Dynamical Locking of the Chiral and the Deconfinement Phase Transition

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Quarks, Gluons, and Hadronic Matter under Extreme Conditions
St. Goar
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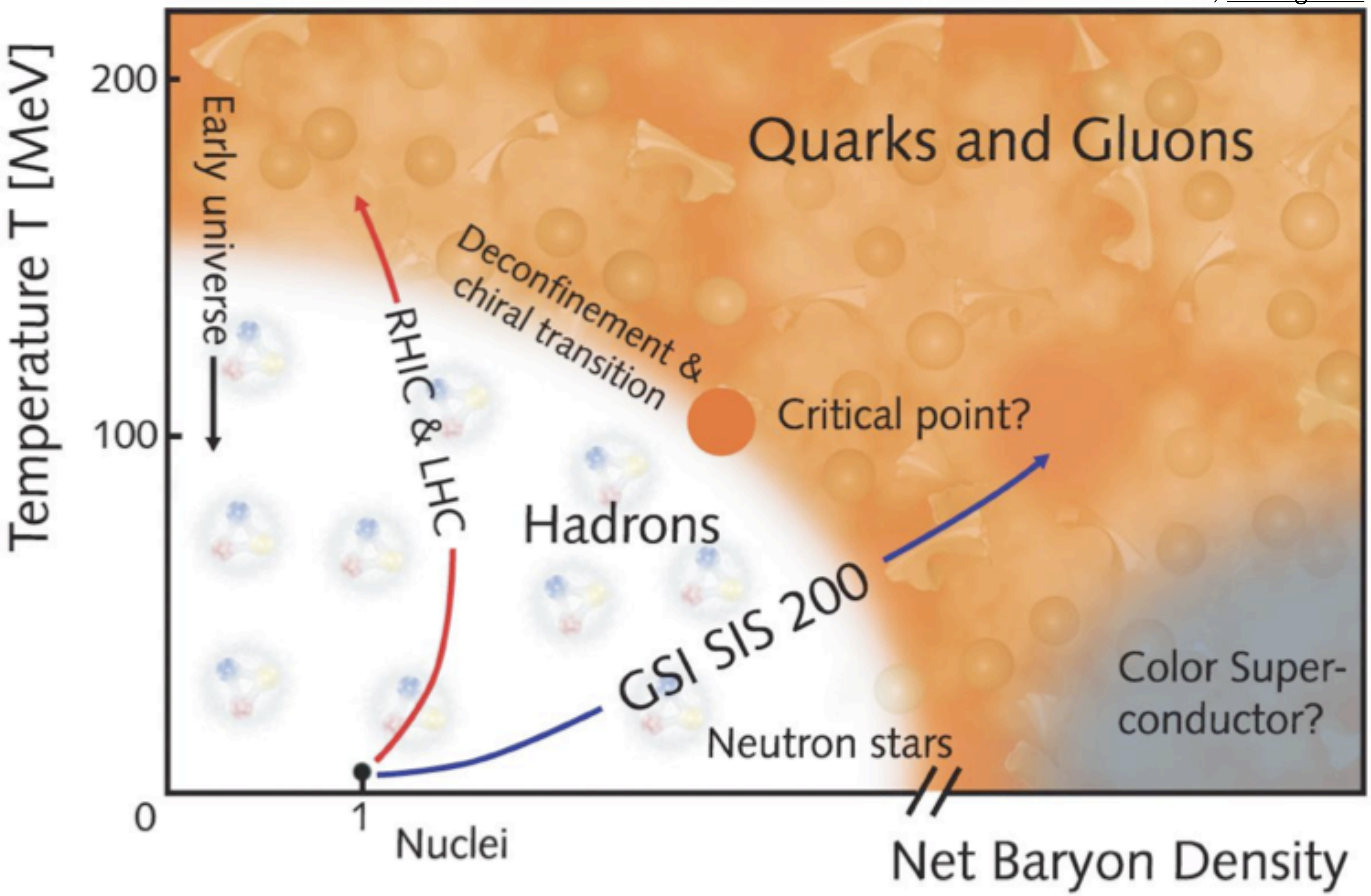
Deutsche Forschungsgemeinschaft DFG

Research Training Group Quantum and Gravitational Fields

J. Braun, A. Janot, arxiv:1102.4841

QCD phase diagram

FAIR, www.gsi.de



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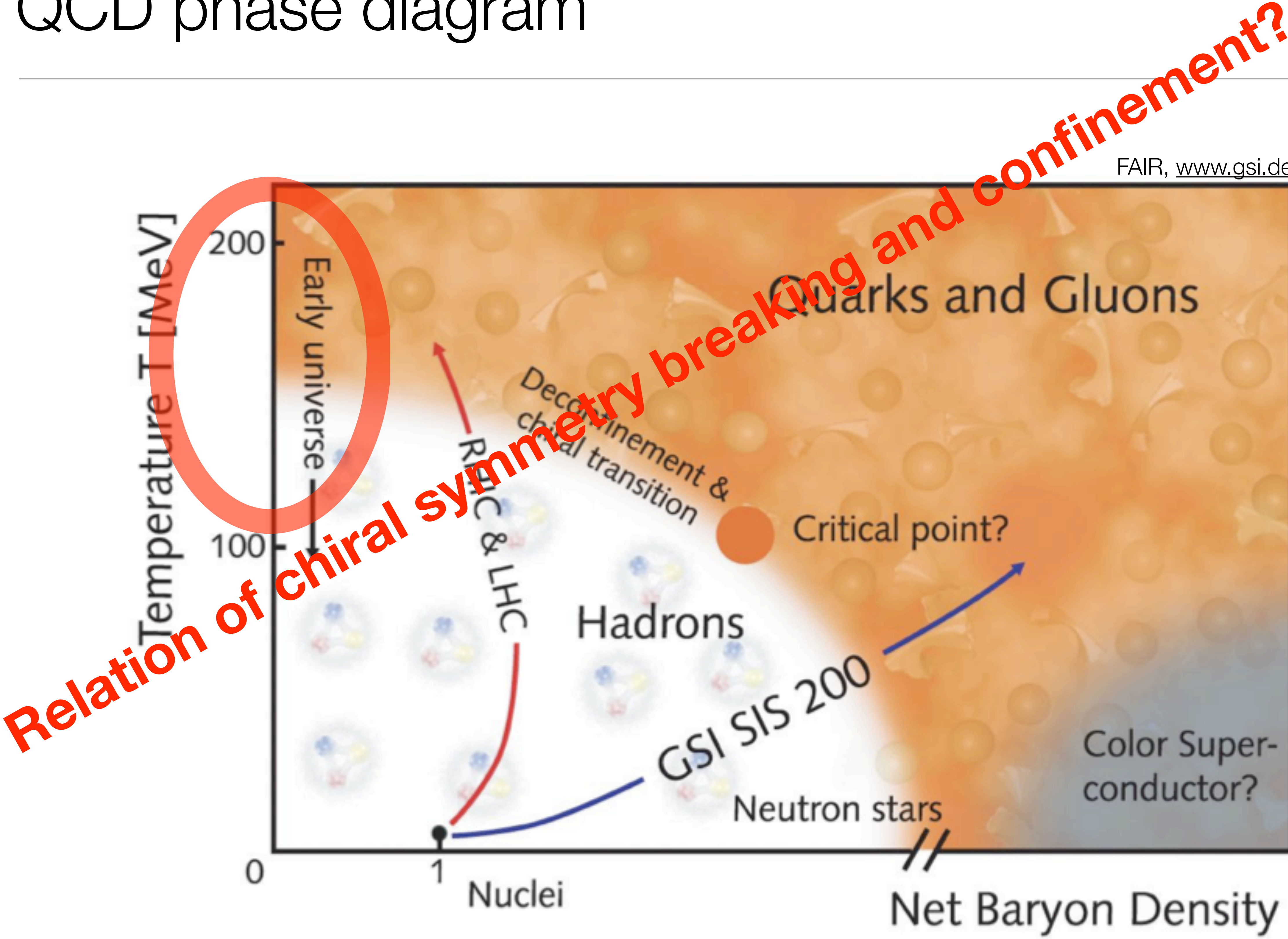
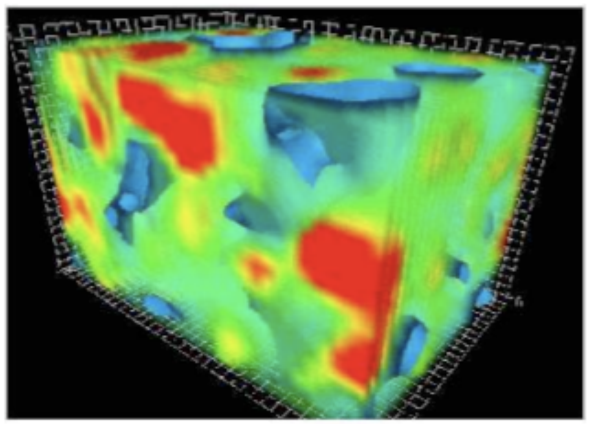


