The strong coupling limit of lattice QCD

Philippe de Forcrand
ETH Zürich and CERN

PhD thesis of Michael Fromm (ETH $\rightarrow$ Frankfurt)

arXiv:0811.1931, 0907.1915 $\rightarrow$ PRL, 0912.2524

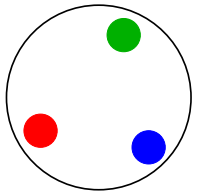
and in progress
with J. Langelage, K. Miura, O. Philipsen and W. Unger

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Scope of lattice QCD simulations

Physics of **color singlets**

- "One-body" physics: **confinement**
  hadron masses
  form factors, etc..



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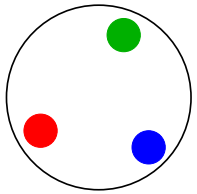
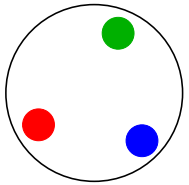
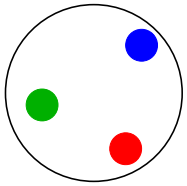
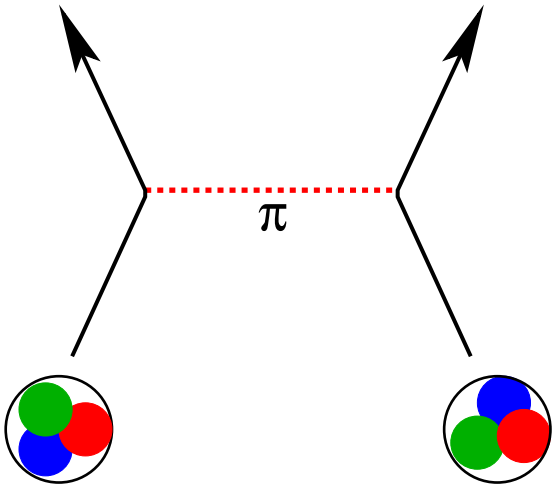


- "Two-body" physics: **nuclear interactions**
  pioneers Hatsuda et al, Savage et al

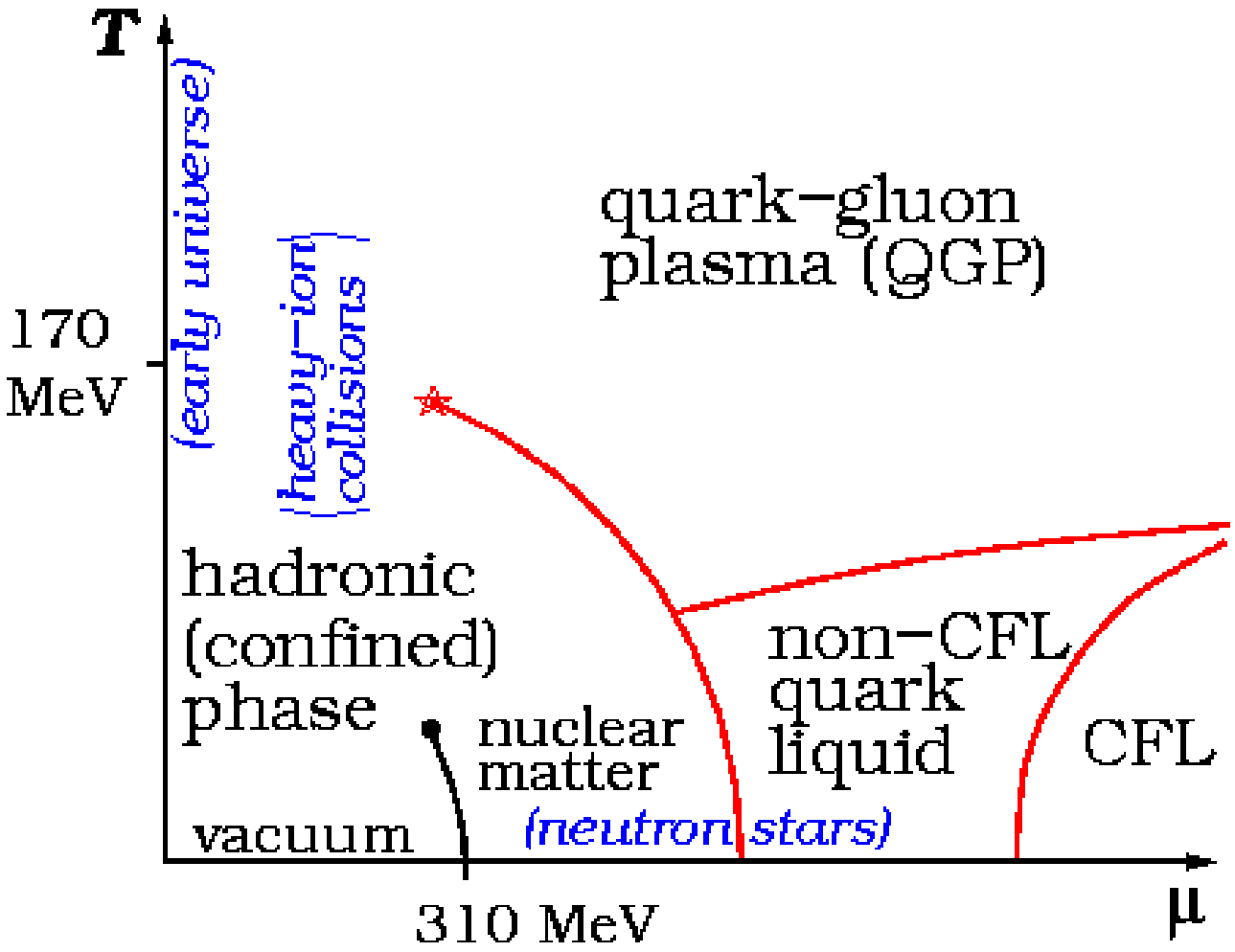






hard-core
+
pion exchange?

QCD phase diagram according to Wikipedia



Here: • "many-body" physics: **hadron ↔ nuclear matter** transition
• "two-body": $T = 0$ **nuclear interactions**

A different approach to the sign problem

$$Z = \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{N_f}\bar{\psi}_i(\not{D} + m_i + \mu_i\gamma_0)\psi_i\right)$$

$\det(\not{D} + m + \mu\gamma_0)$ complex $\rightarrow$ try integrating over the gauge field first!

- Problem: $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \rightarrow \beta_{\text{gauge}}\text{Tr}U_{\text{Plaquette}}$, ie. 4-link interaction 
- Solution: set $\beta_{\text{gauge}} = \frac{2N_c}{g^2}$ to **zero**, ie. $g = \infty$, strong coupling limit
- Then integral over gauge links *factorizes*: $\sim \int \prod dU \exp(\bar{\psi}_x U_{x,\hat{\mu}}\psi_{x+\hat{\mu}})$

- analytic 1-link integral $\rightarrow$ only color singlets survive

- perform Grassmann integration last $\rightarrow$ hopping of color singlets

$\rightarrow$ **hadron worldlines**

- sample gas of worldlines by Monte Carlo

Note: when $\beta_{\text{gauge}} = 0$, quarks are *always* confined $\forall(\mu, T)$, ie. **nuclear matter**

The price to pay: not continuum QCD

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\rm gauge}) \propto \exp(-\frac{\beta_{\rm gauge}}{4N_c b_0})$

ie. $a \to 0$ when $\beta_{\rm gauge} \equiv \frac{2N_c}{g^2} \to +\infty$. Here $\boxed{\beta_{\rm gauge} = 0}$!!

- Lattice "infinitely coarse"
- Physics not universal

Nevertheless:

- Properties similar to QCD: confinement and χSB
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\rm gauge} > 0$

When $\beta_{\rm gauge} = 0$, sign problem is **manageable** $\to$ **complete solution**

Valuable insight? *understand* nuclear interactions

Further motivation

- 25^+ years of analytic predictions:

  80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto

  $$\mu_c(T=0) = 0.66, \quad T_c(\mu=0) = 5/3$$

  90's: Petersson et al., $1/g^2$ corrections

  00's: detailed (μ, T) phase diagram: Nishida, Kawamoto,...

  now: Ohnishi et al. $\mathcal{O}(\beta)$ & $\mathcal{O}(\beta^2)$, Münster & Philipsen,...

  How accurate is mean-field $(1/d)$ approximation?

- Almost no Monte Carlo crosschecks:

  89: Karsch-Mütter $\rightarrow$ MDP formalism $\rightarrow$ $\mu_c(T=0) \sim 0.63$

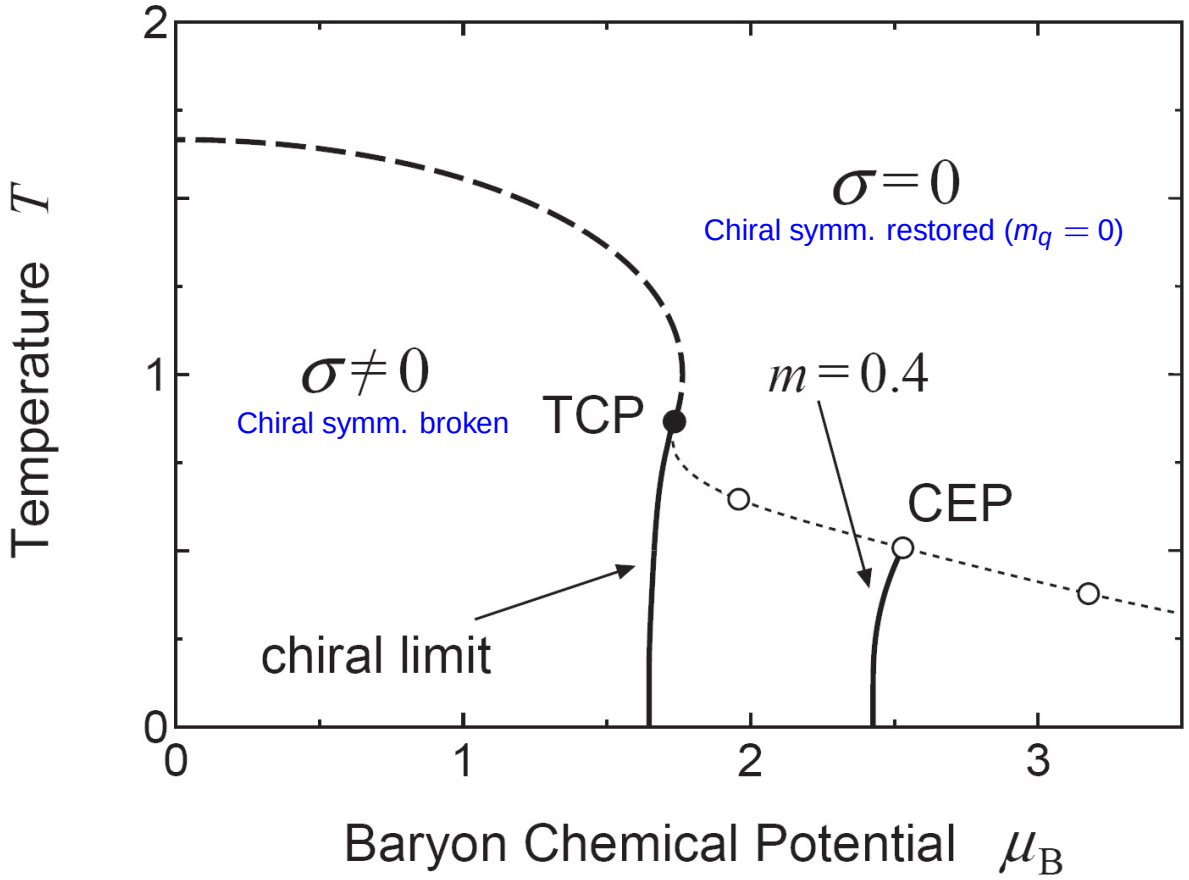
  92: Karsch et al. $T_c(\mu=0) \approx 1.40$

  99: Azcoiti et al., MDP ergodicity ??

