Anisotropic flow far from equilibrium: onset of collectivity

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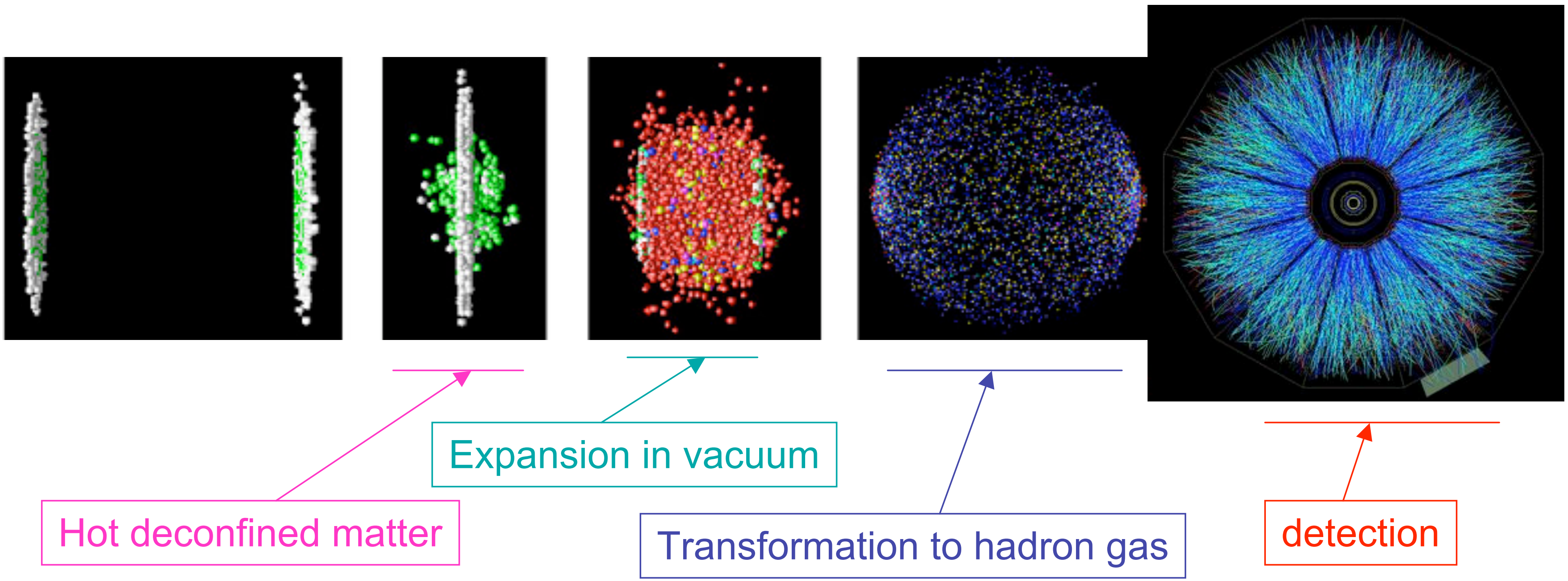
Heavy ions collisions

