Polyakov Loop in a Magnetic Field

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Talk Contents

Relativistic Heavy-Ion Collision and Strong Magnetic Fields $|eB| \sim m_\pi^2 \sim 10^{18}$ gauss

Fermion Propagator and Dimensional Reduction

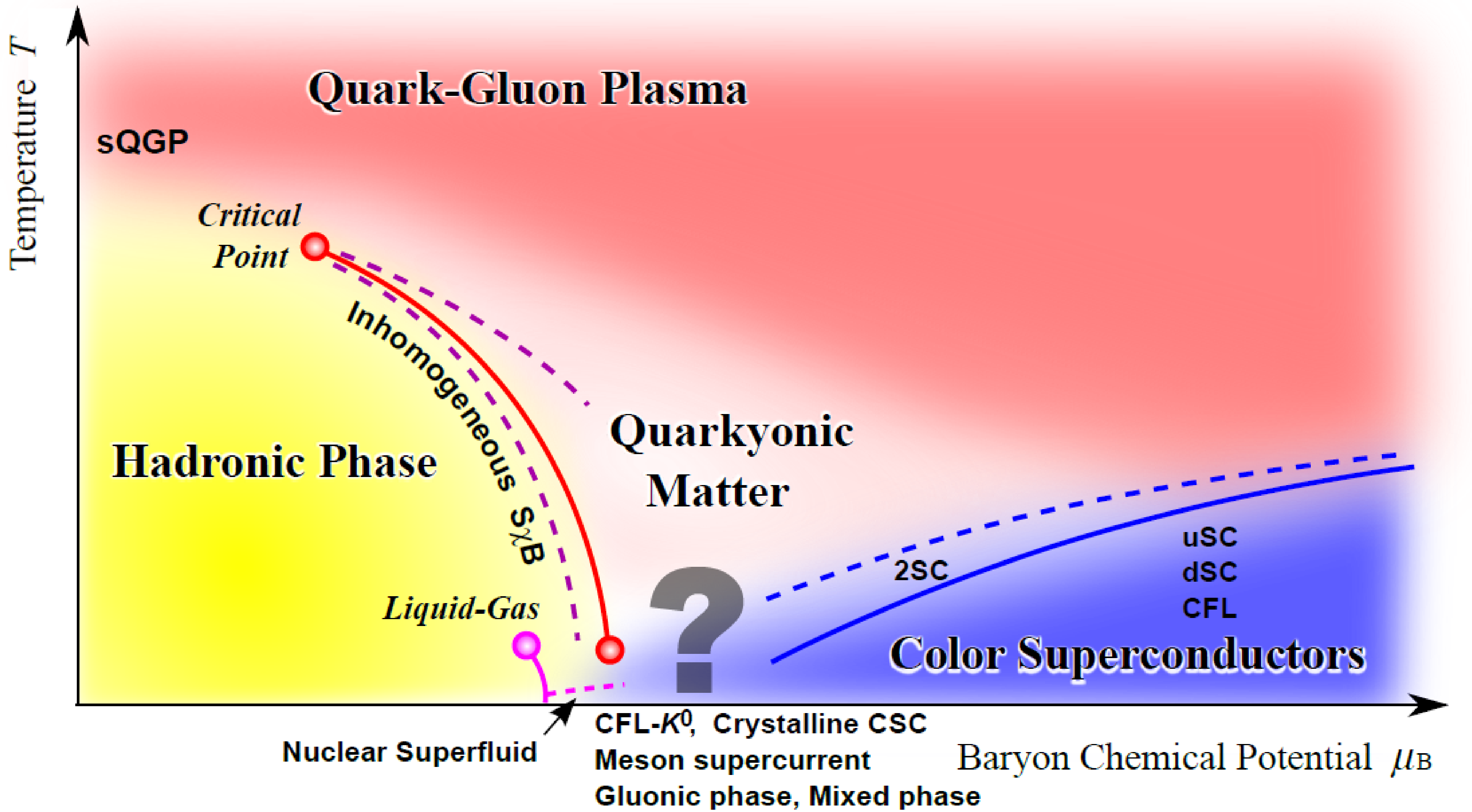
(3+1) → (1+1) in LLLA

One Loop Polarization c.f. two-loop contribution to EH

Magnetic-field Induced Screening Effect and Perturbative Polyakov-loop Potential $M_g^2 \sim g^2|eB|$

Relativistic Heavy-Ion Collision and Strong Magnetic Fields

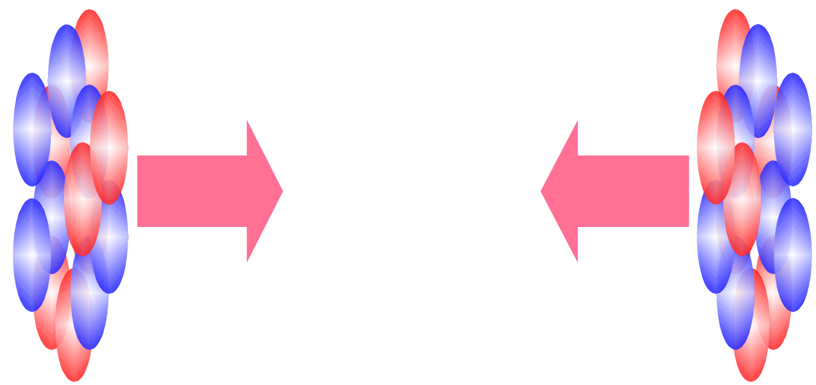
Conventional Starting…



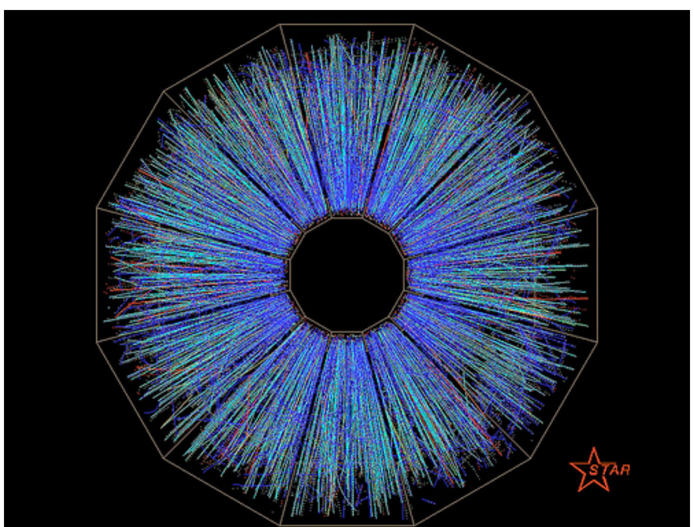
KF-Hatsuda (2010)

Relativistic Heavy-Ion Collisions

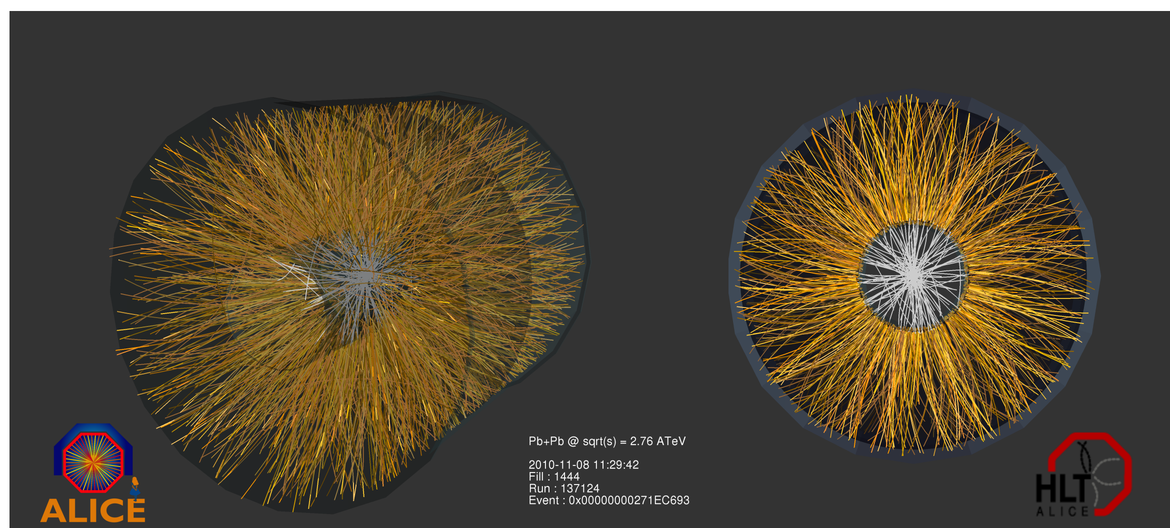
Nucleus (Au,Pb) Collision
Energy per nucleon-nucleon
= 200GeV @ RHIC
2.76TeV @ LHC



