

# Instabilities in anisotropically expanding Quark-Gluon Plasma \*

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\*“ Dans la vie, rien n'est à craindre, tout est à comprendre.” Marie Curie

# Hard Expanding Loops (HEL)

## Hard Expanding Loops (HEL)

Stages of the Little Big Bang

Weibel instabilities

Scales QGP

Hard (Thermal) Loops - Boltzmann - Vlasov

Notations for Bjorken expansion

## Plasma Instabilities

Expanding 1D+3V Abelian plasma

Expanding 3D+3V plasma

Conclusions



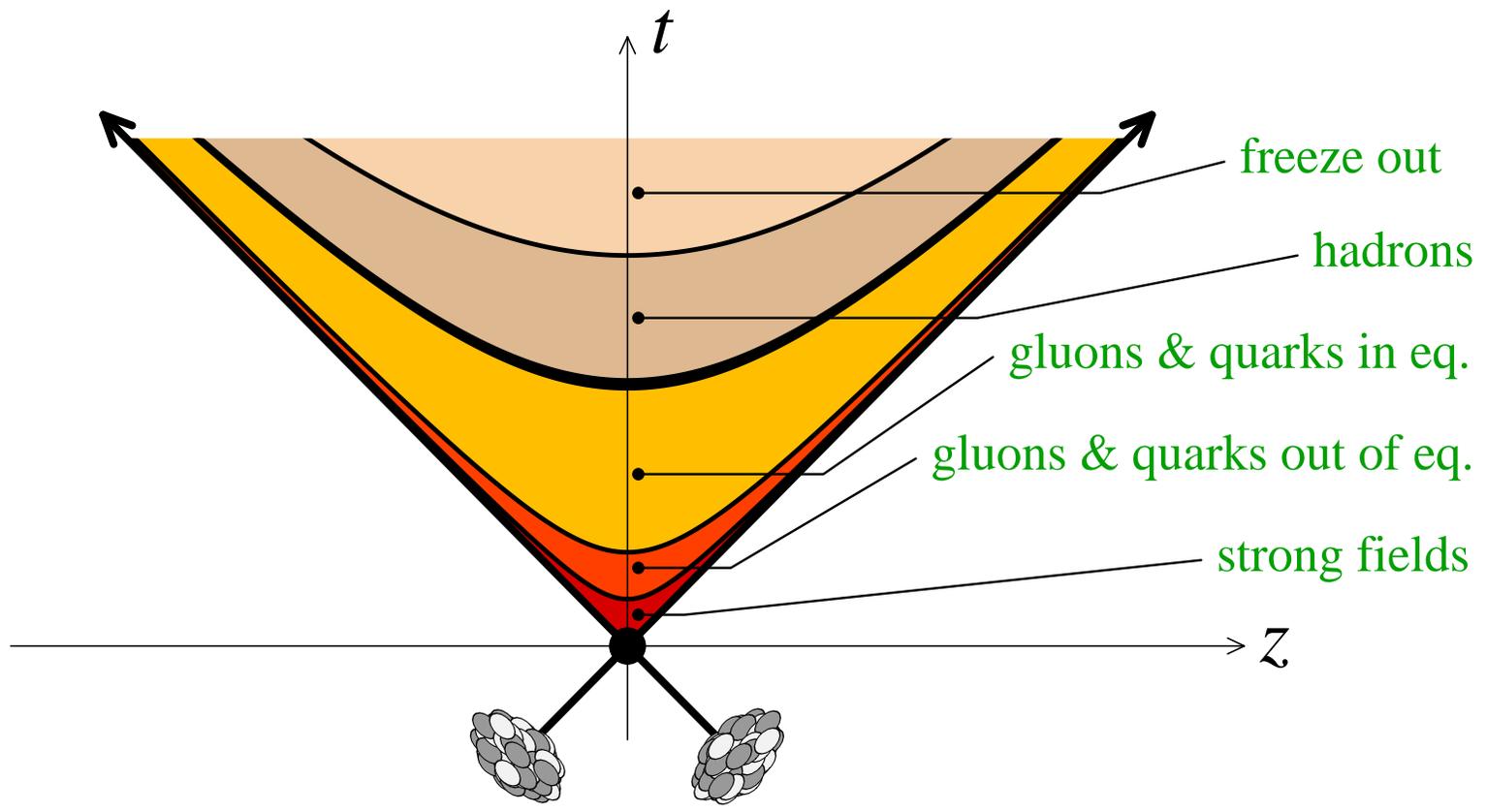
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# Stages of the Little Big Bang

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[Gelis 2010] Illustration of the stages of a heavy ion collision.