  06: PdF-Kim, HMC $\rightarrow$ hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram from Nishida (2004, mean field, cf. Fukushima)



- Very similar to conjectured phase diagram of $N_f = 2$ **QCD**
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass $= M_{\text{proton}}$ $\Rightarrow$ lattice spacing $a \sim 0.6$ fm not universal

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp(-\bar{\psi}(\not{D}(U) + m_q)\psi)$, no plaquette term ($\beta_{\text{gauge}} = 0$)

- **One** complex colored fermion field per site (no Dirac indices, spinless)
- $\not{D}(U) = \frac{1}{2}\sum_{x,\nu} \eta_\nu(x)(U_\nu(x) - U_\nu^\dagger(x - \hat{\nu}))$, $\eta_\nu(x) = (-)^{x_1 + .. + x_{\nu-1}}$
- Chemical potential $\mu \rightarrow \exp(\pm a\mu) U_{\pm 4}$
- $\mathcal{D}U = \prod dU$ *factorizes*$\rightarrow$ integrate over links Rossi & Wolff

$\rightarrow$ **Color singlet** degrees of freedom:

- **Meson** $\bar{\psi}\psi$: *monomer*, $M(x) \in \{0,1,2,3\}$
- **Meson hopping**: *dimer*, non-oriented $n_\nu(x) \in \{0,1,2,3\}$
- **Baryon hopping**: oriented $\bar{B}B_\nu(x) \in \{0,1\}$ $\rightarrow$ *self-avoiding loops C*

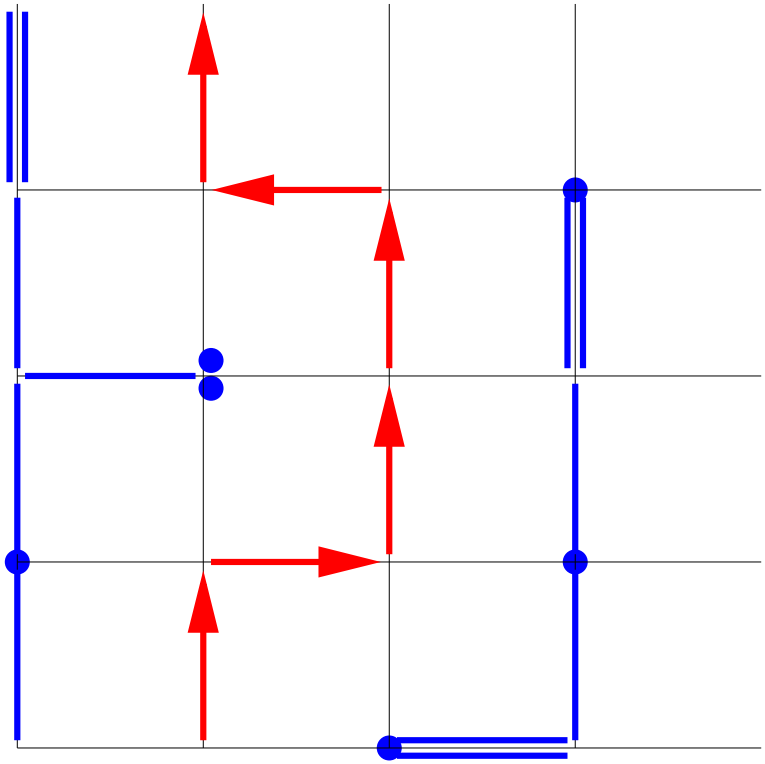
Point-like, hard-core baryons in pion bath

No πNN vertex

MDP Monte Carlo

$$Z(m_q,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{m_q^{M(x)}}{M(x)!} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{n_\nu(x)!} \prod_{\text{loops}\,C} \rho(C)$$

with **constraint** $(M+\sum_{\pm\nu} n_\nu)(x) = 3\ \forall x \notin \{C\}$



Constraint: 3 blue symbols or a baryon loop at every site

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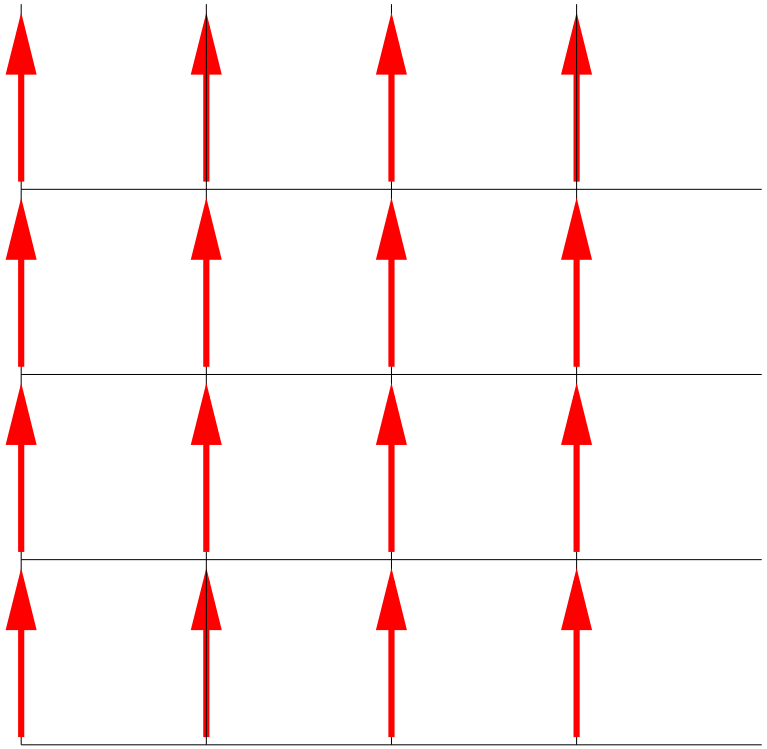

The dense (crystalline) phase: 1 baryon per site; no monomer $\rightarrow$ $\langle\bar\psi\psi\rangle = 0$

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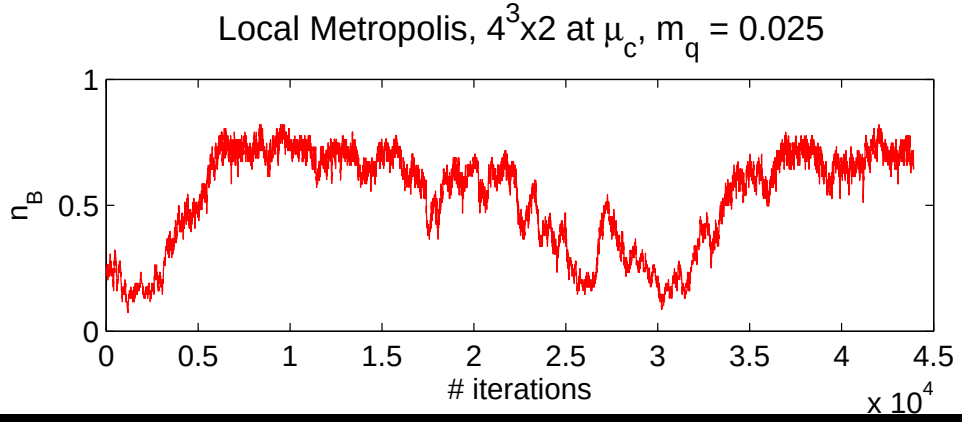
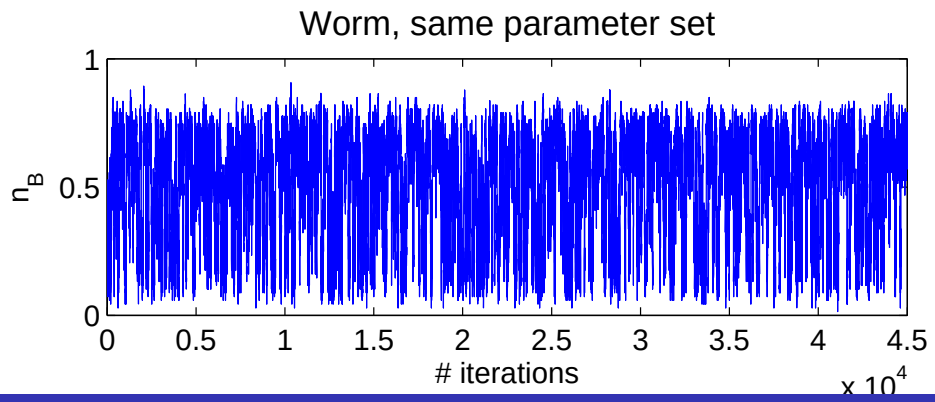
Remaining difficulties:

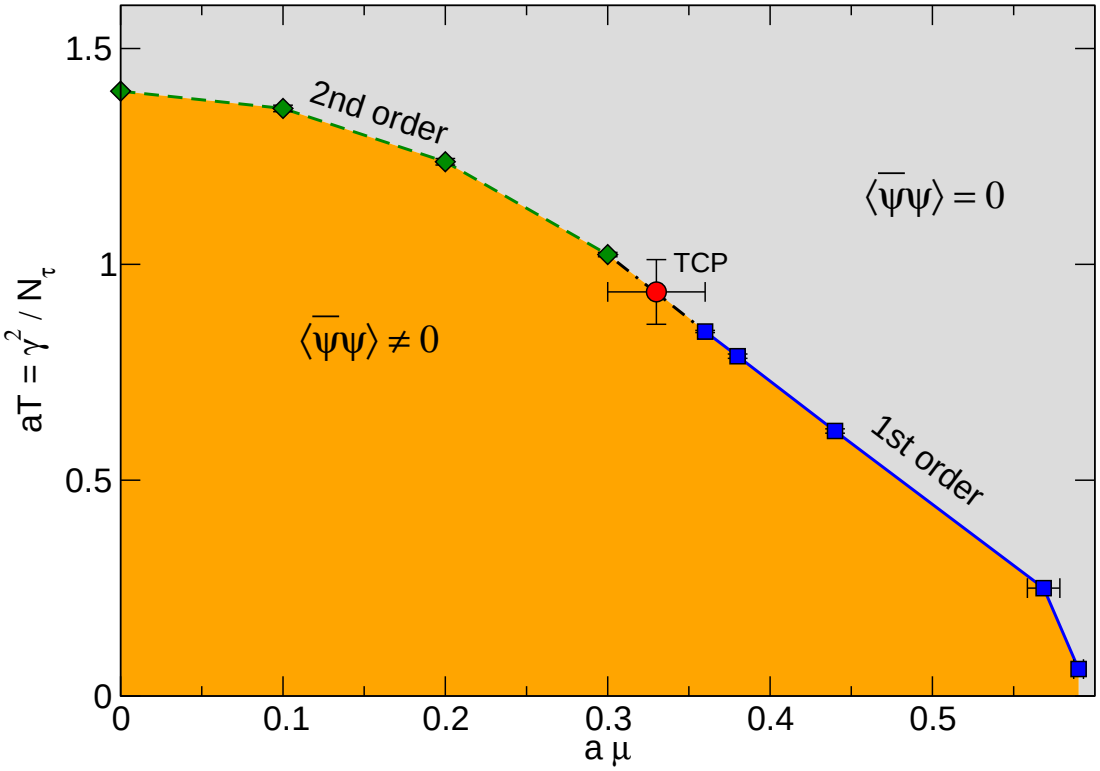
- Baryons are fermions: mild sign problem from $\rho(C)$ Karsch & Mütter
  $\rightarrow$ volumes up to $16^3 \times 4\ \forall\mu$
- tight-packing constraint $\rightarrow$ local update inefficient, esp. as $m \rightarrow 0$
  Solved with **worm algorithm** (Prokof'ev & Svistunov 1998)
  Efficient even when $m_q = 0$





(μ, T) phase diagram in the chiral limit $m_q = 0$, and for $m_q \neq 0$

$m_q = 0$



- Phase boundary for breaking/restoration of $U(1)$ chiral symmetry
- 2nd order at $\mu = 0$: 3d O(2) universality class
- 1rst order at $T = 0$: ρ_B jumps from 0 to 1 baryon per site $\Longrightarrow$ **tricrit. pt. TCP**

Finite-size scaling: $(\mu, T)_{TCP} = (0.33(3), 0.94(7))$ vs $(0.577, 0.866)$ (mean-field)

