Exploring center symmetry with electrically charged quarks

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THE GIST

- Could the *fractional electric charge* of quarks relevant be to the phase diagram? (esp. *deconfinement*)

- We think *‘yes, there might be a way’*

This talk – finite temperature only
OUTLINE

- Lattice based – lean heavily on analogy of Polyakov loops as spins
- Standard picture of (de)confinement
- Inclusion of electromagnetism
- Results from our 2-color model
(De)confinement for Pure Glue

- Center symmetry
  
  ... glue is blind to phases \( z = e^{in2\pi/3} \in \mathbb{Z}_3 \)

- Gauge group \( SU(3)/\mathbb{Z}_3 \)

- Polyakov loop - order parameter for \( Z_3 \) breaking transition
  
  \[ \langle P \rangle = 0 \]
  
  Disordered

  \[ \langle P \rangle \neq 0 \]
  
  Ordered

Center symmetry

\[ z = e^{in2\pi/3} \in \mathbb{Z}_3 \]
(De)confinement for Pure Glue

- Center symmetry

... glue is blind to phases $z = e^{in2\pi/3} \in \mathbb{Z}_3$

c.f. spontaneous magnetization of a spin system
**ADD DYNAMICAL QUARKS (WILSON)**

- explicitly break center symmetry
- Hopping expansion of determinant

\[
\det M = \exp\left( - \sum_j \frac{\kappa_j}{j} \text{Tr} H^j \right), \quad \kappa = \frac{1}{2am + 8}
\]

(1 - \gamma_\mu) U_\mu

Finite temperature

closed loops
- Plaquette-like terms don’t affect symmetry

- Polyakov loop terms pick center sector

\[
\propto -\kappa^{N_t} \sum_{\vec{x}} \text{Re} \ Tr P(\vec{x})
\]

finite temperature

\[
P = 1
\]
- plaquette-like terms don’t affect symmetry

- Polyakov loop terms pick center sector

\[ \alpha - \kappa^{N_t} \sum_{\vec{x}} \Re \text{Tr} \mathcal{P}(\vec{x}) \]

favors

\[ P = 1 \]

Like coupling spins to an ext. magnetic field

Effect of fermions - ordering external field
\[ P_{av} = \frac{1}{V} \sum_{\vec{x}} P \]

**Dynamical Fermions – SU(2)**

\[ 24^3 \times 4, \ \kappa = 0.15, \ 2 \text{ flavors} \]
...BUT QUARKS HAVE ELECTRIC CHARGE

- What if we include electromagnetism?

\[ q_u = +\frac{2}{3}e, \quad q_d = -\frac{1}{3}e \]

- Exactly compensate color center phase by U(1) phase

\[ (e^{i2\pi/3}, e^{i2\pi Q/e}), \quad (e^{-i2\pi/3}, e^{-i2\pi Q/e}) \]

- Symmetry

\[ SU(3) \times U(1)_{em}/\mathbb{Z}_3 \]
**Hidden Symmetry**

- ‘True’ Standard Model symmetry group
  
  \[ SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6 \]
  
- Importance
  - unification, e.g. SU(5), SO(10) GUT
  - topological objects - color-EM monopoles/vortices

A **global center symmetry** with fermions!

- What can it do for us?

  electroweak trans. – Zubhov, Veselov, Bakker
**Toy Model for Simplicity**

- 2 colors, 2 flavors of dynamical Wilson fermions, gauge group

\[ SU(2) \times U(1)_{em}/\mathbb{Z}_2 \]

- u/d quarks with ± ½ charge relative to U(1)\(_{em}\) gauge action

\[
S = - \sum_{\Box} \left( \frac{\beta_{col}}{2} \text{Re} \, \text{Tr} \, \Box_{SU(2)} + \beta_{em} \cos \Box \theta \right) + S_{f,W}
\]

parallel transporters give both color and electromagnetic contribution to quarks – e.g. \(-1 \times -1 = 1\)
COLOR DISORDER THROUGH $U(1)$ DISORDER