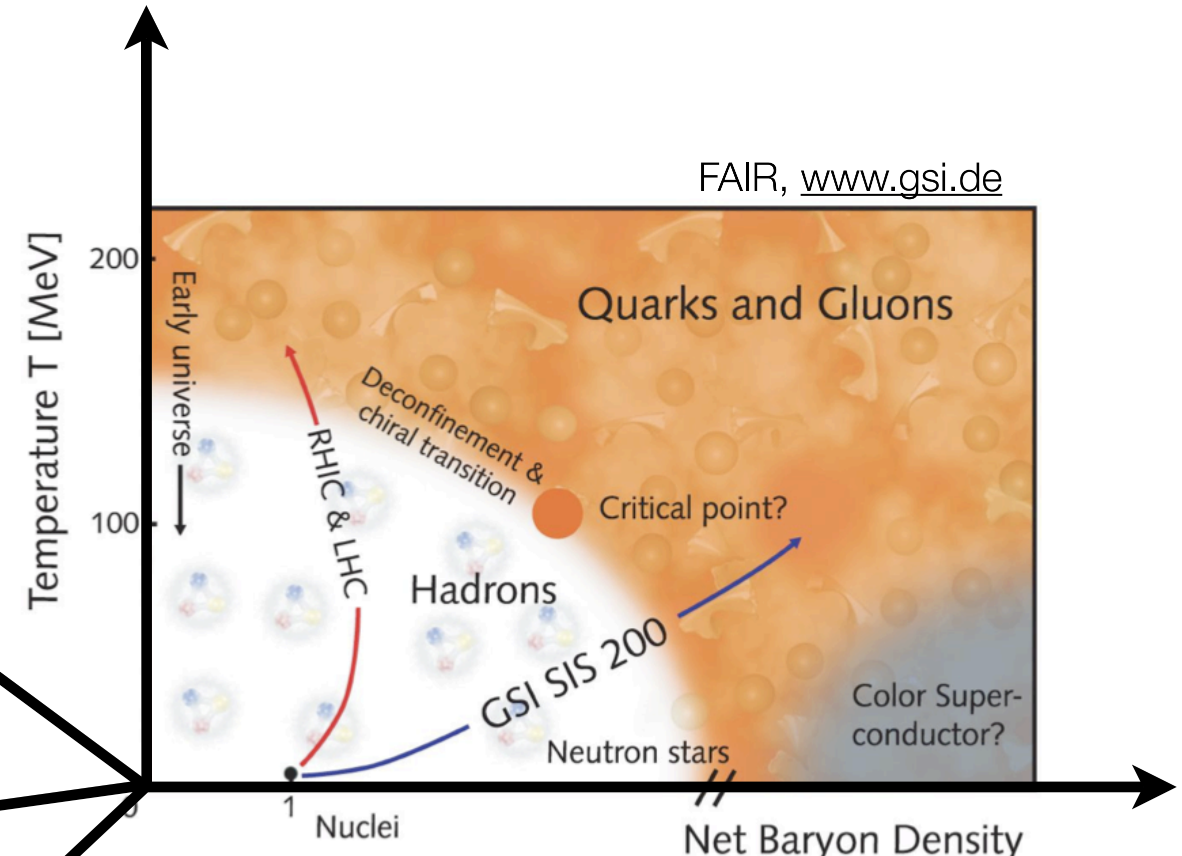
Many ways to learn something about QCD ...

Lattice QCD

D. Leinweber @
www.physics.adelaide.edu.au



finite volume, finite quark masses



Many-color QCD

Many-flavor QCD

(cf. H. Gies' talk)

weak-coupling limit

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \mathrm{i} g A\!\!\!/)\,\psi$$

symmetries:

$$SU(N_c) \times SU_L(N_f) \times SU_R(N_f)$$

Yet another deformation of QCD ...

- λ_ψ-deformed QCD (here two massless flavors and N_c colors)

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\not{\partial} + \mathrm{i} g \not{A})\,\psi + \lambda_\psi (\bar{\psi}\mathcal{O}\psi)^2$$

- **two** parameters: $\lambda_\psi(m_\tau)$ and $\alpha_s(m_\tau)$ $(\Leftrightarrow \Lambda_{\mathrm{QCD}})$

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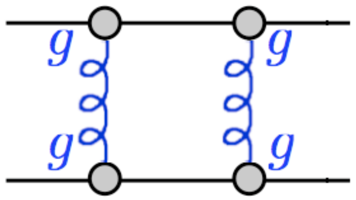
- **two** parameters: $\lambda_\psi(m_\tau)$ and $\alpha_s(m_\tau)$ $(\Leftrightarrow \Lambda_{\rm QCD})$

- **real** QCD: only **one** parameter, namely $\alpha_s(m_\tau)$ $(\Leftrightarrow \Lambda_{\rm QCD})$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \mathrm{i} g A\!\!\!/)\,\psi$$

integrate out fluctuations



(cf. Gies&Jaeckel '05; JB & Gies, '05,'06','09; JB, Fischer, Gies '11)

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quark self-interactions are induced by
the gauge degrees of freedom

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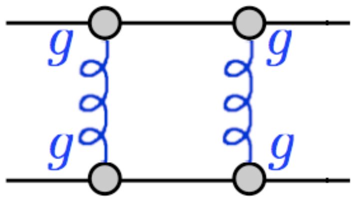

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quark self-interactions are induced by
the gauge degrees of freedom:
onset of chiral symmetry breaking is signaled
by strong quark-self interactions (λ_ψ's become IR-relevant)

Yet another deformation of QCD ...