Beware of quantitative mean-field predictions for phase diagram

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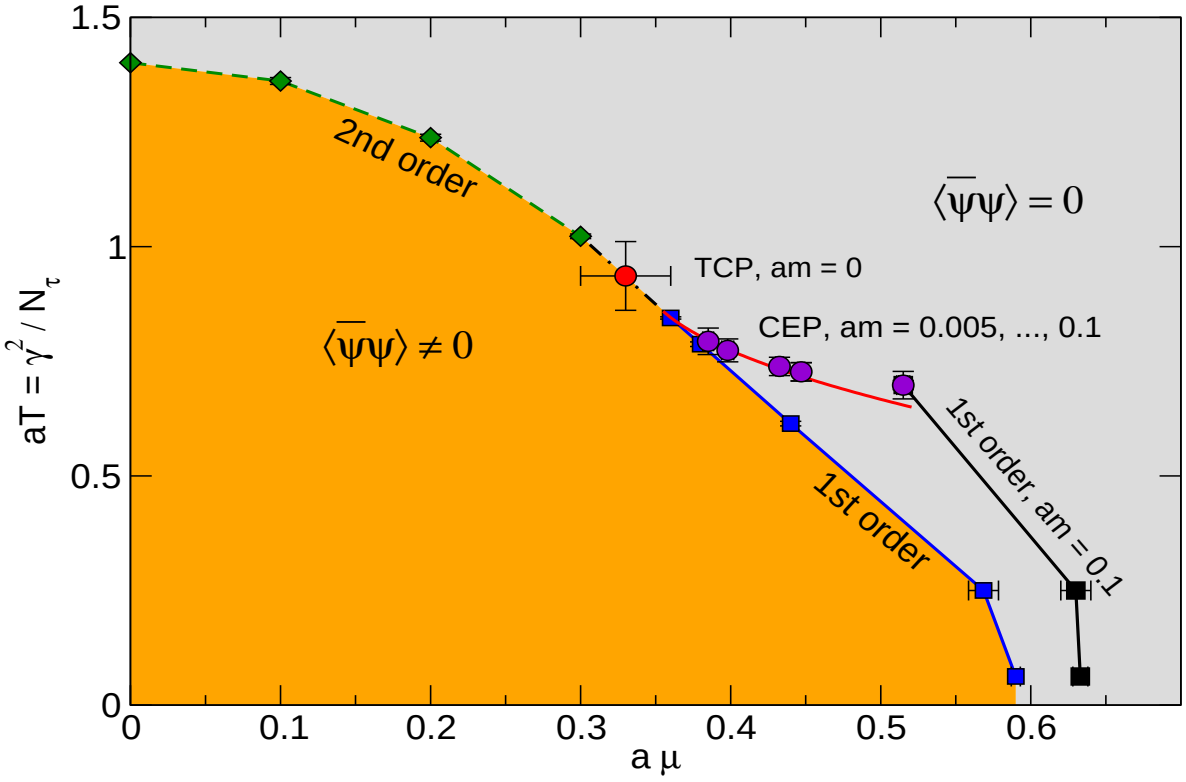

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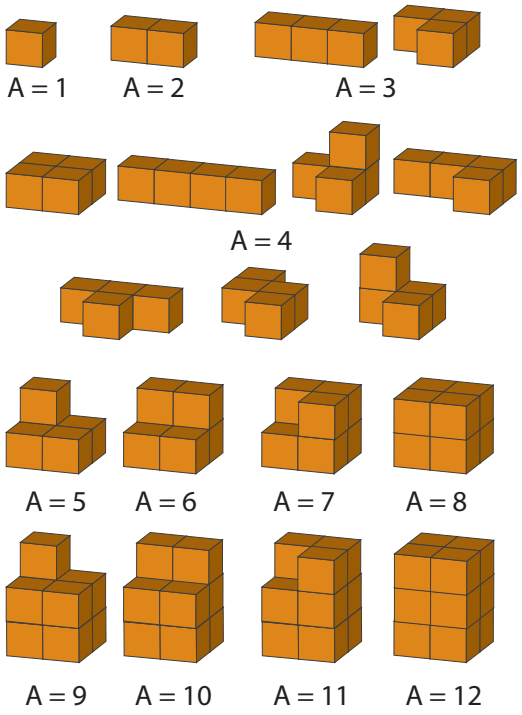
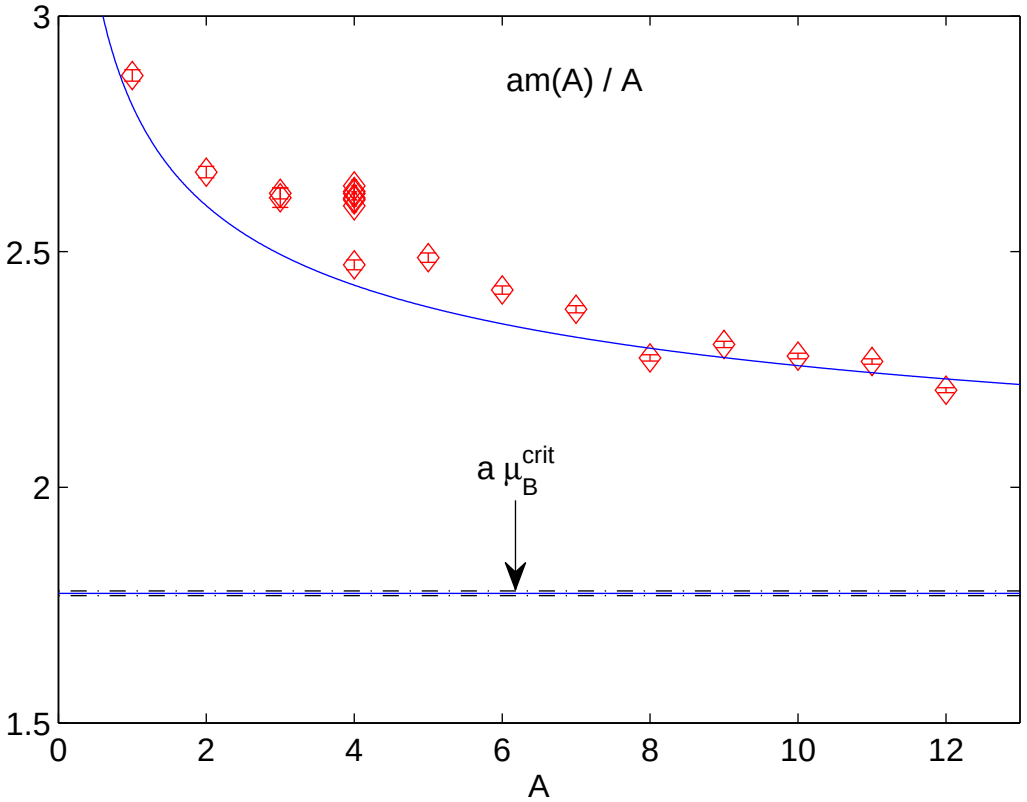
Beware of quantitative mean-field predictions for phase diagram

- $m_q \neq 0$: liquid-gas transition $T_{CEP} \sim 200$MeV – traj. of CEP obeys tricrit. scaling

Nuclear matter: spectroscopy

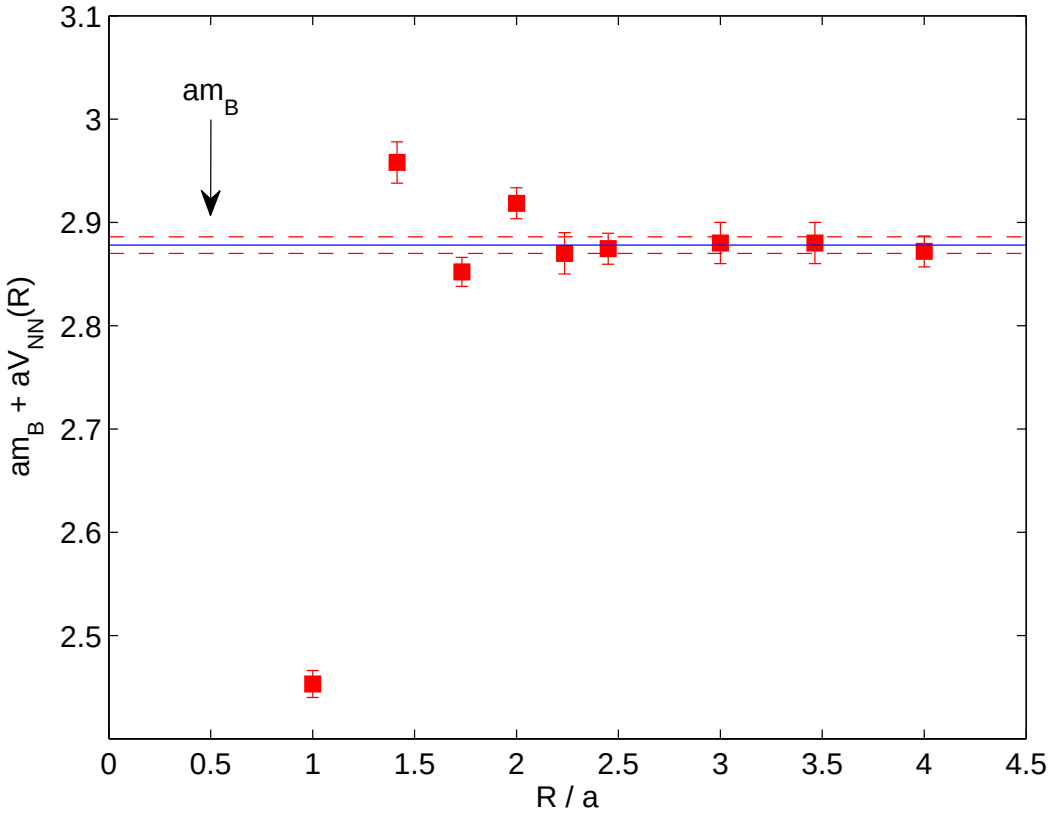




- Can compare masses of differently shaped "isotopes"
- $E(B=2) - 2E(B=1) \sim -0.4$, ie. "deuteron" binding energy ca. 120 MeV
- $am(A) \sim a\mu_B^{\rm crit} A + (36\pi)^{1/3}\sigma a^2 A^{2/3}$, ie. (bulk + surface tension)

  Bethe-Weizsäcker parameter-free ($\mu_B^{\rm crit}$ and σ measured separately)
- "Magic numbers" with increased stability: $A = 4,\ 8,\ 12$ (reduced area)

Nuclear potential: more than hard core

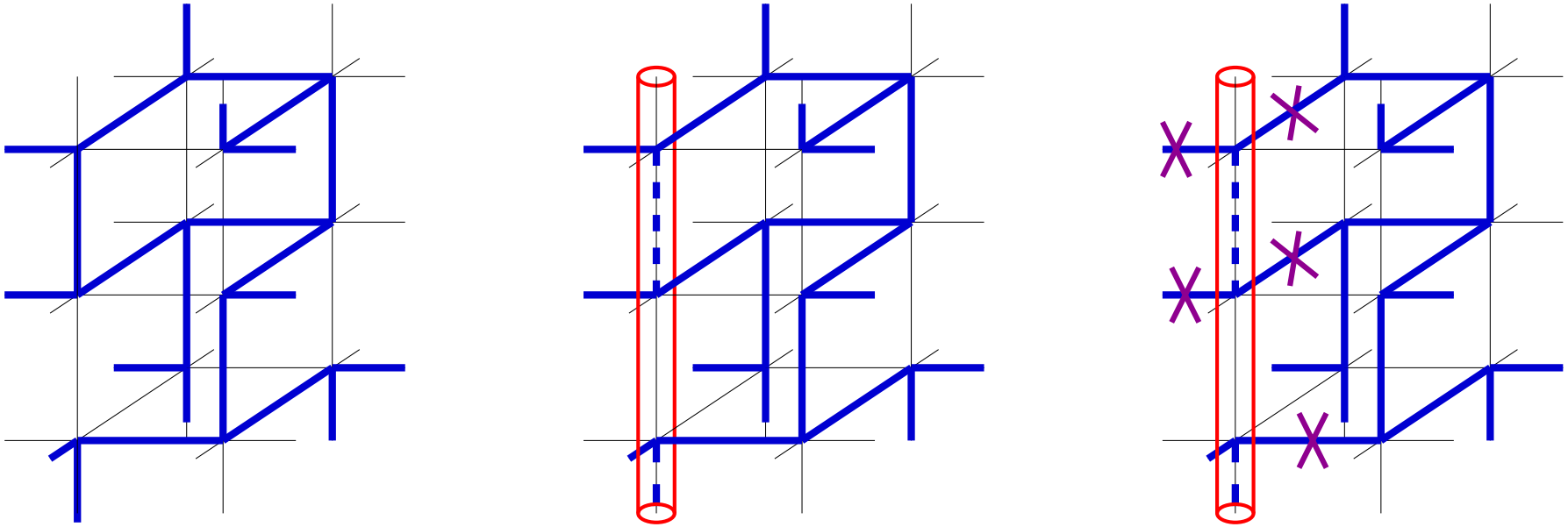


- Nucleons are point-like $\rightarrow$ no ambiguity with definition of static potential
- Nearest-neighbour attraction $\sim$ 120 MeV at distance $\sim$ 0.5 fm: cf. real world
- Baryon worldlines self-avoiding $\rightarrow$ no direct meson exchange (just hard core)

How do baryons interact at non-zero distance?

