Scaling of chiral observables near a many-flavor quantum phase transition

Holger Gies

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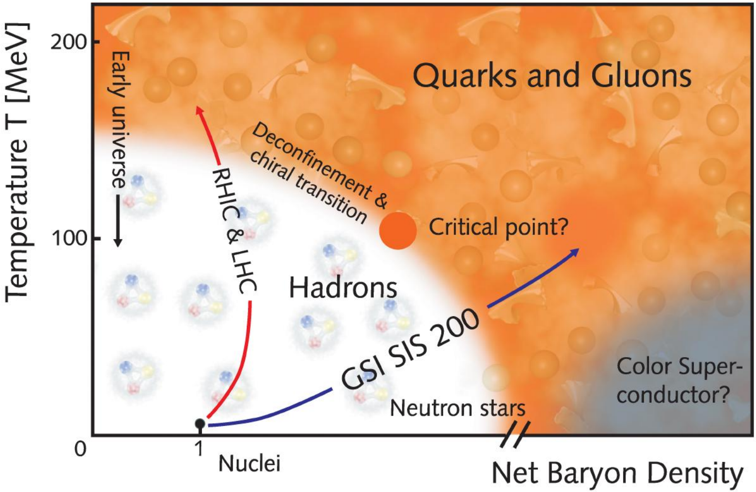
& J. Braun, C.S. Fischer, arXiv:1012.4279

& J. Braun: Phys.Lett.B645:53,2007 [hep-ph/0512085], JHEP 0606:024,2006 [hep-ph/0602226], JHEP 1005:060,2010 [arXiv:0912.416]

& J. Jaeckel: Eur.Phys.J.C46:433,2006 [hep-ph/0507171]

QCD Phase Diagram



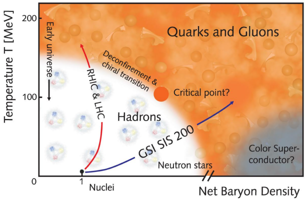
[FAIR@WWW.GSI.DE]

Thermal phase transitions & scaling

$$\xi \sim |T - T_c|^{-\nu}$$

$$C \sim |T - T_c|^{-\alpha}$$

$$\chi \sim |T - T_c|^{-\gamma}$$



▷ scaling relations: $\gamma = \nu(2 - \eta), \ldots$

$\Longrightarrow$ universal behavior
induced by order parameter fluctuations (symmetry!)

Many flavor quantum phase transition

▷ QPT: $T = 0$ phase transition as a function of a

physical control parameter: $\sim |N_{f,cr} - N_f|$

▷ Deformed QCD: study of mechanisms (χSB) $\rightarrow$ J. Braun's talk

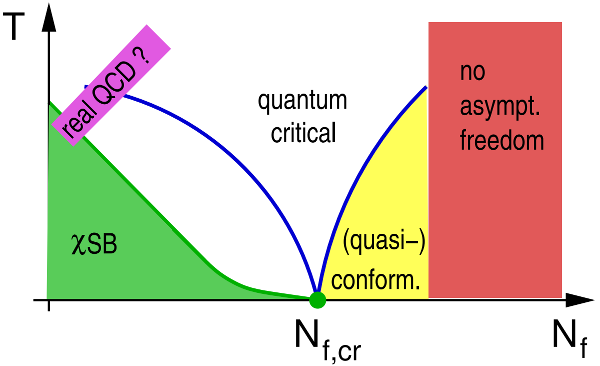
Phenomenological relevance:

- (walking) Technicolor models

  (WEINBERG'79, HOLDOM'81, ... REVIEW:SANNINO'09)
- mechanism@work in QED_3, Thirring ... ?

  (PISARKSI'84, ... FISCHER ET AL.'04)
- quantum critical region $\Longrightarrow$ real QCD ?

