubes on the abelian-projected lattice.

So one guess, at high T, is

(for electric/magnetic pictures of the SQGP, cf. Chernodub & Zakharov and Shuryak)

$$|\mathbf{n}\rangle = \{\text{electric flux tube operators}\} \\ \times \{\text{magnetic flux tube operators}\}|0\rangle$$

Low T: only small electric flux tubes (low-lying glueballs) - would be important. Center vortices are only vacuum fluctuations.

As T increases: longer electric flux tubes (excited glueball states) - are not so much suppressed.

At $T > T_c$, what happens? Do electric flux tubes just disappear? Or percolate? Do they eventually "melt"?

Adjoint Torelon Operators

Ideally, one would like to compute $\langle Y \rangle_{\text{thermal}}$ where

$$Y = (\text{flux tube creation operator}) \times (\text{flux tube destruction operator})$$

for long flux tubes, e.g. winding through the periodic lattice in the z-direction, such that only excited states containing at least one long flux tube would contribute to the thermal average.

I don't know of any operator which does *exactly* this.

Next best thing is the adjoint torelon operator L_A .

where

$$L_A = \mathrm{Tr}[L^\dagger]\mathrm{Tr}[L] - 1$$

and

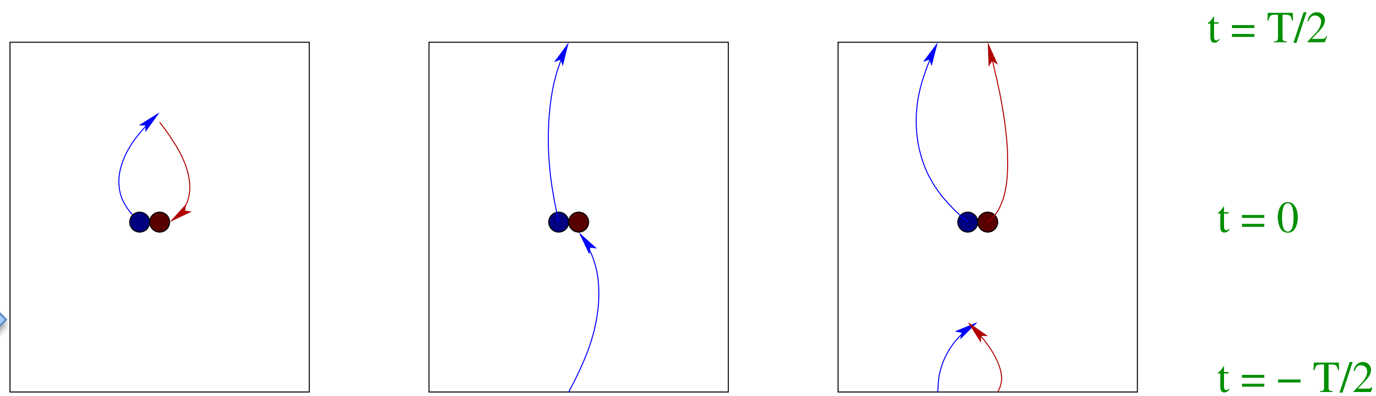
$$L = \mathrm{Tr}\ \mathrm{P}\exp\left[ig\int_0^{l_z} dz A_z(00z0)\right] = \mathrm{Tr}\left[\prod_{n_z=1}^{N_z} U_z(00n_z0)\right]$$

L transforms non-trivially under center transformations. If we (for convenience) let "torelon" refer to a line of electric flux along the positive z-direction, then