Smashing heavy ions together

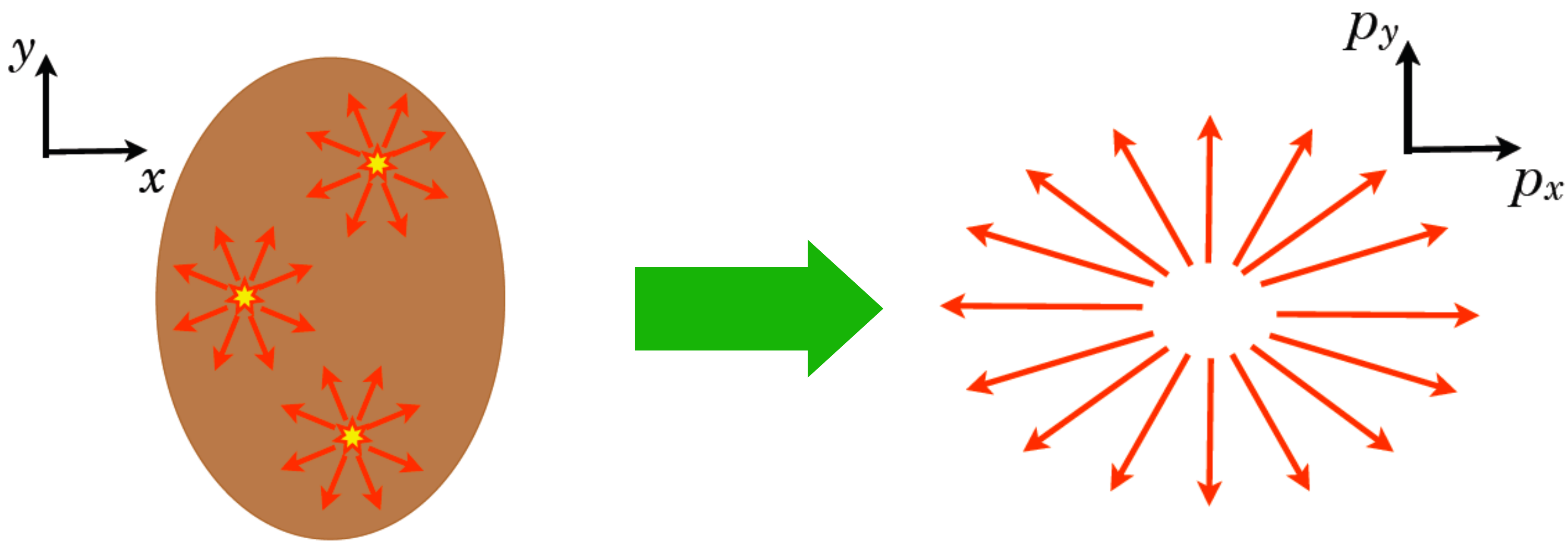
RHIC Au-Au at 200 GeV
LHC Pb-Pb at 2.76 TeV

And studying what comes out



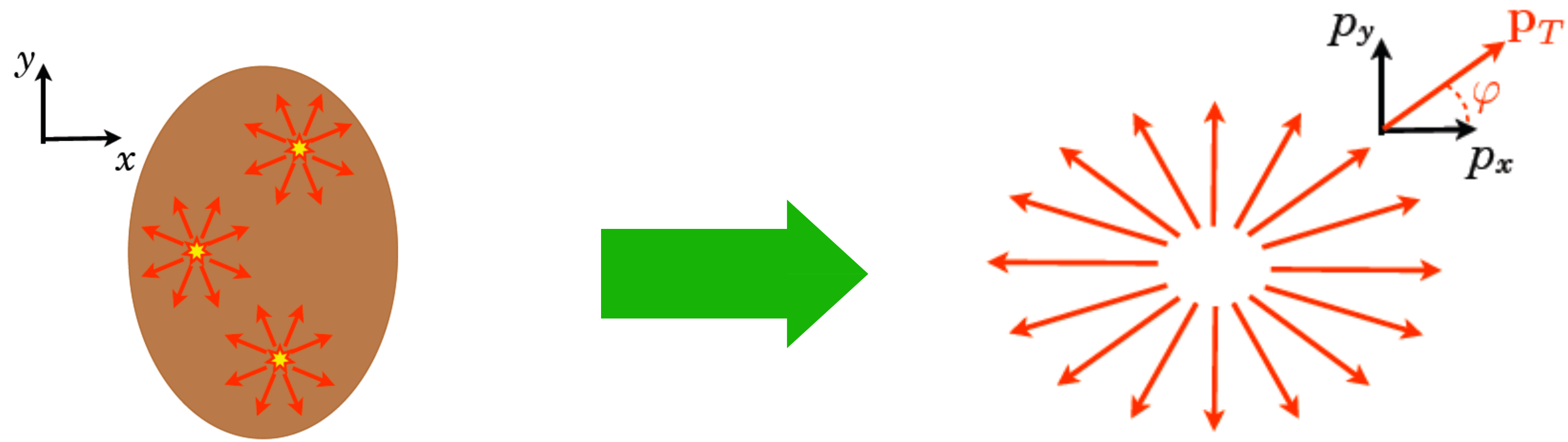
Anisotropic flow

In non-central nucleus-nucleus collisions, the initial spatial asymmetry of the overlap region in the transverse plane is converted by particle rescatterings into an anisotropy in momentum space: anisotropic (transverse) flow



