Dual and dressed quantities in QCD

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Quarks, Gluons, and Hadronic Matter under Extreme Conditions

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Motivation

phase transition/crossover at finite temperature

- Deconfinement: Polyakov loop
  probes center symmetry for infinitely heavy quarks
  order parameter: 0 $\rightarrow$ finite / small $\rightarrow$ large

- Chiral restoration: Quark condensate $\rho(0)$
  probes chiral symmetry for massless quarks
  order parameter: finite $\rightarrow$ 0 / large $\rightarrow$ small

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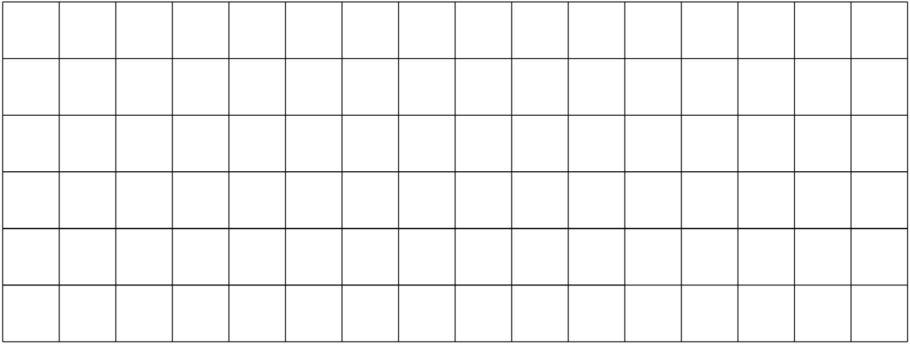
related? Dual quantities

- Dressed Polyakov loop from phase boundary conditions
  - idea and definition — Bilgici, FB, Gattringer, Hagen 08
  - lattice results: quenched
    and dynamical — Zhang, FB, Fodor, Gattringer, Szabo 1012.2314 + Borsanyi
- Dressed Wilson loops from magnetic fields — FB, Endrödi in prep.

Dual quantities: idea

lattice: gauge invariant quantities $\rightleftharpoons$ link products along closed loops



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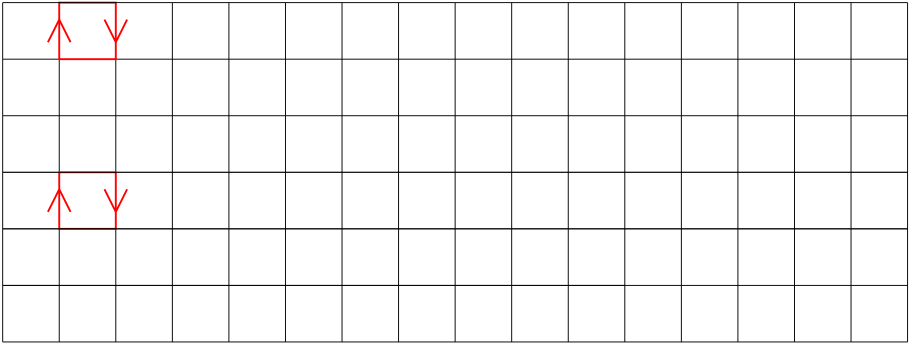

plaquettes

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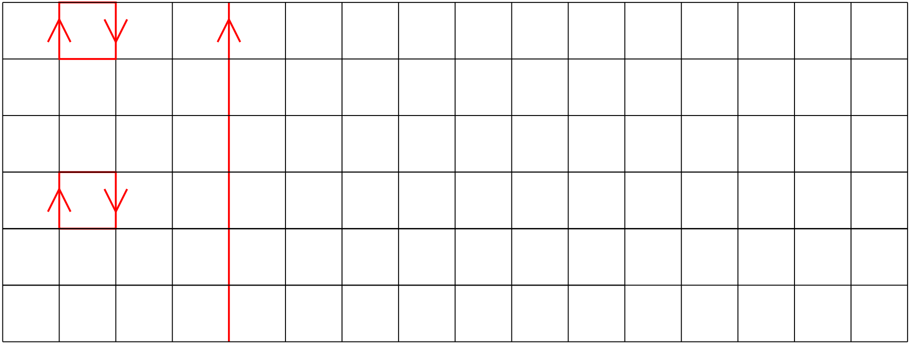