STAR

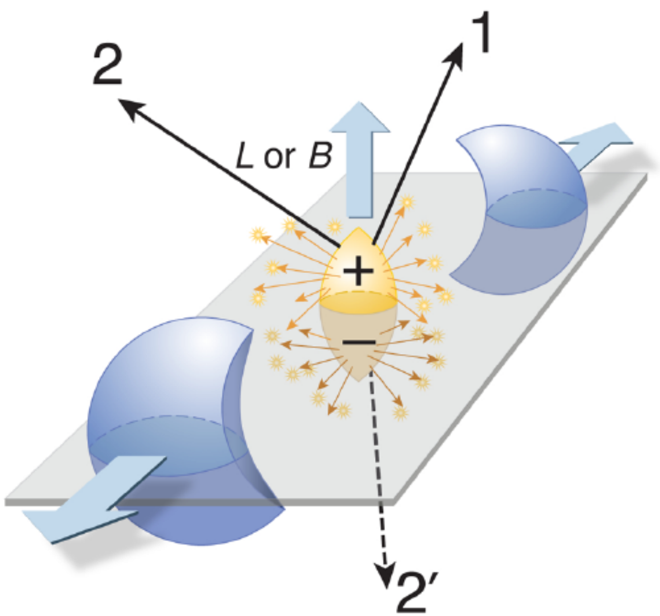


ALICE

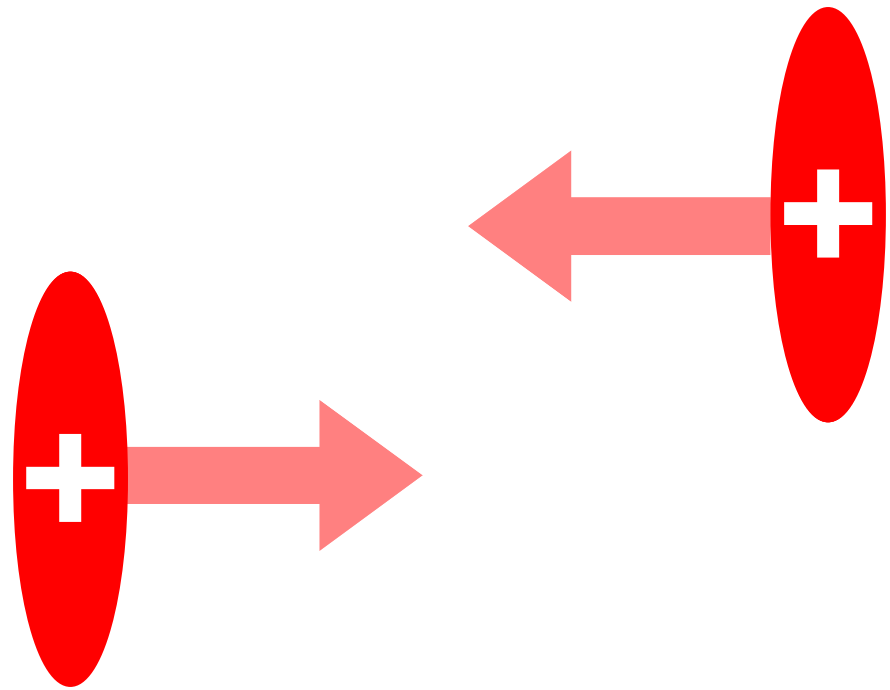


Non-Central Collision

Before Collision (seen from the "upper hemisphere")



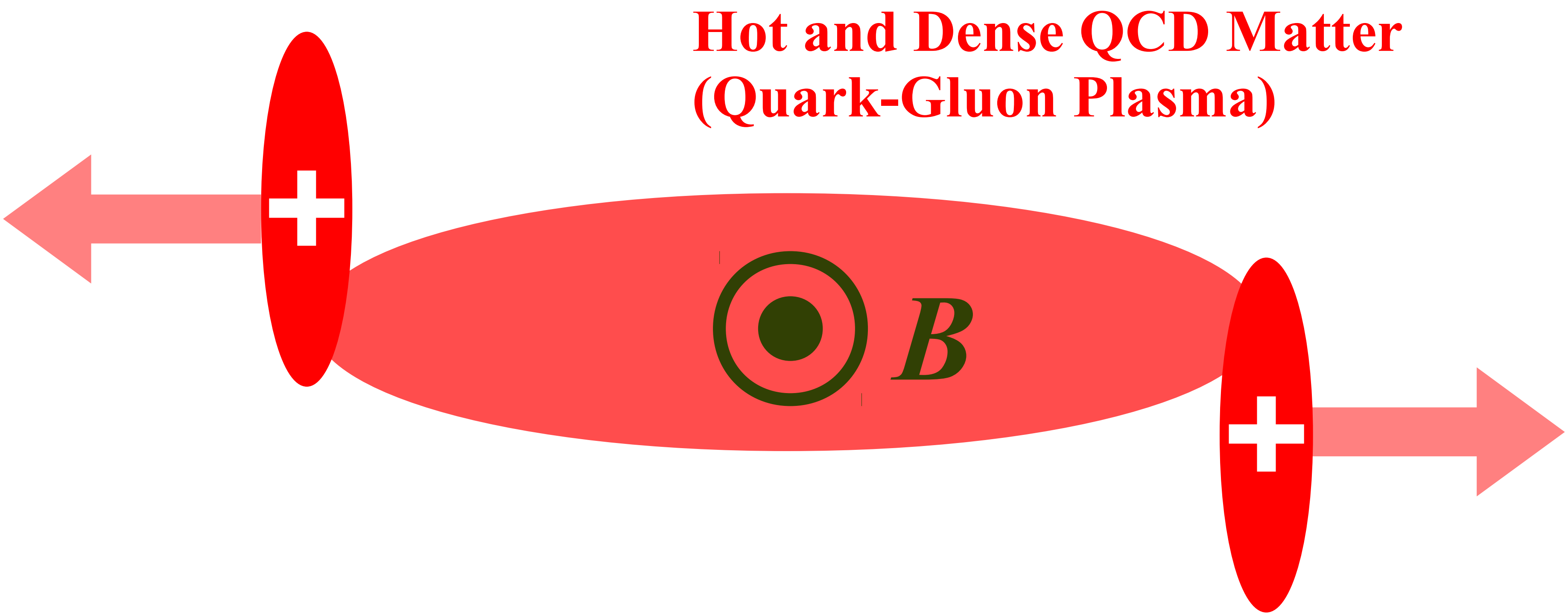
"Looking for parity violation
in heavy-ion collisions"
by Berndt Müller
Physics 2, 104 (2009)