How the nucleon got its mass

- Point-like nucleon **distorts pion bath** cf. Casimir



vacuum static baryon effect on pions

- Energy = nb. time-like pion lines

Constraint: 3 pion lines per site ($m_q = 0$) $\rightarrow$ energy density = 3/4 in vacuum

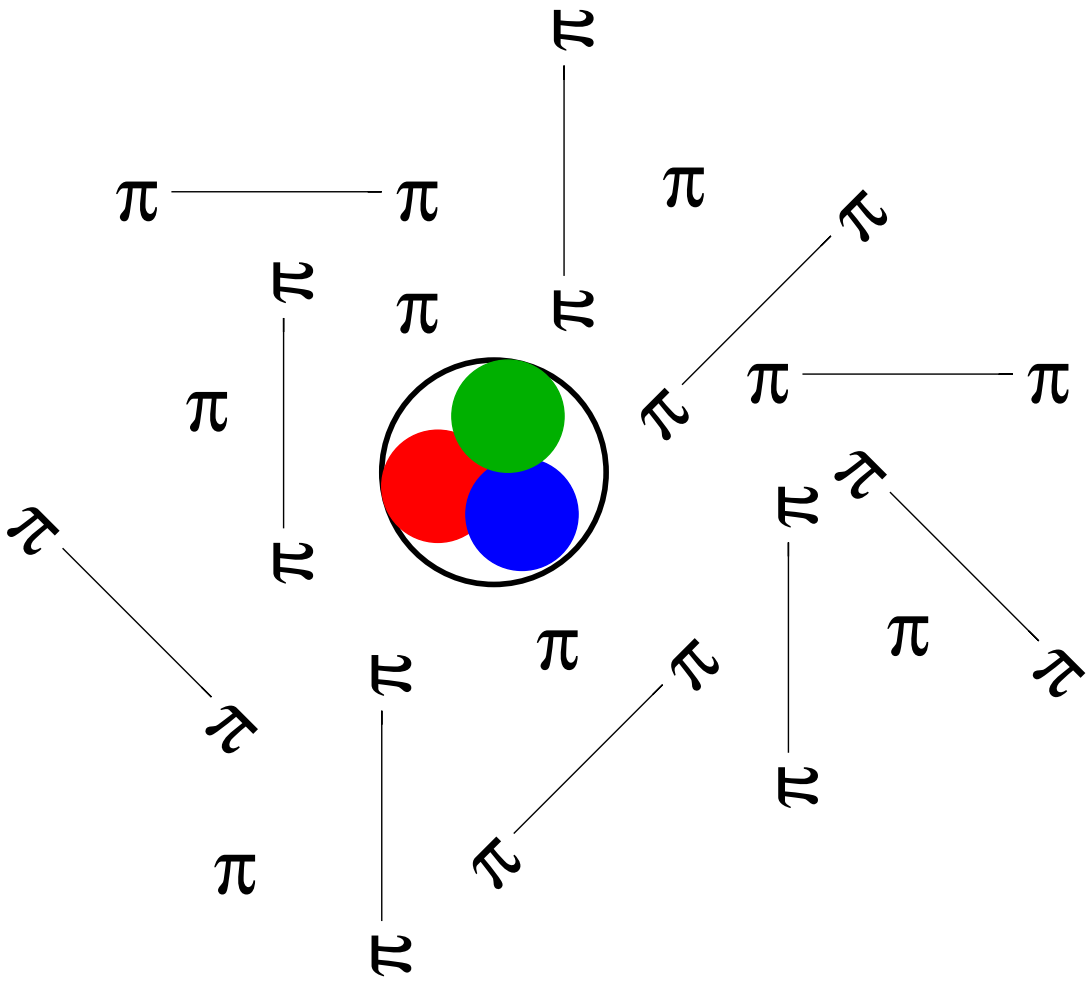
No spatial pion lines connecting to site occupied by nucleon $\rightarrow$ energy increase

Steric effect

- $am_B \approx 2.88 = (3 - 0.75) + \Delta E_\pi$, ie. "valence"(78%) + "**pion cloud**"(22%)

So, in fact, nucleon is *not* point-like

Point-like “bag” of 3 valence quarks → macroscopic disturbance in pion vacuum



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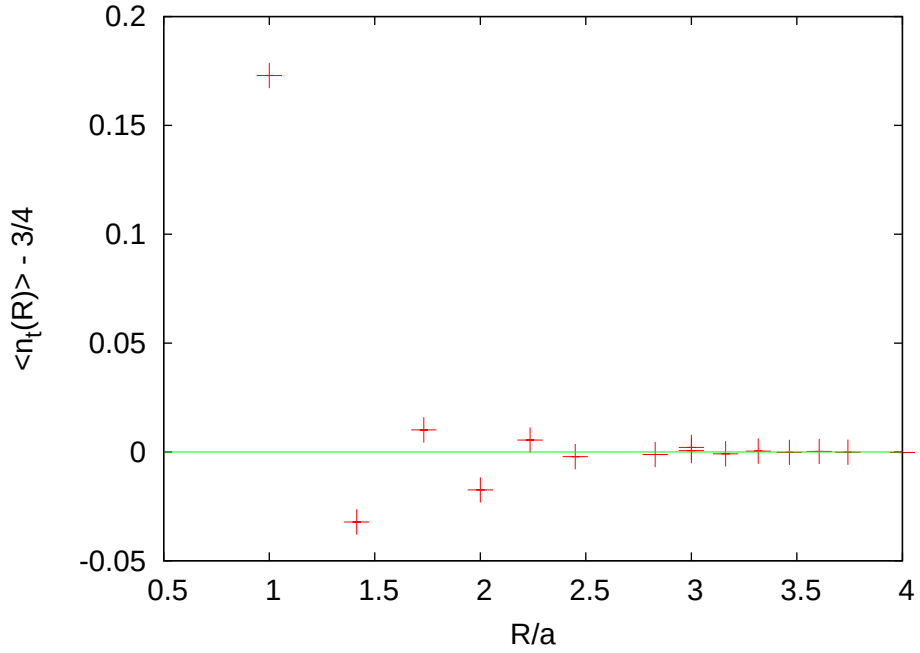
Static baryon prevents monomers = static (t-invariant) monomer "source"

Linear response $\propto$ Green's fct. of lightest t-invariant meson, ie. rho/omega

(pion has factor $(-1)^t$)



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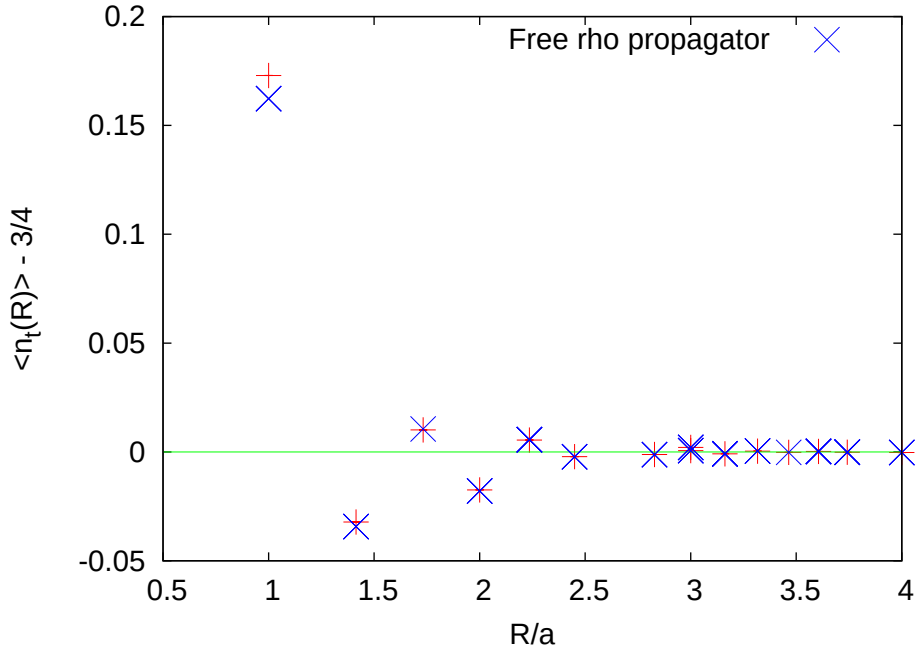
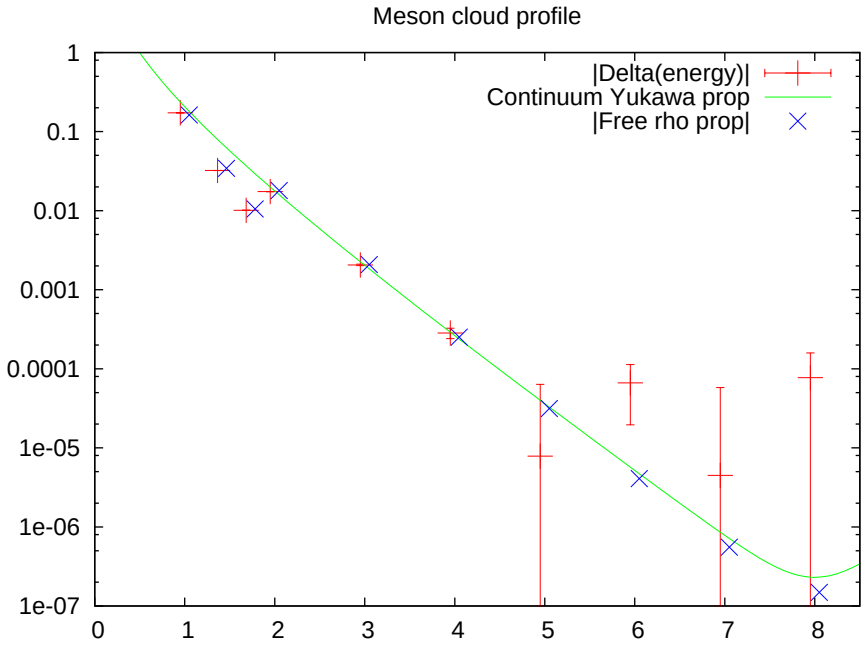




$$\textbf{Pion cloud profile} \propto \frac{\exp(-\mathbf{m}_{\rho/\omega} r)}{r} \times (-1)^{x+y+z}$$