plaquettes   Polyakov loop

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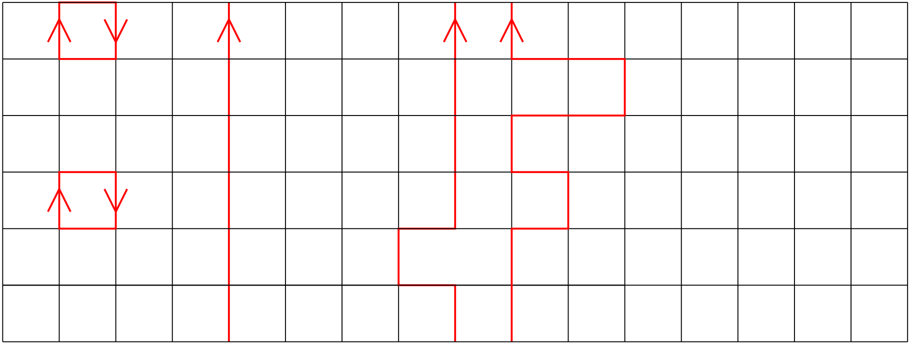

plaquettes Polyakov loop "Polyakov loops with detours"

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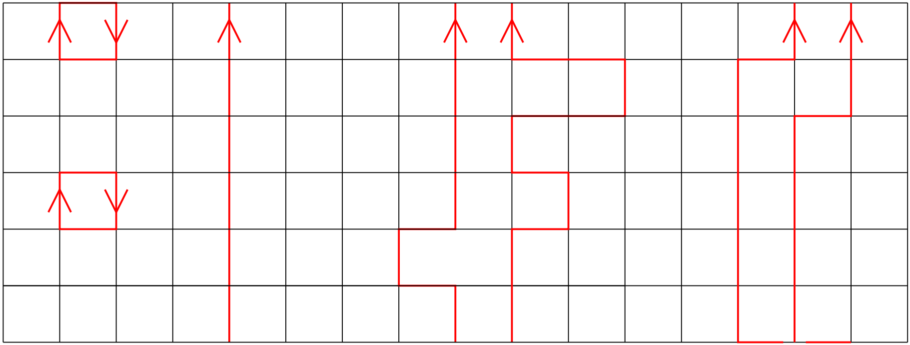

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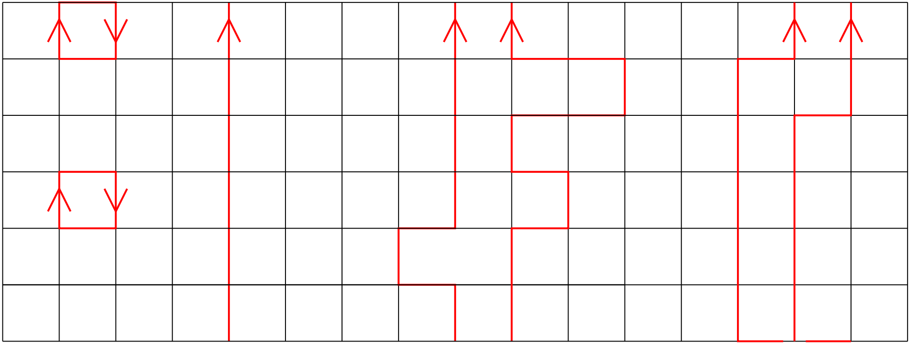

plaquettes | Polyakov loop | "Polyakov loops with detours" | loops winding twice

how to distinguish these classes of loops?