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$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \mathrm{i} g A\!\!\!/)\,\psi + \lambda_\psi (\bar{\psi}\mathcal{O}\psi)^2$$

- **two** parameters: $\lambda_\psi(m_\tau)$ and $\alpha_s(m_\tau)$ $(\Leftrightarrow \Lambda_{\rm QCD})$
- **scales?**

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- **two** parameters: $\lambda_\psi(m_\tau)$ and $\alpha_s(m_\tau)$ $(\Leftrightarrow \Lambda_{\rm QCD})$

- scale dependence of chiral observables in the chiral limit in **real QCD**:

$$T_{\chi\rm SB},\ f_\pi,\ |\langle\bar{\psi}\psi\rangle|^{\frac{1}{3}},\ \dots \sim \Lambda_{\rm QCD}(\alpha_s)$$

- scale dependence of chiral observables in the chiral limit in λ_ψ**-deformed QCD**:

$$T_{\chi\rm SB}(\alpha_s,\lambda_\psi),\ f_\pi(\alpha_s,\lambda_\psi),\ |\langle\bar{\psi}\psi\rangle|^{\frac{1}{3}}(\alpha_s,\lambda_\psi),\ \dots$$

α_s directly related to **gluo**dynamics
λ_ψ directly related to **chiral** observables

Yet another deformation of QCD ...

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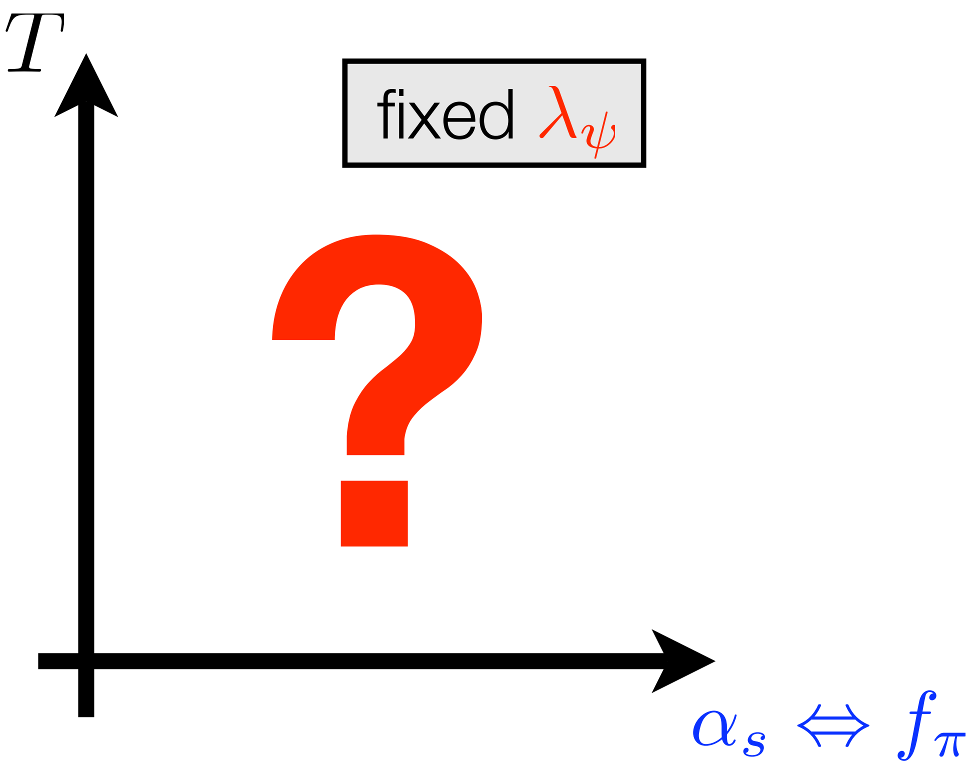
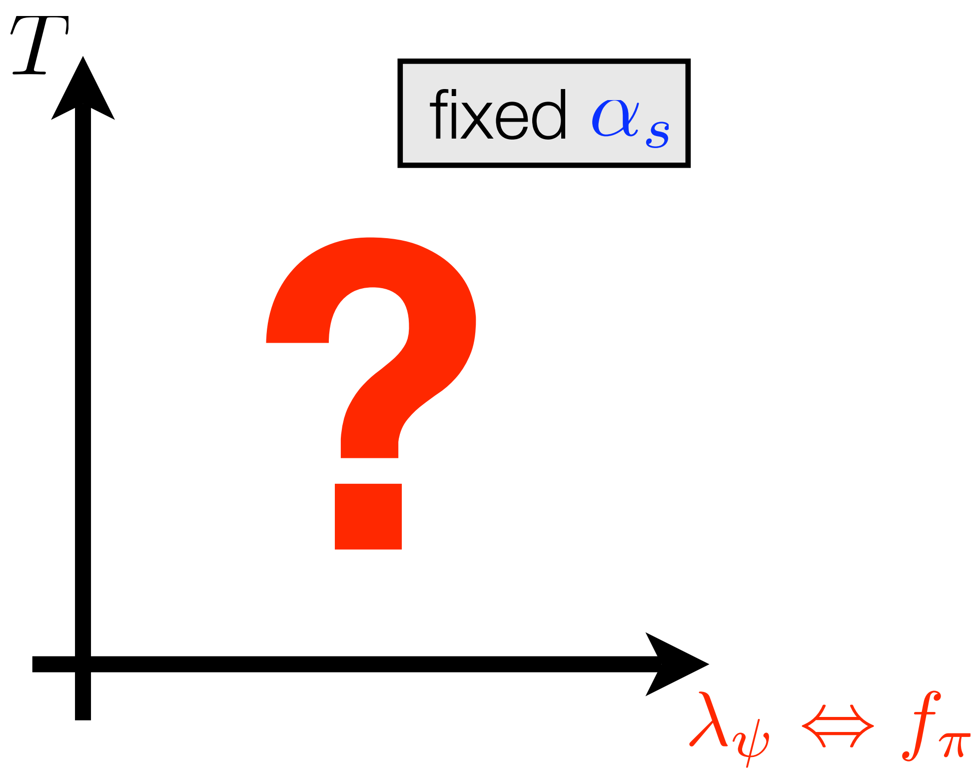
$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \mathrm{i} g A\!\!\!/)\,\psi + \lambda_\psi (\bar{\psi}\mathcal{O}\psi)^2$$

- **two** parameters: $\lambda_\psi(m_\tau)$ and $\alpha_s(m_\tau)$ $(\Leftrightarrow \Lambda_{\mathrm{QCD}})$

- phase diagrams ...





Yet another deformation of QCD ...

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$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \mathrm{i} g A\!\!\!/)\,\psi + \lambda_\psi (\bar{\psi} \mathcal{O} \psi)^2$$