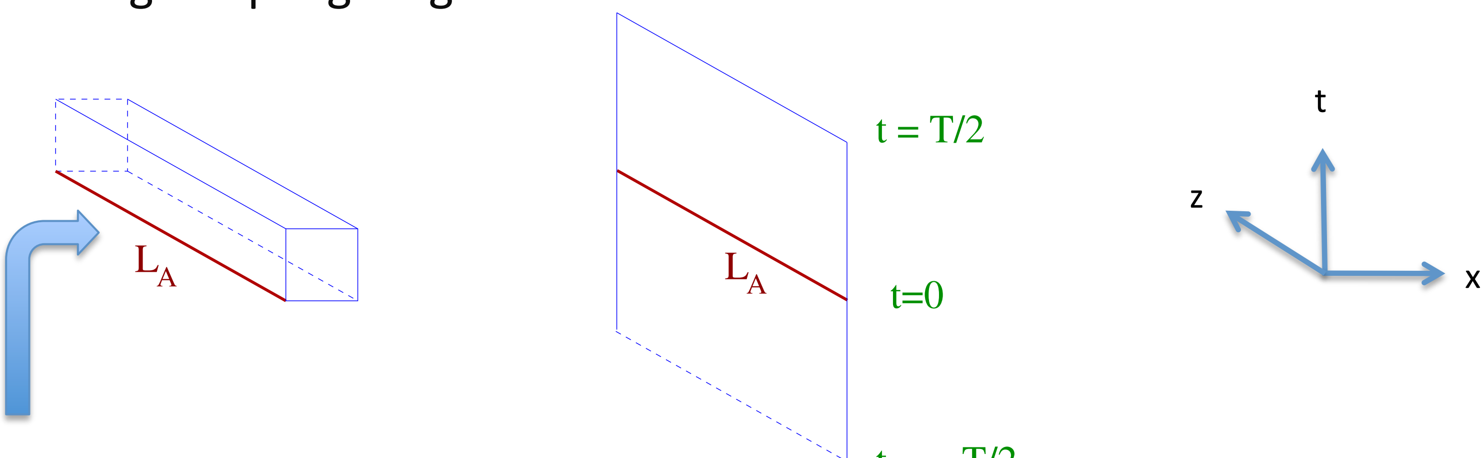
L creates a torelon, destroys an antitorelon

$L^\dagger$ creates an antitorelon, destroys a torelon processes contributing to $\langle L_A \rangle_{\mathrm{thermal}}$:

z=y=0 slice (schematic)



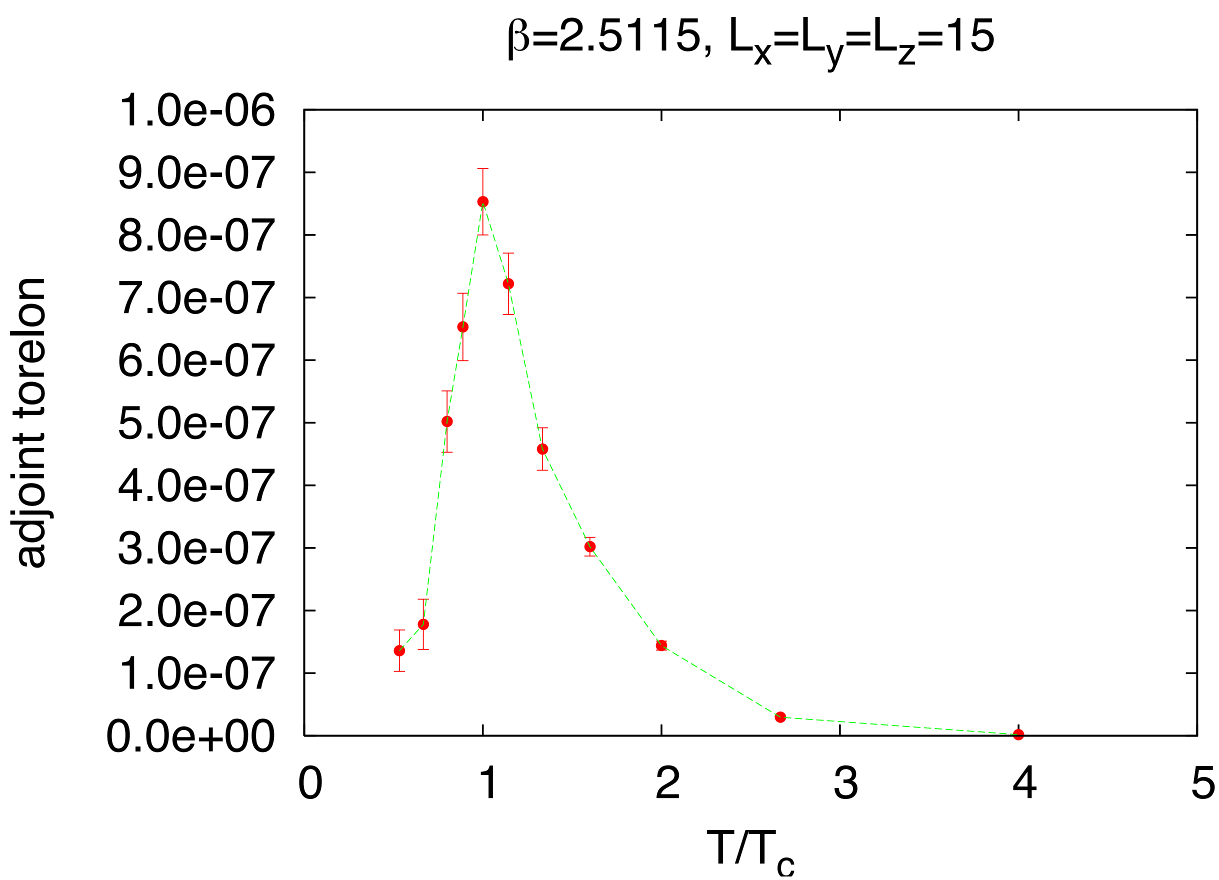
strong-coupling diagrams:



(contribute to $\langle L_A \rangle_{vac}$)

Numerical Result, β=2.5115

Adjoint torelon in the z-direction, fixed L_x, L_y, L_z , vary L_t



peak at $T=T_c$ is an *order of magnitude* higher than at low T.

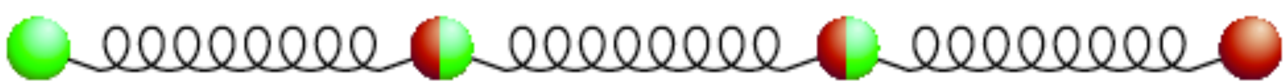
Uses Luscher-Weisz noise reduction, one level, 3 spacings per level.

The peak suggests that the contribution of long flux tubes is enhanced, as T increases, to a maximum at $T=T_C$, *persists* at $T>T_C$, and *disappears* for $T > 3T_C$ or so.

Why disappear?

From the point of view of the gluon chain model, a flux tube seen in Coulomb gauge is a chain of gluons, held together by the color-Coulombic potential

$$V_C(R) = -\left\langle \frac{1}{\nabla \cdot D}(-\nabla^2)\frac{1}{\nabla \cdot D} \right\rangle^{aa}_{|x-y|=R}$$



Maybe the Coulombic force, which is confining at T=0, is non-confining at high T?

Coulomb string tension at high-T

Define the time-like Wilson line of time-extent *t* (<u>not</u> a Polyakov line)

$$\mathcal{L}(\mathbf{x},t) \equiv P\exp\left[i\int_0^t dx_0\; A_0(\mathbf{x},x_0)\right]$$

$$G(R,t) \equiv \left\langle \mathrm{Tr}\left[\mathcal{L}(\mathbf{0},t)\mathcal{L}^\dagger(\mathbf{R},t)\right]\right\rangle$$