- Quark loops get additional $U(1)$ factor
e.g. Polyakov loop terms from Hopping expansion

$$\propto - \sum_x \text{Re} \text{ Tr} \uparrow \cdot \text{Re} \uparrow \text{color} \cdot \text{EM} \exp i \frac{\theta}{2}$$

- if $U(1)$ disordered - c.f. spin model in a random external field

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} s_i s_j - h \sum_i h_i s_i$$

ref. Spin glasses and random fields, 1997
- Compact QED has a confining, disordered phase for $\beta_{\text{em}} < 1$

- Expect a large effect here

\[ \propto -\text{Re} \text{ Tr} \cdot \text{Re} \]

- c.f. Peccei-Quinn mechanism (strong CP)
RESTORATION OF PURE GAUGE BEHAVIOR FOR $P_{SU(2)}$

$SU(2) \times U(1)/Z_2$

$8^3 \times 4$, $\kappa = 0.15$
CRANK UP U(1) COUPLING BEYOND THE U(1) TRANSITION

\[ 8^3 \times 4 \, , \, \kappa = 0.15, \text{ random start} \]
...AND ON BIGGER LATTICES

16^3 \times 4
\kappa = 0.15
random start

...and 24^3 \times 4

Still U(1) disorder for the quarks deep in the Coulomb phase for unit charges...
Quarks have \textit{fractional} charge

See phases that integer charged particles and the gauge action do not

- \( \pi \) for our quarks - \( 2\pi \) in \( U(1) \) action

- \( \beta_{\text{em}} > 1 \), order w.r.t. \( U(1) \) action – links \( \sim 1 \) for int. particles

BUT there is still room for \( Z_2 \) disorder in the links as seen by quarks

\( e^{i\pi} = -1 \) \textit{vs} \( e^{i2\pi} = 1 \)

\( +1 \) link w.r.t. the gauge action could be a \(-1\) link for the quarks

\textit{Frozen in}
**NT=4 Hopping Expansion**

**Fermions through** $\kappa^4$ **terms**

**Poly-Poly**

$\propto \text{Re} \text{Tr} \uparrow \cdot \text{Re} \uparrow \quad \text{color} \quad \text{EM}$

**plaq-plaq**

$\propto \text{Re} \text{Tr} \square \text{Re} \square \theta/2$

- Conspire to order color, U(1) links w.r.t. each other
- Compete with pure gauge terms
- Suggests coupled spin models
  - XY - Ising

Play with relative strength

~Nightingale et al, 1995
So...

- A **center symmetry** recovered when U(1) is added to QCD with dynamical quarks

- **Disordering** effect of U(1)
  - How much can the quarks’ **fractional** electric charge influence **color** dynamics?

  a lot in our model!
TO THINK ABOUT

- Initial conditions – source of $Z_2$ disorder
- Quark mass dependence, lines of constant physics

Speculation for $SU(3) \times U(1) / Z_3$

? first order transition persists for lighter quarks?

? sharpen crossover if it doesn’t reach physical quark masses?

... and if you like vortices...
‘t Hooft’s twisted boundary conditions!

- in presence of dynamical fermions
- combined vortices carrying both color and EM flux

\[
\text{SU}(2) \times \frac{U(1)}{\mathbb{Z}_2}
\]

twist simultaneously
HOT VS COLD START - $P_{SU(2)}$

$16^3 \times 4, \kappa = 0.15$
HOT VS COLD START - $P_{SU(2)}$

All links start at 1 – for large $\beta_{em}$ it’s very tough for algorithm to reach -1 links!

$16^3 \times 4, \, \kappa = 0.15$
CHECKING U(1) POLYAKOV LOOPS — COLD START

\[ \langle |P_{aw}| \rangle \]

\( P_{\text{quark}}, \beta_{\text{col}} = 2.5 \)
Plaq-plaq competition

\[ \propto -\kappa^4 \text{Re} \text{ Tr } \square \text{ Re } \square_{\theta/2} \]

\[ \text{color} \quad \text{EM} \]

\[ \propto -\kappa^4 \text{Re} \text{ Tr } \square \text{ Re } \square_{\theta/2} \]

\[ \text{color} \quad \text{EM} \]