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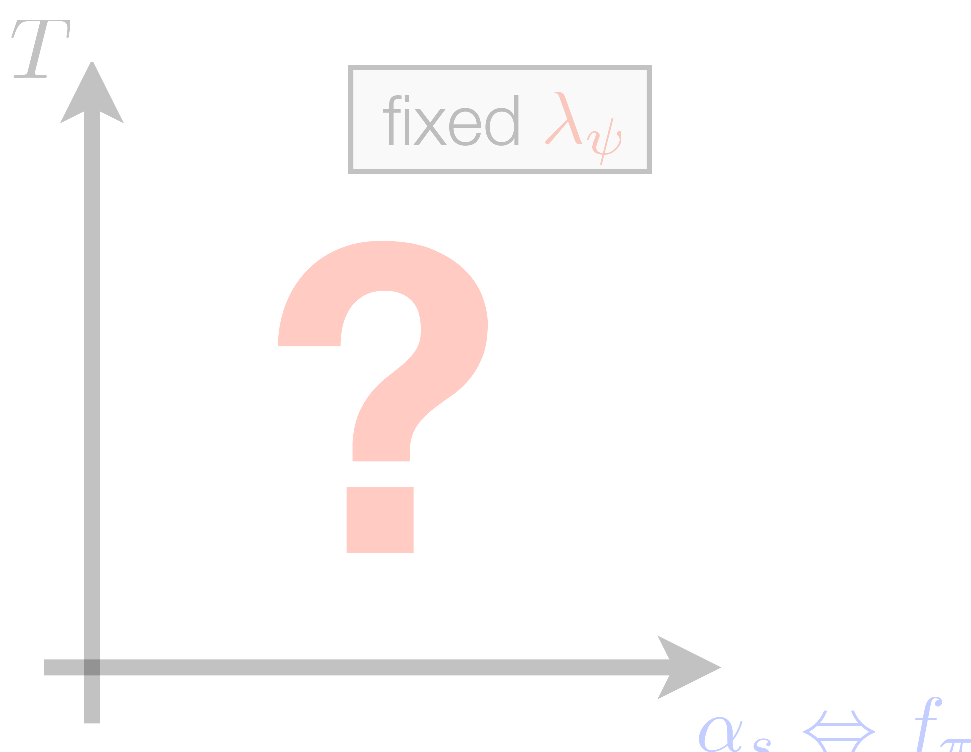
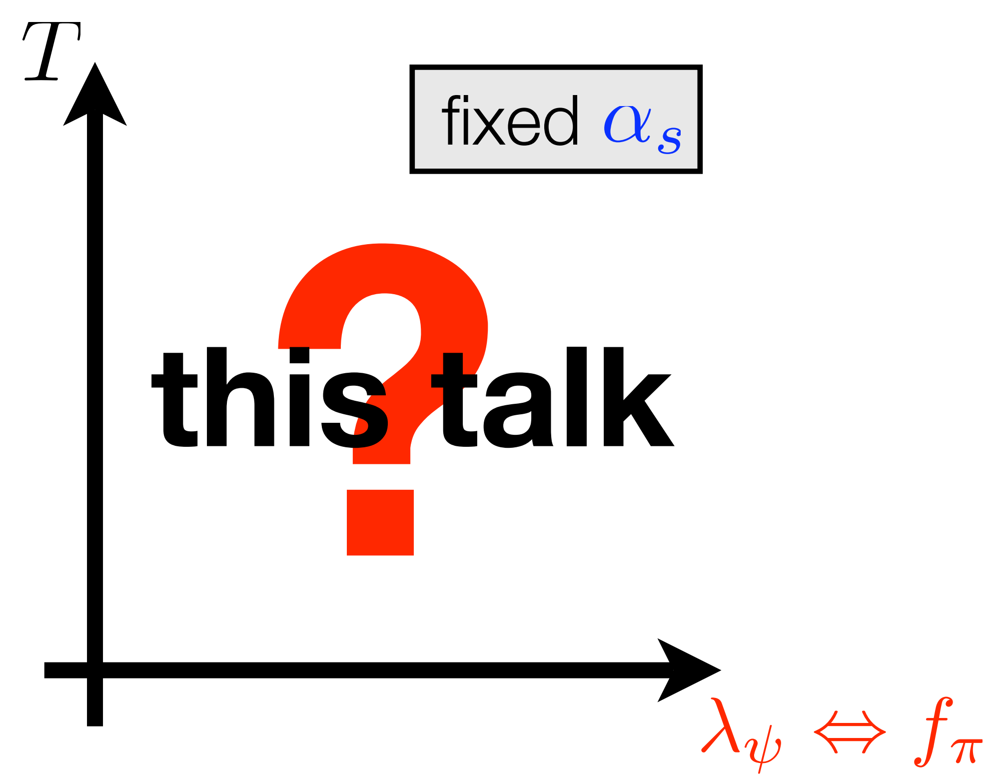




allows to gain insight into the relation of chiral symmetry breaking and confinement

λ_ψ-deformed QCD: our model

- λ_ψ -deformed QCD (model) with two massless flavors and N_c colors:

(cf. matter part of QCD in Landau-DeWitt gauge, e. g. JB, Haas, Marhauser, Pawlowski '09)

$$\mathcal{L} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0 \bar{g}\langle A_0\rangle\right)\psi + \lambda_\psi\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right]$$

- here, the **two** parameters are $\lambda_\psi(\Lambda)$ and $\langle A_0\rangle$ $(\Leftrightarrow \alpha_s)$

- order parameter for deconfinement: (JB, Gies, Pawlowski '07; Pawlowski, Marhauser '08)

$$L[\langle A_0\rangle] = \mathrm{tr_F}\, \mathrm{e}^{\mathrm{i}\beta\bar{g}\langle A_0\rangle} \geq \left\langle \mathrm{tr_F}\, \mathrm{e}^{\mathrm{i}\int_0^\beta \bar{g}A_0} \right\rangle$$

- confer underlying assumption in PNJL/PQM-type models: (e. g. Meisinger, Ogilvie '96; Fukushima '03; Ratti, Thaler, Weise '05; Schaefer, Pawlowski, Wambach '08; ...)

$$L[\langle A_0\rangle] = \mathrm{tr_F}\, \mathrm{e}^{\mathrm{i}\beta\bar{g}\langle A_0\rangle} \stackrel{!}{=} \left\langle \mathrm{tr_F}\, \mathrm{e}^{\mathrm{i}\int_0^\beta \bar{g}A_0} \right\rangle$$

λ_ψ-deformed QCD: our model

(JB, A. Janot '11)

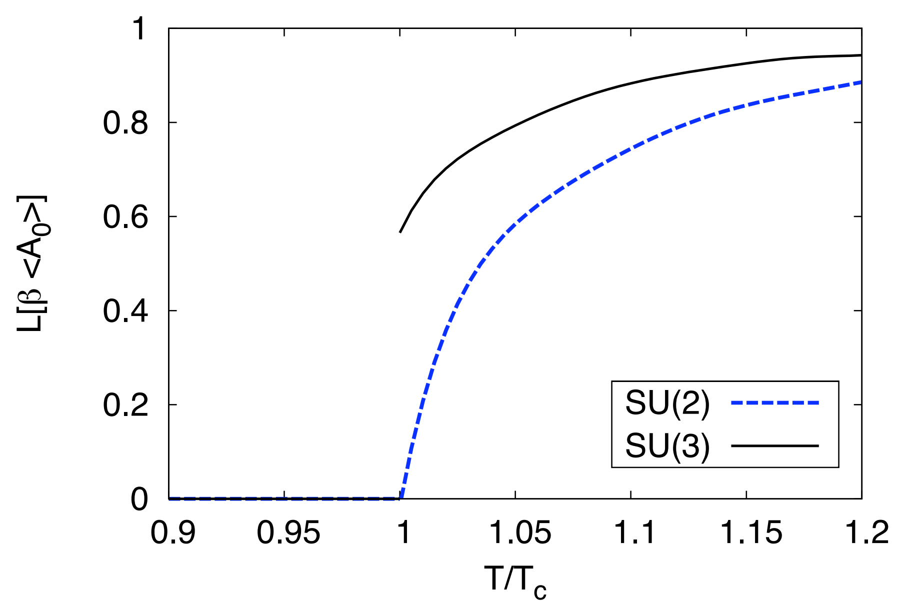
- λ_ψ -deformed QCD (model) with two massless flavors and N_{c} colors:

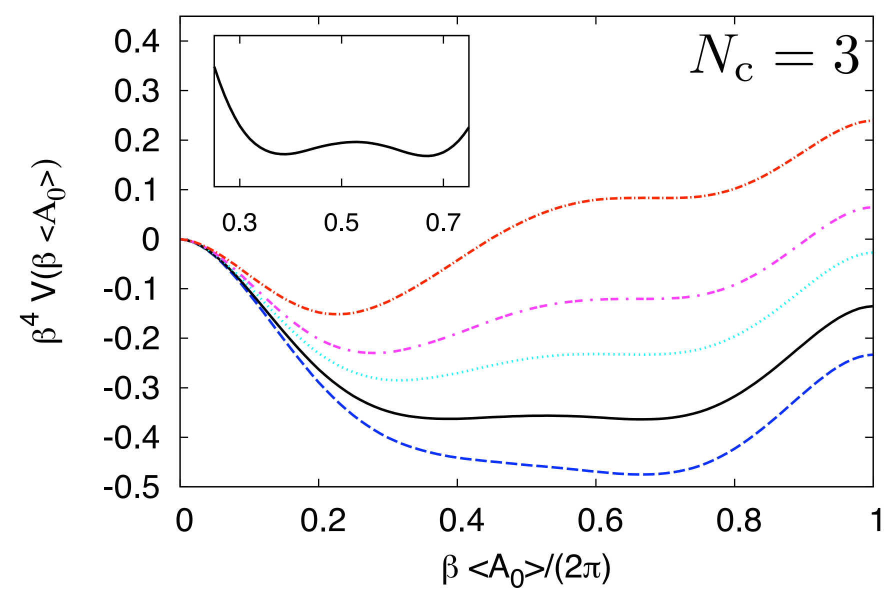
$$\mathcal{L} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0 \bar{g}\langle A_0 \rangle\right)\psi + \lambda_\psi \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right]$$

- **two** parameters: $\lambda_\psi(\Lambda)$ and $\langle A_0 \rangle$ $(\Leftrightarrow \Lambda_{\mathrm{QCD}})$

- data for $L[\langle A_0 \rangle] \neq \langle L[A_0] \rangle$ is available from non-perturbative RG studies:

(JB, Gies, Pawlowski '07; JB, Eichhorn, Gies, Pawlowski '10)





- for our numerical study we employ the RG data: the confinement temperature T_{d} is **fixed**, the chiral transition temperature $T_\chi(\lambda_\psi, \langle A_0 \rangle)$ is a prediction from our model

λ_ψ-deformed QCD: RG fixed-point analysis I

(JB, A. Janot '11)

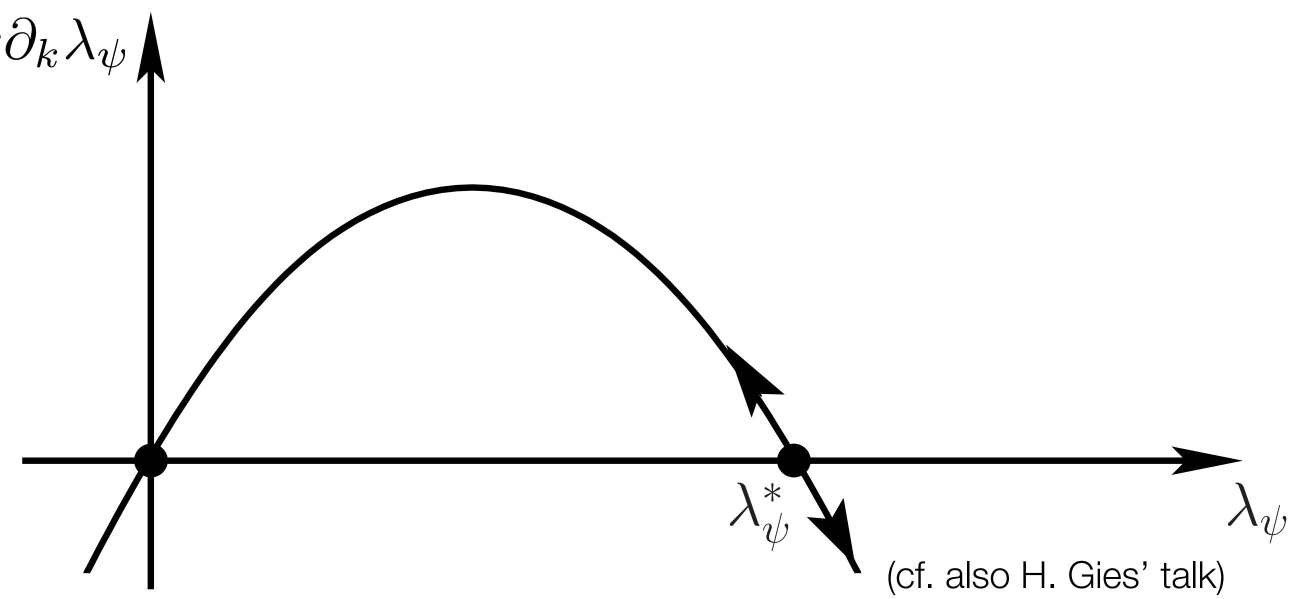
- λ_ψ -deformed QCD (model) with two massless flavors and N_c colors:

$$\mathcal{L} = \bar\psi\,(\mathrm{i}\partial\!\!\!/ \qquad\qquad)\,\psi + \lambda_\psi\left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\vec\tau\psi)^2\right]$$

β-function (RG flow equation):

$$\beta_{\lambda_\psi} \equiv k\partial_k\lambda_\psi = 2\lambda_\psi - \lambda_\psi C \lambda_\psi$$

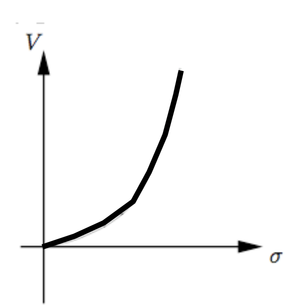
RG scale k $\sim$ momentum scale



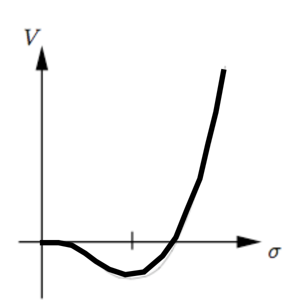
(cf. also H. Gies' talk)

- 'critical' coupling λ_ψ^* :

choose $\lambda_\psi(\Lambda) > \lambda_\psi^*$ $\Longrightarrow$ λ_ψ increases rapidly $\Longrightarrow$ χSB