Gattringer 06

Dual quantities: idea

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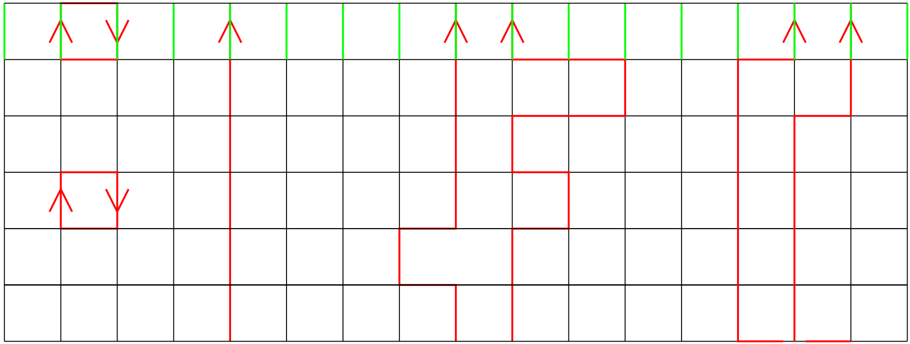

plaquettes Polyakov loop "Polyakov loops with detours" loops winding twice

how to distinguish these classes of loops? Gattringer 06

$\Rightarrow$ phase factor $e^{i\varphi}$ multiplying U_0 at fixed x_0-slice & Fourier component

Dual quantities: idea

phase factor $e^{i\varphi}$ multiplying U_0 at fixed x_0-slice

$\equiv$ quarks with general boundary conditions

$$\psi(x_0+\beta) = e^{i\varphi}\psi(x_0)$$

- physical quarks are antiperiodic: $\varphi = \pi$
- boundary angle φ: tool on the level of observables (valence quarks)
- formally an imaginary chemical potential

Dual condensate: definition

various dual quantities possible

cf. Synatschke, Wipf, Wozar 07

- general quark propagator:

$$\frac{1}{D_\varphi + m}$$

- general quark condensate:

$$\left\langle \operatorname{tr} \frac{1}{D_\varphi + m} \right\rangle \equiv \Sigma_\varphi \qquad \lim_{m \to 0} \Sigma_\pi : \text{chiral condensate}$$

- dual condensate:

$$\tilde{\Sigma}_n \equiv \frac{1}{2\pi} \int_0^{2\pi} d\varphi \, e^{-in\varphi} \Sigma_\varphi$$

Fourier component picks out all contributions that wind n times

Dual condensate: definition

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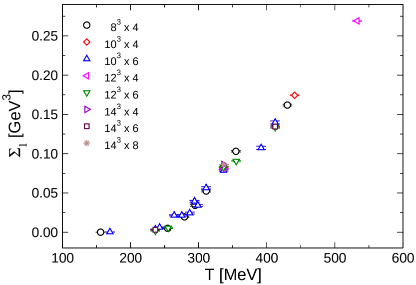
dual quantities: same behavior under center symmetry ($\leftarrow$ special φ's)
$\Rightarrow$ order parameter

Dressed Polyakov loops: order parameter I

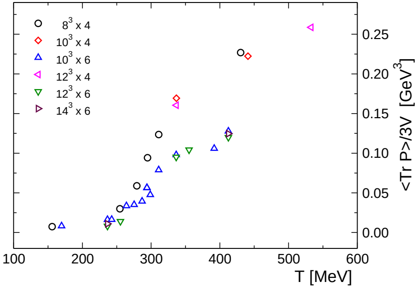
- *SU*(3) quenched

  on lattices with different resolution and volume:

(bare) $\tilde{\Sigma}_1$ with $m = 100\text{MeV}$



(bare) Polyakov loop



less UV renormalisation in $a \sim \frac{1}{N_t}$ $\leftarrow$ 'detours'

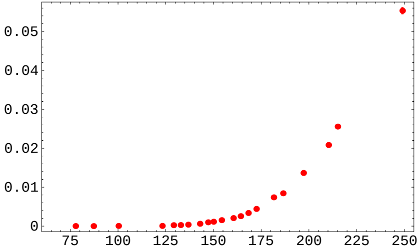
Dressed Polyakov loops: order parameter II

- $SU(3)$ with dynamical quarks

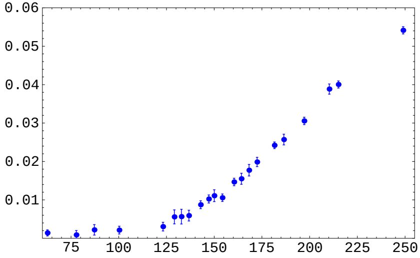
  $2+1$ flavors of improved staggered fermions on $24^3 \cdot 8$ Aoki et al. 06

