Solving the complex phase problem for QCD related systems with chemical potential

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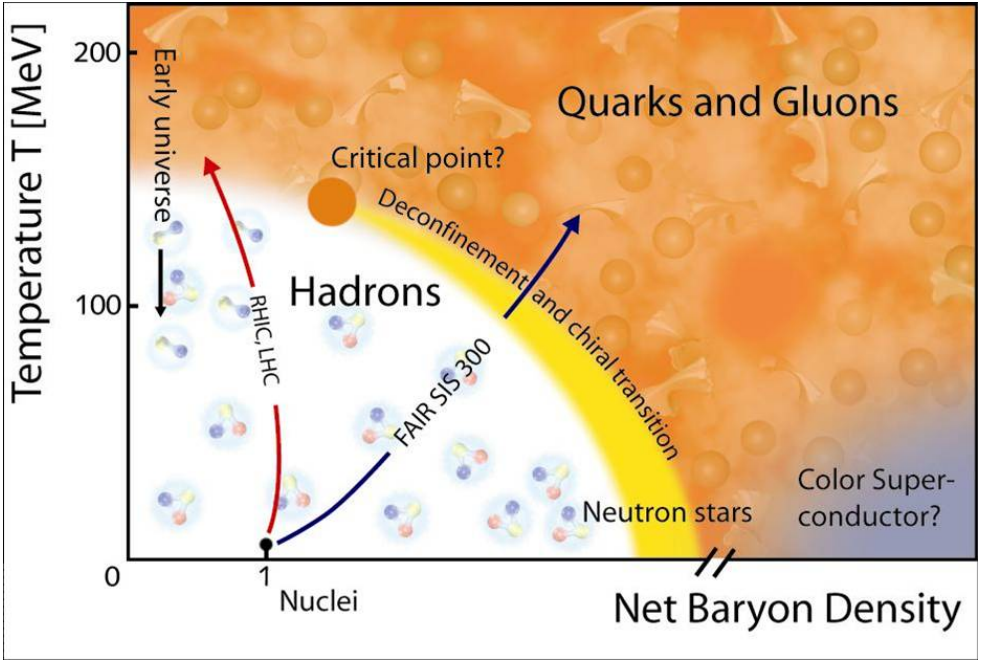
Motivation

Something we would like to understand in detail ...



Lattice QCD with chemical potential

- Grand canonical partition sum

$$Z \;=\; \int D[U]\; e^{-S_G[U]}\; \det D(\mu)^{N_f}$$

- For non-zero chemical potential μ the fermion determinant is complex

$$\det D(\mu) \;\in\; \mathbb{C}$$

  and thus cannot be used as a weight factor in a MC calculation.

- For further progress in the analysis of the QCD phase diagram new approaches would be highly welcome.

- In this project we explore new strategies in simpler models.

An effective theory for QCD with chemical potential

Center symmetry and Svetitsky-Yaffe conjecture

- For pure gauge theory deconfinement can be understood as the spontaneous breaking of center symmetry. An influential idea for understanding this phase transition is the Svetistky-Yaffe conjecture (1981):

- At T_c the critical behavior of SU(3) gauge theory in $d+1$ dimensions can be described by a d - dimensional spin system with a $\mathbb{Z}_3$ - invariant effective action for the local Polyakov loop $P(x) \in$ SU(3).

- Leading term of the effective action from a strong coupling expansion

$$S_c[P] \;=\; -\tau \sum_{\langle x,y \rangle} \Big[\;\mathrm{Tr} P(x)\, \mathrm{Tr} P(y)^\dagger \;+\; \mathrm{Tr} P(y)\, \mathrm{Tr} P(x)^\dagger \;\Big]$$

Center breaking terms from the fermion determinant

- The fermion determinant breaks the center symmetry explicitly.

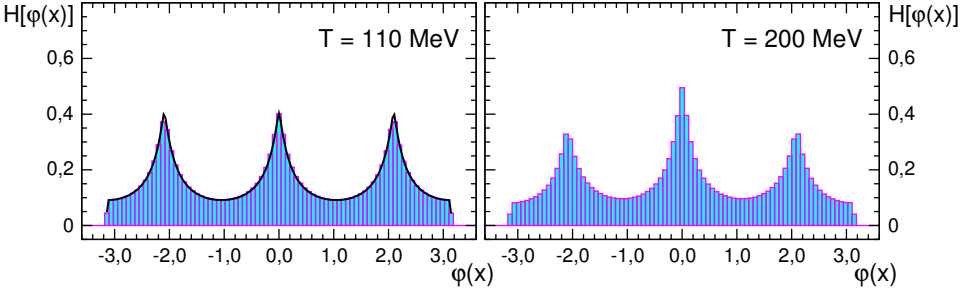
- The leading center symmetry breaking can be calculated using hopping expansion, including the chemical potential μ.