# Weibel instabilities

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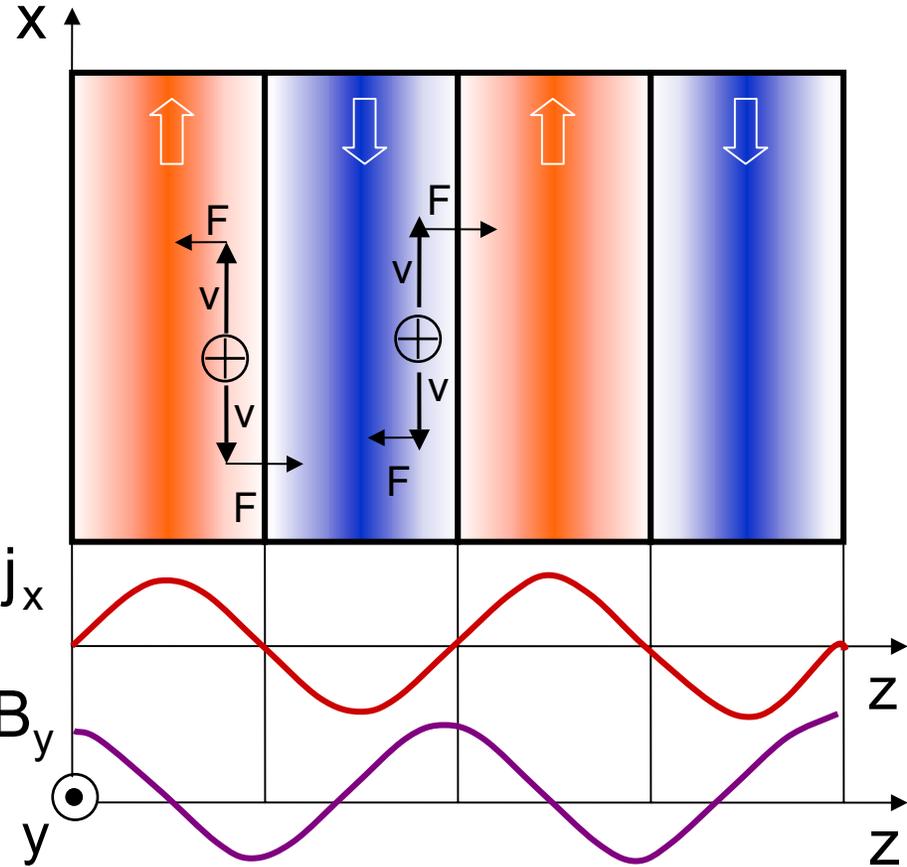
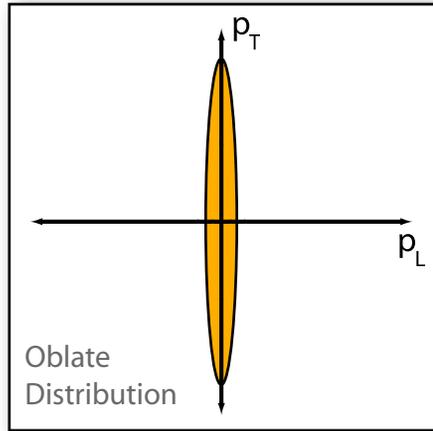
Vlasov

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Induced Current

Magnetic Fluctuation

Illustration of the mechanism of filamentation instabilities.

# QED Plasma

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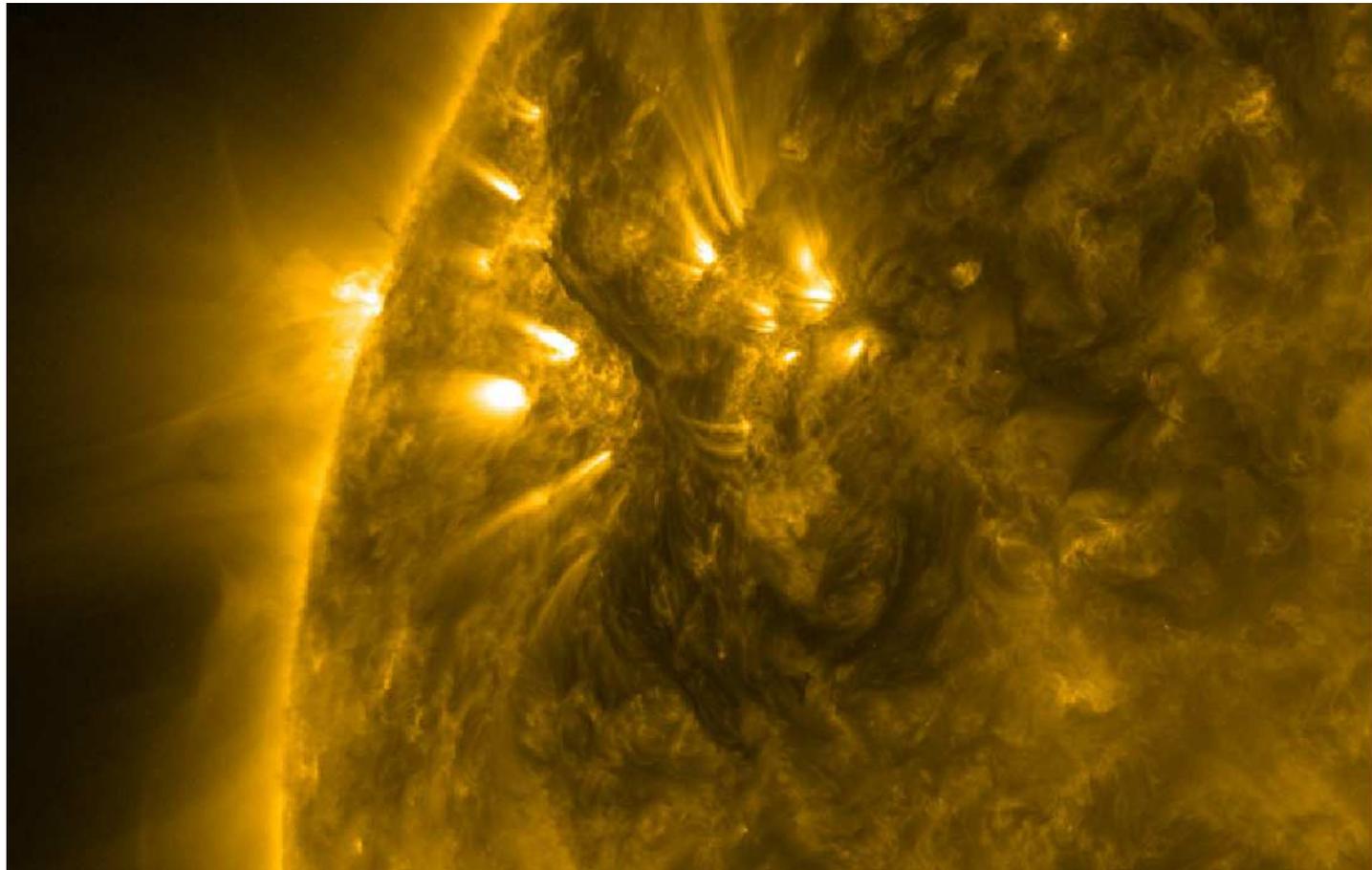
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Filaments and active solar region from NASA's Solar Dynamics Observatory