Many flavor quantum phase transition



$$\text{Scaling} \sim f(|N_{f,cr} - N_f|) \; ?$$

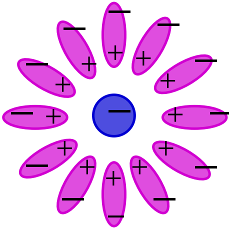
▷ application: $N_{\rm f}$-scaling of PNJL / PQM model parameters

(Schaefer, Pawlowski, Wambach '07)

Many-Flavor QCD

Many-flavor QCD

▷ charge screening:

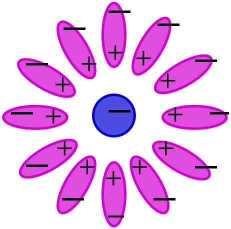


▷ β function

$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} - 2\left(\frac{34{N_c}^3 + 3N_f - 13{N_c}^2 N_f}{3N_c}\right)\frac{g^6}{(16\pi^2)^2} + \dots$$

Many-flavor QCD

▷ charge screening:



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$$\text{for } N_f > \frac{34{N_c}^3}{13{N_c}^2 - 3} \overset{\text{SU(3)}}{\simeq} 8.05$$

Many-flavor QCD

▷ β function

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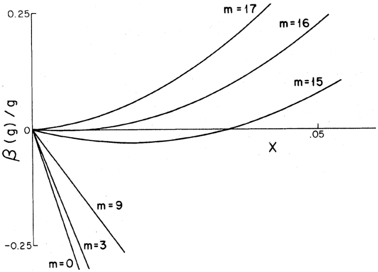
▷ e.g., SU(3): IR fixed point α_* (CASWELL'74; BANKS&ZAKS'82)



[CASWELL@PHYS.REV.LETT.33:244,1974]

Many-flavor QCD

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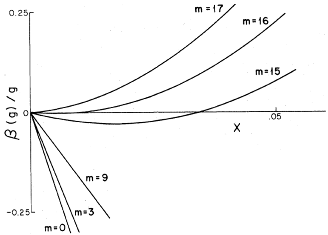
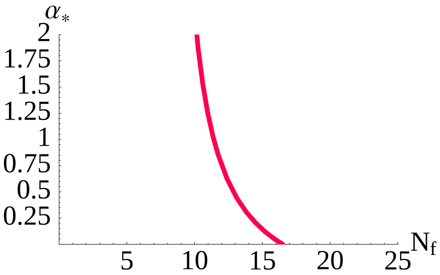
▷ N_f dependence of α_*





Many-flavor QCD

▷ β function

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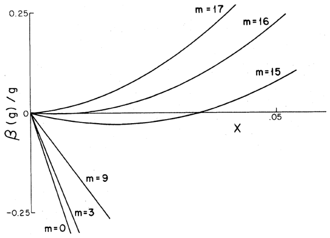
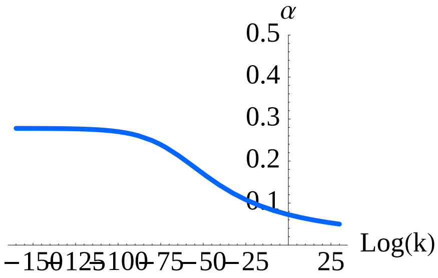
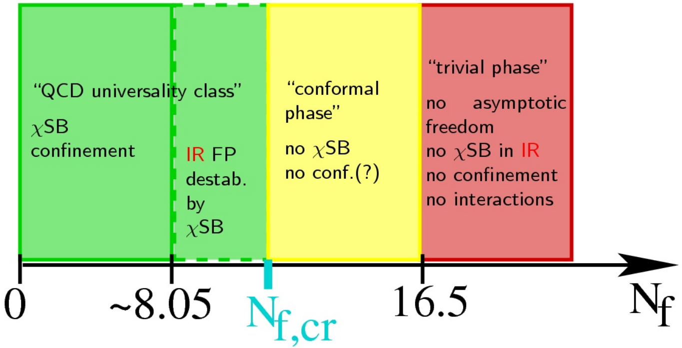
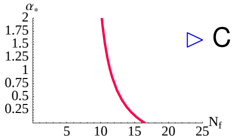
▷ e.g. $N_f = 14$ $\Longrightarrow$ IR fixed point





Many-flavor QCD





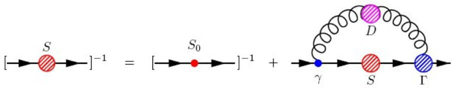
▷ Caswell-Banks-Zaks fixed point destabilized by χSB:

for $g^2 > g^2_{\rm cr}$: fermions decouple

Miransky Scaling ?