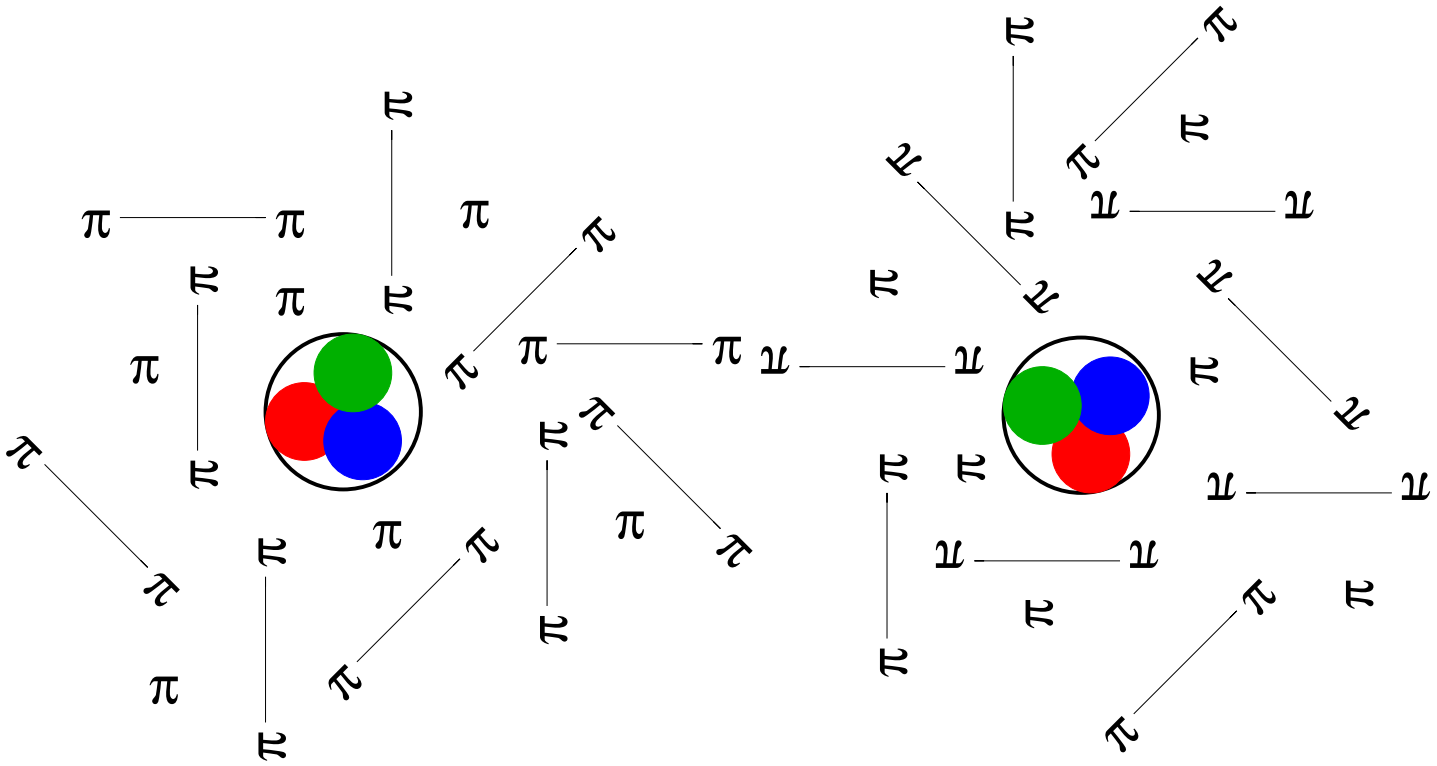
Nuclear interaction via pion clouds (thanks W. Weise)

- Here, baryons make self-avoiding loops $\rightarrow$ no direct meson exchange
- Interaction comes because of **pion clouds**

The two pion clouds can interpenetrate at $\approx$ constant energy (2nd order effect)

But each set of *valence* quarks disturbs pion cloud of other baryon

$$\Longrightarrow \quad V_{NN}(R) \approx -2 \times \Delta E_\pi(R) \;\propto\; \frac{\exp(-\mathbf{m}_{\rho/\omega} R)}{R} \times (-1)^{x+y+z}$$



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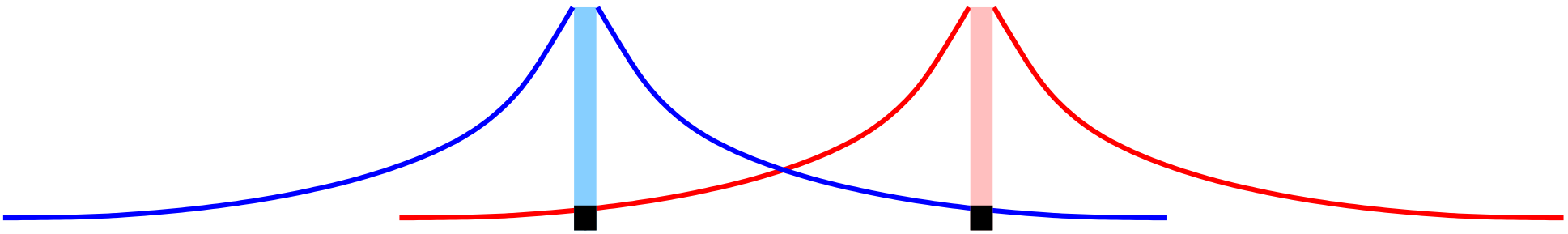

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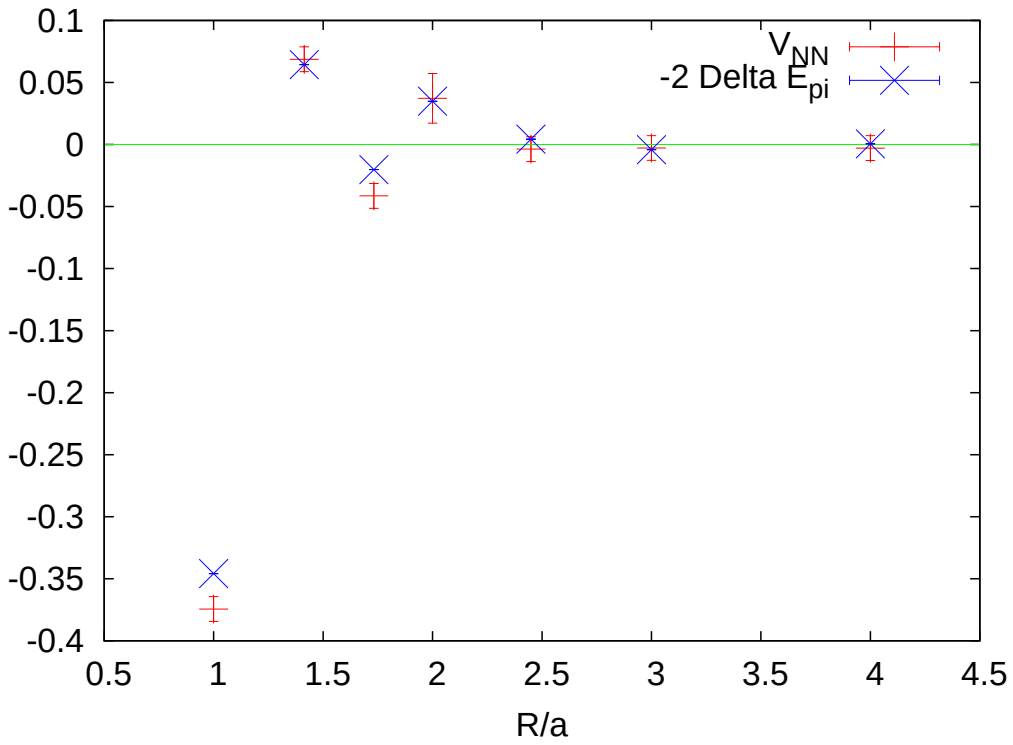
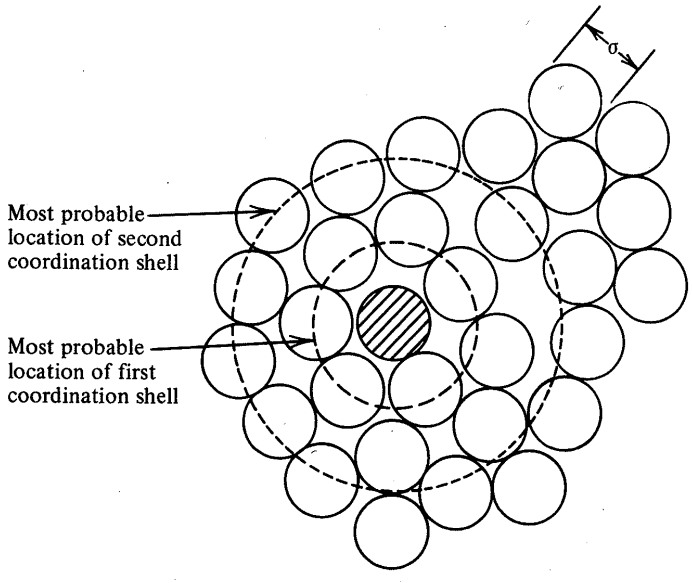
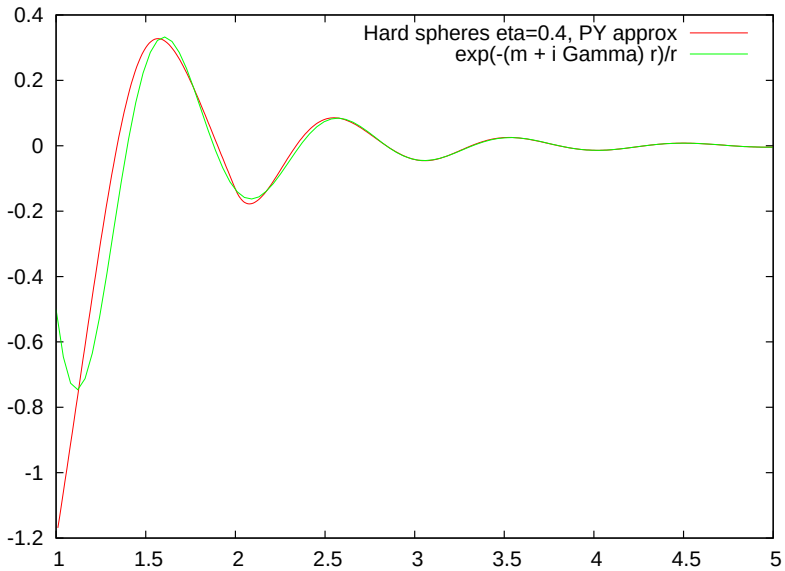


Meson exchange potential without meson exchange!

Recap: baryons and their interactions at $\beta = 0$

- Baryons are not point-like: pion cloud $\sim \exp(-m_{\rho/\omega} r)$
- Nuclear potential:
  - Hard-core from Pauli principle
  - Yukawa potential (times $(-1)^r$) from the two pion clouds
- Precisely like a **classical hard-sphere fluid**:
  - "Pion cloud" from ripples around tagged sphere
  - Density-density correlation: $V_{\rm eff}(r) \equiv -\log(g(r)) \sim \exp(-(m + i\Gamma)r)/r$





Conclusions

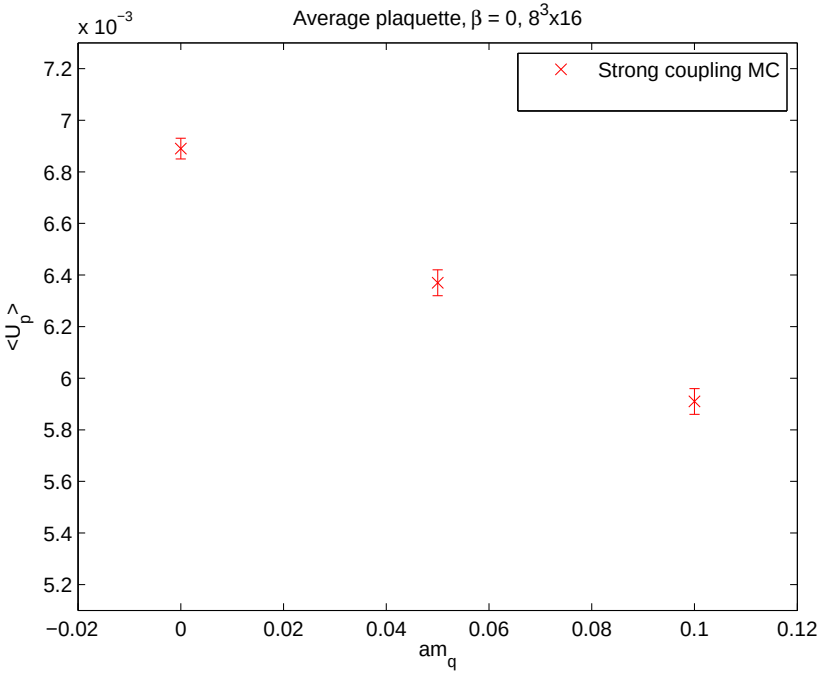
Summary: complete solution of strong coupling limit

- Phase diagram: take mean-field results with a grain of salt
- [Crude, crystalline] nuclear matter from QCD:
  tabletop simulations of first-principles nuclear physics
- Nucleon: point-like "bag" ($\rightarrow$ hard core) + large pion cloud
  cf. "little bag" model (Brown & Rho)
- Yukawa potential without meson exchange