Centrality is to be determined event by event

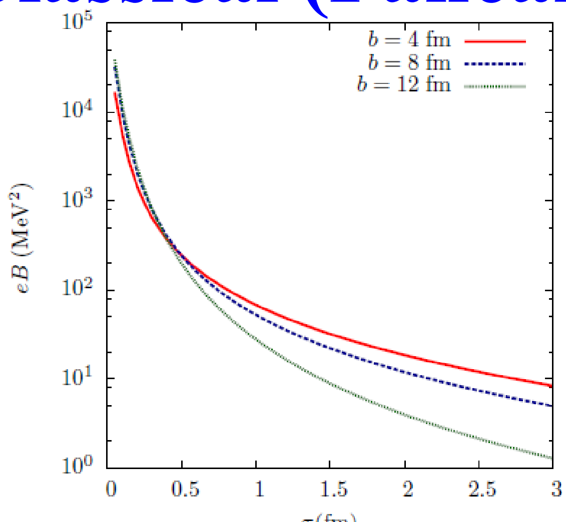
Non-Central Collision

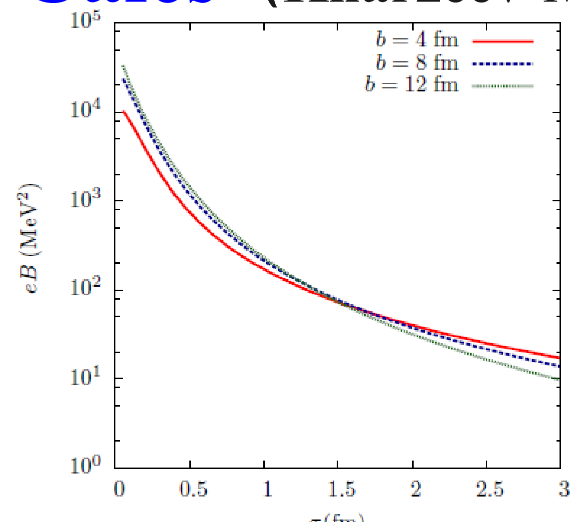
After Collision



Estimated Magnetic Fields

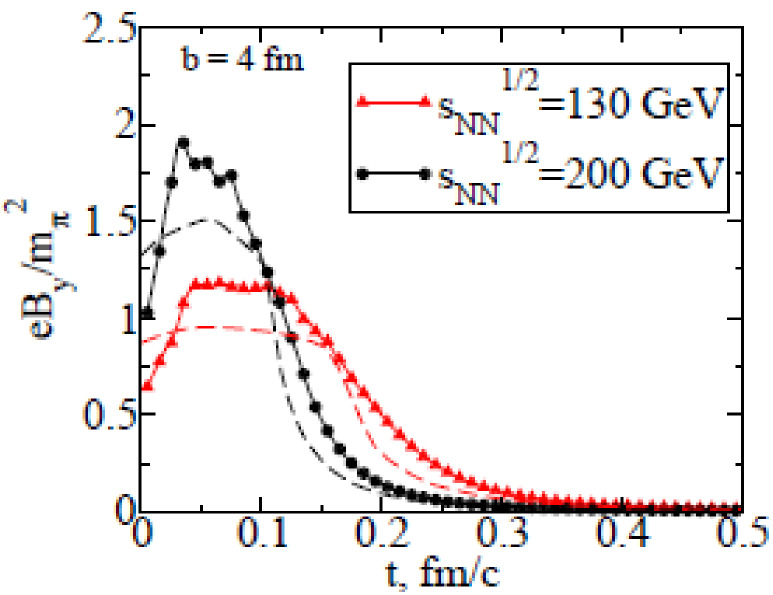
Classical (Pancake) Calcs (Kharzeev-McLerran-Warringa)

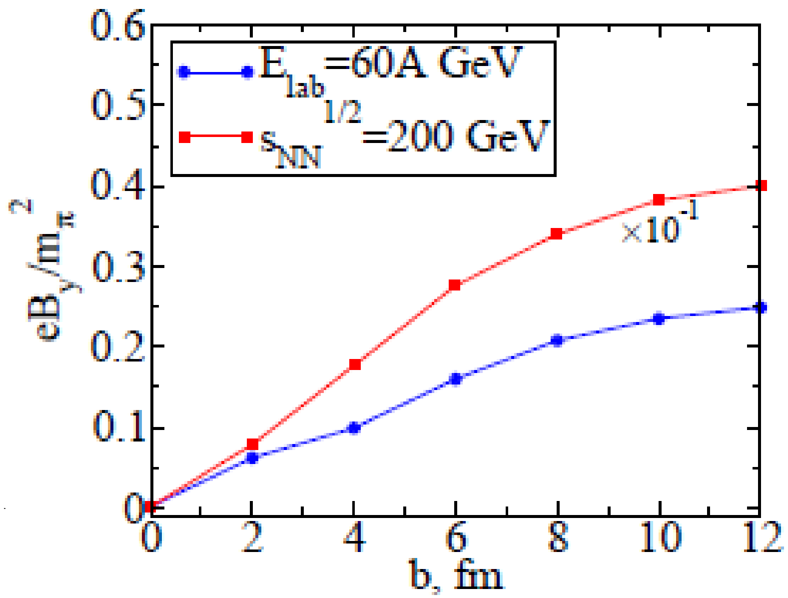




$$eB = 1\,[\,\mathrm{MeV}^2\,]$$
$$\rightarrow B \simeq 1.7 \times 10^{14}\ \text{gauss}$$

UrQMD Calculations (Skokov-Illarionev-Toneev)

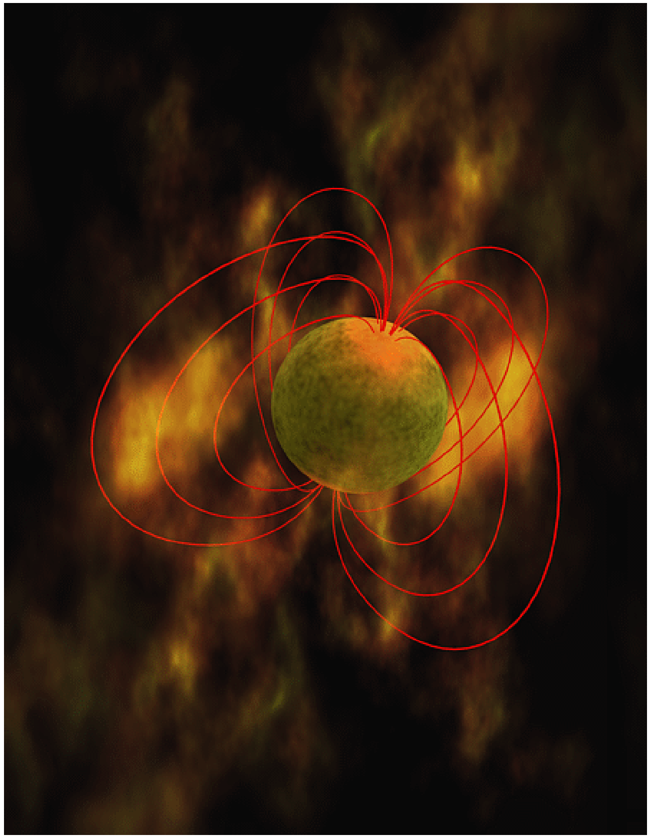




Largest Magnetic Field in the Universe

$|eB| \sim m_\pi^2 \;\rightarrow\; 10^{18}$ **Gauss**

$10^3 \sim 10^6 \times$



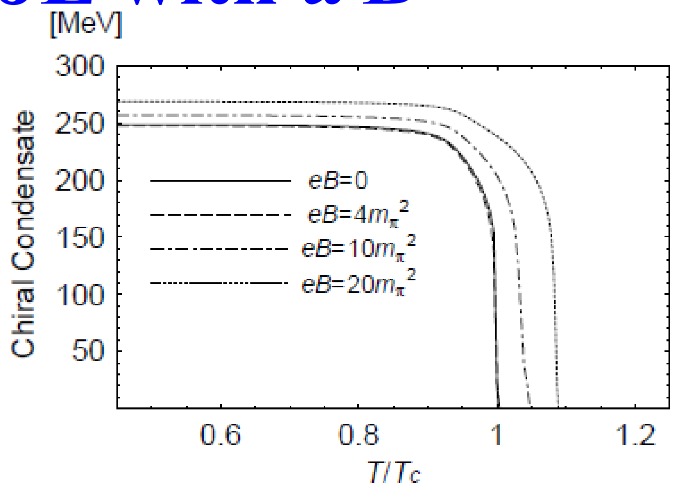
Neutron Star
(Magnetar)

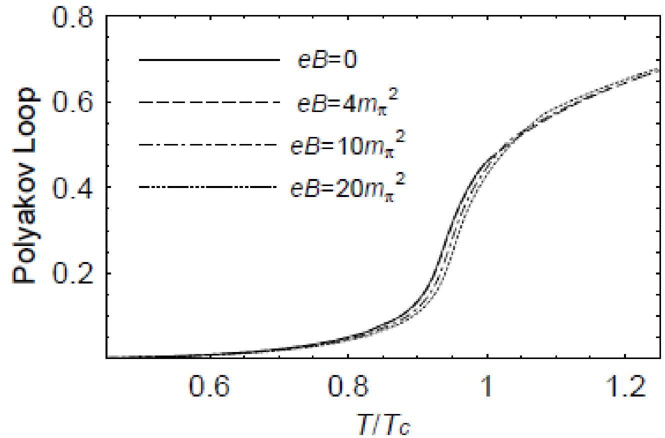
There should be some influence on QCD physics!