Anisotropic flow

$$\frac{d^2N}{d^2\mathbf{p}_T} = \frac{1}{2\pi}\frac{dN}{p_T dp_T}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\cos(n\varphi)\right] \qquad v_n(p_T) = \frac{\int d\varphi \frac{d^2N}{d^2\mathbf{p}_T}\cos(n\varphi)}{\int d\varphi \frac{d^2N}{d^2\mathbf{p}_T}}$$



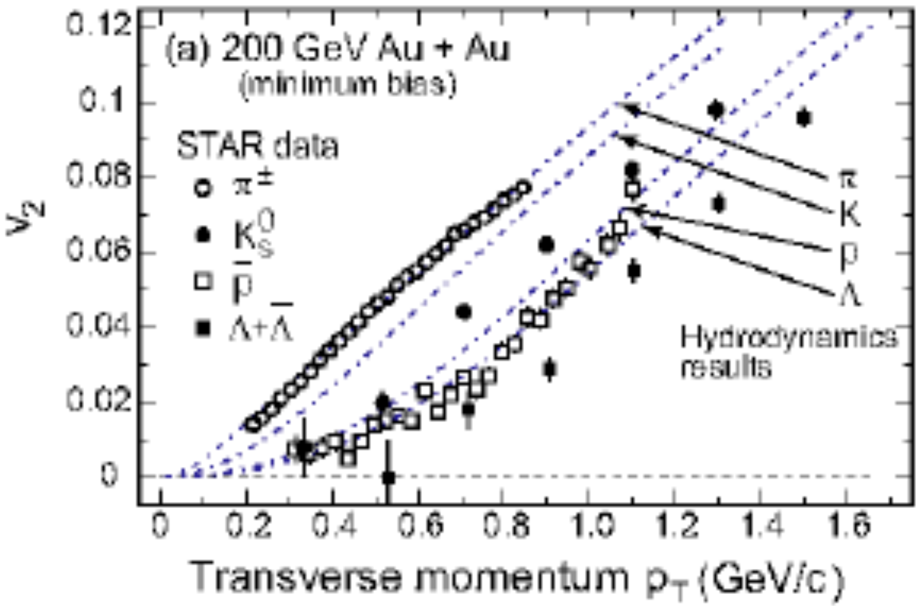
$$\epsilon_2 = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle \qquad \longrightarrow \qquad v_2 \quad \text{(Elliptic flow)}$$

Anisotropic flow

Adams & al Nucl Phys A757,102 (2005)

- The large values of elliptic flow measured at RHIC were initially understood as a signature of ideal hydrodynamics

- Mass ordering also seen in hydrodynamics



✓Do we need many collisions (hydro) to build up "collective flow"?
✓Do we need the presence of a thermalized medium to obtain the "mass ordering" of (elliptic) flow?

The model

- System: A 2-dimensional dilute mixture of several components with different masses which scatter elastically on each other with a constant isotropic differential cross-section σ_d .