$$\frac{1}{m^2} \propto \lambda_\psi \to \infty$$



λ_ψ-deformed QCD: RG fixed-point analysis II

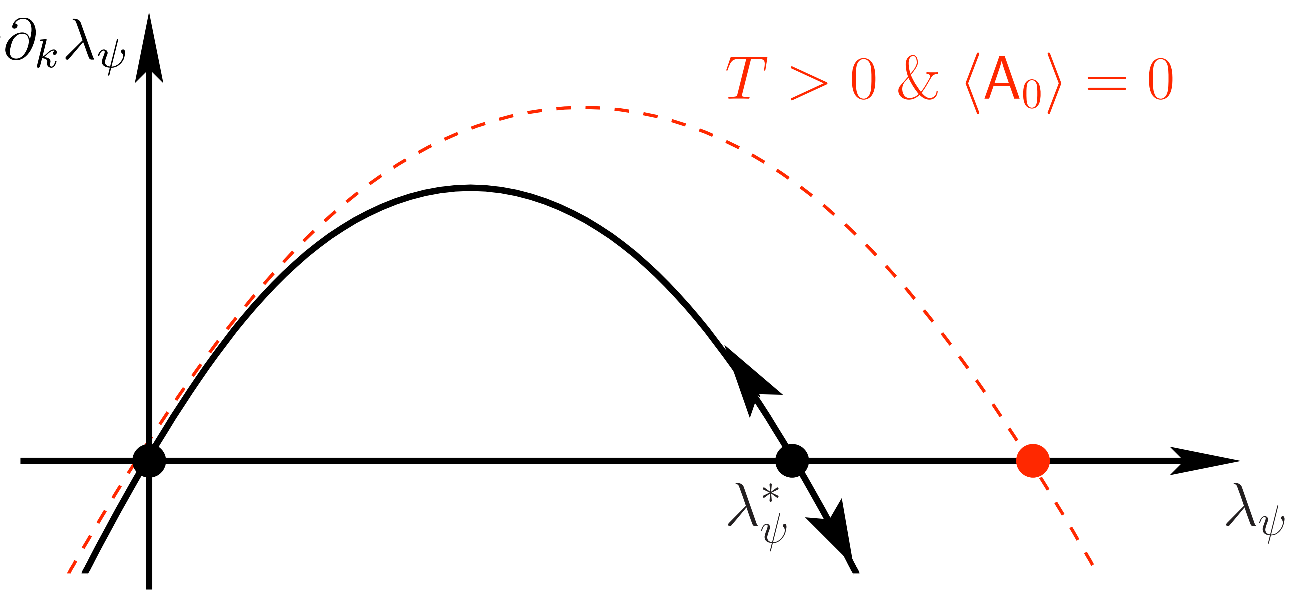
(JB, A. Janot '11)

- λ_ψ -deformed QCD (model) with two massless flavors and N_c colors:

$$\mathcal{L} = \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ \qquad\qquad)\,\psi + \lambda_\psi\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right]$$

β-function (RG flow equation):

$$\beta_{\lambda_\psi} \equiv k\partial_k\lambda_\psi = 2\lambda_\psi - \lambda_\psi C(\tfrac{T}{k})\lambda_\psi$$



- fixed point coupling $\lambda_\psi^*(\frac{T}{k}) \sim (\frac{T}{k})^3$
- for a fixed choice $\lambda_\psi(\Lambda) > \lambda_\psi^*(T=0)$, a critical temperature T_χ exists:

$T > T_\chi$ $\Longrightarrow$ λ_ψ decreases $\Longrightarrow$ no χSB

λ_ψ-deformed QCD: RG fixed-point analysis III

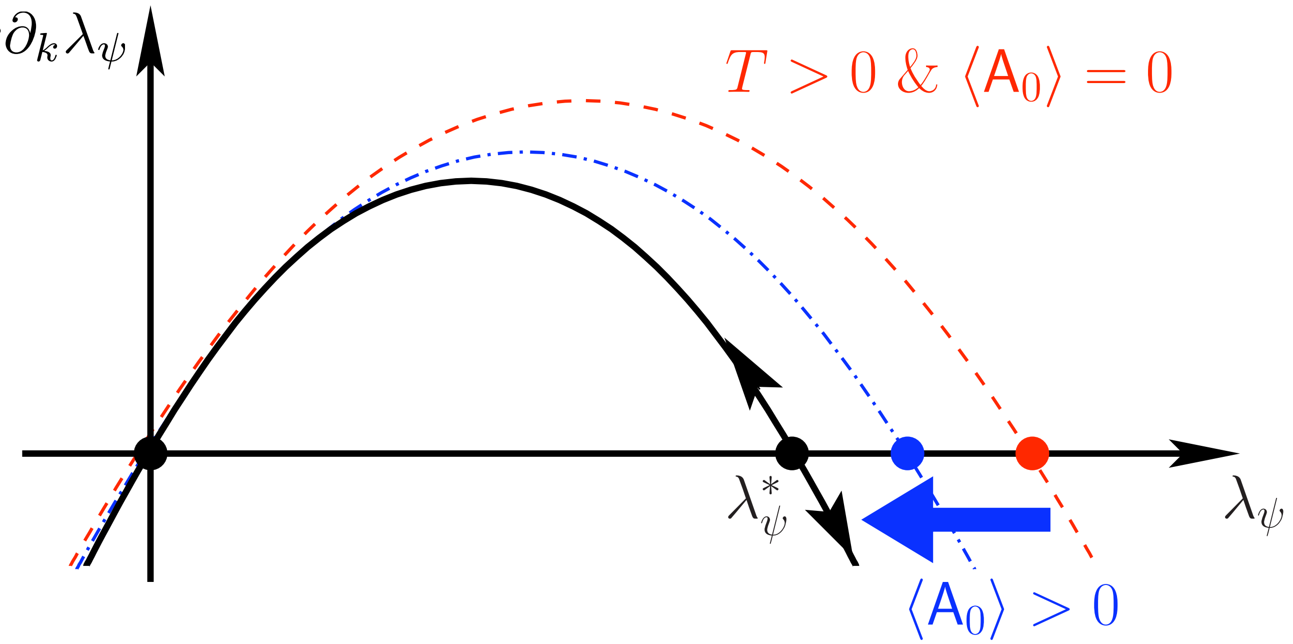
(JB, A. Janot '11)

- λ_ψ -deformed QCD (model) with two massless flavors and N_c colors:

$$\mathcal{L} = \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0 \bar{g}\langle A_0\rangle)\,\psi + \lambda_\psi\,\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right]$$

β-function (RG flow equation):

$$k\partial_k\lambda_\psi = 2\lambda_\psi - \lambda_\psi C(\tfrac{T}{k}, \langle A_0\rangle)\lambda_\psi$$



- for increasing $\langle A_0\rangle$, the fixed point 'moves' to the left:

$$\lambda^*_\psi(\tfrac{T}{k}, \langle A_0\rangle) \sim \lambda^*_\psi(T=0) + (\tfrac{T}{k})^3\, L[\langle A_0\rangle] + \mathcal{O}(1/N_c)$$

λ_ψ-deformed QCD: RG fixed-point analysis III

(JB, A. Janot '11)