Example – Phase Diagram

PNJL with a *B*

Fukushima-Gatto-Ruggieri (2010)





c.f. Fraga et al. (2010) in PQM

Lattice Results

D'Elia et al. (2010)

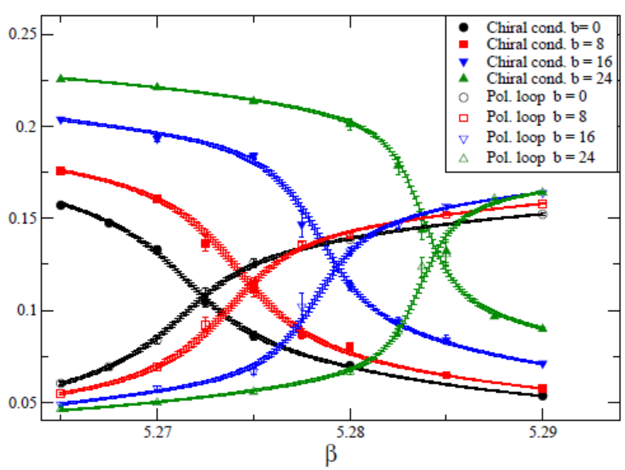


FIG. 3: Same as in Fig. 1 for $am = 0.01335$

Chiral condensate is enhanced in accord with the magnetic catalysis.

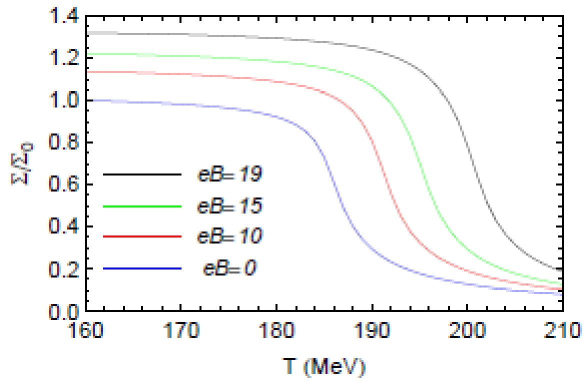
Gusynin-Miransky-Shovkovy

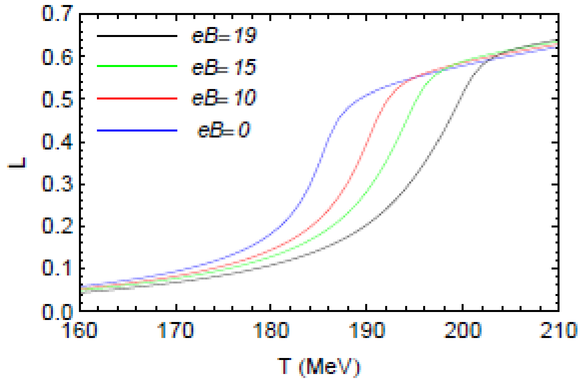
Polyakov loop shows crossover at the same (pseudo-)critical temperature.

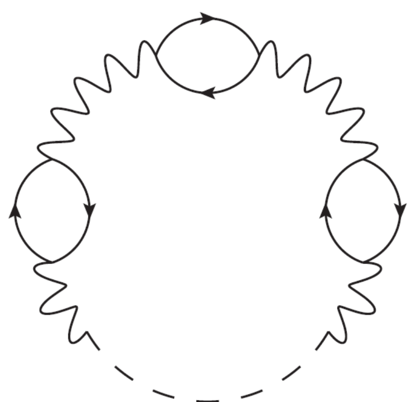
Missing Coupling

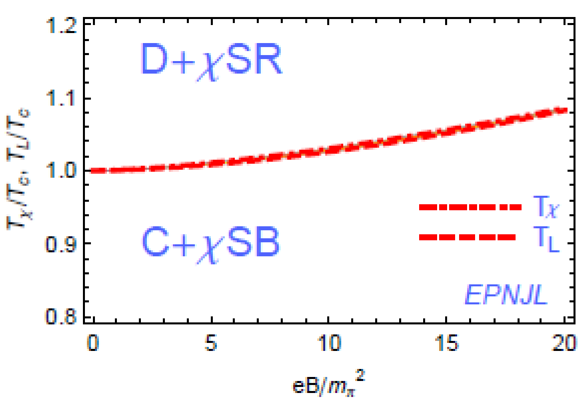
Discrepancy very similar to the problem at finite μ_B

PNJL and PQM (in a mean-field approx.) tend to favor splitting between chiral and deconf. transitions.









tr$L$-dependent G

Ruggieri-Gatto (2010)

Diagrams missing in mean-field PNJL and PQM

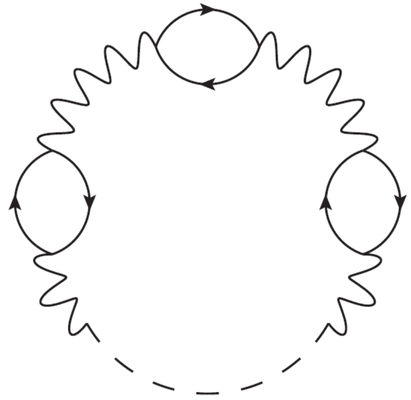
Missing Coupling

Origin of the PNJL (PQM) Coupling

$$\ln\det[i\gamma^\mu\partial_\mu + g\gamma^0 A_0 - m]$$
$$\sim \int \frac{d^3k}{(2\pi)^3}\left(\mathrm{tr}\ln[1+L e^{-(\epsilon_k-\mu)/T}] + \mathrm{tr}\ln[1+L^\dagger e^{-(\epsilon_k+\mu)/T}]\right)$$

Missing Contribution to the tr*L*-Potential

$$\ln\det[i\gamma^\mu D_\mu + g\gamma^\mu \delta A_\mu - m]$$
$$\sim \# + g^2\int dx\, dy\, \delta A_\mu(x)\Pi^{\mu\nu}(x,y)\delta A_\nu(y)$$