  $\Rightarrow$ crossover with $T_c^{\text{chiral}} = 157(4)$ MeV, $T_c^{\text{deconf}} = 170(5)$ MeV

(bare) $\tilde{\Sigma}_1$ with $m = 60$MeV



(bare) Polyakov loop



similar behaviour (center symmetry not an exact symmetry anymore)

mass dependence of $\tilde{\Sigma}_1$?

prelim. plots

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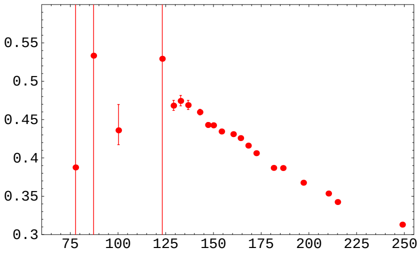
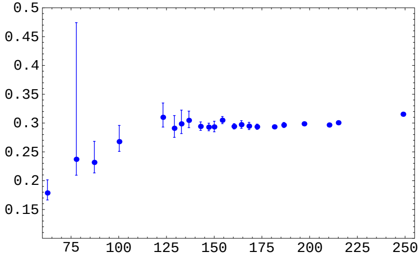
(bare) ‘free energy’ $-\log \tilde{\Sigma}_1/\beta$



(bare) free energy $-\log P/\beta$



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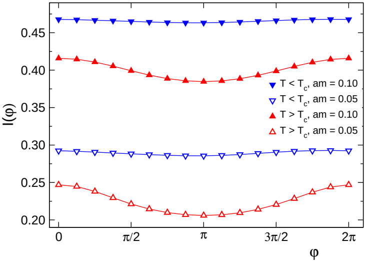
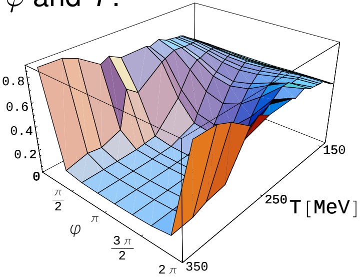
Dressed Polyakov loops: mechanism I

$$\tilde{\Sigma}_1 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \, e^{-i\varphi} \cdot \left\langle \mathrm{tr}\, \frac{1}{D_\varphi + m} \right\rangle$$

Fourier integrand $\langle ... \rangle$ as a function of φ and T:





quenched [real Polyakov loops chosen]   dynamical, $m = 1\mathrm{MeV}$

$\Rightarrow$ depends on φ only at high temperatures $\Rightarrow \tilde{\Sigma}_1 \neq 0$ ✓

in particular: chiral condensate survives at high T for periodic bc.s

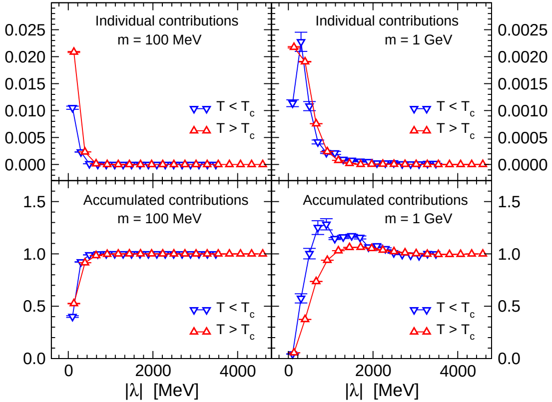
several lattice works, class. dyons FB 09

Dressed Polyakov loops: mechanism II

spectral representation (tr means sum over all eigenmodes):

$$\Sigma_\varphi = \left\langle \mathrm{tr}\, \frac{1}{D_\varphi + m} \right\rangle = \left\langle \sum_{\lambda_\varphi > 0} \frac{2m}{\lambda_\varphi^2 + m^2} \right\rangle \qquad \tilde{\Sigma}_1 = \int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi} \ldots$$