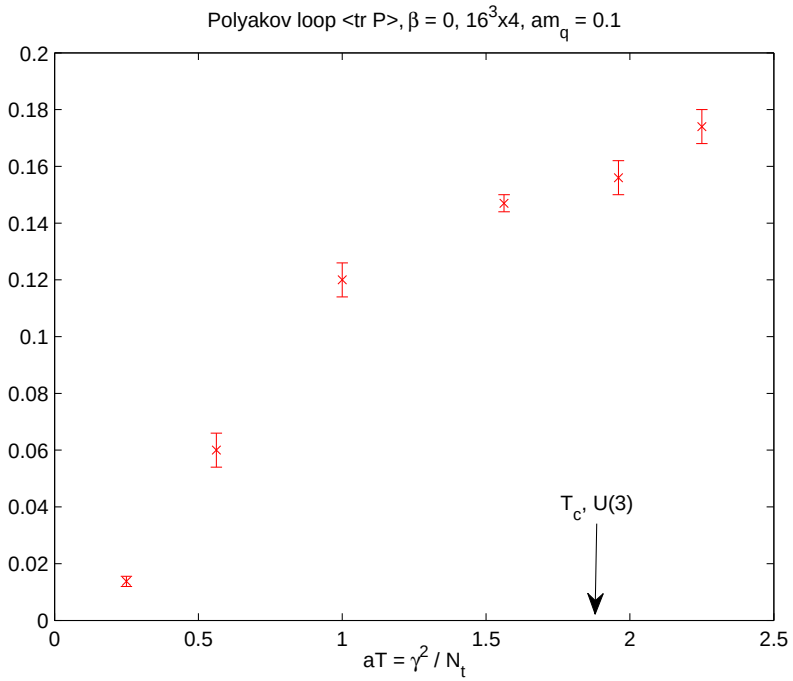
Outlook

- Include $\mathcal{O}(\beta)$ effects
- Take continuous Euclidean time
- Include second quark species $\rightarrow$ isospin ?

First step towards $\mathcal{O}(\beta)$: gauge observables at $\beta = 0$

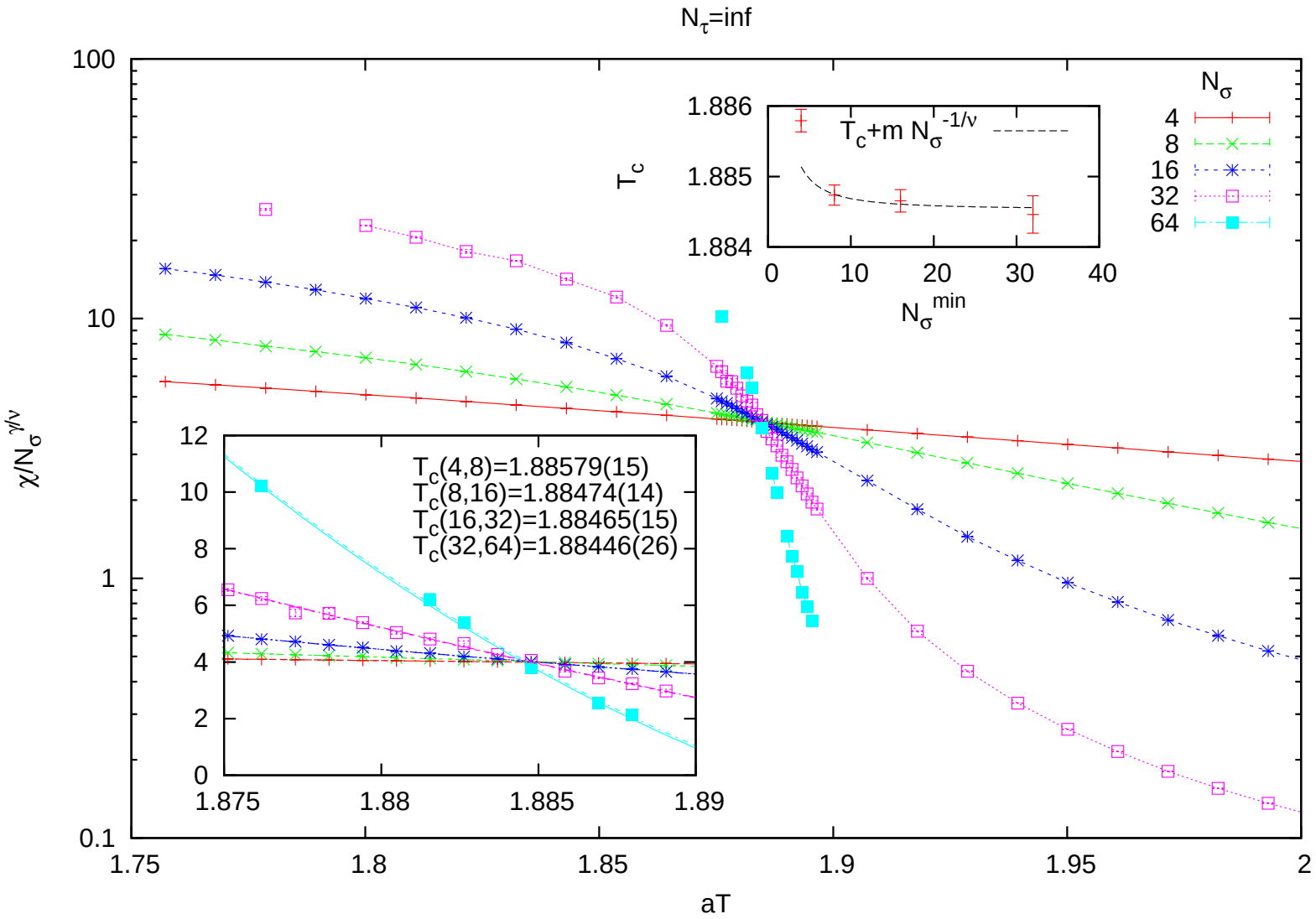


Plaquette vs m_q



Polyakov loop vs T ($m_q = 0.1$)

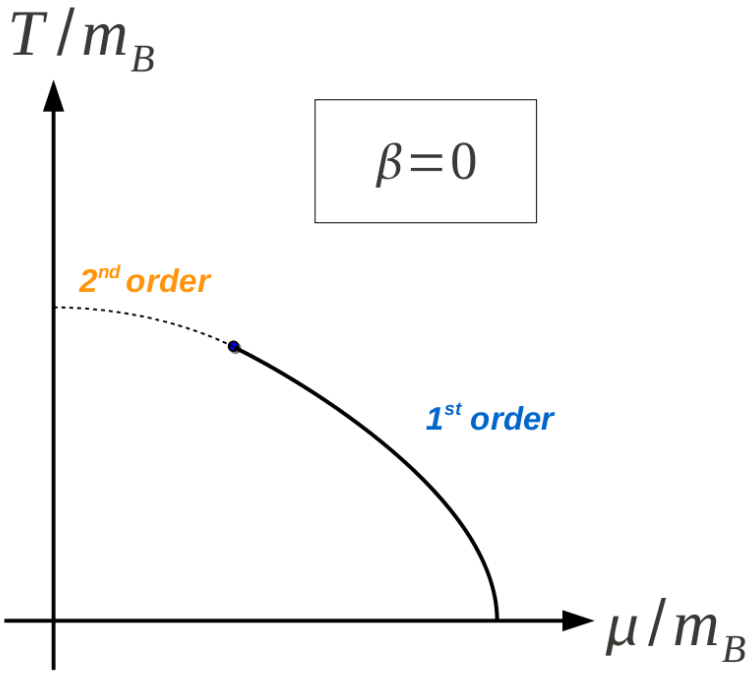
Continuous time: finite-T transition ($m_q = 0, \mu = 0$)



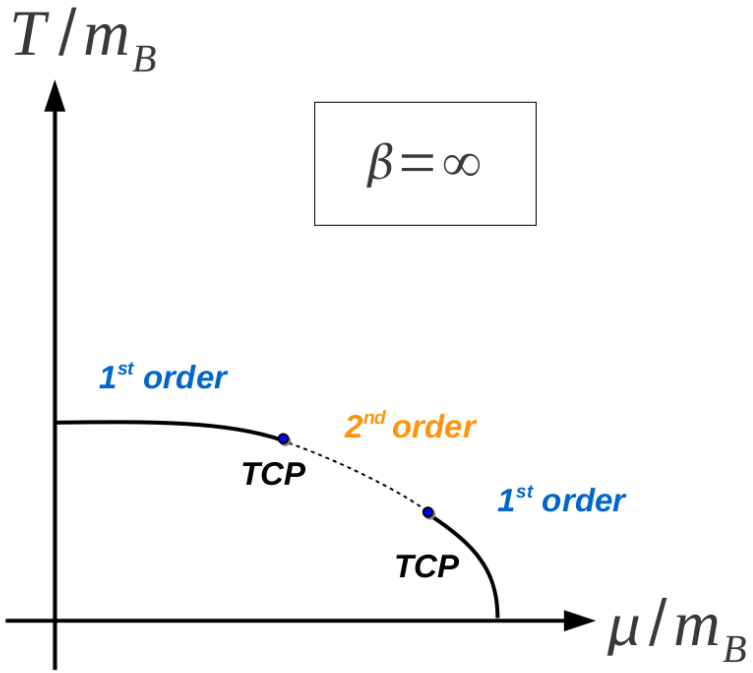
Avoid anisotropy γ $\rightarrow$ no error/ambiguity on T

Relating the strong-coupling and continuum phase diagrams

$N_f = 4$ **continuum flavors,** $m_q = 0$



Strong coupling (known)



Continuum (minimal)

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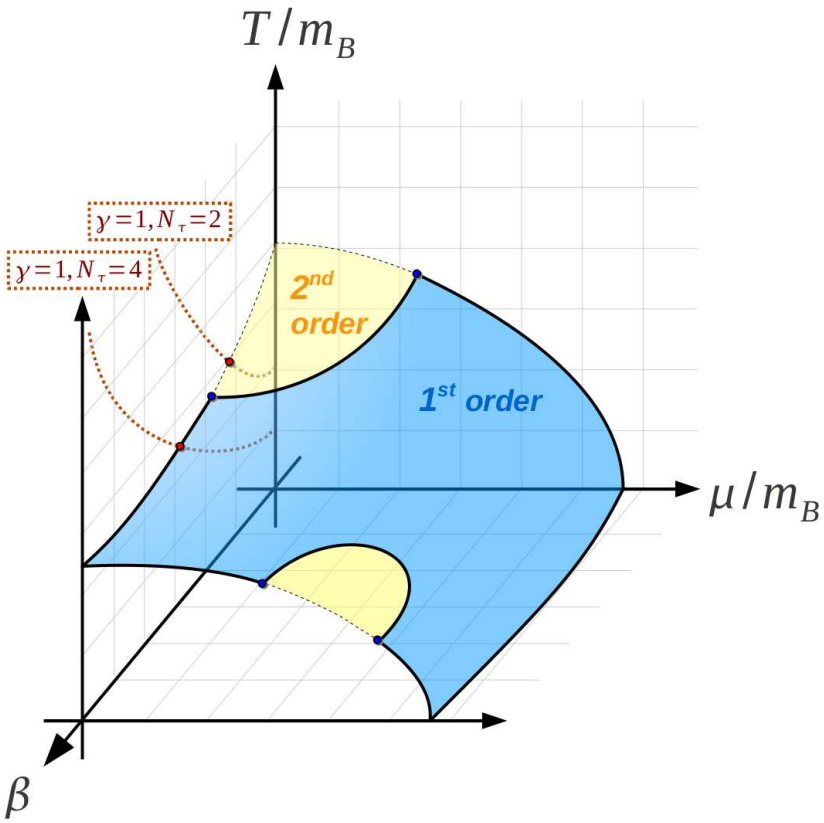
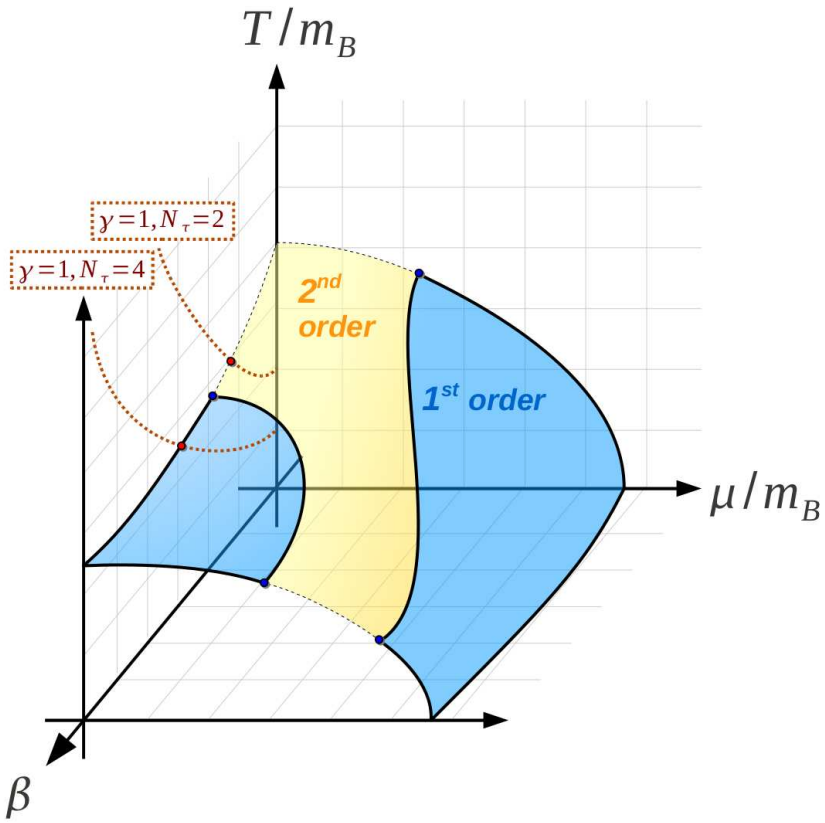
$N_f = 4$ **continuum flavors,** $m_q = 0$

