Electromagnetic superconductivity **of vacuum** induced by strong magnetic field

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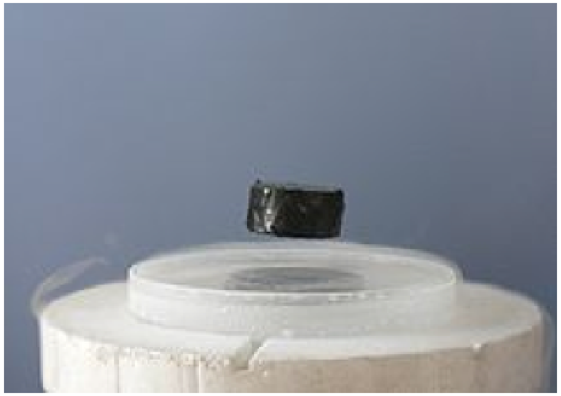
Based on:

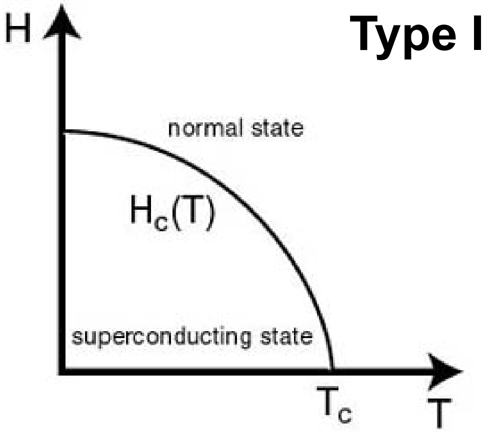
Phys. Rev. D82, 085011 (2010) [arXiv:1008.1055]

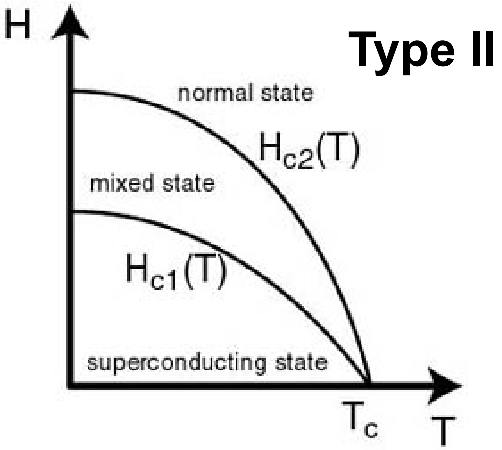
arXiv:1101.0117 → Phys. Rev. Lett. (2011)

Superconductivity

Discovered by Kamerlingh Onnes at the Leiden University
100 years ago, at 4:00 p.m. April 8, 1911 (Saturday).







I. Any superconductor has zero electrical DC resistance

II. Any superconductor is an enemy of the magnetic field:

1) weak magnetic fields are expelled by all superconductors (the Meissner effect)

2) strong enough magnetic field always kills superconductivity

Our claim:

In a background of strong enough magnetic field vacuum becomes an electromagnetic superconductor.

The superconductivity emerges in empty space.
Literally, "nothing becomes a superconductor".

Some features of the superconducting state of vacuum:
1. spontaneously emerges above critical magnetic field
$$B_c \simeq 10^{16}\ \text{Tesla} = 10^{20}\ \text{Gauss} \simeq m_\rho^2 \simeq 31\, m_\pi^2 \simeq 0.6\ \text{GeV}^2$$
2. conventional Meissner effect does not exists

The claim contradicts textbooks which state that:
1. Superconductor is a material (form of matter)
2. Weak magnetic fields are suppressed by superconductivity
3. Strong magnetic fields destroy superconductivity

1+4 approaches to the problem:

0. Handwaving arguments; (this talk)

1. Effective bosonic model for electrodynamics of ρ mesons
   based on vector meson dominance [M.Ch., arXiv:1008.1055]; (this talk)