▷ gap equation

[PHYSIK.UNI-GRAZ.AT/ITP/SICQFT]



▷ approximation: $\Gamma = \gamma \quad \leftrightarrow \alpha = const.$

$$k_{SB} \sim \Lambda \exp\left(-\frac{const.}{\sqrt{g^2 - g_{cr}^2}}\right)$$

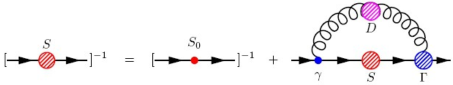
(MIRANSKY'94; MIRANSKY,YAMAWAKI'97)

▷ chiral observables:

$$T_c,\ f_\pi,\ \langle\bar{\psi}\psi\rangle^{1/3},\ m_{c.q.}, \cdots \sim k_{SB}$$

Miransky Scaling ?

▷ gap equation [PHYSIK.UNI-GRAZ.AT/ITP/SICQFT]



▷ leading singularity near the QPT: $g^2 - g^2_{cr} \sim |N_{f,cr} - N_f|$

$$k_{SB} \sim \Lambda \exp\left(-\frac{const.}{\sqrt{|N_{f,cr} - N_f|}}\right)$$

▷ NB: similar to essential BKT scaling in 2d XY model:

$$\xi^{-1}_{BKT} \sim \Lambda \exp\left(-\frac{const.}{\sqrt{|T - T_c|}}\right)$$

(KAPLAN,LEE,SON,STEPHANOV'09)

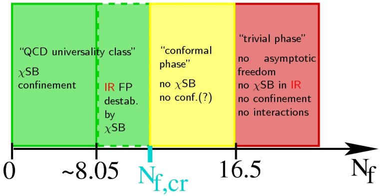
▷ CAVE: α=const.

Powerlaw Scaling ?

▷ lower end of conformal window

=

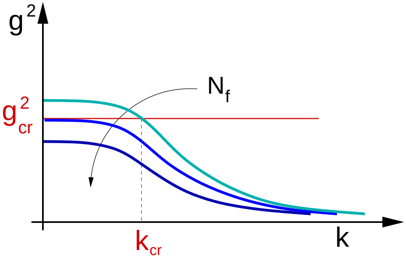
onset of χSB



▷ assumption:

onset of χSB requires

$g^2 > g^2_{cr}$



$\Longrightarrow$ fixed-point regime is relevant

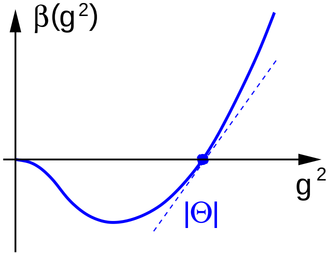
Powerlaw Scaling ?

▷ RG flow in the fixed-point regime:

governed by universal

critical exponent Θ

$$\beta(g^2) \simeq -\Theta\,(g^2 - g_*^2) + \dots$$



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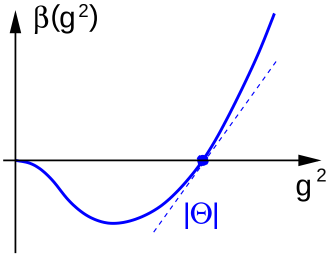
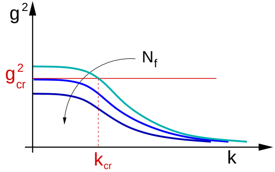


▷ solution in the fixed-point regime:

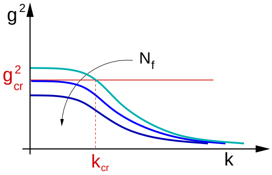
$$g^2(k) = g_*^2 - \left(\frac{k}{\Lambda}\right)^{|\Theta|}$$



Powerlaw Scaling ?