c.f. "renormalization" of T_0

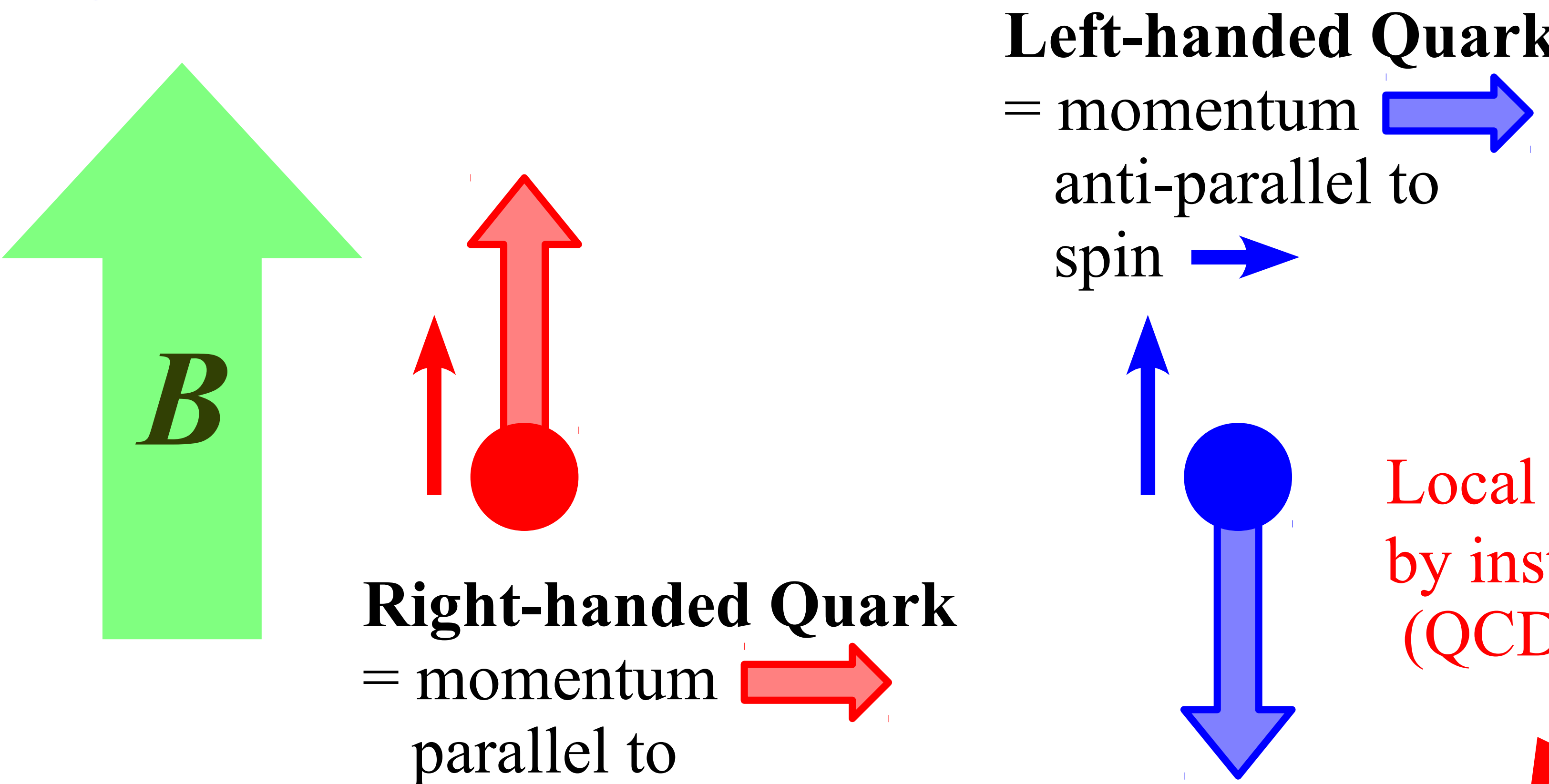
Pawlowski, Schaefer, Wambach

Vaccum-polarization → β-function

Two-loop Weiss potential?

Example – Chiral Magnetic Effect

Classical Picture



Local $\mathcal{P}$ violation
by instantons
(QCD Physics)

$$\boldsymbol{J} \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Kharzeev-McLerran-Warringa (2007)

Key Equations

Induced N_5 by Topological Effects

$$\frac{dN_5}{dt} = -\frac{g^2 N_f}{8\pi^2}\int d^3x\ \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

QCD Anomaly

Introduce μ_5 to describe induced N_5

Induced J by the presence of N_5 and B

$$\boldsymbol{j} = \frac{e^2\mu_5}{2\pi^2}\boldsymbol{B}$$

QED Anomaly

$$\left(\boldsymbol{j} = \sum_{i=\mathrm{flavor}} \frac{q_i^2\mu_5}{2\pi^2}\boldsymbol{B} \quad \text{in QCD}\right)$$

Metlitski-Zhitnitsky (2005)
Fukushima-Kharzeev-Warringa (2008)

Derivation (naïve calculation)

Thermodynamic Potential (*UV divergent*)

$$\Omega = -V N_c \sum_f \frac{|q_f B|}{2\pi} \sum_{s=\pm} \sum_{n=0}^{\infty} \alpha_{n,s}^f \int \frac{dp_3}{2\pi} \left[\omega_{n,s} + 2T \ln\left(1 + e^{-\beta\omega_{n,s}}\right) \right]$$

$$\omega_{n,s}^2 = \left(\sqrt{p_3^2 + 2|q_f B| n} + \mathrm{sgn}(p_3) s \mu_5 \right)^2 + m^2$$

$$\alpha_{n,s} = \begin{cases} 1 & n > 0, \\ \delta_{s+} & n = 0, \quad eB > 0 \\ \delta_{s-} & n = 0, \quad eB < 0 \end{cases}$$

Current (*UV finite*)

$$j_3 = \left. \frac{\partial \Omega}{\partial A_3} \right|_{A_3 = 0} \quad \Leftarrow \quad \frac{\partial}{\partial A_3} = e \frac{\mathrm{d}}{\mathrm{d}p_3}$$

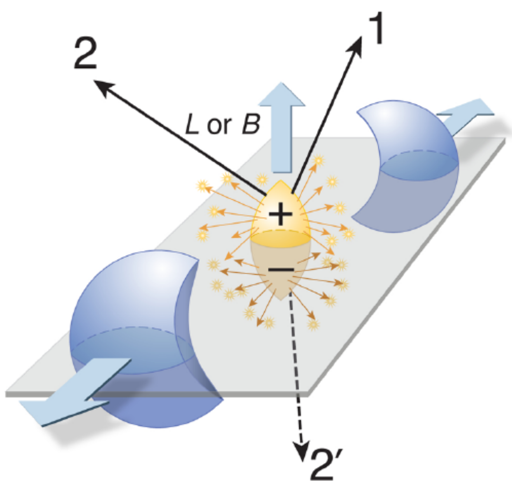
Only surface terms!

$$j_3 = e \frac{|eB|}{4\pi^2} \sum_{s,n} \alpha_{n,s} \left[\omega_{n,s}(p_3 = \Lambda) - \omega_{n,s}(p_3 = -\Lambda) \right]$$

$$= e \frac{|eB|}{2\pi^2} \sum_{s,n} \alpha_{n,s} s \mu_5 = \frac{e^2 B \mu_5}{2\pi^2}$$

Observable on Average

What can be measured in heavy-ion collision experiments is not the current $j_3 \sim B$ ($\mathcal{P}$-odd) but the current-current fluctuations $\langle j_3 j_3 \rangle$ ($\mathcal{P}$-even).

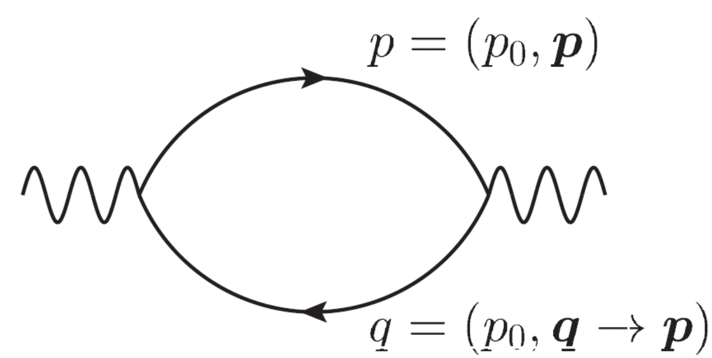


$$\langle j_3 j_3 \rangle = \langle j_3 \rangle \langle j_3 \rangle + \langle j_3 j_3 \rangle_{\text{connected}}$$

Background

As long as *e* is small;

$$\langle j_3 j_3 \rangle \sim$$



Fukushima-Kharzeev-Warringa (2009)

Very naïve calculation – Wrong!?

$$\chi \sim \frac{\partial^2 \Omega}{\partial A^2} \sim \frac{e^2|eB|}{2\pi^2}\left(1+\frac{2\Lambda^2}{3|eB|}\right)$$

One is tempted to drop the UV divergence by hand.
Dangerous calculation... but... $\chi_L - \chi_T = \text{(UV fintie)}$

But again, if this is accepted, the anomaly equation for <j^3> should receive a correction...

Fukushima-Ruggieri (2010)
c.f. Miransky-Shovkovy

Some confusions...

