Coherent Center Clusters and their Percolation at the Deconfinement Transition

Julia Danzer[1]

S. Borsanyi[2], Z. Fodor[2], C. Gattringer[1]

[1]Institut of Physics, Karl-Franzens University Graz

[2]Department of Physics, University of Wuppertal

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Center Symmetry of Gluodynamics

- Local Polyakov loop:

$$L(\vec{x}) = \mathrm{Tr}\Big[\prod_{t=0}^{N_t-1} U_4(\vec{x},t)\Big]$$

- SU(3) gauge theory: Center elements $z \in \{\mathbb{1}, e^{i2\pi/3}, e^{-i2\pi/3}\}$
- Center transformation: Acts on temporal links at time slice $t = t_0$

$$U_4(\vec{x},t_0) \longrightarrow z\ U_4(\vec{x},t_0)$$

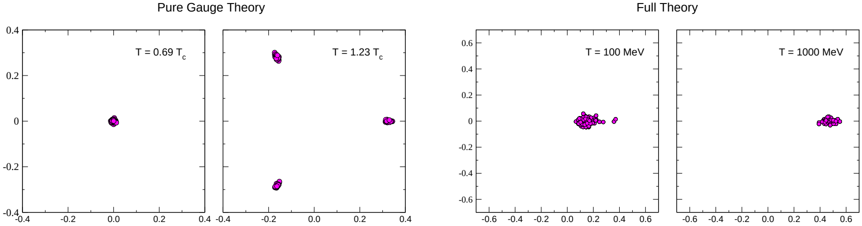
- Action and gauge measue are invariant
- Polyakov loop transforms non-trivially under a center transformation

$$L(\vec{x}) \longrightarrow z\ L(\vec{x})$$

- Non-vanishing $\langle L(\vec{x})\rangle$ signals spontaneous breaking of center symmetry

INCLUSION OF DYNAMICAL FERMIONS

- When quarks are included the fermion determinant acts as an additional weight factor
- Fermions play the role of an external magnetic field which break the center symmetry explicitly
- Although the Polyakov loop P is no true order parameter anymore it signals the crossover to deconfinement



EFFECTIVE SPIN SYSTEM FOR THE QCD TRANSITION

- At T_c the critical behavior of SU(N) gauge theory in $d+1$ dimensions can be described by a d - dimensional spin system with a $\mathbb{Z}_N$ - invariant effective action (Svetitsky-Yaffe Conjecture, 1981)
- The spins are related to the local loops $L(\vec{x})$
- In spin systems one can define clusters of parallel spins (e.g. Fortuin-Kasteleyn clusters in Ising systems)
- At T_c these clusters start to percolate
- Can we identify characteristic properties of such clusters directly in QCD?
- Here we focus on clusters with coherent phases of the Polyakov loop and their percolation properties near T_c.

Setting and Goal of our Analysis

- We study clusters and critical percolation directly in quenched and dynamical $SU(3)$ gauge theory
- For that purpose we analyze properties of the local loops $L(\vec{x})$
- Technicalities – Quenched case:
  - Lüscher-Weisz gauge action
  - Lattice sizes: $20^3 \times 6$ ... $40^3 \times 12$
  - Temperatures: $T \in [\,0.63\,T_c\,,\,1.32\,T_c\,]$
  - C. Gattringer, Phys.Lett.B690:179-182,2010
- Technicalities – Dynamical case:
  - Symanzik improved gauge and stout-link improved staggered action
  - $2+1$ flavors with physical quark masses
  - Lattice sizes: $18^3 \times 6, 24^3 \times 6, 36^3 \times 6, 24^3 \times 8, 32^3 \times 10$
  - Temperatures: $T \in [\,50$ MeV ... , 1000 MeV$\,]$
  - Y. Aoki et al., Phys. Lett. B643 (2006) 46, JHEP 0906:088 (2009)

The local Polyakov Loop $L(\vec{x})$

- For the analysis of local properties of $L(\vec{x})$ we define:

$$L(\vec{x}) = \rho(\vec{x}) e^{i\varphi(\vec{x})}$$

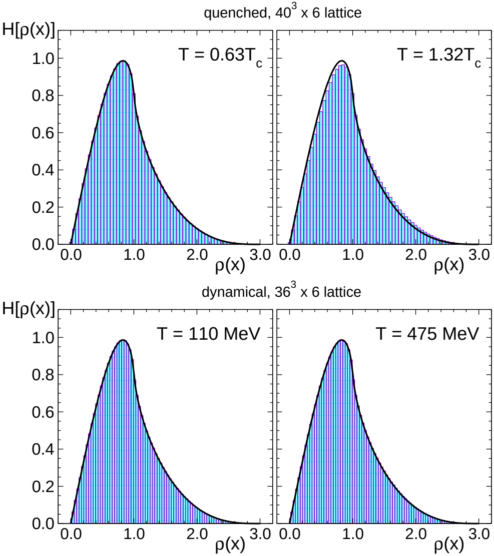
- We analyze properties of the modulus $\rho(\vec{x})$ and the phase $\varphi(\vec{x})$