2. Nambu-Jona-Lasinio model [M.Ch., arXiv:1101.0117]; ~~(this talk)~~

3. AdS/CFT [Callebaut, Dudal, Verschelde, arXiv:1102.3103];
   [Erdmenger, Kerner, Strydom, PoS(FacesQCD)004];

4. From first principles? [M.Ch., in preparation, not guaranteed]

Key players: ρ mesons and vacuum

- ρ mesons:

- electrically charged ($q = \pm e$) and neutral ($q=0$) particles
- spin: $s=1$, vector particles
- quark contents: $\rho^+ = u\bar{d}$, $\rho^- = d\bar{u}$, $\rho^0 = (u\bar{u} - d\bar{d})/2^{1/2}$
- mass: $m_\rho = 775.5$ MeV
- lifetime: $\tau_\rho = 1.35$ fm/c (very short)
- main known decay modes of charged mesons are

$$\rho^{\pm} \to \pi^{\pm} X\,, \qquad X = \pi^0,\ \eta,\ \gamma,\ \pi\pi\pi$$

- QCD+QED vacuum: zero tempertature and density

Charged relativistic particles in magnetic field

- Energy of a relativistic particle in the external magnetic field B_{ext}:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

momentum along the magnetic field axis

nonnegative integer number

projection of spin on the magnetic field axis

(the external magnetic field is directed along the *z*-axis)

- Masses of ρ mesons and pions in the external magnetic field

$$m^2_{\pi^\pm}(B_{\text{ext}}) = m^2_{\pi^\pm} + eB_{\text{ext}}$$ becomes heavier

$$m^2_{\rho^\pm}(B_{\text{ext}}) = m^2_{\rho^\pm} - eB_{\text{ext}}$$ becomes lighter

- Masses of ρ mesons and pions:

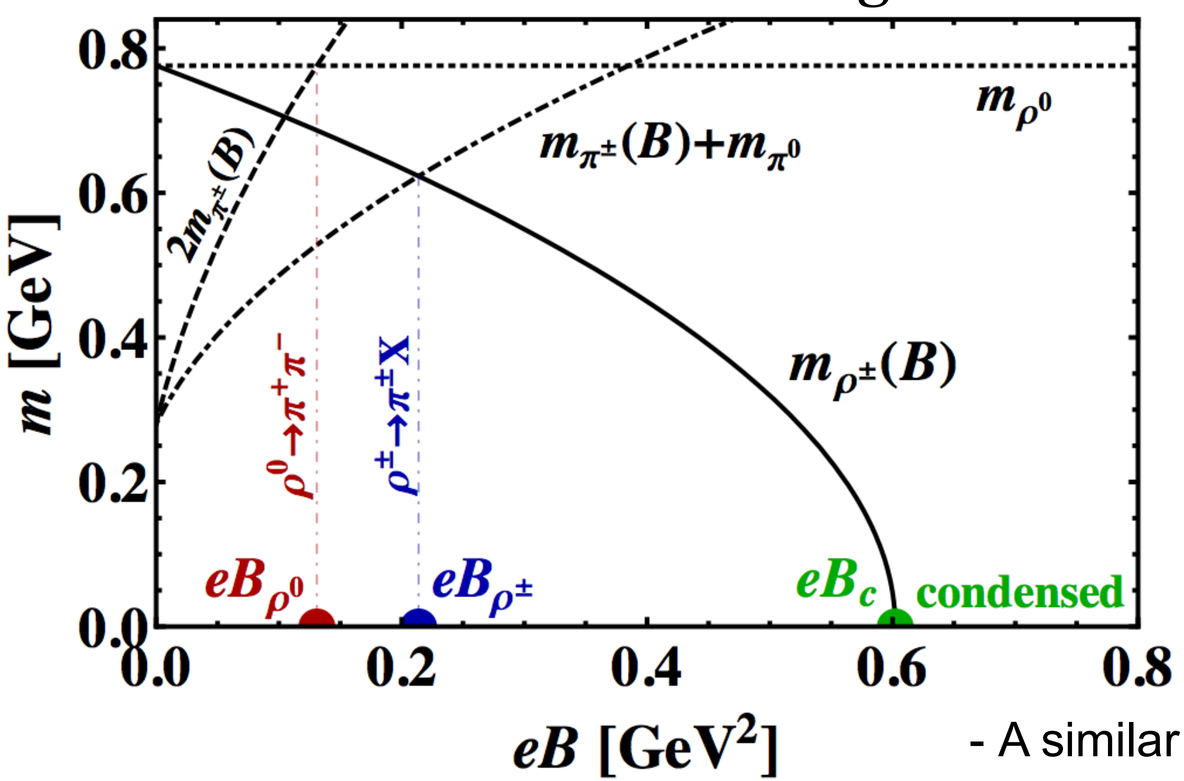
$$m_\pi = 139.6\,\text{MeV}\,, \qquad m_\rho = 775.5\,\text{MeV}$$