# Scales of weakly coupled QGP



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- $T$ : energy of hard particles
- $gT$ : thermal masses, Debye screening mass, Landau damping, **plasma instabilities** [Mrowczynski 1988, 1993, ..]
- $g^2T$ : magnetic confinement, color relaxation, rate for small angle scattering
- $g^4T$ : rate for large angle scattering,  $\eta^{-1}T^4$

# Hard (Thermal) Loops - Boltzmann - Vlasov

Assuming free streaming

$$v \cdot \partial f_0(\mathbf{p}, \mathbf{x}, t) = 0, \quad v^\mu = p^\mu / p^0 \quad (1)$$

solve the gauge covariant Boltzmann-Vlasov equation

$$v \cdot D \partial f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) \quad (2)$$

coupled to Yang-Mills equation

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t) \quad (3)$$

in the HTL approximation

$$g A_\mu \ll |\mathbf{p}_{hard}|. \quad (4)$$

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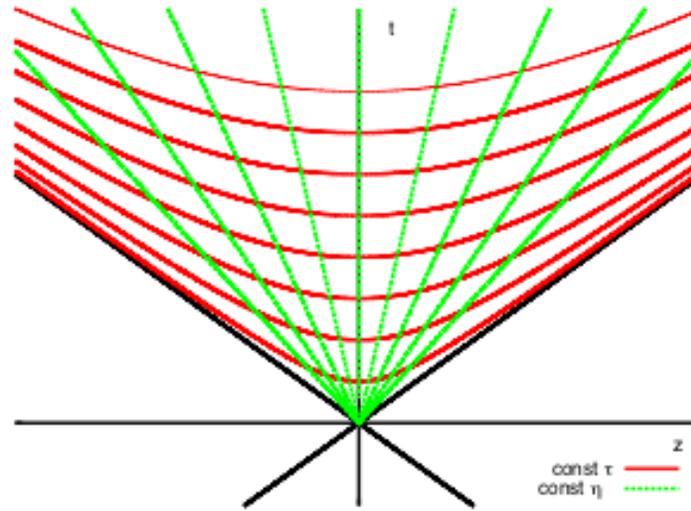
# Notations for Bjorken expansion

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It is convenient to switch to comoving coordinates

$$\begin{aligned} t &= \tau \cosh \eta, & \beta &= \tanh \eta, \\ z &= \tau \sinh \eta, & \gamma &= \cosh \eta, \end{aligned} \quad (5)$$

with corresponding metric  $ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2$ .