Fermion Propagator
and Dimensional Reduction

Construction of the Propagator

No Magnetic Field Background

Particles

$$\langle \psi(x)\bar{\psi}(y)\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{e^{-i\omega_p(x^0-y^0)+i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}}{2\omega_p}\left(\omega_p\gamma^0-\boldsymbol{p}\cdot\boldsymbol{\gamma}+m\right)$$

$$\sum_s u_s(p)\bar{u}_s(p)=p_\mu\gamma^\mu+m$$

Anti-Particles

$$\langle \bar{\psi}(y)\psi(x)\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{e^{i\omega_p(x^0-y^0)+i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}}{2\omega_p}\left(\omega_p\gamma^0+\boldsymbol{p}\cdot\boldsymbol{\gamma}-m\right)$$

$$\sum_s v_s(p)\bar{v}_s(p)=p_\mu\gamma^\mu-m$$

$$\rightarrow\ \langle T\psi(x)\bar{\psi}(y)\rangle = \int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}\frac{i}{p_\mu\gamma^\mu-m+i\epsilon}$$

Ritus' Method

Landau Wave Function ($A^2 = Bx$ gauge)

$$(iD_\mu \gamma^\mu - m) P_k(x) e^{-ip^0 x^0 + ip^2 x^2 + ip^3 x^3} = P_k(x) e^{-ip^0 x^0 + ip^2 x^2 + ip^3 x^3} (\tilde{p}_\mu \gamma^\mu - m)$$

$$P_k(x) = \begin{bmatrix} f_{k+}(x) & 0 & 0 & 0 \\ 0 & f_{k-}(x) & 0 & 0 \\ 0 & 0 & f_{k+}(x) & 0 \\ 0 & 0 & 0 & f_{k-}(x) \end{bmatrix}$$

$$f_{k+}(x) = \phi_k(x^1 - p^2/eB)$$
$$f_{k-}(x) = \phi_{k-1}(x^1 - p^2/eB)$$

Wave function of the harmonic oscillator

$$\tilde{p} = (p^0, 0, -\sqrt{2\,eBk}, p^3)$$

Landau quantization

Construction of the Propagator

Magnetic Field Background

Particles

$$\langle \psi(x)\bar{\psi}(y)\rangle = \int \frac{d p^2 d p^3}{(2\pi)^2} \sum_k \frac{e^{-i\omega_p(x^0-y^0)+ip^2(x^2-y^2)+ip^3(x^3-y^3)}}{2\omega_p} \times P_k(x)\left(\omega_p\gamma^0 - \tilde{\boldsymbol{p}}\cdot\boldsymbol{\gamma} + m\right)P_k(y)$$

Anti-Particles

$$\langle \bar{\psi}(y)\psi(x)\rangle = \int \frac{d p^2 d p^3}{(2\pi)^2} \sum_k \frac{e^{i\omega_p(x^0-y^0)+ip^2(x^2-y^2)+ip^3(x^3-y^3)}}{2\omega_p} \times P_k(y)\left(\omega_p\gamma^0 + \tilde{\boldsymbol{p}}\cdot\boldsymbol{\gamma} - m\right)P_k(x)$$

LLLA

Lowest Landau-Level Approximation

P_0 commutes with $\omega_p \gamma^0 - p^3 \gamma^3$

$$\rightarrow \langle T\psi(x)\bar{\psi}(y)\rangle = \int \frac{d\,p^0\,dp^2\,dp^3}{(2\pi)^3} e^{-i\,p^0(x^0-y^0)+i\,p^2(x^2-y^2)+i\,p^3(x^3-y^3)}$$

$$\times\sqrt{\frac{eB}{\pi}}\,e^{-\frac{1}{2}eB[(x^1-p^2/eB)^2+(y^1-p^2/eB)^2]}\,P_0\frac{i}{p^0\gamma^0-p^3\gamma^3-m+i\epsilon}$$

$$P_0=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}$$

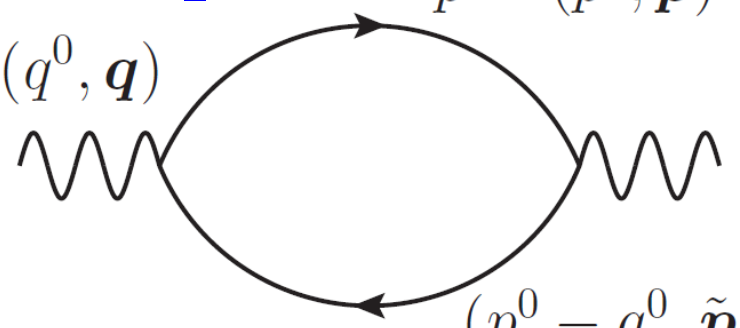
Landau zero-mode has only one spin state. ***s*** // ***B*** preferred.

The momentum conservation is highly non-trivial. $A^2 = Bx$

One Loop Polarization

Polarization (Self-energy)

In configuration space



$$i\Pi^{\mu\nu,ab}(x,y) = -(ig)^2 \,\mathrm{tr}[t^a t^b]\, \mathrm{Tr}[\gamma^\mu S_0^F(x,y)\gamma^\nu S_0^F(y,x)]$$

Note that this is not a function of $x - y$ apparently

In momentum space

$$i\Pi^{\mu\nu,ab}(k,q) = \int d^4x\, d^4y\, e^{iq\cdot x + ik\cdot y}\, i\Pi^{\mu\nu,ab}(x,y)$$

$$\rightarrow \quad \Pi^{\mu\nu,ab}(k,q) = (2\pi)^4 \delta^{(4)}(k+q)\, \delta^{ab}\, \Pi^{\mu\nu}(q)$$

(1+1) Dimensional System

Transverse Directions (to the Magnetic Field)

$$\Pi^{1\nu} = \Pi^{2\nu} = \Pi^{\mu 1} = \Pi^{\mu 2} = 0$$

Longitudinal Directions (μ,ν either 0,3)

$$\Pi^{\bar{\mu}\bar{\nu}} = i\frac{g^2|eB|}{2\pi}\int\frac{d^2p}{(2\pi)^2}\frac{p^{\bar{\mu}}(p^{\bar{\nu}}-q^{\bar{\nu}})+(p^{\bar{\mu}}-q^{\bar{\mu}})p^{\bar{\nu}}-g^{\bar{\mu}\bar{\nu}}[\bar{p}\cdot(\bar{p}-\bar{q})-m^2]}{[\bar{p}^2-m^2+i\epsilon][(\bar{p}-\bar{q})^2-m^2+i\epsilon]}$$

This is an ordinary expression for the one-loop polarization diagram in (1+1) dimensions

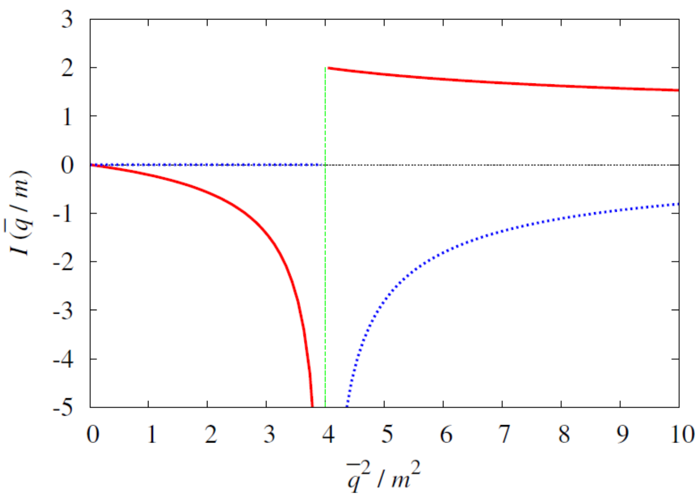
Gauge Invariance

Results from the Dimensional Regularization

$$\Pi^{\bar{\mu}\bar{\nu}}(q) = \left(g^{\bar{\mu}\bar{\nu}} - \frac{q^{\bar{\mu}} q^{\bar{\nu}}}{\bar{q}^2} \right) \frac{g^2 |eB|}{4\pi^2} \int_0^1 dx \, \frac{x(1-x)}{x(1-x) - (m/\bar{q})^2}$$

c.f. Naïve Integration

Fukushima-Kharzeev-Warringa (2009)