Condensation of ρ mesons

The $\rho^{\pm}$ mesons become massless and condense at the critical value of the external magnetic field

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\text{Tesla}$$

masses in the external magnetic field



Kinematical impossibility of dominant decay modes

The pion becomes heavier while the rho meson becomes lighter

- The decay $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ stops at certain value of the magnetic field

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

- A similar statement is true for $\rho^0 \to \pi^+\pi^-$

Condensed state of ρ mesons in the external magnetic field

The condensation of the rho mesons is due to large gyromagnetic ratio of these particles, $g=2$. In external magnetic field the vacuum becomes unstable.

This effect is similar to
1) unstable Nielsen-Olesen mode in the gluonic vacuum
2) condensation of W bosons in the Standard Electroweak model in strong magnetic field (Ambjorn-Olesen effect)

Our strategy:
1. Take simplest model of the rho mesons
2. Solve classical equations of motion in the magnetic field
3. Find instability, describe the rho meson condensates
4. <u>Show that the vacuum becomes a superconductor</u>

Electrodynamics of ρ mesons

- Lagrangian (based on vector dominance models):

$$\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\, \rho^{\dagger}_{\mu\nu}\rho^{\mu\nu} + m_\rho^2\, \rho^{\dagger}_{\mu}\rho^{\mu}$$
$$-\frac{1}{4}\, \rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{m_\rho^2}{2}\, \rho^{(0)}_{\mu}\rho^{(0)\mu} + \frac{e}{2g_s}\, F^{\mu\nu}\rho^{(0)}_{\mu\nu}$$

Nonminimal coupling leads to *g=2*

- Tensor quantities

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$
$$f^{(0)}_{\mu\nu} = \partial_\mu \rho^{(0)}_\nu - \partial_\nu \rho^{(0)}_\mu\,,$$
$$\rho^{(0)}_{\mu\nu} = f^{(0)}_{\mu\nu} - i g_s(\rho^{\dagger}_\mu \rho_\nu - \rho_\mu \rho^{\dagger}_\nu)$$
$$\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu\,,$$

- Gauge invariance

$$U(1):\quad \begin{cases} \rho^{(0)}_\mu(x) & \rightarrow\ \rho^{(0)}_\mu(x)\,, \\ \rho_\mu(x) & \rightarrow\ e^{i\omega(x)}\rho_\mu(x)\,, \\ A_\mu(x) & \rightarrow\ A_\mu(x) + \partial_\mu\omega(x) \end{cases}$$

- Covariant derivative

$$D_\mu = \partial_\mu + i g_s \rho^{(0)}_\mu - ieA_\mu$$

- Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2} f_\pi} = 5.88$$
$$g_s \gg e \equiv \sqrt{4\pi\alpha_{\rm e.m.}} \approx 0.303$$

[D. Djukanovic, M. R. Schindler, J. Gegelia, S. Scherer, PRL (2005)]

Homogeneous approximation (I)