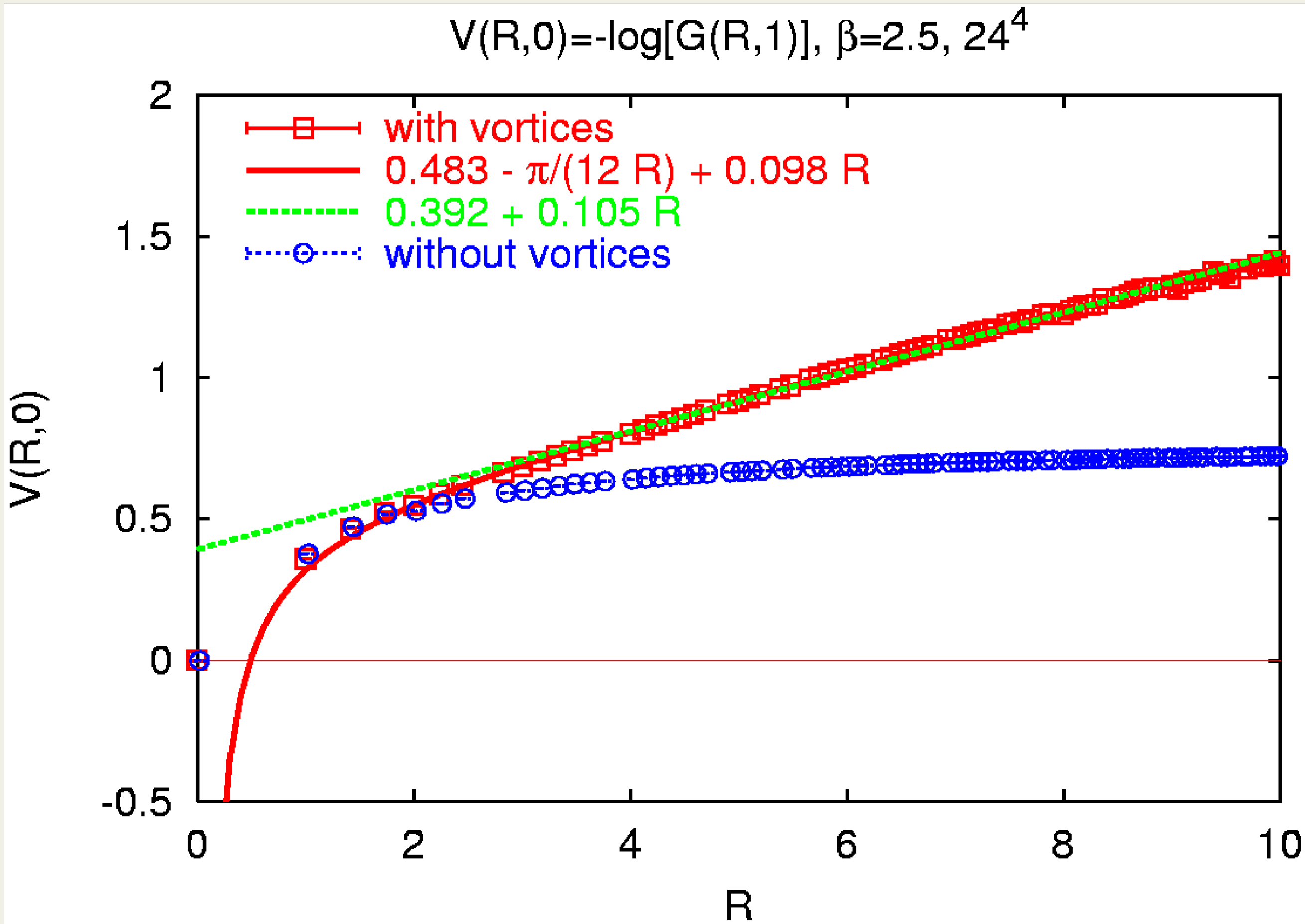
It is not hard to show that (also at finite temperature)