$$
\begin{aligned}
S[P] \;\; = \;\; & -\tau \sum_{\langle x,y \rangle} \Big[\; \mathrm{Tr} P(x) \, \mathrm{Tr} P(y)^\dagger \; + \; \mathrm{Tr} P(y) \, \mathrm{Tr} P(x)^\dagger \; \Big] \\
& -\kappa \sum_{x} \Big[\; e^{\,\mu} \, \mathrm{Tr} P(x) \; + \; e^{-\mu} \, \mathrm{Tr} P(x)^\dagger \; \Big]
\end{aligned}
$$

- For non-vanishing chemical potential μ the action is complex and the model has a complex phase problem.

Reduction to the center degrees of freedom

- The distribution of $\mathrm{Tr}P(x)$ is dominated by the center.



- Reduction to the center elements: $\mathrm{Tr}P(x) \to L(x) \in \{1, e^{\pm i2\pi/3}\}$

$$S = -\tau \sum_{\langle x,y \rangle} \left[L(x)L(y)^\star + L(y)L(x)^\star \right] - \kappa \sum_x \left[e^\mu L(x) + e^{-\mu} L(x)^\star \right]$$

Contains leading center symmetric and center symmetry breaking terms.

Flux representation for the effective theory

Flux representation - I

- Identity for the nearest neighbor term:

$$e^{\tau[L(x)L(x+\hat{\nu})^* + c.c.]} \;=\; C \sum_{b_{x,\nu}=-1}^{+1} B^{|b_{x,\nu}|} \; \left(L(x)L(x+\hat{\nu}) \right)^{b_{x,\nu}}$$

- Identity for the magnetic term:

$$e^{\kappa[e^{\mu} L(x) + e^{-\mu} L(x)^*]} \;=\; \sum_{s_x=-1}^{+1} M_{s_x} \, L(x)^{s_x}$$

- C, B and M_s are real and positive functions of τ, κ and μ.

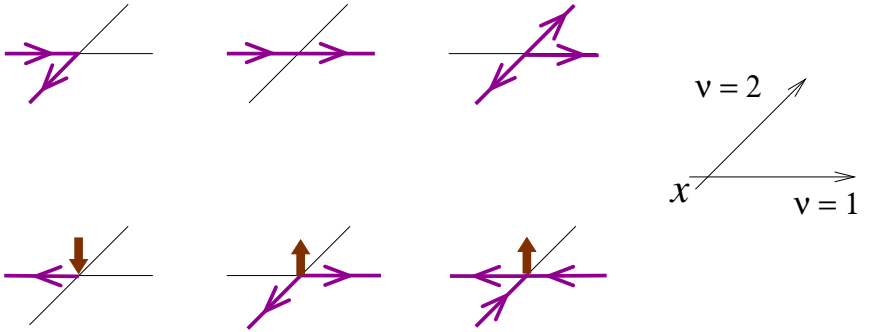
- Remaining sums over the center elements at each site:

$$\sum_{L \in \{1, e^{\pm i 2\pi/3}\}} L^n \;=\; 3\,T(n) \qquad \text{with} \qquad T(n) \;=\; \delta_{0\,,\,n \bmod 3}$$

Flux representation - II

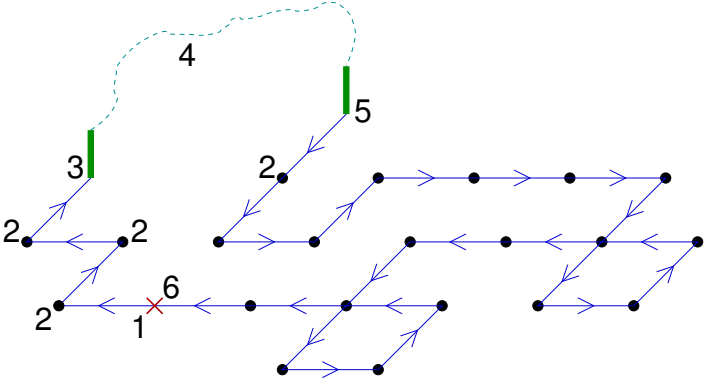
- Representation with dimers $b_{x,\nu} \in \{-1, 0, 1\}$ and monomers $s_x \in \{-1, 0, 1\}$

$$Z = \sum_{\{b,s\}} W(b,s) \prod_x T\left(\sum_\nu [b_{x,\nu} - b_{x-\hat{\nu},\nu}] + s_x\right) \quad \text{with} \quad W(b,s) > 0$$



Update with a Prokof'ev – Svistunov worm algorithm

- We start the worm at some point (Position 1).
- The worm may decide to change dimers (Pos. 2) or insert monomers (3).
- Insertion of a monomer is followed by a random hop (4) and another monomer insertion (5).
- The worm closes when it reaches its starting point (6).