- Energy density:

$$\epsilon \equiv T_{00} = \frac{1}{2}F_{0i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}\left(\rho_{0i}^{(0)}\right)^2 + \frac{1}{4}\left(\rho_{ij}^{(0)}\right)^2$$
$$+\frac{m_\rho^2}{2}\left[\left(\rho_0^{(0)}\right)^2 + \left(\rho_i^{(0)}\right)^2\right] + \rho_{0i}^\dagger\rho_{0i} + \frac{1}{2}\rho_{ij}^\dagger\rho_{ij}$$
$$+m_\rho^2(\rho_0^\dagger\rho_0 + \rho_i^\dagger\rho_i) - \frac{e}{g_s}F_{0i}\rho_{0i}^{(0)} - \frac{e}{2g_s}F_{ij}\rho_{ij}^{(0)}$$

- Disregard kinetic terms (for a moment) and apply B_{ext}:

$$\epsilon_0^{(2)}(\rho_\mu) = ieB_{\text{ext}}(\rho_1^\dagger\rho_2 - \rho_2^\dagger\rho_1) + m_\rho^2\rho_\mu^\dagger\rho_\mu$$
$$= \sum_{a,b=1}^{2}\rho_a^\dagger\mathcal{M}_{ab}\rho_b + m_\rho^2(\rho_0^\dagger\rho_0 + \rho_3^\dagger\rho_3)$$

$$\mathcal{M} = \begin{pmatrix} m_\rho^2 & ieB_{\text{ext}} \\ -ieB_{\text{ext}} & m_\rho^2 \end{pmatrix}$$

mass matrix

$$\vec{B} = (0,0,B)$$

- Eigenvalues and eigenvectors of the mass matrix:

$$\mu_\pm^2 = m_\rho^2 \pm eB_{\text{ext}}\,, \qquad \rho_\pm = \frac{1}{\sqrt{2}}(\rho_1 \pm i\rho_2)$$

At the critical value of the magnetic field: imaginary mass (=condensation)!

Homogeneous approximation (II)

- The condensate of the rho mesons: $\rho_1 = -i\rho_2 = \rho$

- The energy of the condensed state:

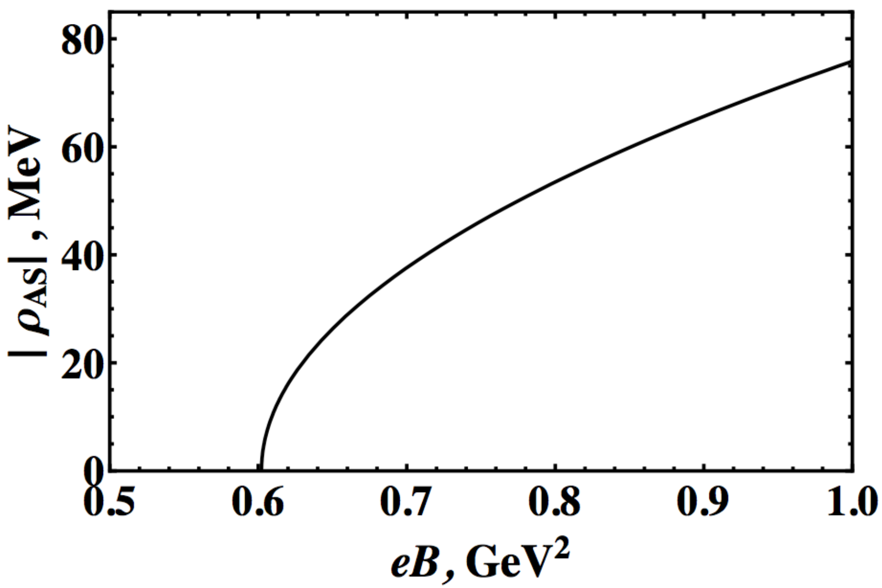
$$\epsilon_0(\rho) = \frac{1}{2}B_{\text{ext}}^2 + 2(m_\rho^2 - eB_{\text{ext}})\,|\rho|^2 + 2g_s^2\,|\rho|^4$$

(basically, a Ginzburg-Landau potential for an s-wave superconductivity!)