The following plots are taken from:

C. Gattringer, Phys.Lett.B690:179-182,2010
C. Gattringer and A. Schmidt, JHEP 1101:051,2011
J. Danzer, C. Gattringer, S. Borsanyi, Z. Fodor; PoS LATTICE2010:176,2010
and work in preparation

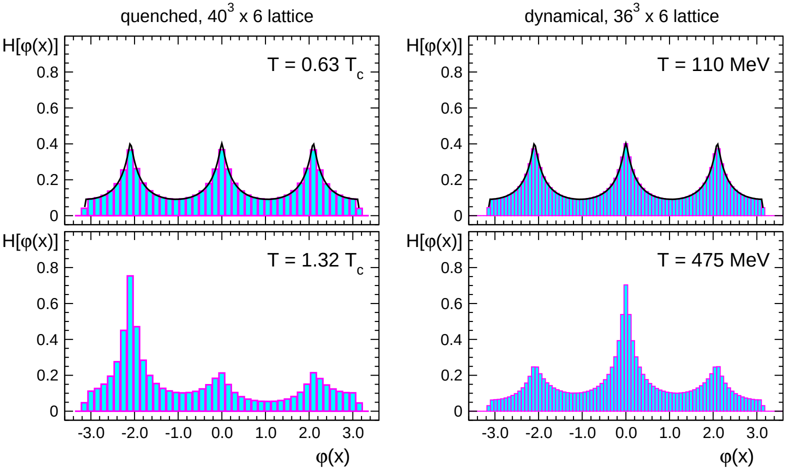
Modulus $\rho(\vec{x})$:

(full curve = Haar measure distribution)

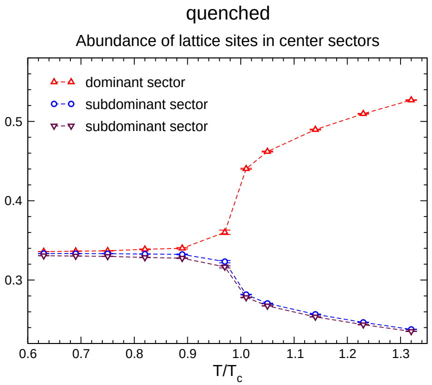


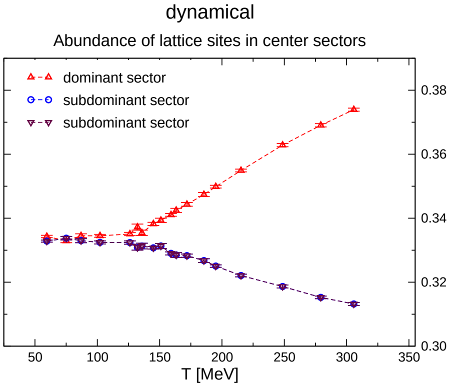
PHASE $\varphi(\vec{x})$:

(full curve = Haar measure distribution)



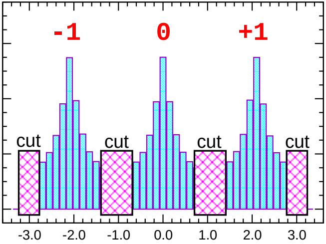
CENTER SECTORS ACROSS THE PHASE TRANSITION





CONSTRUCTION OF CLUSTERS

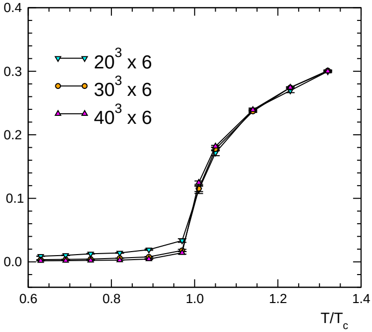
- We assign the sector numbers $-1, 0, 1$ according to the phase of the Polyakov loop
- We also introduce a cut for sites far from center elements
- A similar cut is necessary for percolating clusters in spin systems, e.g. Bonds for Fortuin-Kasteleyn clusters with $p_B = 1 - e^{-2\beta}$
- Neighboring sites with same sector number are put in the same cluster
- In 3 dimensions the critical site percolation probability $p_c = 0.3116$
- Without cut we thus always find percolating clusters below T_c