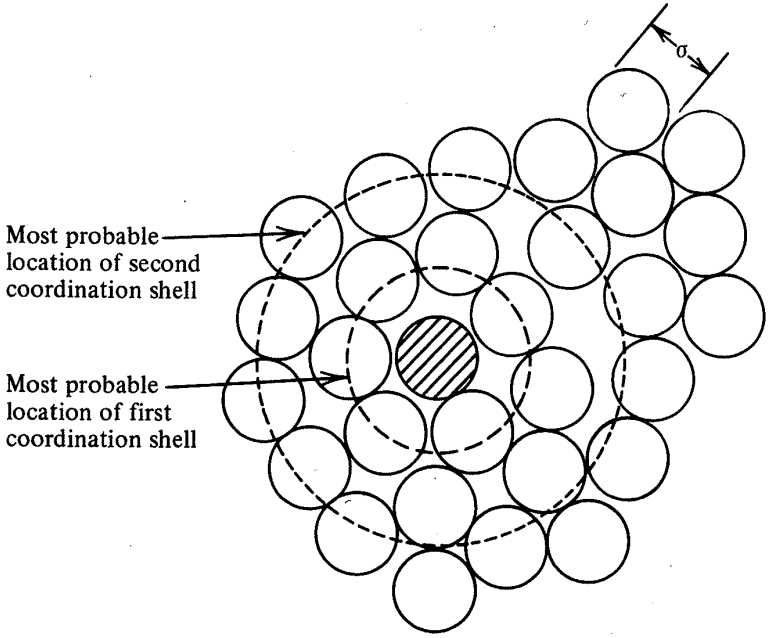


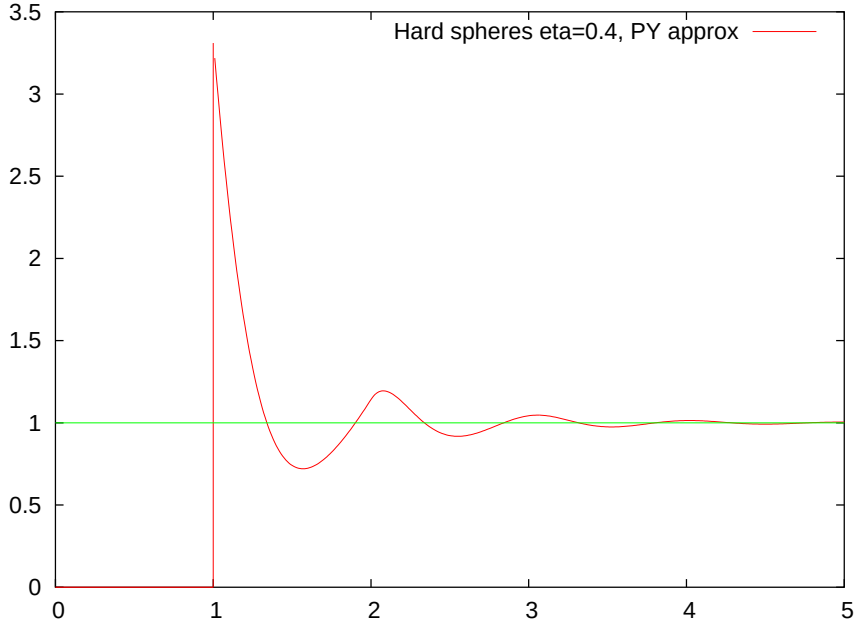


Can differentiate between the two cases

Case *right* favorable for quantitative continuum predictions

Backup: is pion bath essential? Classical hard spheres

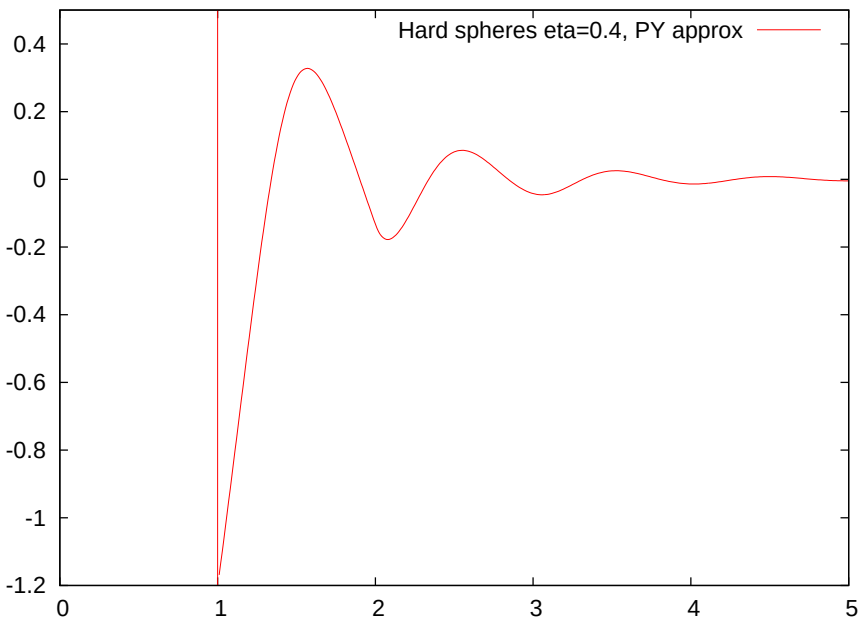




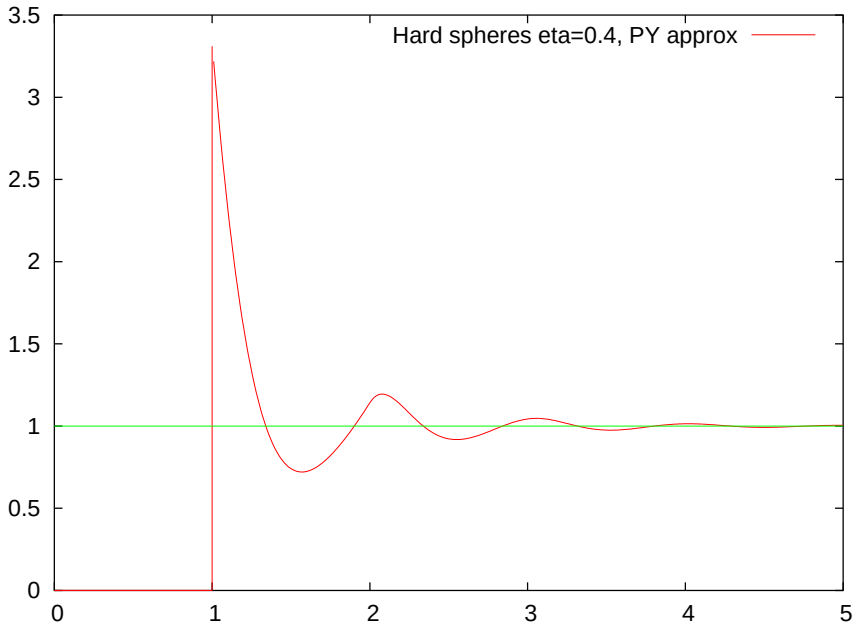
Density-density correlation
(Percus-Yevick approximation)

$g(r) \equiv \langle \rho(0)\rho(r) \rangle$ relaxes to $\langle \rho \rangle^2$ with damped oscillations $\rightarrow$ liquid

Backup: is pion bath essential? Classical hard spheres



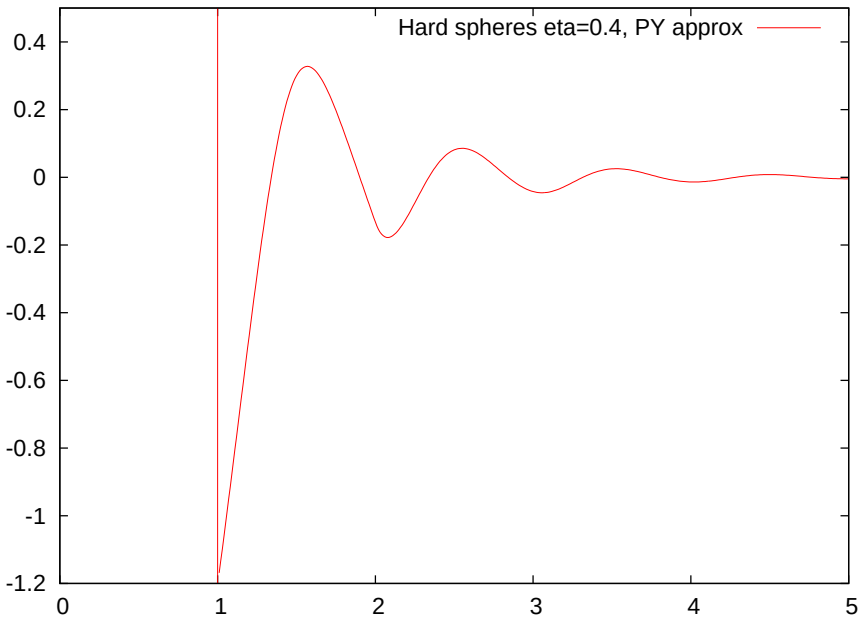
"Potential of mean force"



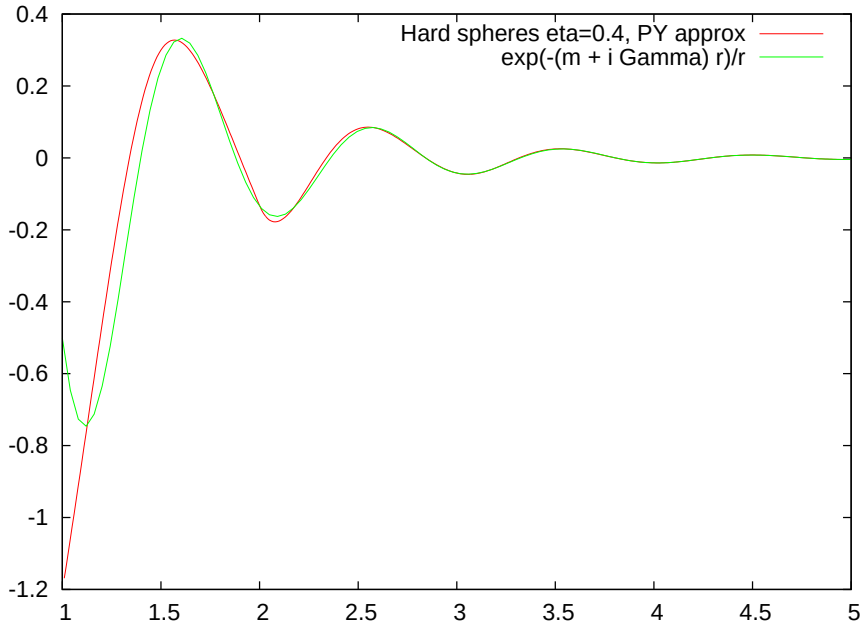
Density-density correlation

"Potential of mean force" $V_{\rm eff}(r) \equiv -\log(g(r))$ is hard-core + damped oscillatory