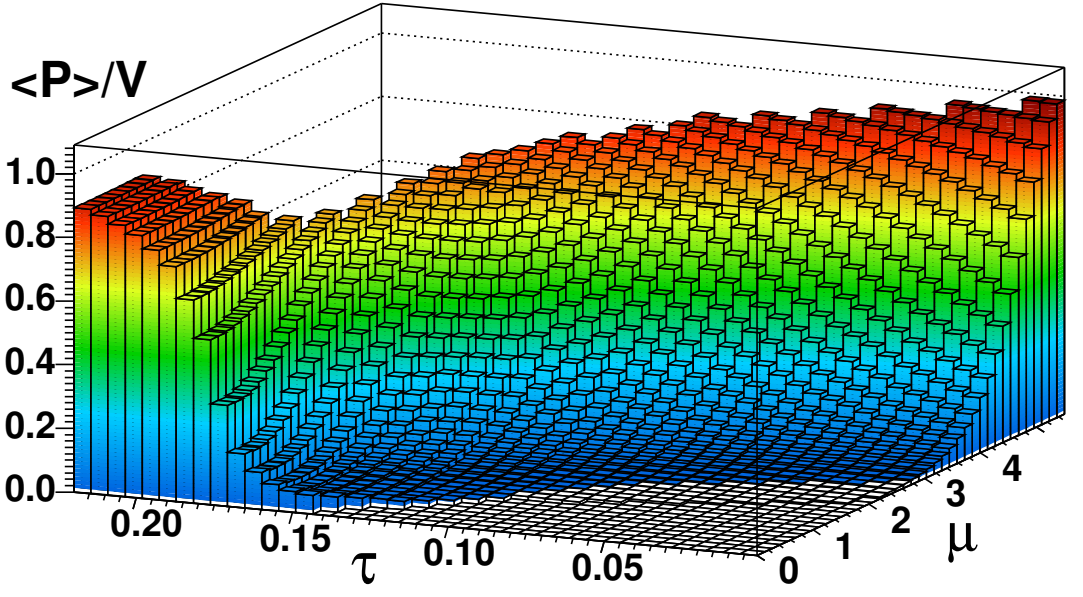


Numerical analysis

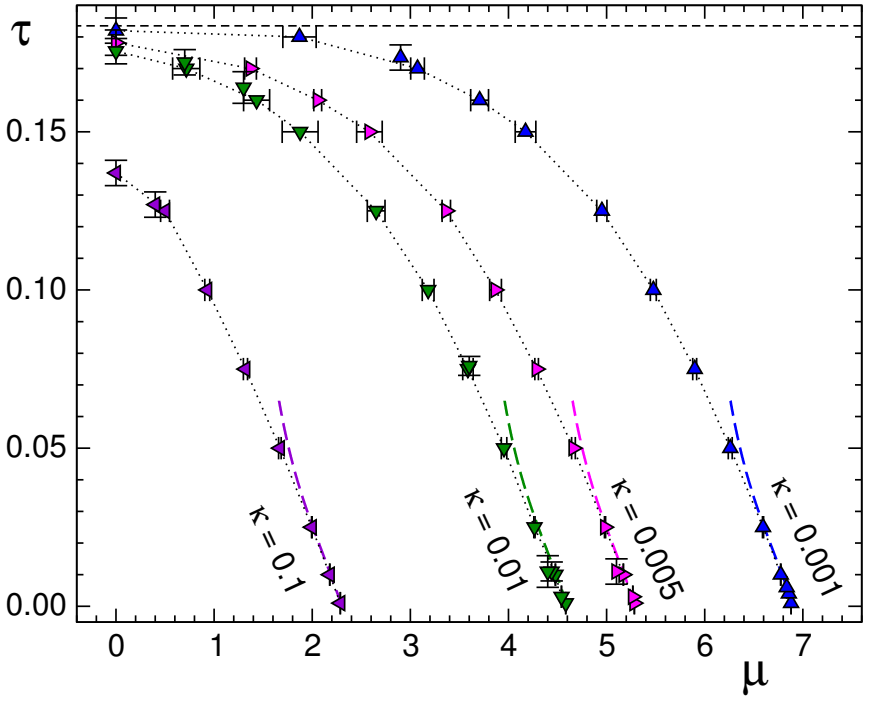
- The worm algorithm is ergodic and efficient also for large κ and μ.
- As observables we consider the internal energy E, the heat capacity C, the expectation value of the Polyakov loop $\langle P \rangle$ and the corresponding susceptibility χ_P.
- All our observables can be expressed in terms of the dimer- and monomer occupation numbers and their fluctuations.
- We work on lattice with sizes 16^3 to 72^3 with statistics of 10^5 to 10^6 worms.
- Comparison of the results for small τ with low temperature expansion.