▷ χSB dynamics sets in at:

$$k_{\rm cr} \simeq \Lambda\,(g_*^2 - g_{\rm cr}^2)^{\frac{1}{|\Theta|}}$$



▷ criticality scale k_{cr}: upper bound for χSBscale

$$k_{\rm cr} \gtrsim k_{SB} \sim T_{\rm c},\ f_\pi,\ \langle\bar\psi\psi\rangle^{1/3},\ m_{c.q.},\dots$$

$\Longrightarrow$ powerlaw scaling (Braun, HG'06&'09)

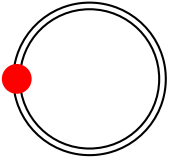
$$k_{\rm cr} \sim \Lambda\big|N_{\rm f,cr} - N_{\rm f}\big|^{\frac{1}{|\Theta|}}$$

Many-Flavor QCD with Functional RG

▷ RG flow equation

(WILSON'71; WEGNER&HOUGHTON'73; POLCHINSKI'84; WETTERICH'93)
(COMPENDIUM: PAWLOWSKI, ANN.PHYS.**322** 2831, 2007)

$$\partial_t \Gamma_k \equiv k\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k =$$



[cf. talks by B.J. Schaefer, J.M. Pawlowski, N. Strodthoff, B. Klein, J. Braun]

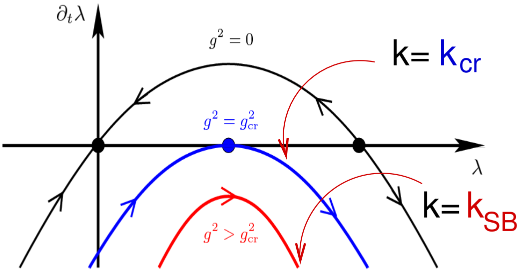
▷ computation of effective action in a gauge-covariant derivative expansion: SU(N_c), $\mathrm{SU}(N_f)_\mathrm{L} \times \mathrm{SU}(N_f)_\mathrm{R}$

$$\begin{aligned}\Gamma_k \quad = \quad \int \frac{Z_\mathrm{F}}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi\,(\mathrm{i}Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/)\,\psi \\ + \frac{1}{2}\frac{\lambda_\sigma}{k^2}\,(\text{S-P}) + \frac{1}{2}\frac{\lambda_\mathrm{VA}}{k^2}\left[2(\text{V-A})^{\text{adj.}} + (1/N_c)(\text{V-A})\right] \\ + \frac{1}{2}\frac{\lambda_+}{k^2}\,(\text{V+A}) + \frac{1}{2}\frac{\lambda_-}{k^2}\,(\text{V-A})\end{aligned}$$

(HG,JAECKEL,WETTERICH'04)

Beyond Miransky Scaling

(Braun, Fischer, HG'10)



$$k_{SB} \simeq \Lambda \left| N_{f,cr} - N_f \right|^{\frac{1}{|\Theta|}} \exp\left(- \frac{const.}{\sqrt{|N_{f,cr} - N_f|}} \right)$$

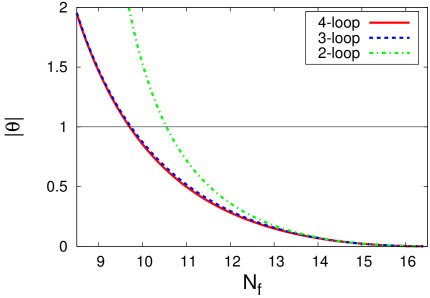
Miransky vs. Powerlaw Scaling

(Braun,Fischer,HG'10)

$$k_{SB} \simeq \Lambda \left| N_{f,cr} - N_f \right|^{\frac{1}{|\Theta|}} \exp\left(- \frac{const.}{\sqrt{|N_{f,cr} - N_f|}} \right)$$

$$|\Theta| \gg 1 \implies \text{Miransky scaling}$$

$$|\Theta| \ll 1 \implies \text{powerlaw scaling}$$



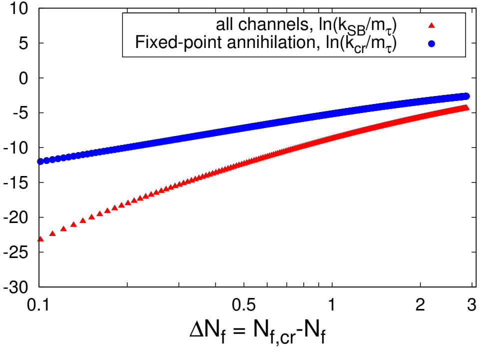
(van Rittbergen et al'97)

(Czakon'05)

Miransky vs. Powerlaw Scaling

▷ Example: 2-loop β function, all Fierz channels in pointlike limit

(Braun,Fischer,HG'10)