$$V_C(R) = -\lim_{t\to 0}\frac{d}{dt}\log[G(R,t)]$$

On the lattice

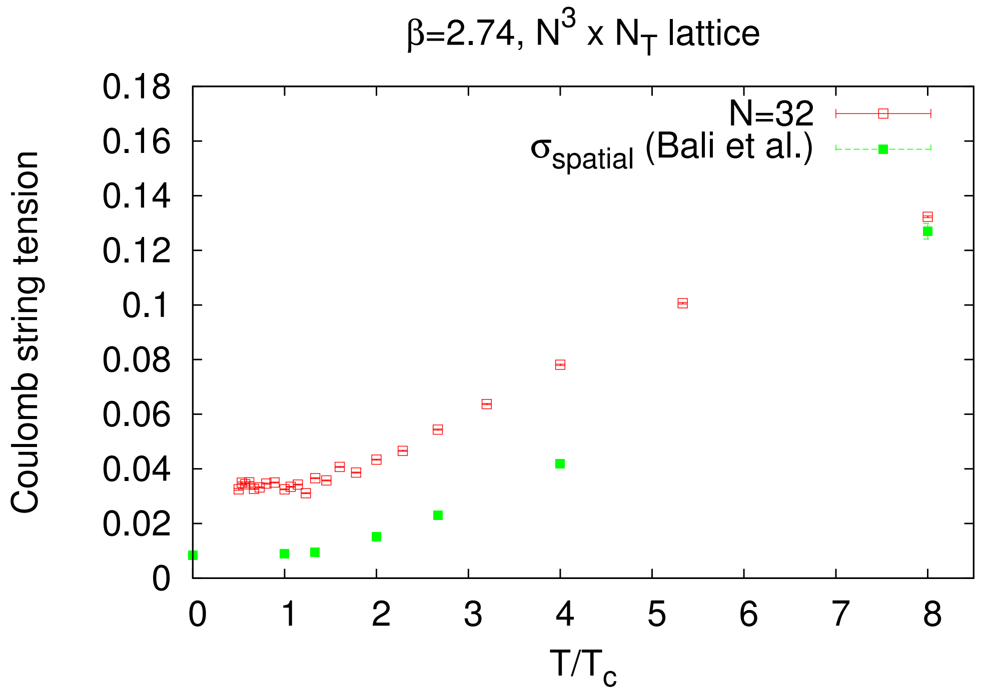
$$V_C(R) = -\log\left\langle \mathrm{Tr}\left[U_0(\mathbf{0},0)U_0(\mathbf{R},0)\right]\right\rangle$$

zero-temperature result: the color-Coulomb potential is linearly confining, with a string tension about three times larger than the asymptotic string tension.



Olejnik & JG (2003)

This is the result at finite temperatures, with the spatial string tension shown for comparison:



Like the spatial string tension, the Coulomb string tension $\boldsymbol{\sigma_c(T)}$ <u>increases</u> with temperature.

Conclusions at High T

- The adjoint torelon observable has a pronounced peak at $T=T_C$, and runs to zero around $T = 3T_C$ or so.

- The suggestion is that long electric flux tubes persist, in thermal ensembles, beyond the critical temperature, and then disappear.

- The color-Coulomb string tension, like the spacelike string tension, actually *increases* with $T>T_C$. So this does not explain flux tube disappearance.

Perhaps gluon chains "melt" into their constituent gluons at high T, as the overall density of gluons becomes large. That means: many gluons, but the organization of gluons into chains (i.e. correlation of color indices with spatial position) is lost.

IFTT #1: color screening at strong coupling

Ψ_0 has the dimensional reduction form at large scales

$$\Psi_0^{eff}[A] = \mathcal{N} \exp\left[-\frac{1}{2}\mu \int d^3x \ \mathrm{Tr}[F_{ij}^2]\right]$$

2D Yang-Mills --> Casimir scaling.

However, Casimir scaling in D=2+1 and 3+1 does not hold asymptotically for all group representations. At large enough scales, the string tension should depend only on the N-ality of the static color charges, due to color screening by gluons.

So - what about color screening?

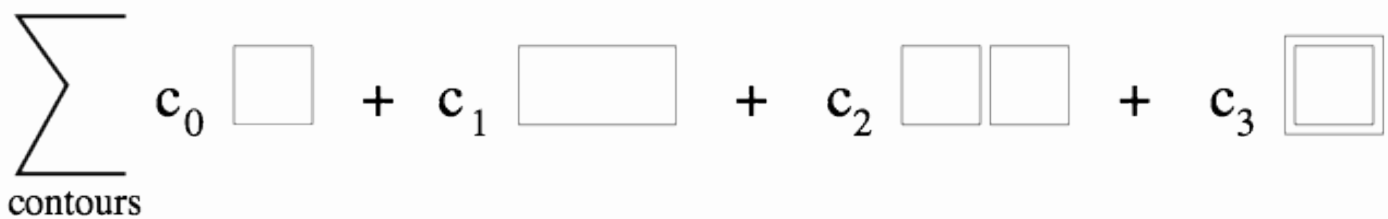
How is this problem solved at strong couplings, where we can solve for the ground state analytically? *(JG, 1980)*