# Plasma Instabilities

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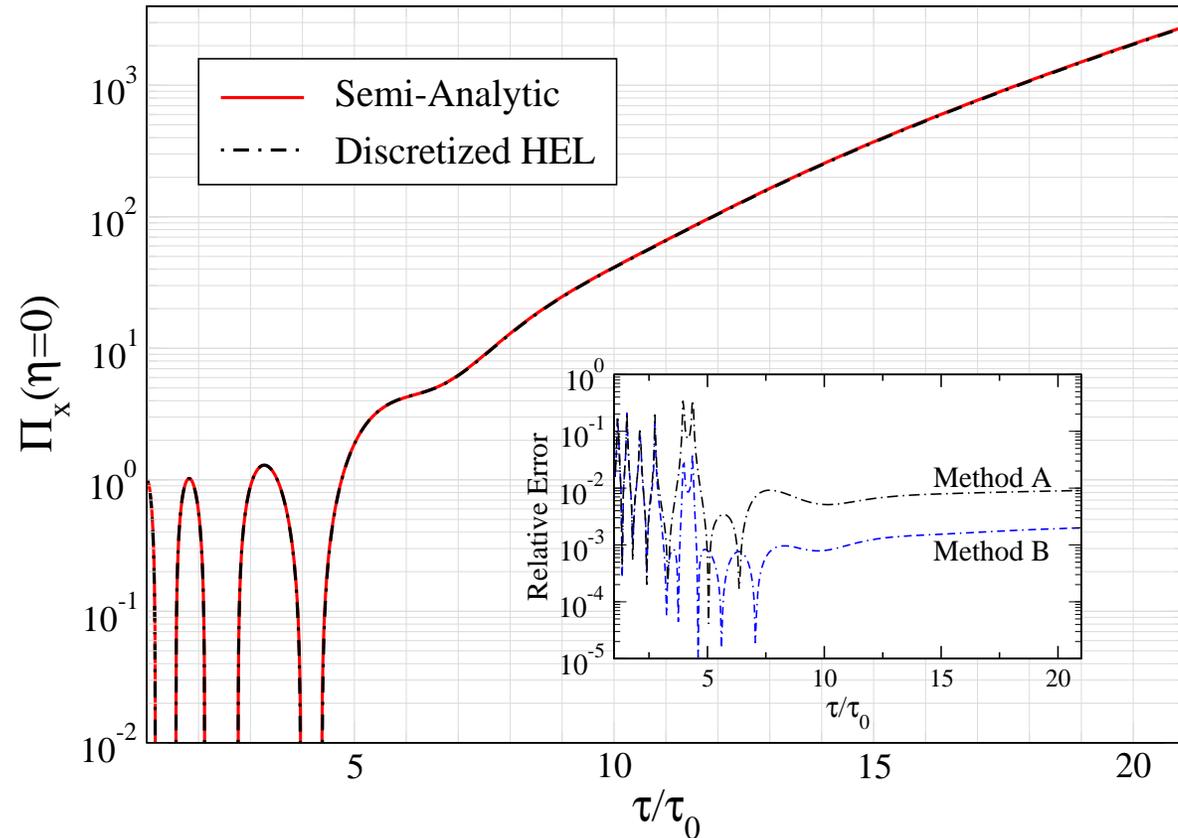
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# Expanding 1D+3V Abelian plasma

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[Rebhan, Strickland, Attems 2008] The proper-time evolution of the canonical field momentum of a single Abelian mode in comparison with [Romatschke, Rebhan 2006].

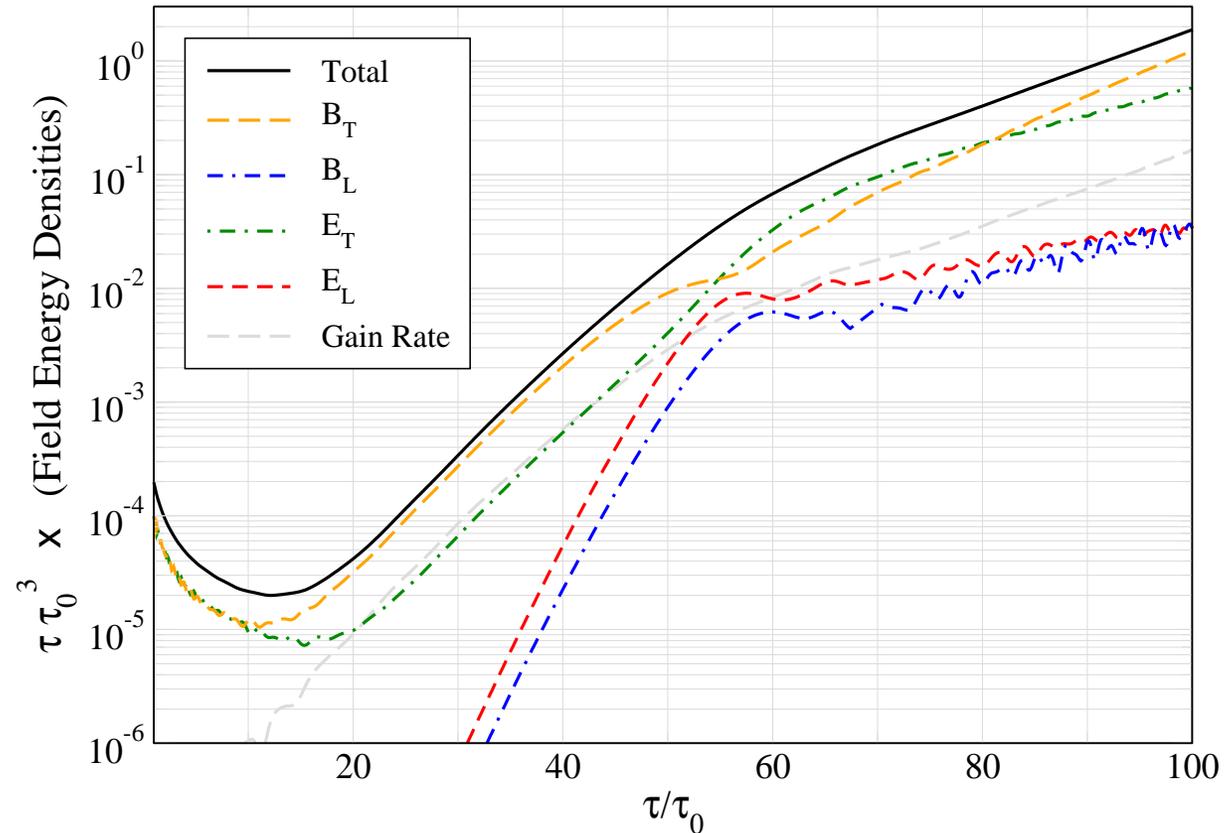
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[Rebhan, Strickland, Attems 2008] The proper-time dependence of the chromo-field energy densities and the energy gain rate times resulting from non-Abelian run initialized with [Fukushima, Gelis, and McLerran (FGM) 2007] initial conditions.