- The amplitude of the condensate:

$$|\rho|_0 = \begin{cases} \sqrt{\frac{e(B_{\text{ext}} - B_c)}{2g_s^2}}, & B_{\text{ext}} \geqslant B_c \\ 0, & B_{\text{ext}} < B_c \end{cases}$$

(qualitatively the same picture in NJL)



Second order (quantum) phase transition, critical exponent = 1/2

Structure of the condensates

In terms of quarks, the state $\rho_1 = -i\rho_2 = \rho$ implies

$$\langle \bar{u}\gamma_1 d\rangle = \rho(x_\perp)\,, \qquad \langle \bar{u}\gamma_2 d\rangle = i\rho(x_\perp)$$

Depend on transverse coordinates only

(the same results in NJL)

$$\vec{B} = (0, 0, B)$$

$U(1)_{\mathrm{e.m.}}:$ $\rho(x) \to e^{i\omega(x)}\rho(x)$ Abelian gauge symmetry

$O(2)_{\mathrm{rot}}:$ $\rho(x) \to e^{i\varphi}\rho(x)$ Rotations around B-axis

- The condensate "locks" rotations around field axis and gauge transformations:

$$U(1)_{\mathrm{e.m.}} \times O(2)_{\mathrm{rot}} \to U(1)_{\mathrm{locked}}$$

Basic features of ρ meson condensation, results
(now we are solving the full set of equations of motion)

- The condensate of the ρ mesons appears in a form of an inhomogeneous state, analogous to the Abrikosov lattice in the mixed state of type-II superconductors.

A similar state, the vortex state of W bosons, may appear in Electroweak model in the strong external magnetic field [Ambjorn, Olesen (1989)]

- The condensate forms a lattice, which is made of the new type of topological defects, the ρ vortices.

- The emergence of the condensate of the charged ρ mesons induces spontaneous condensation of the neutral ρ mesons.

- The condensate of charged ρ mesons implies <u>superconductivity</u>.

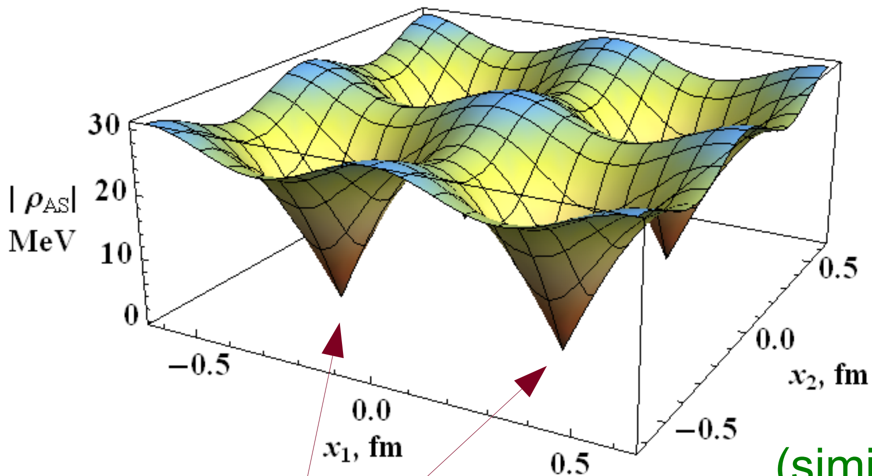
- The condensate of neutral ρ mesons implies <u>superfluidity</u>.