Results

Order parameter

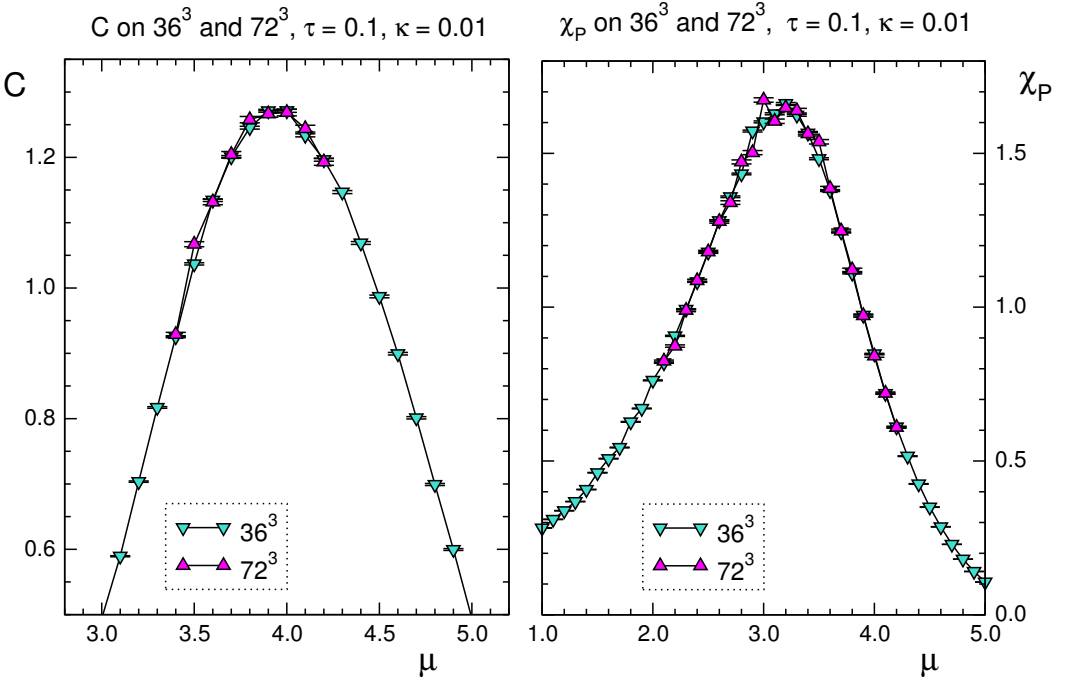


Exploring the phase diagram

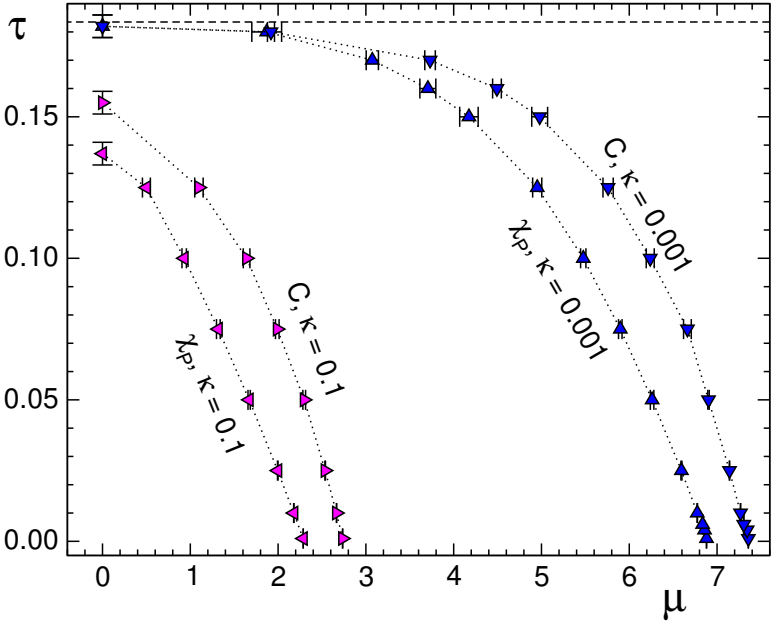


Volume scaling

For all values of κ we studied, no volume scaling of the second derivatives of the free energy C and χ_P was observed $\Rightarrow$ Crossover.