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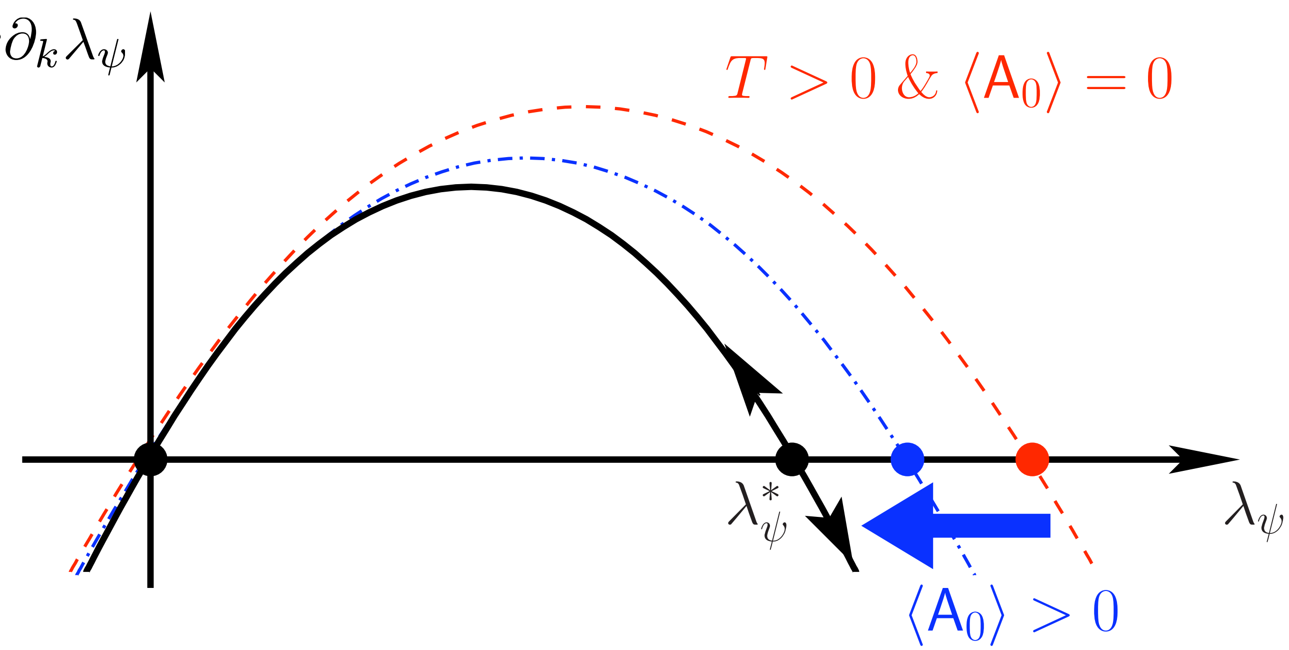

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$$\lambda_\psi^*(\tfrac{T}{k}, \langle A_0\rangle) \sim \lambda_\psi^*(T=0) + (\tfrac{T}{k})^3\, L[\langle A_0\rangle] + \mathcal{O}(1/N_\mathrm{c})$$

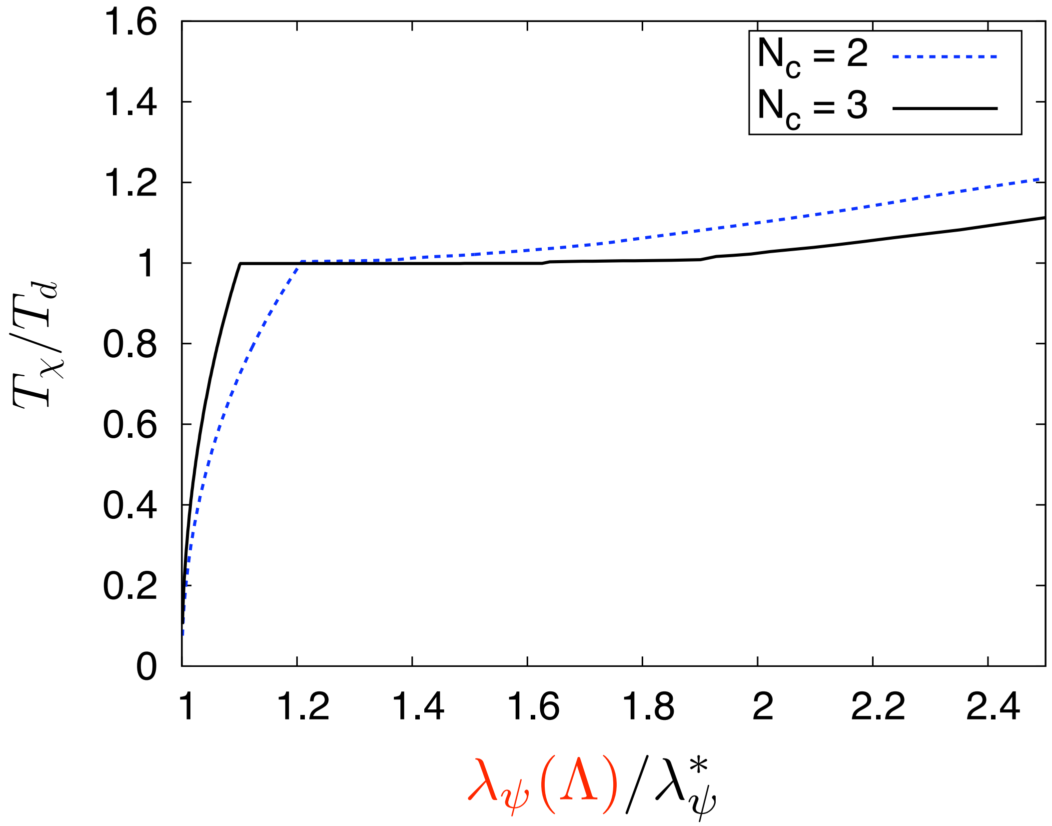
- fixed choice $\lambda_\psi(\Lambda) > \lambda_\psi^*(T=0)$:

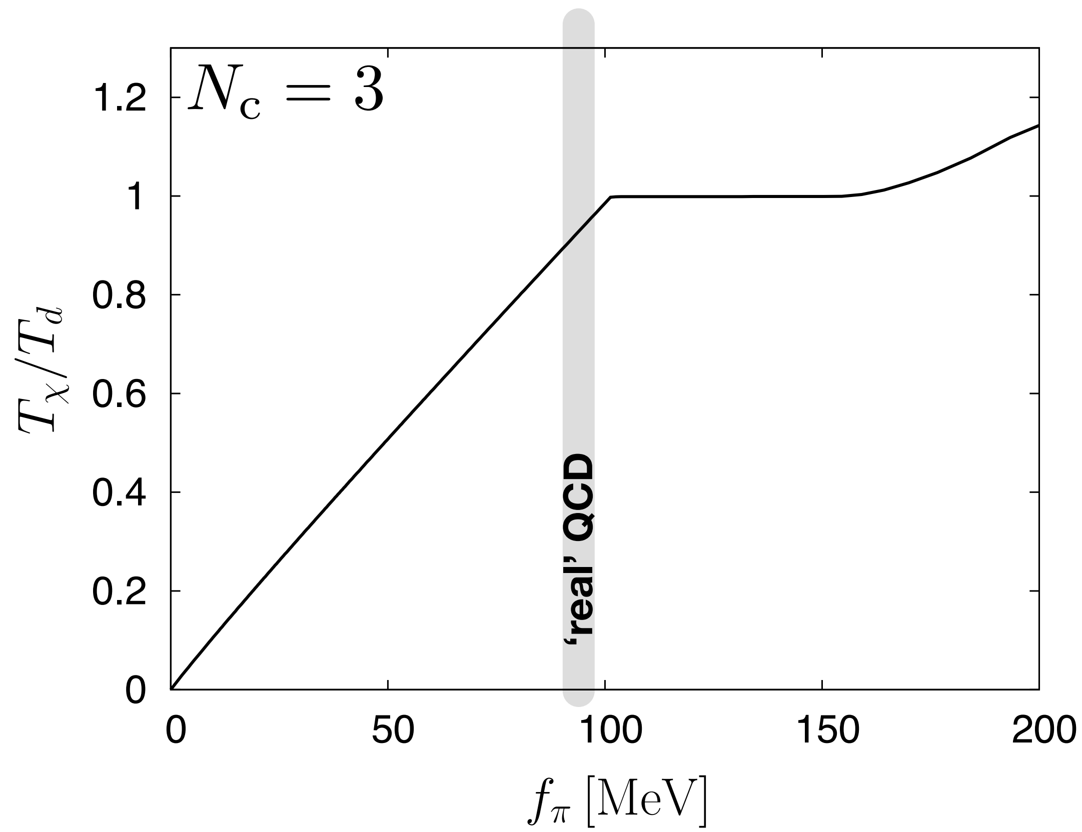
confinement order parameter $L[\langle A_0\rangle] \to 0$ $\Rightarrow$ χSB $\Rightarrow$ $T_\chi \gtrsim T_\mathrm{d}$