  - 2-dimensional: we are interested in transverse expansion
  - σ_d isotropic, constant, p_T-independent: a single parameter
  - dilute system: kinetic description “à la Boltzmann” meaningful

- Initial condition: isotropic in momentum space
  asymmetric in position space $R_y > R_x$

$$f_{i,k}(0, \mathbf{x}, \mathbf{p}_{i,k}) = \frac{N}{4\pi^2 R_x R_y} \tilde{f}_{i,k}(\mathbf{p}_{i,k}) exp\left(-\frac{x^2}{2R_x^2} - \frac{y^2}{2R_y^2}\right)$$

The model: evolution equation

Boltzmann equation $$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = \int d^2\mathbf{p}_k d\theta \left(f_i' f_k' - f_i f_k \right) v_{ik} \sigma_d$$

We have neglected the mean field term

- Distributions before the collision $f_i \equiv f_i(t, \mathbf{x}, \mathbf{p}_i)$ and $f_k \equiv f_k(t, \mathbf{x}, \mathbf{p}_k)$
- Distributions after the collision $f_i' \equiv f_i(t, \mathbf{x}, \mathbf{p}_i')$ and $f_k' \equiv f_k(t, \mathbf{x}, \mathbf{p}_k')$
- Relative velocity $v_{ik} = \sqrt{(\mathbf{v}_i - \mathbf{v}_k)^2 - \frac{(\mathbf{v}_i \times \mathbf{v}_k)^2}{c^2}}$
- θ angle between $\mathbf{p}_T$ and $\mathbf{p'}_T$ $\left(\text{irrelevant: } \sigma_d \text{ isotropic } \rightarrow \int d\theta = 2\pi \right)$

Once the distribution function is known, the transverse momentum distribution is given by: $$\frac{d^2N}{d^2\mathbf{p}_T}(t, \mathbf{p}_T) = \int d^2\mathbf{x} f(t, \mathbf{x}, \mathbf{p}_T)$$

The model: evolution equation

Integrating the Boltzmann equation over x,

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = \int d^2\mathbf{p}_k d\theta (f_i' f_k' - f_i f_k) v_{ik} \sigma_d$$

The gradient term (odd function of x) disappears

$$\frac{\partial}{\partial t} \frac{d^2 N_i}{d^2 \mathbf{p}_i} = \int d^2\mathbf{x} \int d^2\mathbf{p}_k d\theta (f_i' f_k' - f_i f_k) v_{ik} \sigma_d$$

Just remind that $v_n(p_T) = \frac{\int d\varphi \frac{d^2N}{d^2\mathbf{p}_T} \cos(n\varphi)}{\int d\varphi \frac{d^2N}{d^2\mathbf{p}_T}}$

Then multiplying with $\cos(n\varphi_i)$ and averaging over the azimuth φ_i yields the time derivative of the anisotropic flow coefficient $v_n(t, p_i)$

The model: resolution

First consider the free streaming regime $\sigma_d = 0$

The Boltzmann equation reads: $\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = 0$

It admits the "free streaming" distribution as solution

$$f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) = f_i^{(0)}(0, \mathbf{x} - \mathbf{v}_i t, \mathbf{p}_i)$$

In the free streaming regime, because there are no scattering, the momentum distribution remains unchanged during the evolution:

- Initially isotropic, it remains isotropic

No collective behavior is built up: $v_n^{FS}(t, p_T) \equiv 0$

Let’s turns on the scattering

We allow only few collisions:

- New solution to the Boltzmann equation

$$f_i(t,\mathbf{x},\mathbf{p}_i) = f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) + f_i^{(1)}(t,\mathbf{x},\mathbf{p}_i) \quad \text{with} \quad f_i^{(1)} \ll f_i^{(0)}$$

- Corresponding to an expansion in the number of particle scatterings, number given by:

$$N_{coll} = \int_0^\infty dt \frac{dN_{coll}}{dt} = \int_0^\infty dt \int d^2\mathbf{x} \int d^2\mathbf{p}_k d^2\mathbf{p}_i d\theta f_i^{(0)} f_k^{(0)} v_{ik} \sigma_d$$

This number has to be kept small to ensure the convergence of our expansion. This is achieved by fixing σ_d