The vacuum is

$$\Psi_0[U] = e^{R[U]}$$

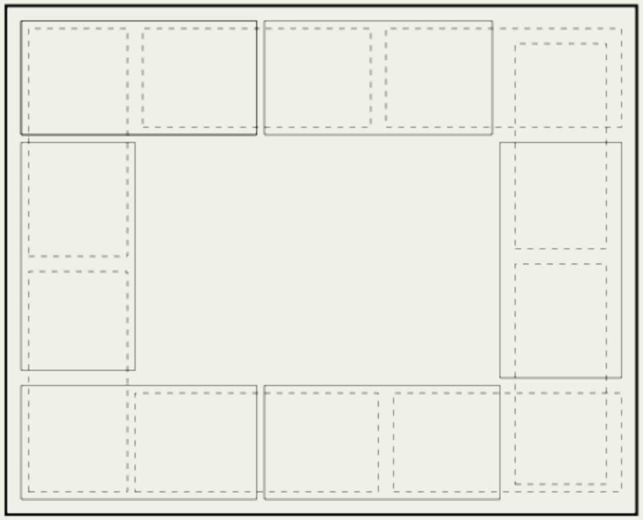
where, up to O(β^4)

$$R[U] =$$



$+$ larger contours

The 1X2 rectangle c_1  screens adjoint loops.



It also gives the leading correction to dimensional reduction.

Expansion in powers of lattice spacing:

$$\Psi_0[U] = \exp\left[-\frac{2}{\beta}\int d^2x\ (a\kappa_0 B^2 - a^3\kappa_2 B(-D^2)B + \ldots)\right]$$

where *(Guo et al, 94)*

$$\begin{aligned}\kappa_0 &= \frac{1}{2}c_0 + 2(c_1 + c_2 + c_3)\\ \kappa_2 &= \frac{1}{4}c_1\end{aligned}$$

Note! Leading correction comes from the rectangle ($\propto c_1$)

IFTT #2: Probability distribution of Polyakov lines

Do the eigenvalues of Polyakov lines, in the confined phase, attract or repel? Some extremes:

1. ***Center projected SU(N) gauge theory***:
   P(x) = a center element. <Tr[P]>=0 because of fluctuations among center elements.
2. ***Dilute gas of dyons*** (Diakonov & Petrov), ***trace deformations*** (Unsal) :
   Polyakov lines are mostly near Tr[P]=0 in each configuration.

In the actual vacuum state there is a broad probability distribution for P(x). But we can ask whether that distribution is (slightly) peaked at zero - ***eigenvalues "repel"*** - or at center elements **-** ***eigenvalues "attract"***.

Let ρ(g) = prob density that P(x)=g . Then in SU(2)

$$\begin{aligned}\rho(g) &= \Big\langle \delta[P(x)-g] \Big\rangle \\ &= \sum_j \langle \chi_j[P(x)] \rangle \chi_j(g) \\ &\approx 1 + \langle \mathrm{Tr}_A[P(x)] \rangle \mathrm{Tr}_A[g]\end{aligned}$$

where $\mathrm{Tr}_A[g] = \chi_1[g]$

But if $\rho(g) \approx 1 + \langle \mathrm{Tr}_A[P(x)] \rangle \mathrm{Tr}_A[g]$

it means that:

1. probability is peaked at center elements if $\langle \mathrm{Tr}_A[P(x)] \rangle > 0$

2. probability is peaked at Tr[P]=0 if $\langle \mathrm{Tr}_A[P(x)] \rangle < 0$

The numerical result is that $\langle \mathrm{Tr}_A[P(x)] \rangle > 0$, and therefore the first alternative ("eigenvalues attract") is the correct one.