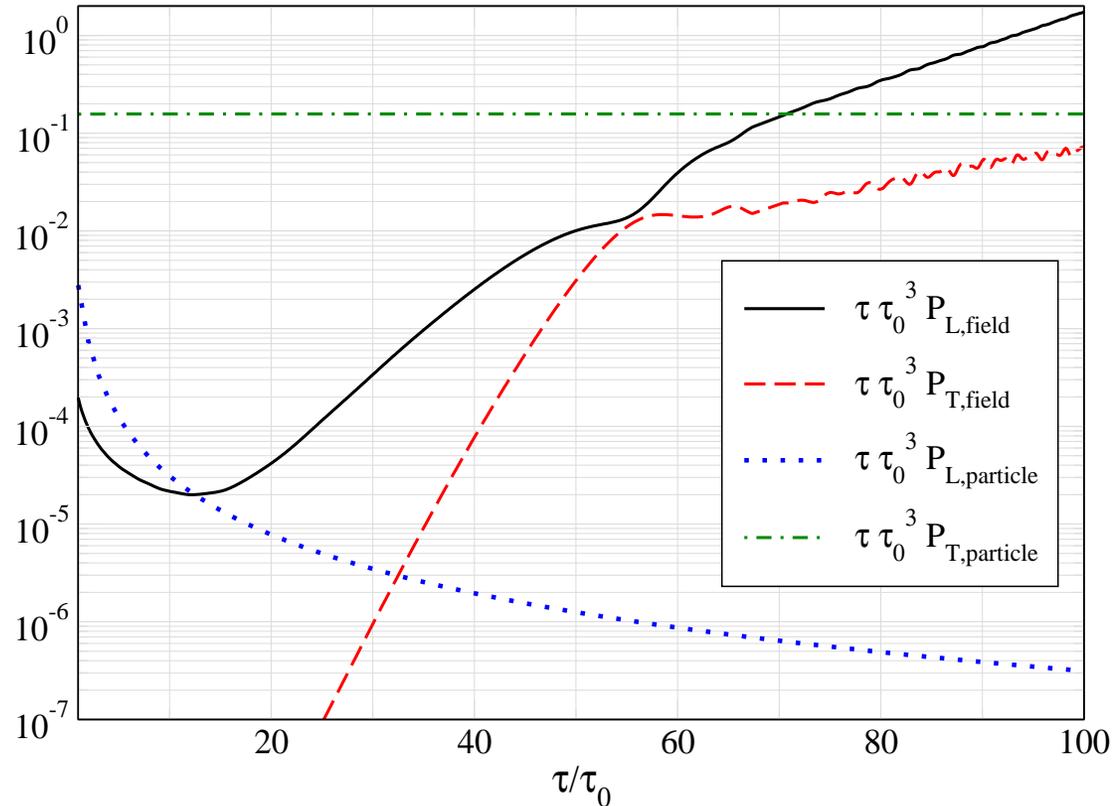
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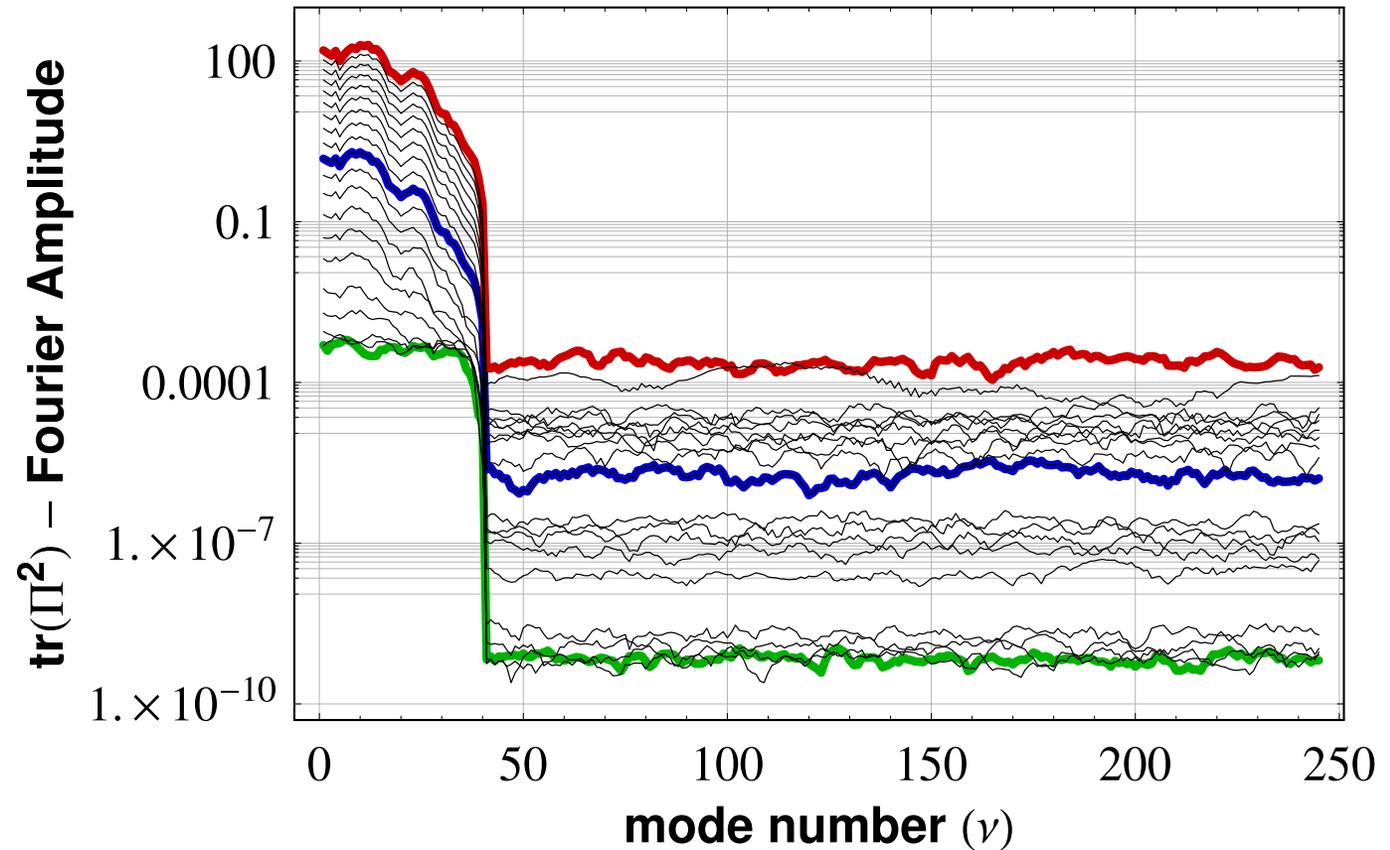