The momentum anisotropies of f_i are those of $f_i^{(1)}$

Momentum anisotropies

In the presence of few collisions, Boltzmann equation reads :

$$\frac{\partial}{\partial t}(f_i^{(0)} + f_i^{(1)}) + \mathbf{v}_i \cdot \nabla_{\mathbf{x}}(f_i^{(0)} + f_i^{(1)}) = \int d^2\mathbf{p}_k d\theta \left(f_i^{\prime(0)} f_k^{\prime(0)} - f_i^{(0)} f_k^{(0)} \right) v_{ik}\sigma_d$$

A priori; both gain and loss terms may contribute to building an anisotropy in momentum space. We have considered both separately:

The gain term involves the “free streaming” particle distributions after collision. Thanks to the “molecular chaos” in Boltzmann theory, the “memory” of the distributions extends only back to their last interaction time.

$$f_i^{\prime(0)}(t, \mathbf{x}, \mathbf{p}_i) = f_i^{\prime(0)}(t_{coll}, \mathbf{x}(t_{coll}) - \mathbf{v}'_i(t - t_{coll}), \mathbf{p}'_i)$$

The “free streaming” particle distribution after collision does not carry any information about the initial geometry, they are independent of the momentum azimuths φ_i and φ_k .

$$\Rightarrow \quad \left\langle f_i^{\prime(0)} f_k^{\prime(0)} v_{ik}\sigma_d \cos(n\varphi_i) \right\rangle_{\varphi_i, \varphi_k} = 0$$

The gain term does not contribute to developing momentum anisotropies

Momentum anisotropies

In the presence of collisions, one should solve :

$$\frac{\partial}{\partial t}(f_i^{(0)} + f_i^{(1)}) + \mathbf{v}_i \cdot \nabla_{\mathbf{x}}(f_i^{(0)} + f_i^{(1)}) = \int d^2\mathbf{p}_k d\theta \left(f_i^{\prime(0)} f_k^{\prime(0)} - f_i^{(0)} f_k^{(0)} \right) v_{ik}\sigma_d$$

A priori; both gain and loss terms may contribute to building an anisotropy in momentum space. We have considered both separately:

The loss term of the Boltzmann equation does lead to anisotropies:
The number of particles with azimuth φ_i lost in a scattering is directly related to the initial geometry

To access the flow coefficients one should then compute:

$$\frac{\partial}{\partial t} v_n(t, p_i) \propto -\int d^2\mathbf{x} d\varphi_i d^2\mathbf{p}_k d\theta f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) f_k^{(0)}(t, \mathbf{x}, \mathbf{p}_k) v_{ik}\sigma_d \cos(n\varphi_i)$$

Back on the model

We must still focus on the relative velocity term:

$$v_{ik} = \sqrt{(\mathbf{v}_i - \mathbf{v}_k)^2 - \frac{(\mathbf{v}_i \times \mathbf{v}_k)^2}{c^2}}$$

For a gas of massive particles, v_{ik} is a non-trivial function of the particles azimuths and velocities.

$$v_{ik} = c\sqrt{[1 - \beta_i \beta_k \cos(\varphi_i - \varphi_k)]^2 - (1 - \beta_i^2)(1 - \beta_k^2)}$$

With $\beta_i = \frac{|\mathbf{v}_i|}{c}$ and $\beta_k = \frac{|\mathbf{v}_k|}{c}$

Integrating over the azimuths is non trivial, one has first to integrate over the time. The usually, experimentally accessible harmonic $v_n(p_T)$ corresponds to the large time limit of the momentum anisotropy coefficient $v_n(t \to \infty, p_T)$

Mixture of massive components:
number of collisions

The elastic cross-section is the key parameter of our model, it is fixed such a way that the total number of collisions per particle is at maximum equal to 1.

$$N_{coll} = \int_0^\infty dt \frac{dN_{coll}}{dt} = \int_0^\infty dt \int d^2\mathbf{x} \int d^2\mathbf{p}_k d^2\mathbf{p}_i d\theta f_i^{(0)} f_k^{(0)} v_{ik} \sigma_d$$