Solution for condensates of ρ mesons

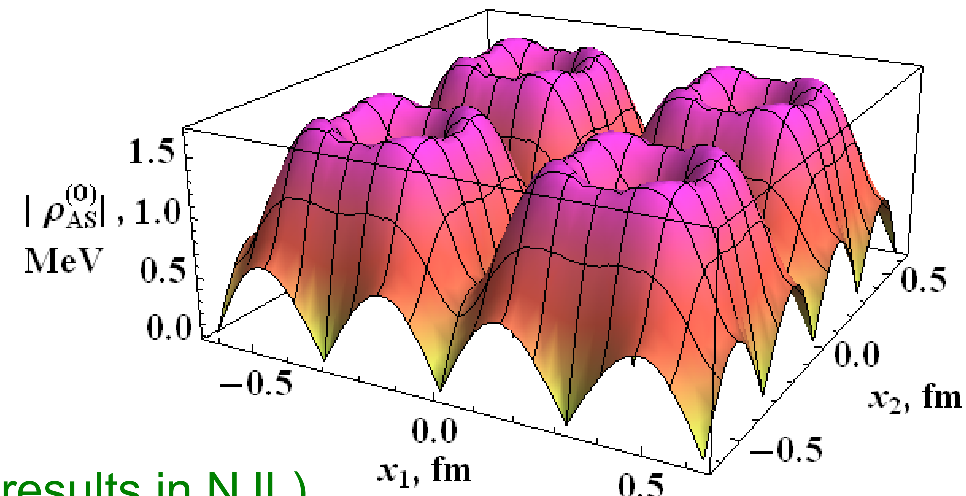
[an example in the external magnetic field eB=(800 MeV)^2]

Four elementary lattice cells of the vortex lattice are shown

Superconducting condensate
(charged rho mesons)



Superfluid condensate
(neutral rho mesons)

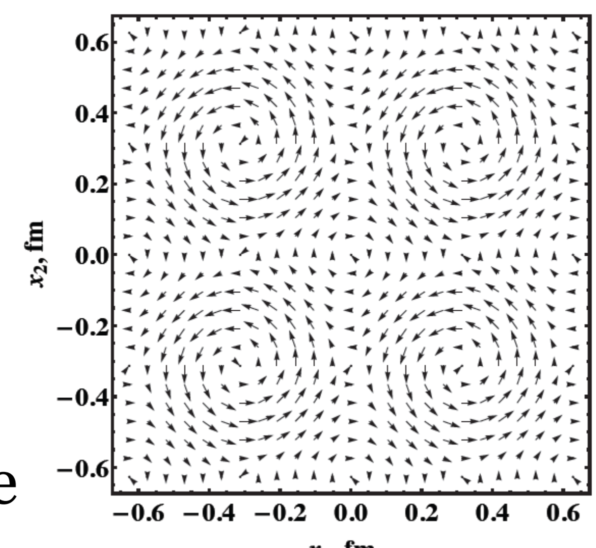


(similar results in NJL)

New objects, topological vortices,
made of the rho-condensates
(the phase of the rho-field winds around the rho-vortex center, the rho-condensate vanishes)

$$\boxed{B = 1.06\, B_c}$$

Local electric currents in the transverse plane



Anisotropic superconductivity
(an analogue of the London equations)

- Apply a weak electric field ***E*** to an ordinary superconductor (described, say, by the Ginzburg-Landau model).

- Then one gets accelerating electric current along the electric field:

$$\frac{\partial \vec{J}_{\mathrm{GL}}}{\partial t} = m_A^2 \vec{E} \qquad \text{[London equation]}$$

- In the QCD vacuum, we get an accelerating electric current iff the electric field ***E*** is directed along the magnetic field ***B***:

$$\frac{\partial}{\partial t}(\overline{J}_3)_{\mathcal{A}} = -\frac{2e^3}{g_s^2}(B_{\mathrm{ext}} - B_c)E_3 \qquad \frac{\partial}{\partial t}(\overline{J}_i)_{\mathcal{A}} = 0$$

$i = 1, 2$

Electric current averaged over one elementary rho-vortex cell

(similar results in NJL)

Anisotropic superconductivity
(Lorentz-covariant form of the London equations)

We are working in the vacuum, thus the transport equations may be rewritten in a Lorentz-covariant form:

Electric current averaged over one elementary rho-vortex cell

$$\partial_{[\mu,} j_{\nu]} = \kappa \frac{(F \cdot \widetilde{F})}{(F \cdot F)} \widetilde{F}_{\mu\nu}$$

A scalar function of Lorentz invariants.
In this particular model:

$\kappa = (e^3/g_s^2)(\sqrt{(F \cdot F)/2} - B_c)$

(slightly different form of κ function in NJL)

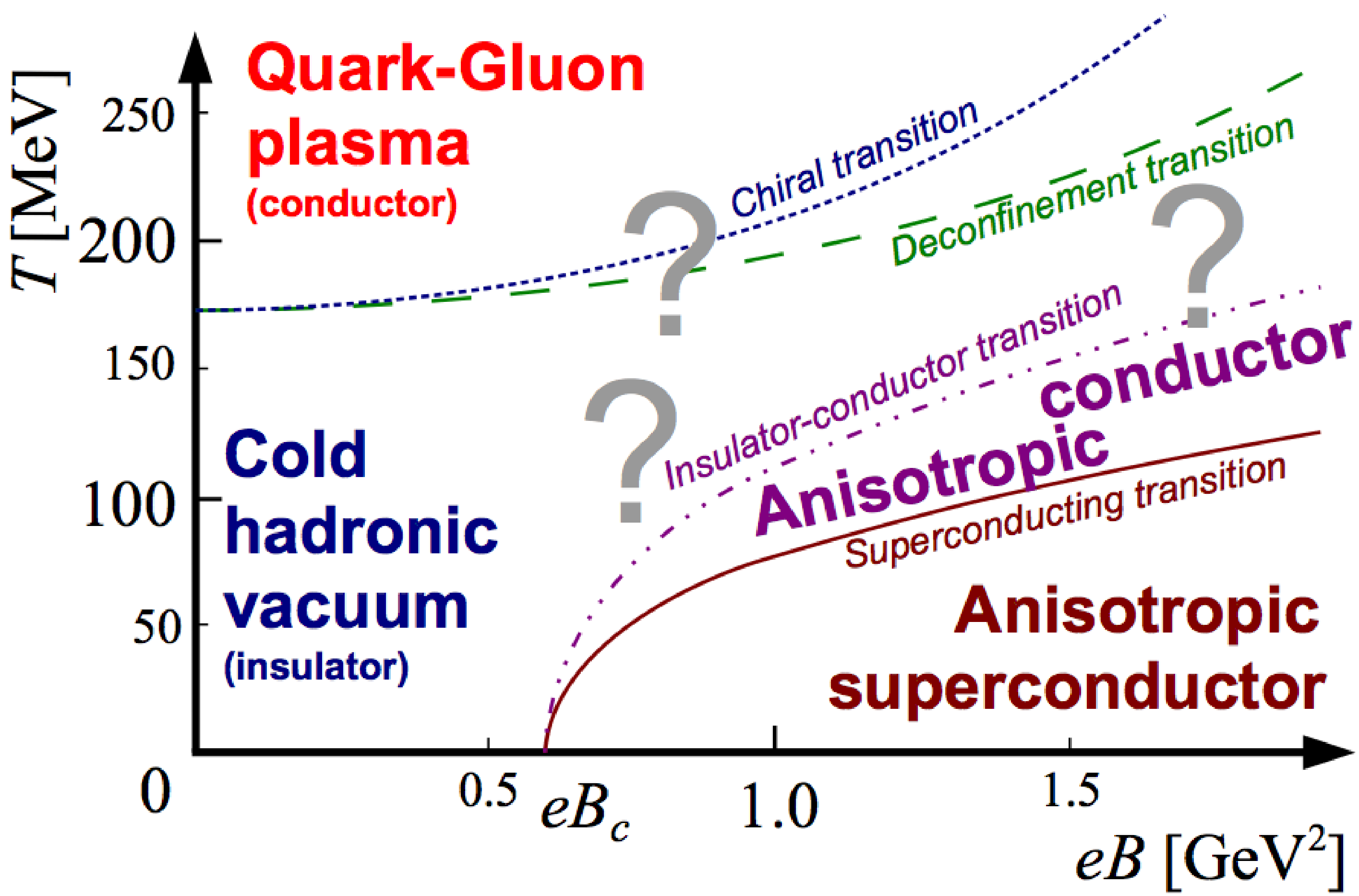
Lorentz invariants:

$$(F \cdot \widetilde{F}) = F^{\mu\nu}\widetilde{F}_{\mu\nu} \equiv 4(\vec{B} \cdot \vec{E})$$

$$(F \cdot F) = F^{\mu\nu}F_{\mu\nu} \equiv 2(\vec{B}^2 - \vec{E}^2)$$

$$\widetilde{F}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

Possible phase diagram

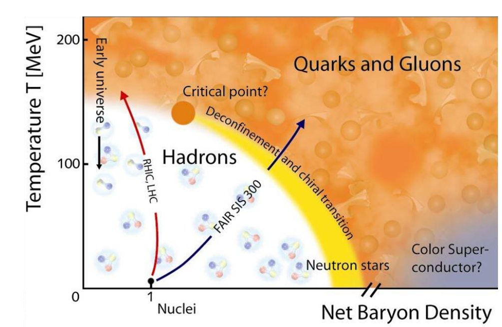


Fukushima, Ruggieri, Gatto, PRD81, 114031 (2010);
Ruggieri, Gatto, PRD82, 054027 (2010); PRD83, 034016 (2011);
D'Elia, Mukherjee, Sanfilippo, PRD82, 051501 (2010);
Mizher, Fraga, M.Ch., PRD82, 105016 (2010).

Buividovich, Kharzeev, Kalaydzhyan, Luschevskaya, Polikarpov, M.Ch., PRL 105, 132001 (2010)
+ talk by Polikarpov 24 minutes ago

M.Ch., PRD82, 085011 (2010); arXiv:1101.0117 → PRL (2011).

ADVERTISEMENT: no sign problem!
And many interesting question marks!



Too strong magnetic field?

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\mathrm{Tesla}$$

Very strong magnetic fields (with a typical strength of the QCD scale) may be generated in heavy-ion collisions and in Early Universe (duration is short, however).

A bit of dreams (in verification stage):

Signatures of the superconducting state of the vacuum could possibly be found in ultra-periferal heavy-ion collisions at LHC.

[ultra-periferal: cold vacuum is exposed to strong magnetic field]

Conclusions

- In a sufficiently strong magnetic field condensates with $\rho^{\pm}$ meson quantum numbers are formed spontaneously via a second order phase transition with the critical exponent 1/2.

- The vacuum (= no matter present, = empty space, = nothing) becomes electromagnetically superconducting.

- The superfluidity of the neutral ρ^0 mesons emerges as well.

- The superconductivity is anisotropic: the vacuum behaves as a superconductor only along the axis of the magnetic field.

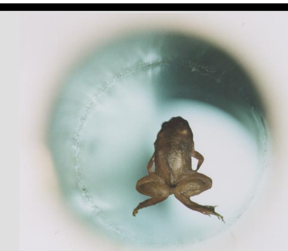
- New type of tological defects,"ρ vortices", emerge.

- The ρ vortices form Abrikosov-type lattice in transverse directions.

- The Meissner effect is absent.

What is «very strong» field? Typical values:

- Thinking — human brain: 10^{-12} Tesla
- Earth's magnetic field: 10^{-5} Tesla
- Refrigerator magnet: 10^{-3} Tesla
- Loudspeaker magnet: 1 Tesla
- Levitating frogs: 10 Tesla
- Strongest field in Lab: 10^{3} Tesla
- Typical neutron star: 10^{6} Tesla
- Magnetar: 10^{9} Tesla
- Heavy-ion collisions: 10^{15} Tesla (and higher)



IgNobel 2000 by
A.Geim (Nobel 2010)

Destructive explosion