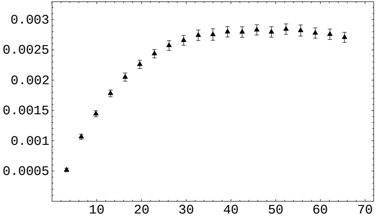
smallest $\lambda_\varphi \lesssim m$ contribute (denominator): IR dominance

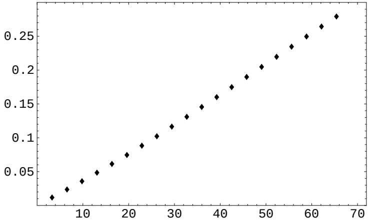


Dressed Polyakov loops: mechanism II

accumulated condensates as a function of λ

dual condensate vs. physical condensate





$\Rightarrow$ dual condensate has converged at even higher masses, where the conventional one has not yet (at our resolution)

additional effect: high modes respond to bc.s only weakly, no contribution after Fourier transform in $\tilde{\Sigma}_1$

Dressed Polyakov loops: mechanism III

geometric series for lattice D hopping once (like staggered we use):

$$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi} \Big\langle \mathrm{tr}\, \frac{1}{D_\varphi + m} \Big\rangle \rightsquigarrow \mathrm{tr} \sum_{k=0}^{\infty} (-1)^k \frac{(D_\varphi)^k}{m^k} \rightsquigarrow \mathrm{tr} \Big(\frac{U - U^\dagger}{2am} \Big)^k$$

k = length of loop

$\Rightarrow$ large mass suppresses long loops (many U's $\Rightarrow$ many $1/m$'s)

$m \to \infty$: conventional Polyakov loop reproduced since shortest

but less IR dominated FB, Gattringer, Hagen 06

$\Rightarrow$ mass gives inv. width of dressed Polyakov loops

- through Schwinger-Dyson equations — Fischer, Mueller 09
- from Random matrix models — FB in prep.

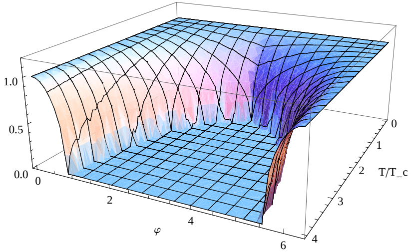
Gaussian integrals replace QCD dynamics

modified Matsubara frequencies: $\omega = \varphi T$

$\omega = \pi T$ for physical bc.s

$\omega = 0$ for periodic bc.s $\leftarrow$ 'no twist'

Σ_φ from saddle point method:



Dressed Polyakov loops: other applications

- at imag. μ through functional RG — Braun, Haas, Marhauser, Pawlowski 09
- in Polyakov loop extended NJL model

  Kashiwa, Kouno, Yahiro 09, Mukherjee, Chen, Huang 09

  incl. magnetic field — Gatto, Ruggieri 10
- in gauge group G_2 (no center) — Danzer, Gattringer, Maas 08
- in $SU(2)$ with adjoint quarks ($T_{\text{chiral}} \simeq 4T_{\text{deconf}}$)

  Bilgici, Gattringer, Ilgenfritz, Maas 09
- canonical determinants — Bilgici et al. 11, Nagata, Nakamura 10

‘Wilson Xloop 800’

Dressed Wilson loops: idea and definition

Wilson loops of fixed area (in some plane)

$\Rightarrow$ phase factor e^{ifS} according to area S

Dressed Wilson loops: idea and definition

Wilson loops of fixed area (in some plane)

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- magnetic / electric field: constant and abelian $f = d\bar{a}$ (charge=1)

$$U_\mu \;\to\; U_\mu e^{ia\bar{a}_\mu} \qquad W \;\to\; W\, e^{i\oint_C \bar{a}} = W\, e^{i\iint f} = W\, e^{ifS} \quad \text{[Stokes]}$$

Dressed Wilson loops: idea and definition

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- again in quark condensate (or other observables):

$$\left\langle \mathrm{tr}\,\frac{1}{D_f + m}\right\rangle \equiv \Sigma_f = \# \cdot \mathrm{tr}1 \cdot e^0 + \# \cdot \mathrm{tr}\,W_{1\times 1} \cdot e^{if} + \ldots$$

- dual condensate:

$$\tilde{\Sigma}_S \equiv \sum\!\!\!\!\!\!\int_f \; e^{-iSf}\Sigma_f$$

  picks out all Wilson loops of area S and any circumference

  $\equiv$ dressed Wilson loops

- constant magn. fields $\Rightarrow$ Wilson loops $\quad\leftarrow$ beyond lattice!?