$\Big\downarrow$

K : Elliptical function of first kind
$F_D^{(3)}$: Lauricella function
x_i : functions of the velocities

$$N_{coll} = \int d^2\mathbf{p}_k d^2\mathbf{p}_i \frac{N_i N_k \sigma_{\mathrm{d}}}{\sqrt{\pi} R} \tilde{f}_i(p_i)\, \tilde{f}_k(p_k) \sqrt{1-\epsilon}\, K\left(\sqrt{\frac{2\epsilon}{1+\epsilon}}\right) F_D^{(3)}\left(\frac{1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}, 1; x_1, x_2, x_3\right)$$

To compute N_{coll}, we need to make some assumptions for the initial spectra

Mixture of massive components:
number of collisions

The particles spectra are normalized

The Lauricella function is smooth and takes values in the range $\left[\frac{2}{\pi}, 1\right]$

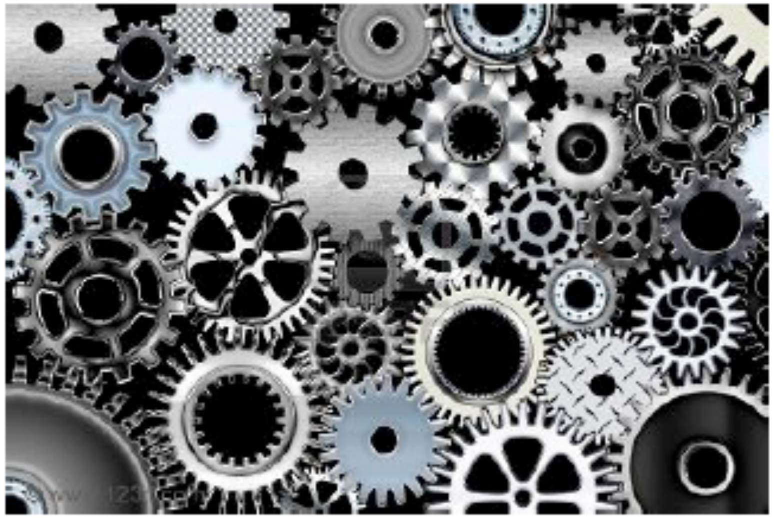
This defines an upper bound for N_{coll} $$N_{coll} \leq \frac{N_i N_k \sigma_d}{\sqrt{\pi} R} \sqrt{1-\epsilon^2} K\left(\sqrt{\frac{2\epsilon}{1+\epsilon}}\right)$$

The number of interactions is maximum in central collisions $(\epsilon = 0)$

Allowing at most 1 interaction per particle leads to $$\sigma_d = \frac{2R}{N_k \sqrt{\pi}}$$

Mixture of massive components:
momentum anisotropy

$$v_n(p_i) \quad \propto \quad -\int dt d^2\mathbf{x} d^2\mathbf{p}_k d\varphi_i d\theta f_i^{(0)} f_k^{(0)} v_{ik}\sigma_d \cos(n\varphi_i)$$



$$v_n(p_i) = \mathcal{N}_n \mathcal{K}_n(\epsilon) \int dp_k d_k N_k \tilde{f}_k(p_k) \mathcal{F}_n(\beta_i, \beta_k)$$

Mixture of massive components:
momentum anisotropy

$$v_n(p_i) = \mathcal{N}_n \mathcal{K}_n(\epsilon) \int dp_k d_k N_k \tilde{f}_k(p_k) \mathcal{F}_n(\beta_i, \beta_k)$$

Centrality dependence $\mathcal{K}_n(\epsilon)$

Momentum distribution of "scattering centers" $\tilde{f}_k(p_k)$

Velocity dependent term $\mathcal{F}_n(\beta_i, \beta_k)$

The anisotropic flow coefficients are functions of the particles velocities rather than their momenta