(exact in the limit $N_\mathrm{c} \to \infty$)

λ_ψ-deformed QCD: phase diagram

(JB, A. Janot '11)





- window in which the chiral phase transition is **locked** to the deconfinement phase transition increases with increasing N_c
- for finite N_c, we have three regimes: $T_\chi < T_d$, $T_\chi \approx T_d$ and $T_\chi > T_d$
- below the locking window, we have $T_\chi \sim (\lambda_\psi(\Lambda) - \lambda_\psi^*)^{1/2} \sim f_\pi$

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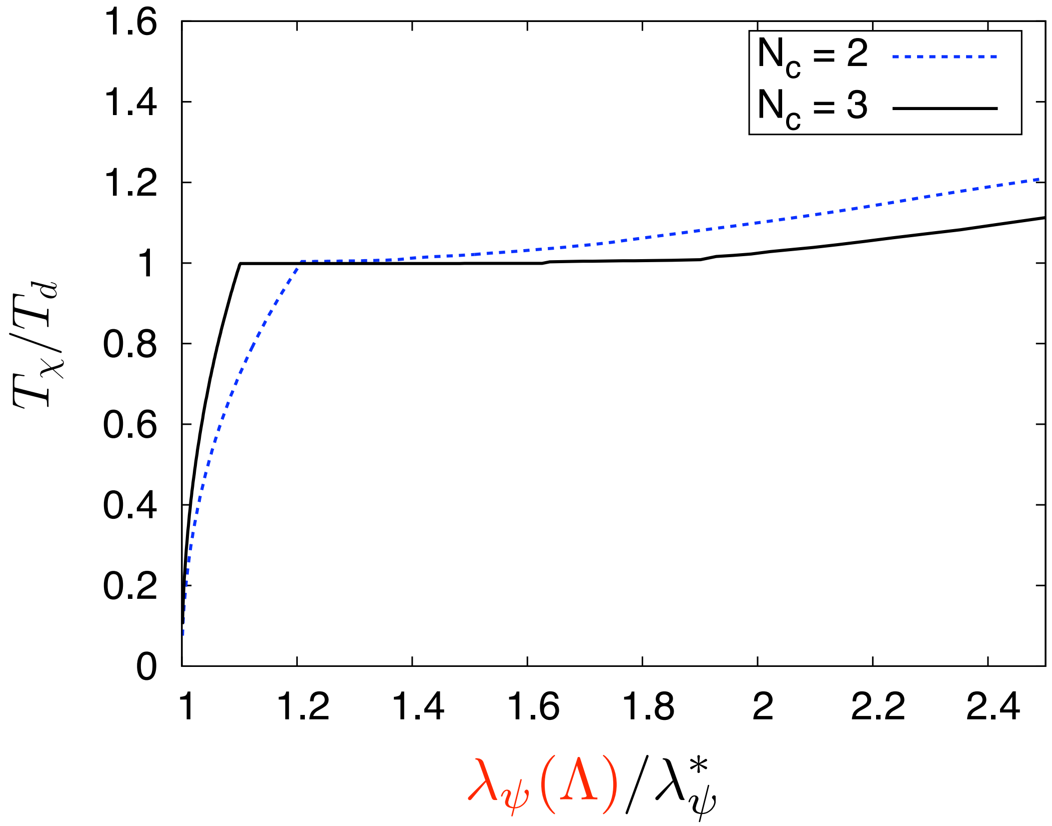

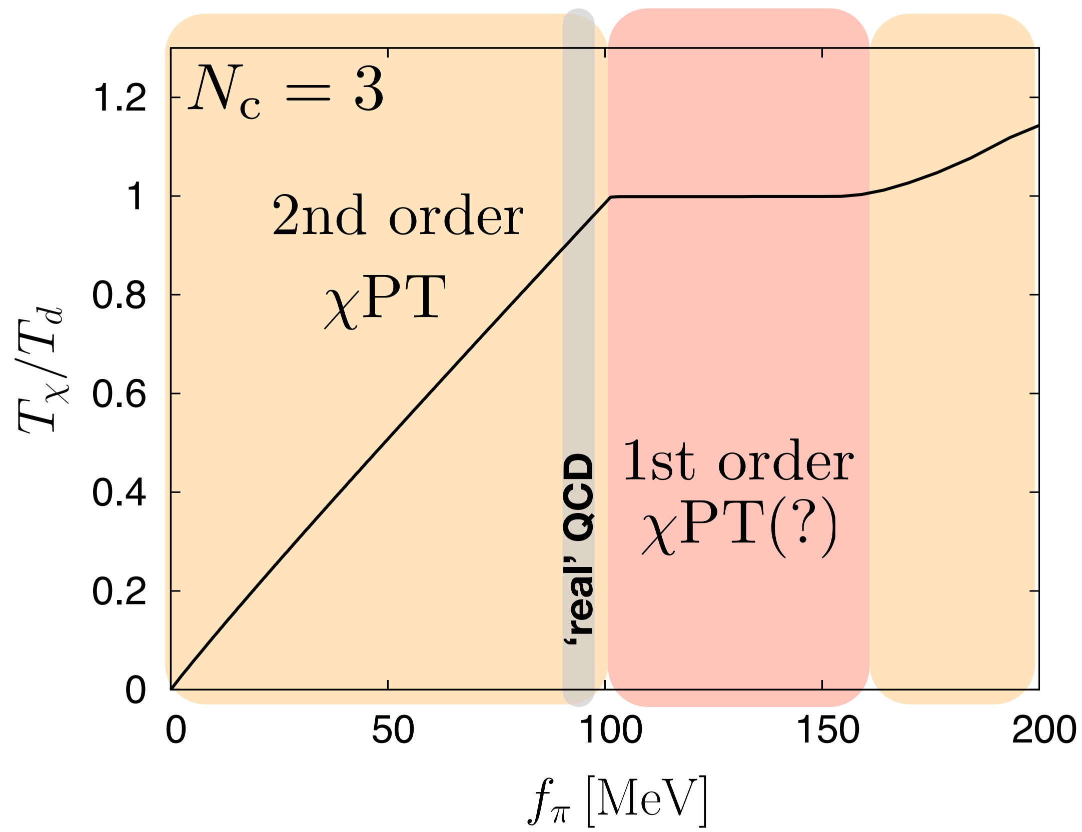

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- for finite N_c, we have three regimes: $T_\chi < T_d$, $T_\chi \approx T_d$ and $T_\chi > T_d$
- below the locking window, we have $T_\chi \sim (\lambda_\psi(\Lambda) - \lambda_\psi^*)^{1/2} \sim f_\pi$
- window with a chiral phase transition of first order **may** open up due to a rapidly rising deconfinement order parameter

Conclusions

- locking mechanism for the phase boundary from a study of the (non-perturbative) fixed-point structure
- chiral phase transition is locked to the deconfinement phase transition for $N_c \to \infty$: $T_\chi \gtrsim T_{\mathrm{conf.}}$
- locking mechanism works also beyond the large N_c-limit
- testable prediction; does not suffer from problems present at finite quark chemical potential

Outlook

- more deformations: finite (current) quark masses, finite quark chemical potential, ...