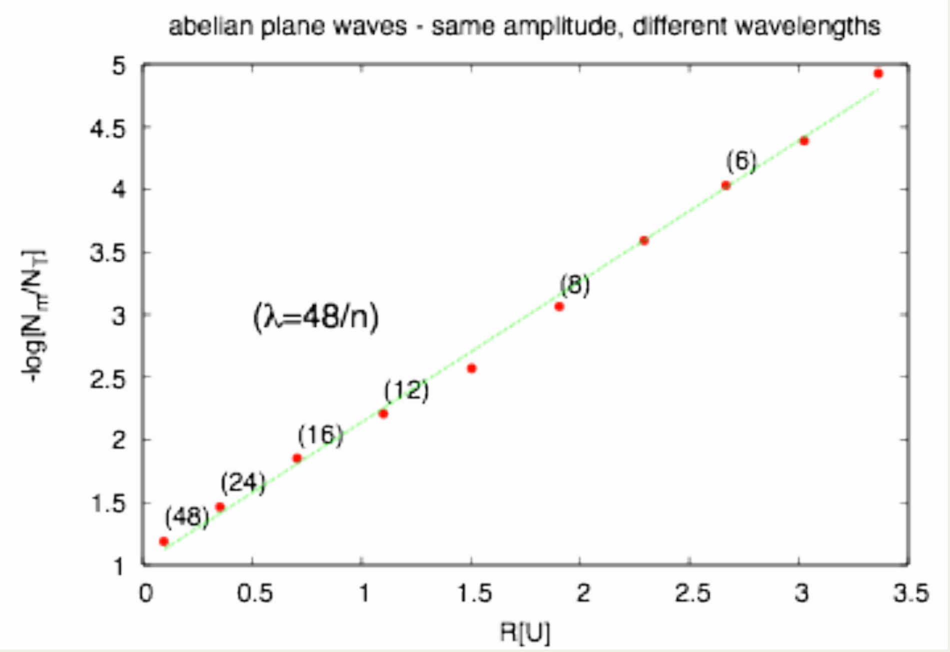
IIb. Abelian plane-wave configurations (same amplitude, different λ)

$$
\begin{aligned}
U_1^{(m)}(n_1,n_2) &= \sqrt{1-a_m^2(n_1,n_2)}I + ia_m(n_1,n_2)\sigma_3 \\
U_2^{(m)}(n_1,n_2) &= I \\
a_m(n_1,n_2) &= \sqrt{\frac{\kappa}{L^2}}\cos\frac{2\pi m n_2}{L}
\end{aligned}
$$

parameters κ=1 , β=6, L=48, m=1-10

wavelengths λ = 4.8 - 48

the slope is 1.1 ± 0.02 .



Dimensional Reduction: Expand B(x) in eigenmodes of the covariant Laplacian:

$$
\begin{aligned}
(-D^2)^{ab}\phi_n^b(x) &= \lambda_n \phi^a(x) \\
B^a(x) &= \sum_{n=0}^{\infty} b_n \phi_n^a(x) \\
B^{a,\mathrm{slow}}(x) &= \sum_{n=0}^{n_{max}} b_n \phi_n^a(x)
\end{aligned}
$$

The cutoff mode sum defines the "slowly varying" B-field. Choosing n_{max} such that

$\lambda_{n_{max}} - \lambda_0 \ll m^2$

$$
\begin{aligned}
&\int d^2x d^2y \; B^{a,\mathrm{slow}}(x) \left(\frac{1}{\sqrt{-D^2 - \lambda_0 + m^2}} \right)^{ab}_{xy} B^{b,\mathrm{slow}}(y) \\
&\approx \frac{1}{m} \int d^2x \; B^{a,\mathrm{slow}}(x) B^{a,\mathrm{slow}}(x)
\end{aligned}
$$

So the part of the squared wavefunctional that involves B^{slow} is

$$|\Psi_0|^2 = \exp\left[-\frac{1}{m}\int d^2x \; B^{slow} B^{slow}\right]$$

which is the probability distribution of D=2 dimensional Yang-Mills (i.e. dimensional reduction). The string tension σ can be calculated analytically; in lattice units it is

$$\sigma = \frac{3}{4}\frac{m}{\beta}$$