Dressed Wilson loops: details

- Dirac operator restricted to some 2D plane (cheap)
  e.g. magn. field in z:
  $\Rightarrow D$ in (x, y)-plane, for different z and t (many)
  $\Rightarrow$ Wilson loops in (x, y)-plane = spatial (area law, too)

- finite volume:
  magn. flux quantised: $f = 2\pi \frac{n}{\mathrm{Vol}_2} \quad n \in \mathbb{Z}$ (like momentum)
  area bounded, i.e. periodic
  $\Rightarrow$ Fourier series

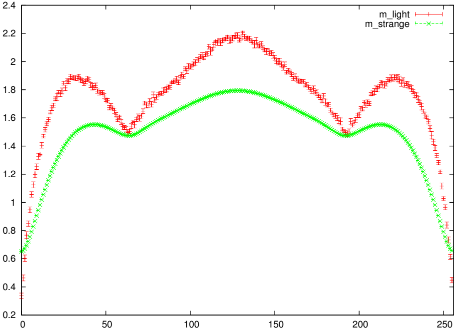
- discrete lattice:
  flux bounded: $f \leq 2\pi \frac{1}{a^2}$ (like momentum)
  area quantised
  $\Rightarrow$ discrete Fourier transform (both variables discrete)

- for Wilson loops: need all magn. fields

Dressed Wilson loops: results

$2 + 1$ staggered fermions on a $16^3 \cdot 4$ lattice ($T = 146$ MeV)

- condensate Σ_f over magn. field $f = 2\pi \frac{n}{256}$, $n = 0, 1, .., 256$



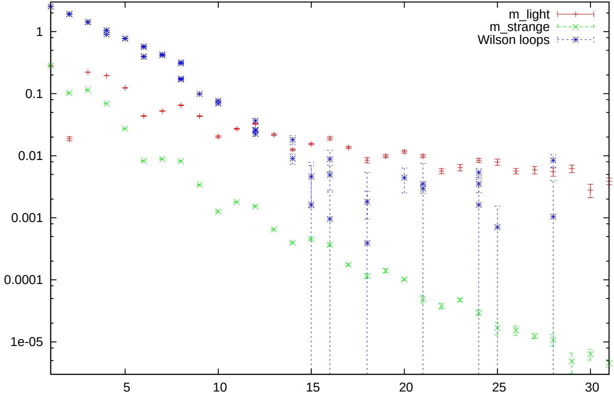
(averaged over 64 spatial planes and 5 configurations)

grows with magn. field ✓ shoulders?

heavy quarks: effect washed out ✓

not flat $\overset{\text{Fourier components}}{\Rightarrow}$ dual condensate = dressed Wilson loop

- dressed Wilson loop $\tilde{\Sigma}_S$ ($S_{\max} = 256$)



together with conventional Wilson loops

decays with area ✓

pattern for small sizes? geometry of lattice loops?

Dressed Wilson loops: mechanism I

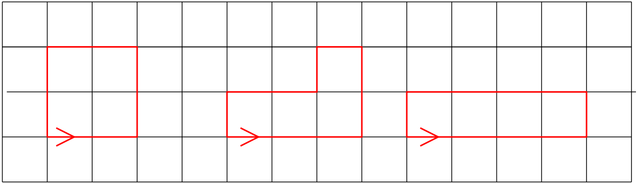
geometric series as before (on the lattice):

$$\tilde{\Sigma}_S \equiv \sum\!\!\!\!\!\!\int_f \; e^{-iSf}\left\langle \mathrm{tr}\,\frac{1}{D_f+m}\right\rangle \rightsquigarrow \mathrm{tr}\sum_{k=0}^{\infty}(-1)^k\frac{(D_f)^k}{m^k} \rightsquigarrow \mathrm{tr}\left(\frac{U-U^\dagger}{2am}\right)^k$$

k = length of loop

large mass suppresses long loops

e.g. loops with area $S=4$:

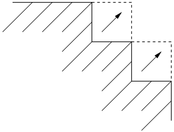


circumference $k=8$ circumference $k=10$

Some geometry of lattice loops

- ideal lattice loop

  $\equiv$ max. area at fixed circumference:

  rectangles, not circle-like

but entropy = number of loops of different shape!?