LARGEST CLUSTER

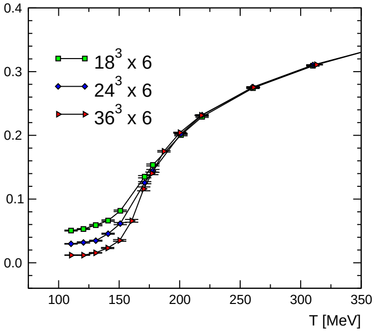
quenched

Size of largest cluster normalized with the volume

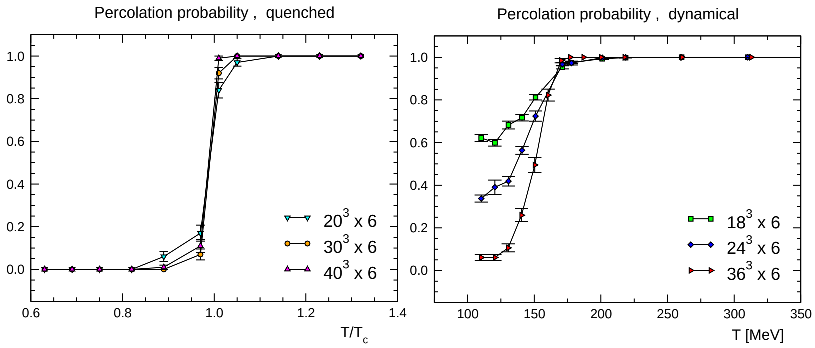


dynamical

Size of largest cluster normalized with the volume



Percolation Probability



Physical Diameter of the Clusters

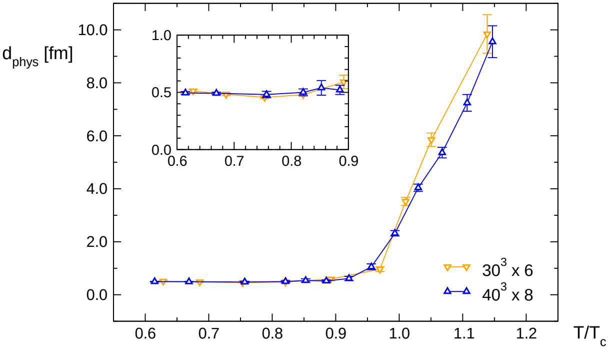
- We study 2-point correlation functions $C(r)$ within a cluster of a fixed center sector
- Clusters decay exponentially with distance r:

$$C(r) \propto e^{\frac{-r}{\rho}}$$

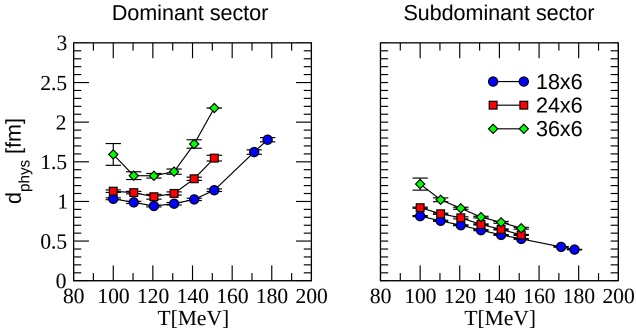
- Physical diameter is given by:

$$d_{phys} = 2 \cdot \rho \cdot a$$

Physical Diameter - Quenched



PHYSICAL DIAMETER - DYNAMICAL



$$d_{phys} = C \cdot L = C \cdot N_s \cdot a = C \frac{N_s}{N_t} \frac{1}{T}$$

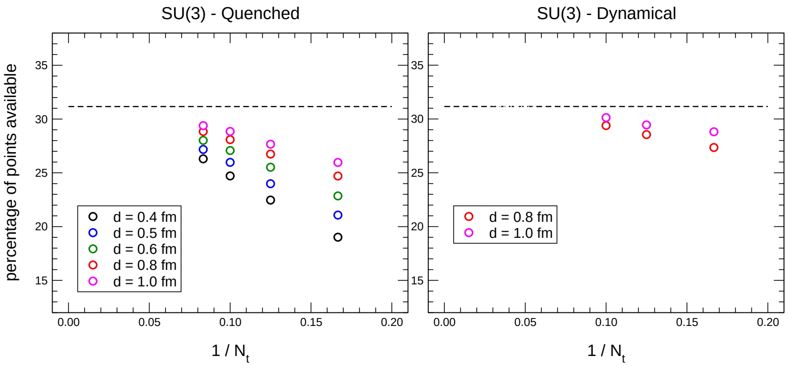
Continuum Limit of Center Clusters

- We determine the physical diameter $d_{phys} = a \cdot d_{lat}$ of the cluster
- The diameter clearly depends on the value of the cut parameter
- For a given value of d (e.g. 1.0 fm, 0.8 fm) we compare the cut parameters on lattices with different lattice spacing a
- The number of lattice points which are not cut and thus are available for the clusters is

$$f = \frac{100\% - \text{cut}\%}{N_c}$$

- f scales linearly to a value very close to the critical percolation density $p_c = 0.3116$

CONTINUUM LIMIT



$$d_{phys} = a \cdot d_{lat}$$

- Same behaviour for SU(2)!

SUMMARY AND OUTLOOK

- We analyze the behavior of local Polyakov loops $L(\vec{x})$
- We find that below T_c they are distributed according to Haar measure
- The phases always have preferred values near the center angles $0, \pm i2\pi/3$
- The phases form spatially localized clusters
- For pure gauge theory the deconfinement transition can be characterized by the onset of percolation of suitably defined clusters
- For the full theory, due to the crossover, the transition is much smoother
- We studied the physical diameter of the clusters
- The clusters can be shown to have a continuum limit