Strips of crossover behavior



If QCD is reduced to center degrees of freedom $\Rightarrow$ Crossover transitions.

Developments and perspectives

More realistic models:

- Action for effective theory with SU(3)-valued Polykov loops:

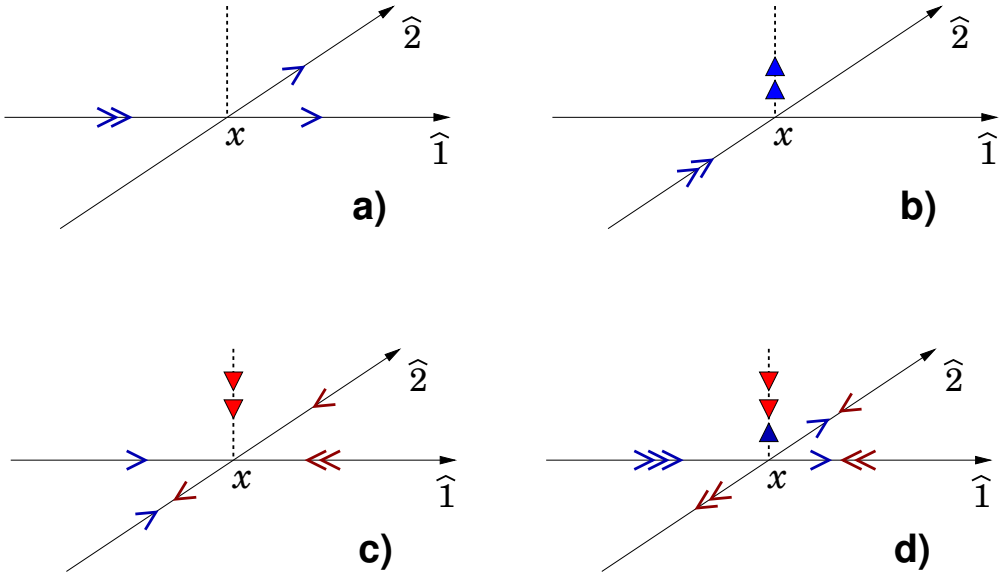
$$\begin{aligned} S[P] \;\; = \;\; & -\tau \sum_{\langle x,y \rangle} \Big[\; \mathrm{Tr} P(x) \, \mathrm{Tr} P(y)^\dagger \; + \; \mathrm{Tr} P(y) \, \mathrm{Tr} P(x)^\dagger \; \Big] \\ & -\kappa \sum_{x} \Big[\; e^{\,\mu} \, \mathrm{Tr} P(x) \; + \; e^{-\mu} \, \mathrm{Tr} P(x)^\dagger \; \Big] \end{aligned}$$

- Flux representation:

$$Z \; = \sum_{\{l,\bar{l},s,\bar{s}\}} \mathcal{W}[l,\bar{l},s,\bar{s}] \; \prod_x T\Big(\sum_\nu [(l_{x,\nu} - \bar{l}_{x,\nu}) - (l_{x-\hat{\nu},\nu} - \bar{l}_{x-\hat{\nu},\nu})] + (s_x - \bar{s}_x) \Big)$$

$\mathcal{W}[l,\bar{l},s,\bar{s}]$ real and positive ; $\quad l_{x,\nu}\,,\, \bar{l}_{x,\nu}\,,\, s_x\,,\, \bar{s}_x \; \in \; \mathbb{N}_0$

Examples of admissible flux and monomer vertices



Generalized PS worm algorithms can be applied for a Monte Carlo calculation.

Summary

- We study QCD reduced to the leading center symmetric and center symmetry breaking terms.
- The complex phase problem is solved by using a flux representation with dimers and monomers, which is suitable for a PS worm algorithm.
- We map out the phase diagram using E, $\langle P \rangle$, C and χ_P.
- If QCD is reduced to the center degrees of freedom only crossover transitions remain.
- Work in progress: Generalizations to theories closer to QCD.
- The new models can and should be used for testing various approaches to finite μ (expansions, complex Langevin, reweighting ...).