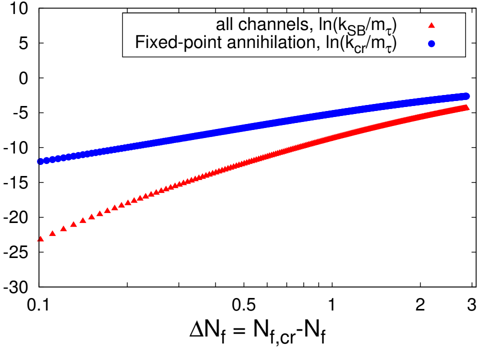
k_{cr} : powerlaw dominated

$k_{SB} \leq k_{cr}$: superposition of Miransky+powerlaw

Miransky vs. Powerlaw Scaling

▷ Example: 2-loop β function, all Fierz channels in pointlike limit

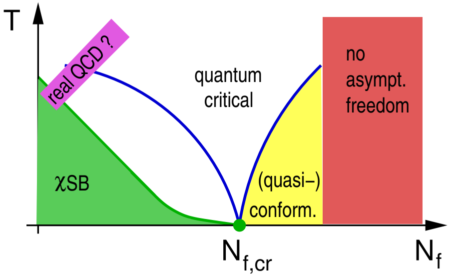
(BRAUN,FISCHER,HG'10)



▷ Lattice: probe at $N_f \in \mathbb{N}$

- Miransky scaling may not be visible
- powerlaw fit may overestimate $|\Theta|$

Many-Flavor QCD, Quantitatively . . . ?



LGT: (Kogut&Sinclair'88; Brown et al.'92; Iwasaki et al.'96; Damgaard et al.'97)

$$N_{\mathrm{f,cr}}(SU(3), < 2003) = \begin{cases} 5 & \text{(Harada\&Yamawaki'00)} \\ 6 & \text{(Iwasaki et al.'03)} \\ \gtrsim 6 & \text{(Velkovsky\&Shuryak'97, Appelquist\&Selipsky'97)} \\ \gtrsim 10 & \text{(Sannino\&Schechter'99)} \\ \simeq 12 & \text{(Miransky\&Yamawaki'96, Appelquist et al.'96)} \end{cases}$$

▷ FRG: $N_{\mathrm{f,cr}} = 10.0 \pm 0.29(\text{fermion})\,^{+1.55}_{-0.63}(\text{gluon}) \lesssim N_{\mathrm{f}} < 16.5$

(HG, Jaeckel'05)

Recent Results from the Lattice

(DEUZEMANN,LOMBARDO,PALLANTE'08; APPELQUIST,FLEMING,NEIL'08&'09)

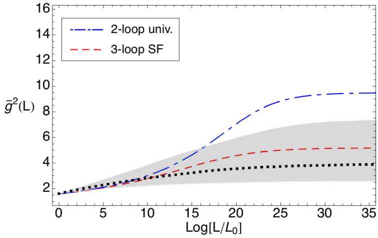
(JIN, MAWHINNEY'09; FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'09)

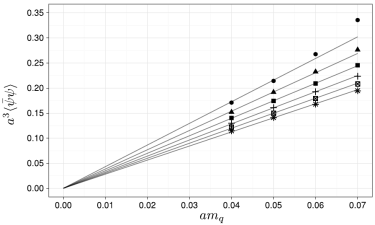
(FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'09)

$$SU(3): \qquad 9 \lesssim N_{f,cr} \lesssim 13$$

▷ e.g. evidence for (quasi-) conformal phase of $N_f = 12, N_c = 3$ QCD:

(APPELQUIST,FLEMING,NEIL'08&'09; DEUZEMANN,LOMBARDO,PALLANTE'09)





▷ other fermion representations: (CATTERALL,SANNINO'07; MAAS'11; . . .)

Conclusions

▷ Scaling near the many-flavor QPT:

. . . lessons on chiral structure

. . . N_f scaling of model parameters

. . . applications to walking technicolor, QED_3, Thirring, . . .

▷ universal aspects:

shape of the phase boundary $\Longleftrightarrow$ IR critical exponent

Miransky scaling $\Longleftrightarrow$ powerlaw scaling

▷ functional RG for $\Gamma[\phi]$

- systematic and consistent expansion schemes for QCD
- chiral symmetry ✓
- calculations “from first principles”

▷ Quantum critical regime $\Longleftrightarrow$ real QCD ?