[Rebhan, Strickland, Attems 2008] The comparison of the longitudinal and transverse pressures for the fields and particles resulting from a typical non-Abelian run initialized with FGM (CGC inspired) initial conditions.

# Expanding 1D+3V Abelian plasma

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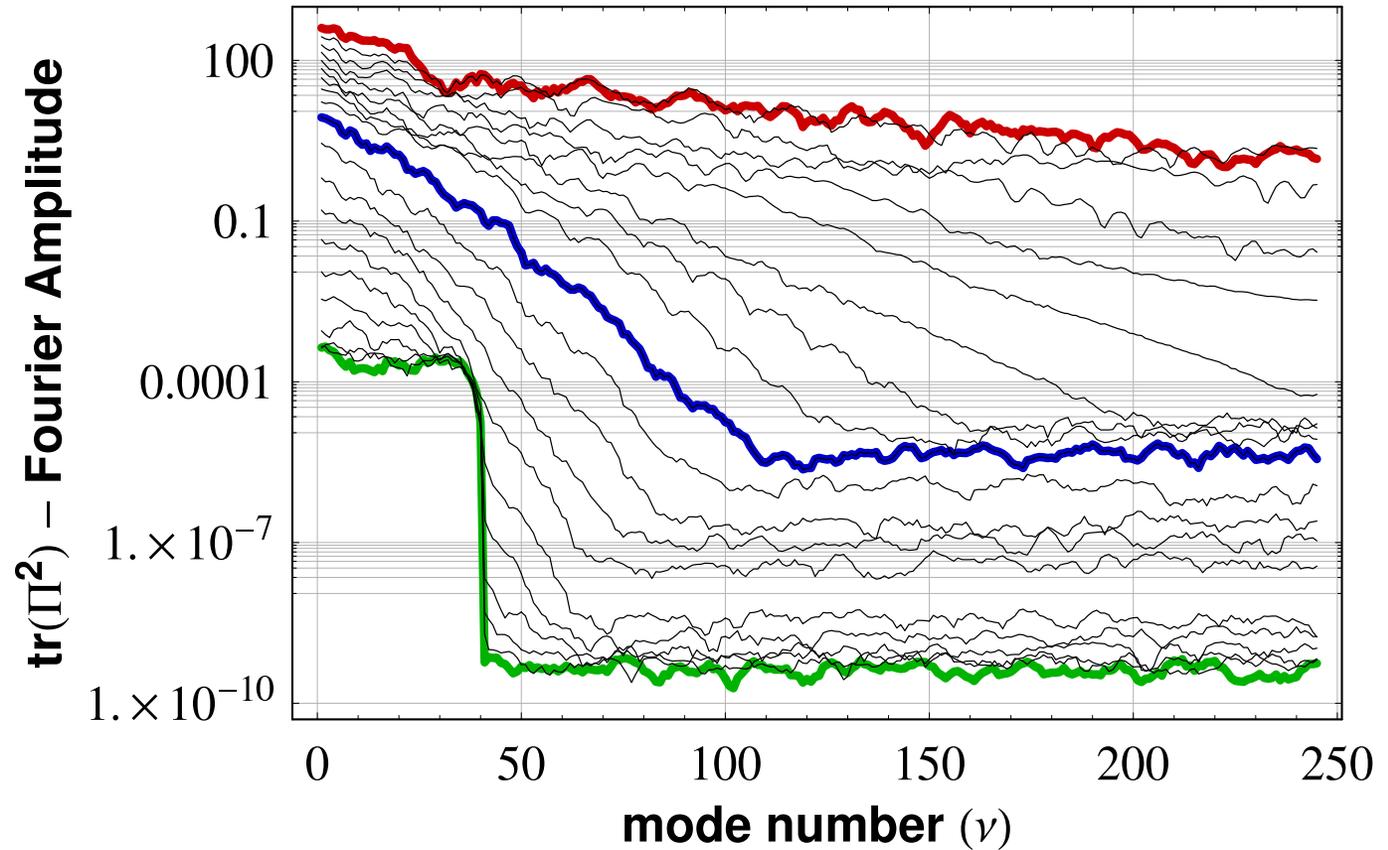


[Rebhan, Strickland, Attems 2008] Fourier spectrum of the color-traced conjugate field momentum obtained from Abelian run with FGM initial conditions.