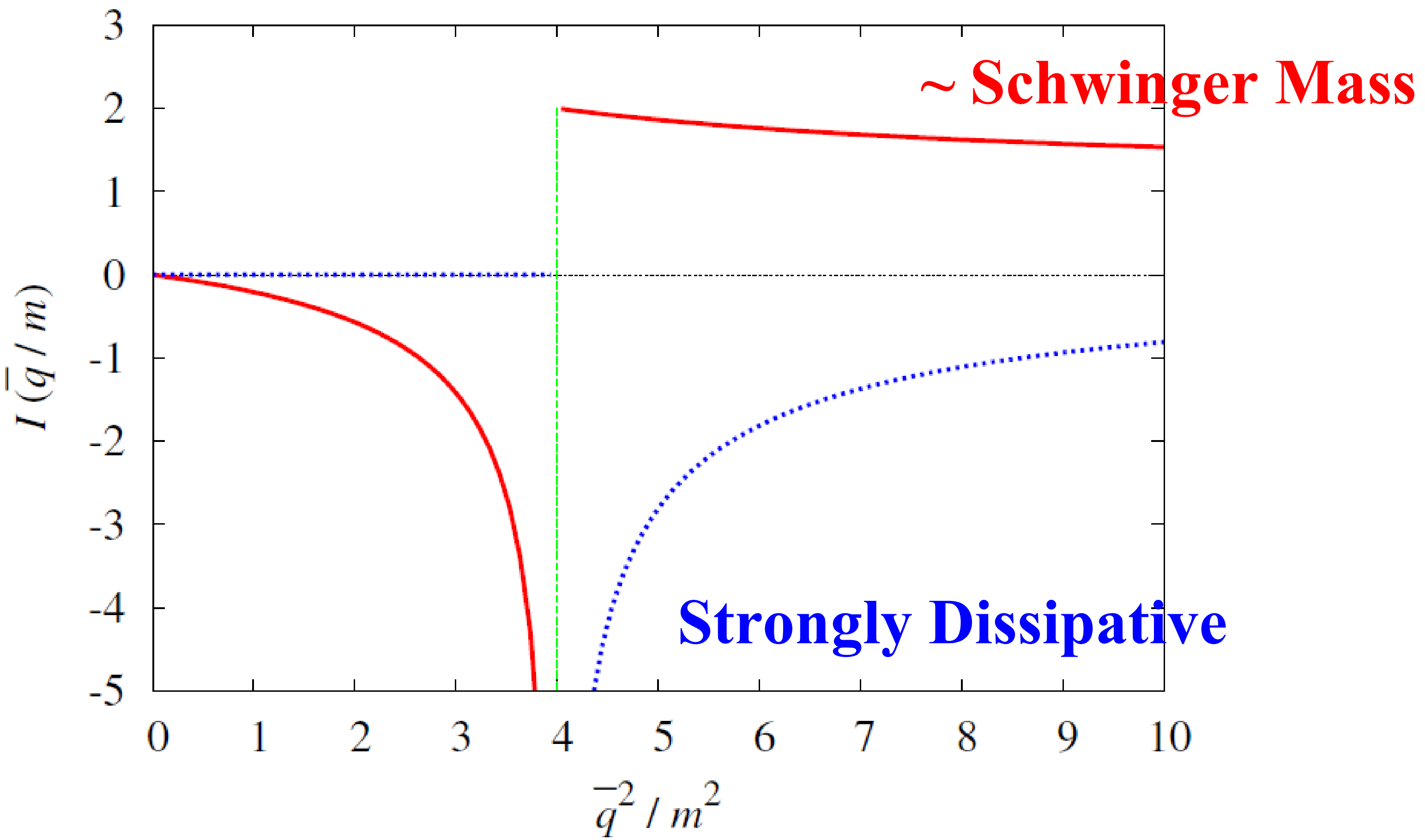
0-0 component

$$\int \frac{dp^0}{2\pi} \frac{(p^0)^2 + \omega_p^2}{[(p^0)^2 - \omega_p^2]^2} = 0 \;\rightarrow\; \Pi^{00} = 0$$

3-3 component

$$\int \frac{dp^0}{2\pi} \frac{(p^0)^2 + \omega_p^2 - 2m^2}{[(p^0)^2 - \omega_p^2]^2} = \frac{-i\, m^2}{2\omega^3} \;\rightarrow\; \int \frac{dp^3}{2\pi} \# = \frac{-i}{2\pi} \;\rightarrow\; \Pi^{33}(0) = \frac{g^2 |eB|}{4\pi^2}$$

Threshold Behavior in (1+1)-Dim



Different from (3+1)-dim in which no divergence appears.

Magnetic-field Induced Screening Effect and Perturbative Polyakov-loop Potential

Polyakov-loop Potential

Background A_4=diag(a,-a) Field (in Euclidean)

$$\partial_4 \;\rightarrow\; \partial_4 \pm iga \;\rightarrow\; q_{\pm 4} = q_4 \pm ga$$

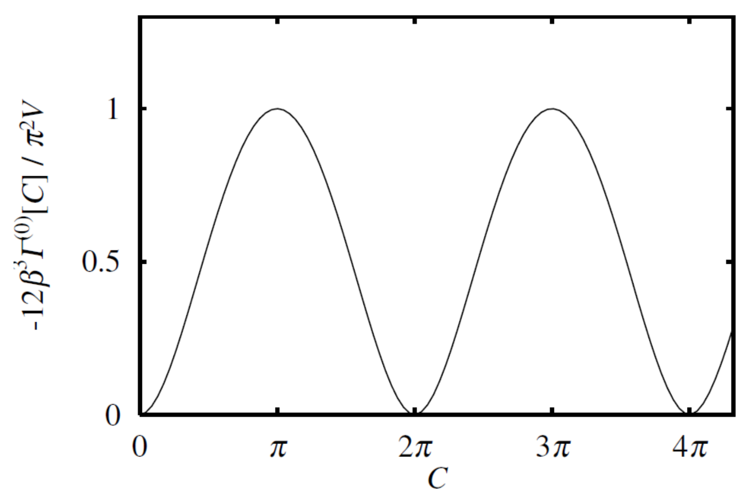
$$\ln Z = -\frac{1}{2}\mathrm{Tr}\ln[q_\pm^2\delta^{ij} - q^i q^j] + \mathrm{Tr}\ln[q_{\pm 4}]$$

Haar measure

$$= -\frac{1}{2}\mathrm{Tr}\ln[(q_{\pm 4})^2 + \boldsymbol{q}^2]^2[q_{\pm 4}^2] + \mathrm{Tr}\ln[q_{\pm 4}]$$

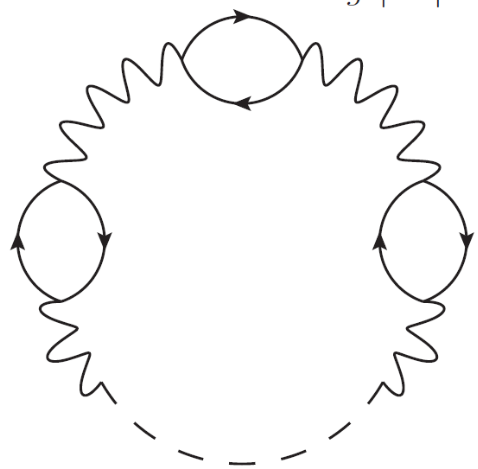
Weiss Potential (in color SU(2))

$$U_{\mathrm{Weiss}} = \frac{V\pi^2}{12\beta^3}\left(\frac{g\beta a}{\pi}\right)^2_{\mathrm{mod}\,2}\left[\left(\frac{g\beta a}{\pi}\right)_{\mathrm{mod}\,2} - 2\right]^2$$



Magnetic-field Induced Screening

Effect of the Magnetic Field

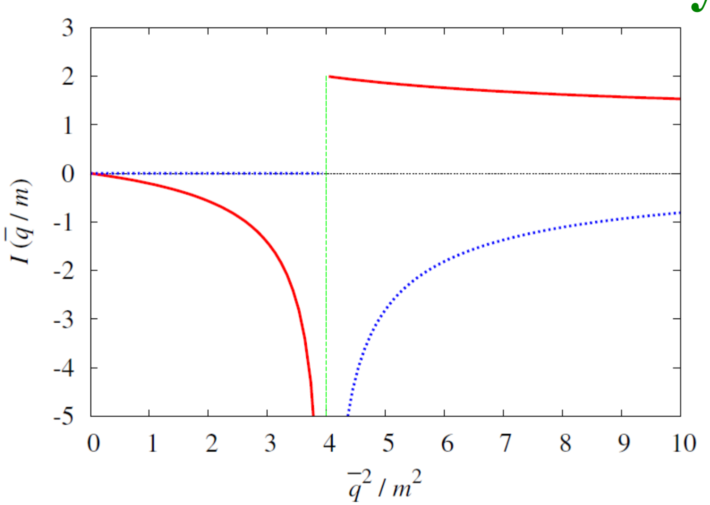


$$-\frac{1}{2}\mathrm{Tr}\ln[\,q_{\pm}^2\,\delta^{ij}-q^i q^j\,]$$

$$\to -\frac{1}{2}\mathrm{Tr}\ln\left[q_{\pm}^2\,\delta^{ij}-q^i q^j+\delta^{i3}\delta^{j3}\frac{(q_{\pm 4})^2}{(\bar{q}_{\pm})^2}M_g^2\right]$$

$$= -\frac{1}{2}\mathrm{Tr}\ln[(q_{\pm 4})^2+\boldsymbol{q}^2][\underline{(q_{\pm 4})^2+\boldsymbol{q}^2+M_g^2}][(q_{\pm 4})^2]$$