- no disconnected loops since $\mathrm{tr} D^k = D(x,y)D(y,z)\ldots D(w,x)$
- extreme cases for area $\tilde{\Sigma}_{S=p^2}$:
  - ideal loop = square: factor $\frac{4}{(2am)^{4p}}$
  - many plaquettes after each other: factor $\frac{4^{p^2}}{(2am)^{4p^2}}$
  - for large mass square loop favored

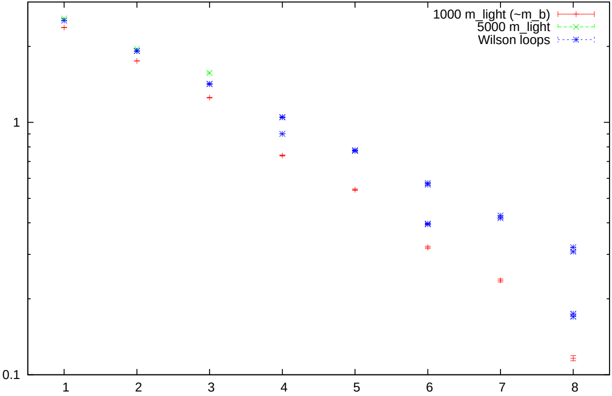
for large mass $\tilde{\Sigma}_S$ contains mainly rectangular Wilson loops

how many such Wilson loops $\rightsquigarrow$ combinatorical factor
(for given area at miminal circumference)

Dressed Wilson loops: results

- again: dressed Wilson loop $\tilde{\Sigma}_S$

  now for large mass and incl. combinatorical factor



agrees with Wilson loops ✓ ← used for scale setting

at large area S precision limited: tiny $\tilde{\Sigma}_S$ × huge combin. factor

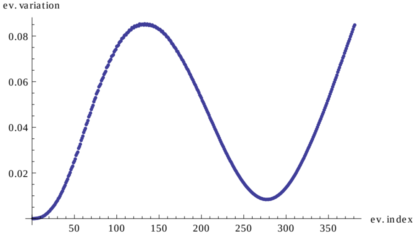
Dressed Wilson loops: mechanism II

IR dominance? only the lowest modes respond to the magnetic field?

NO:

eigenvalue variation at fixed index

= standard deviation wrt. all magn. fields



high modes do respond to magnetic field

(in contrast to phase bc.s for dressed Polyakov loops)

$\Rightarrow$ dressed Wilson loops converge slowly in spectral representation, especially at large masses

Summary and Outlook

dressed Polyakov loops | dressed Wilson loops

center symmetry $\tilde{\Sigma}_1 \neq 0$ | string tension $\tilde{\Sigma}_S \sim e^{-\sigma S}$

related to quark condensate: chiral symmetry

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winding from phase bc.s φ | area from magn. field f

loops with long circumference suppressed by large mass

IR dominance | no IR dominance, mass range limited

$\log \tilde{\Sigma}_{1,\mathrm{bare}} \not\sim \frac{1}{a} + \ldots$? | $\log \tilde{\Sigma}_{S,\mathrm{bare}} \not\sim \frac{c}{a} + \ldots$?

mass: growing T_c? | what is the meaning of $\log \tilde{\Sigma}_S(m < \infty)$? (sensitive to lattice geometry?)

beyond lattice! | beyond lattice?

dual condensates $\tilde{\Sigma}_1$ and its 'free energies' $-\log\tilde{\Sigma}_1/\beta$ as a function of temperature for masses $m = 1$ MeV, 10 MeV, 100 MeV (smeared)