# Expanding 1D+3V non-Abelian plasma

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[Rebhan, Strickland, Attems 2008] Fourier spectrum of the color-traced conjugate field momentum obtained from non-Abelian run with FGM initial conditions.

# Expanding 3D+3V plasma

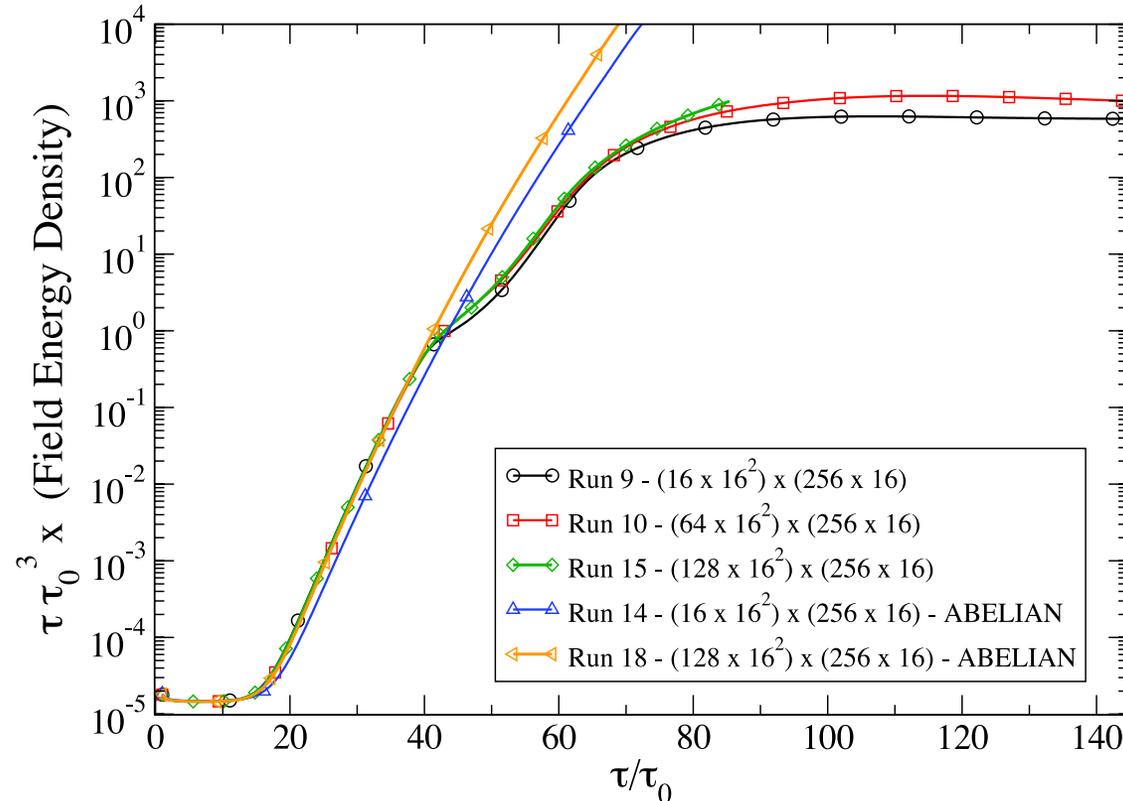
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[Attems, Rebhan, Strickland forthcoming] Preliminary runs from the HEL 3d codes in Abelian and non-Abelian setup with different lattice sizing's in the longitudinal  $\eta$  direction, but identical transverse size and  $\mathcal{W}$  auxiliary field numbers.