Backup: is pion bath essential? Classical hard spheres



"Potential of mean force"

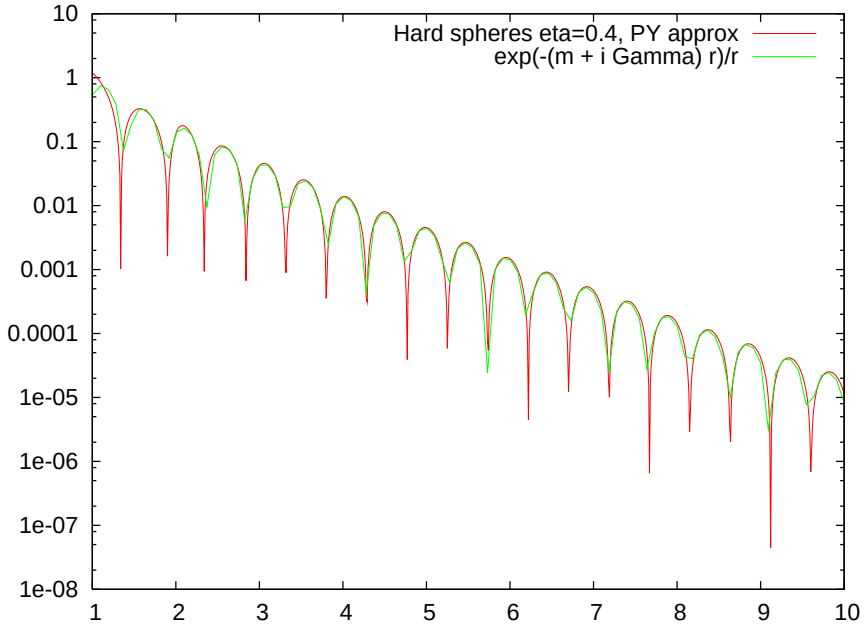


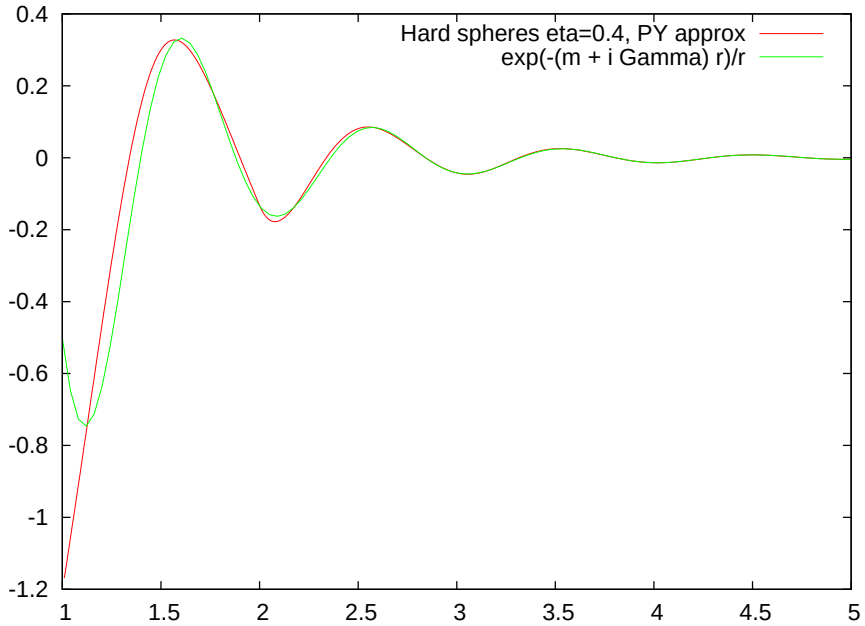
Compare with Yukawa potential

"Potential of mean force" $V_{\rm eff}(r) \equiv -\log(g(r))$ is hard-core + damped oscillatory

Consistent with Yukawa form $\frac{\exp(-mr)}{r} \times \cos(\Gamma r)$

Backup: is pion bath essential? Classical hard spheres





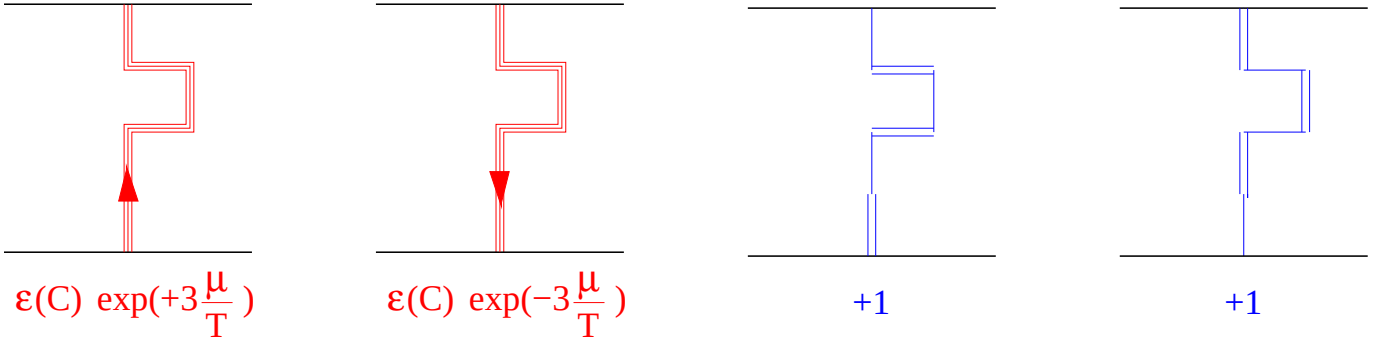
$\log(|V_{\rm eff}(r)|)$ + Yukawa fit

Perfect fit at large distance

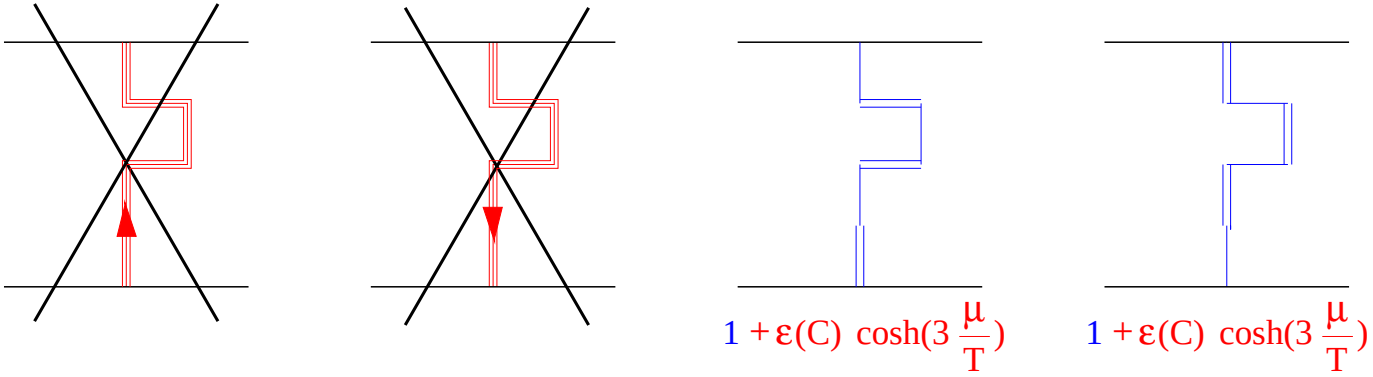
Hard-sphere "potential of mean force" is of Yukawa form

$$V_{\rm eff}(r) = {\rm Re}\left[\frac{e^{-(m+i\Gamma)r}}{r}\right]$$

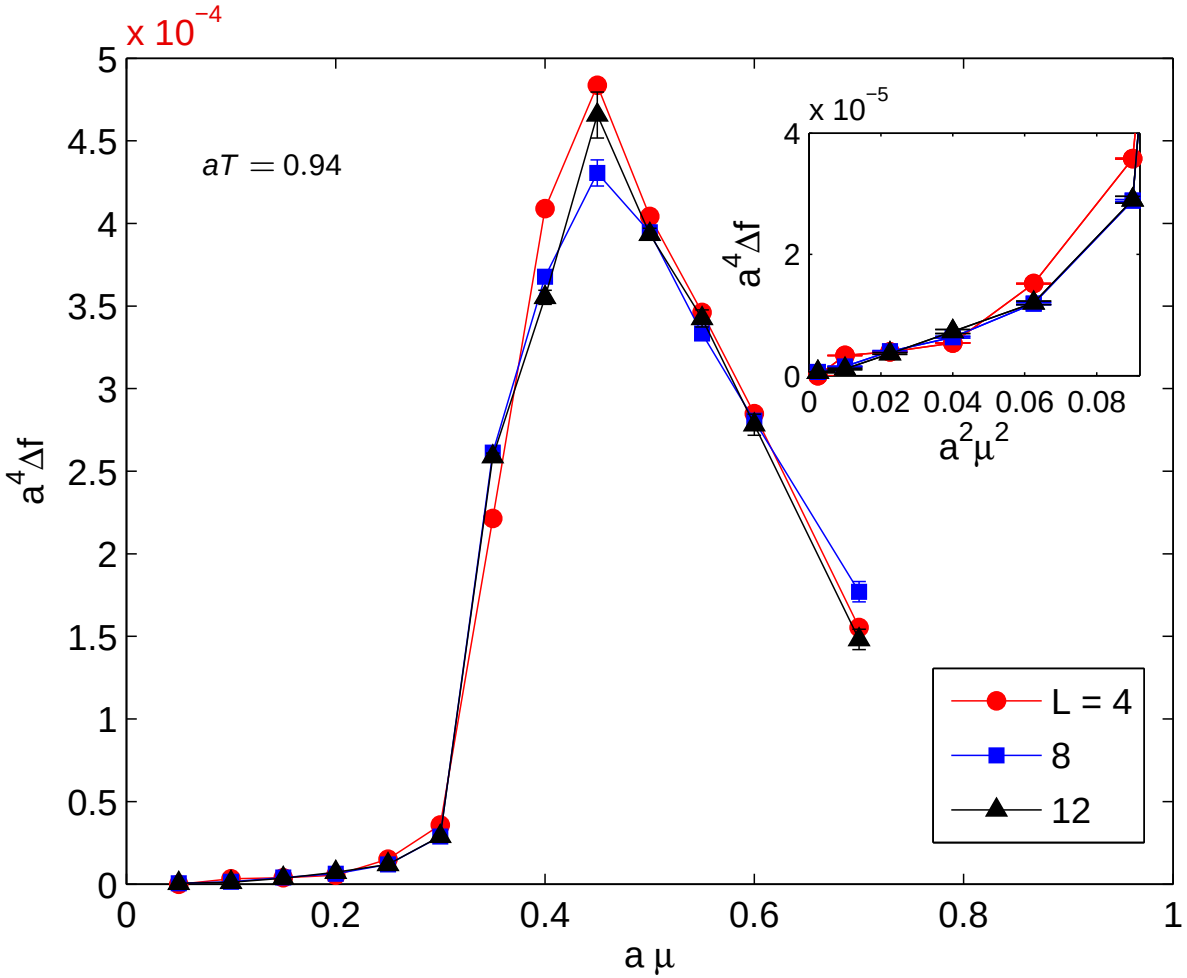
Backup: Karsch & Mütter's resummation



Karsch & Mütter: Resum into "MDP ensemble" $\rightarrow$ sign pb. eliminated at $\mu = 0$



Backup: Sign problem? Monitor $-\frac{1}{V}\log\langle\text{sign}\rangle$



- $\langle\text{sign}\rangle = \frac{Z}{Z_{||}} \sim \exp(-\frac{V}{T}\Delta f(\mu^2))$ as expected; $\Delta f \sim \mu^2 + O(\mu^4)$
- Can reach $\sim$ **$16^3 \times 4$** $\forall\mu$, ie. adequate