At fixed momenta, heavier particles have smaller velocities

\+ v_2 increasing function of the velocities

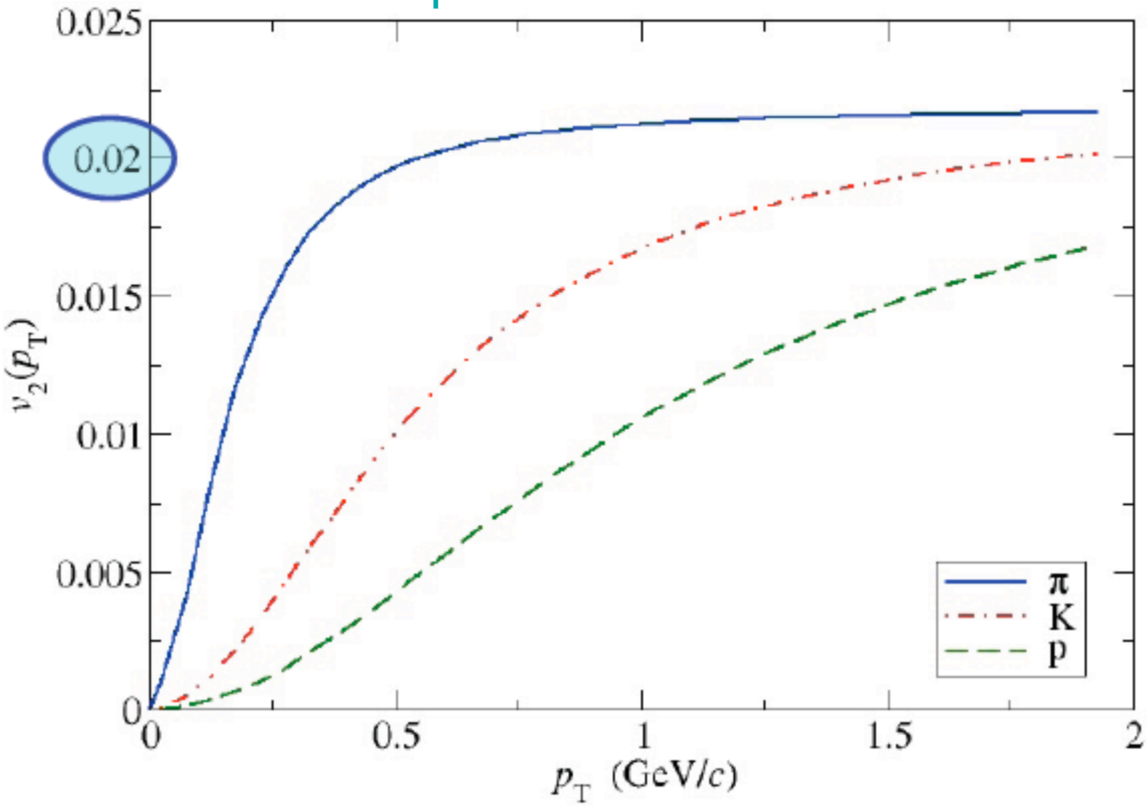
Ellitpic flow mass-ordering is not related to thermalization

Mixture of massive components:
Elliptic flow

Thermal like momentum distribution assumed $\left(\langle p_T^{(\pi)} \rangle = 420 \text{ MeV}\right)$

more plots: arXiv:1012.0899

$\epsilon = 0.1$



mixture :
80% pions
12.5% kaons
7.5% protons

Sizeable elliptic flow generated by only 1 interaction per particle

Anisotropic flow far from equilibrium: onset of collectivity

✓Do we need many collisions (hydro) to built up “collective flow”?

- NO: With only 1 interaction per particle we already get:

$$\frac{v_2}{\epsilon} = 0.2$$

✓Do we need the presence of a thermalized medium to obtain the “mass ordering” of (elliptic) flow?

- NO: v_n function of the velocities rather than momenta

Intrinsic properties of kinetic equation

Anisotropic flow far from equilibrium:
phenomenological relevance

- Really simple “Toy model”
  - No longitudinal dilution
  - Unique, constant, p_T-independent cross-section

- Considering a single interaction may however be relevant for particle destroyed after one collision
  - High-p_T particle which lose enough of their energy when interacting and thus change p_T-bin.
  - Fragile states (Quarkonia)