Screened by the *B*-induced Gluon Mass



$$M_g^2=\frac{g^2|eB|}{4\pi^2}I(\bar{q}/m)$$

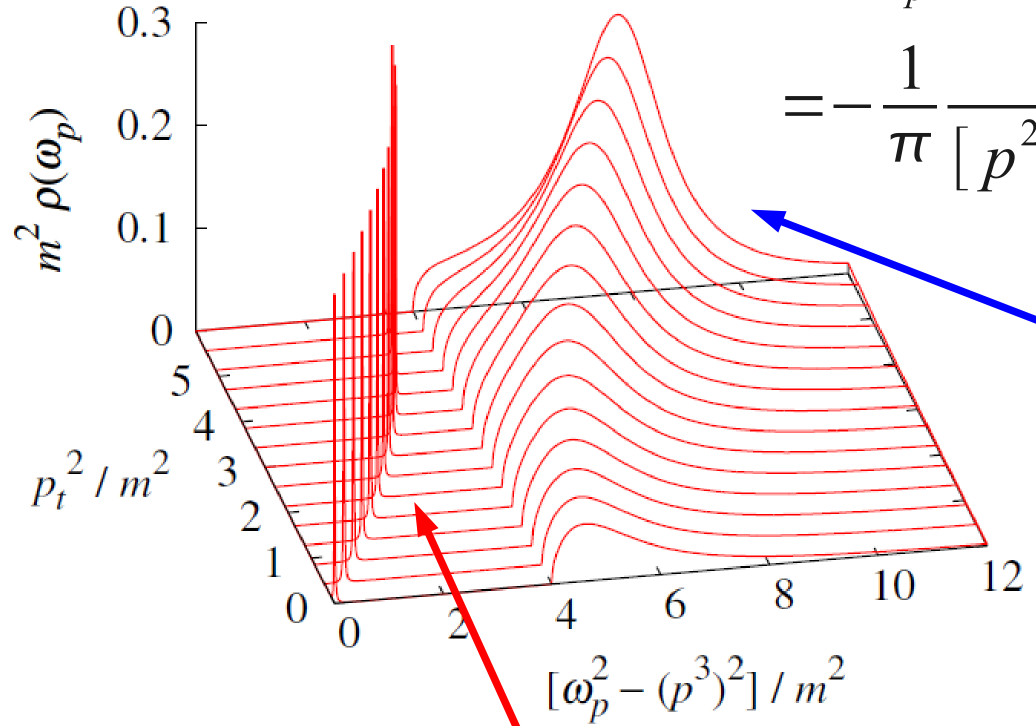
$$\to \frac{g^2|eB|}{4\pi^2}\quad (m\to 0)$$

Two-particle threshold at $|q|=2m$

Spectral Function

Collective Excitation in a Strong *B*

$$\rho(\omega_p) = -\frac{1}{\pi}\frac{\mathrm{Im}\, M_g^2(p)}{[p^2 - \mathrm{Re}\, M_g^2(p)]^2 + [\mathrm{Im}\, M_g^2(p)]^2}$$



Massive and Dissipative Transverse Photon (Gluon)

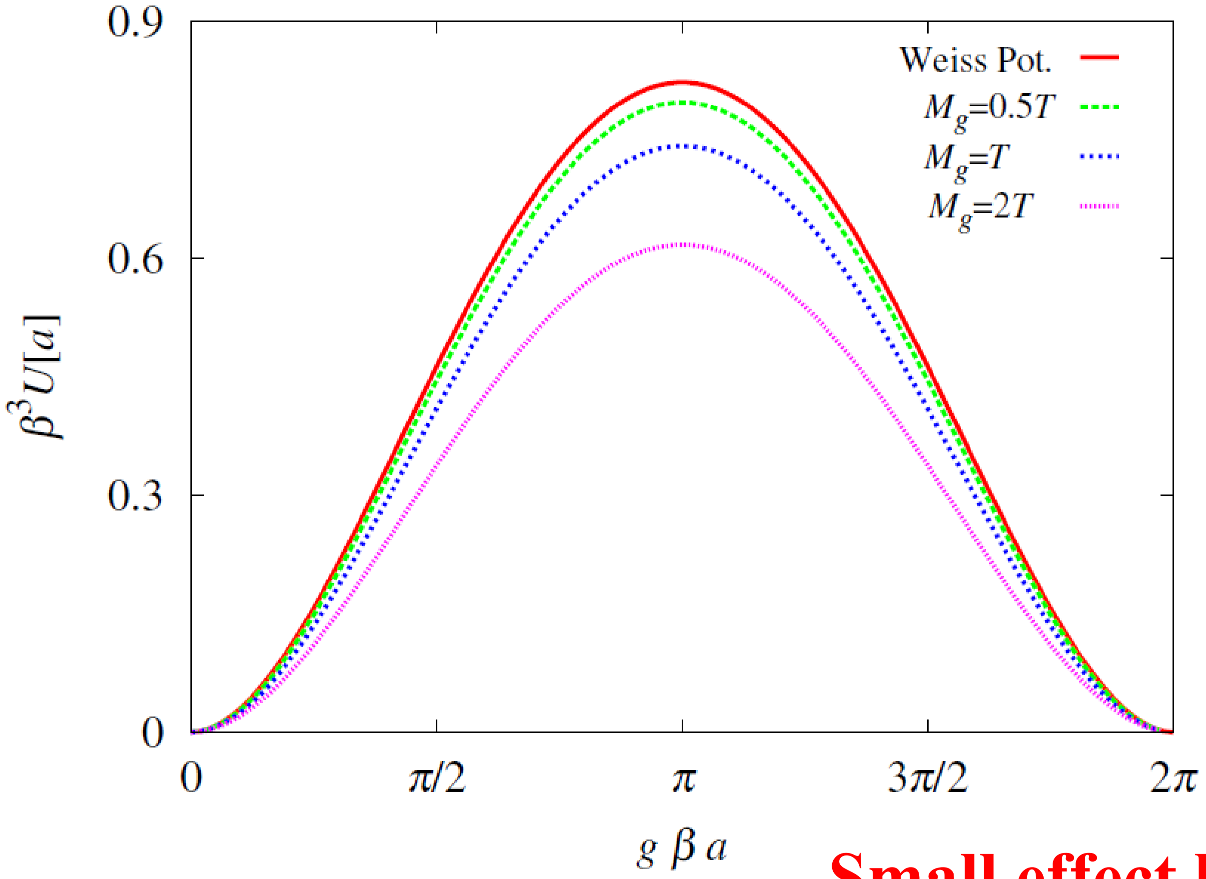
(1+1)-dim Analogue to (3+1)-dim "Zero Sound"

c.f. followking talks by Misha and Maxim

c.f. "chiral magnetic wave" Kharzeev-Yee (2010)

Screened Weiss Potential

Change in the Weiss Potential as M_g increases



Calculation at m=0

Perturbative vacuum at $g\beta a = 0$ (tr$L = 1$) is less stabilized (slightly).

A barrier at the confining vacuum at $g\beta a = \pi$ (tr$L = 0$) is (slightly) suppressed.

Small effect but "correct" direction to have a larger "effective" *T*

Remarks and Conclusions

- Magnetic-field induced screening effects has a similar structure as the finite-T temperature.

- The one-loop polarization diagram is computed in the LLLA in a gauge-invariant procedure.

- Magnetic screening effect in (1+1) dimensions is embedded in the (3+1)-dim transverse part in the gluon (photon) propagation → collective excitation

Physics with a high B ~ Physics at a high μ_B
[effective (1+1)-dim systems]