# Expanding 3D+3V non-Abelian plasma

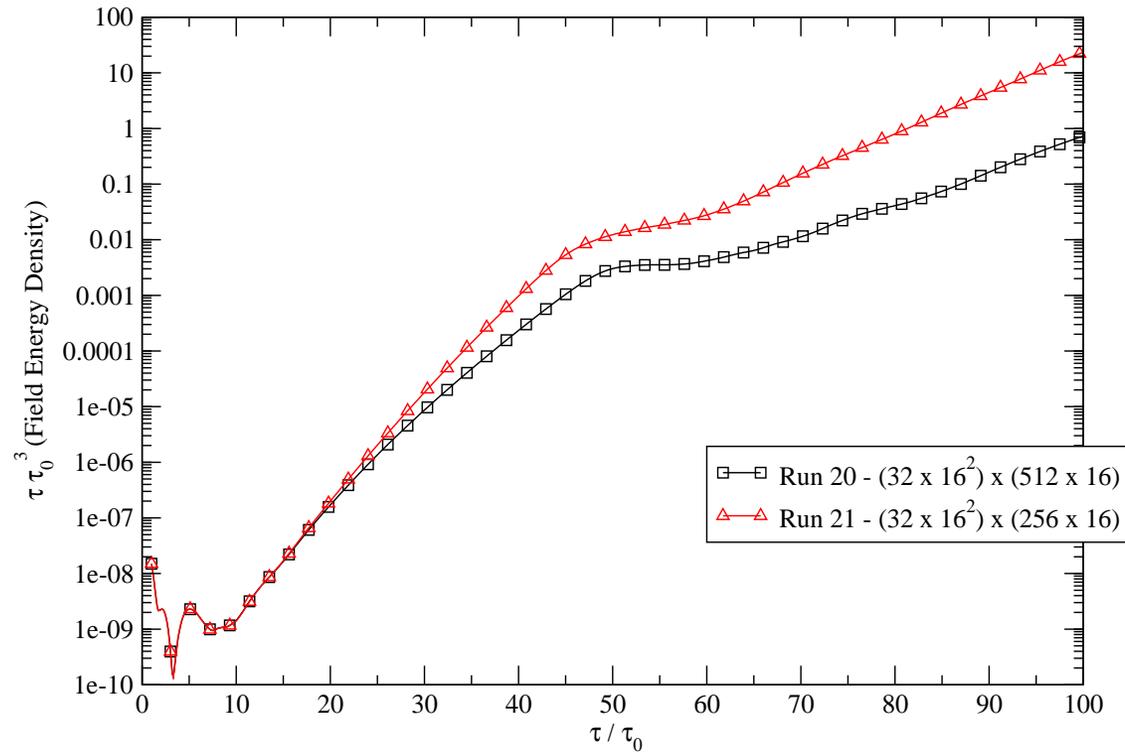
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[Attems, Rebhan, Strickland forthcoming] Preliminary runs from the HEL 3d parallel code of a non-Abelian single mode setup with identical longitudinal and transverse lattice sizing, but different  $\mathcal{W}$  auxiliary field numbers.

# Conclusions

Non-abelian plasma instabilities accelerate isotropization and thermalization of the Quark Gluon Plasma.

Large amplitude turbulent field configurations can have an important effect on Quark Gluon Plasma transport such as momentum broadening, energy loss, plasma viscosity, ...

In the 1D+3V Hard Expanding Loop (HEL) 1D we found that the exponential (in  $\sqrt{\tau}$ ) growth in the Abelian (weak-field) phase is only mildly weakened when nonlinearities through non-Abelian self-interactions of the collective fields set in.

The previous 1D HEL code has been extended to full 3D+3V. Final results including different initial conditions are being computed.



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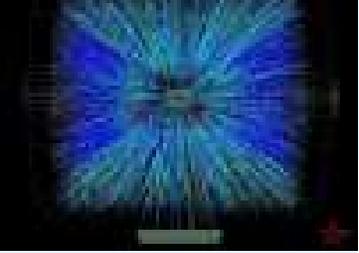
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# Backup - Expanding 1D+3V non-Abelian plasma



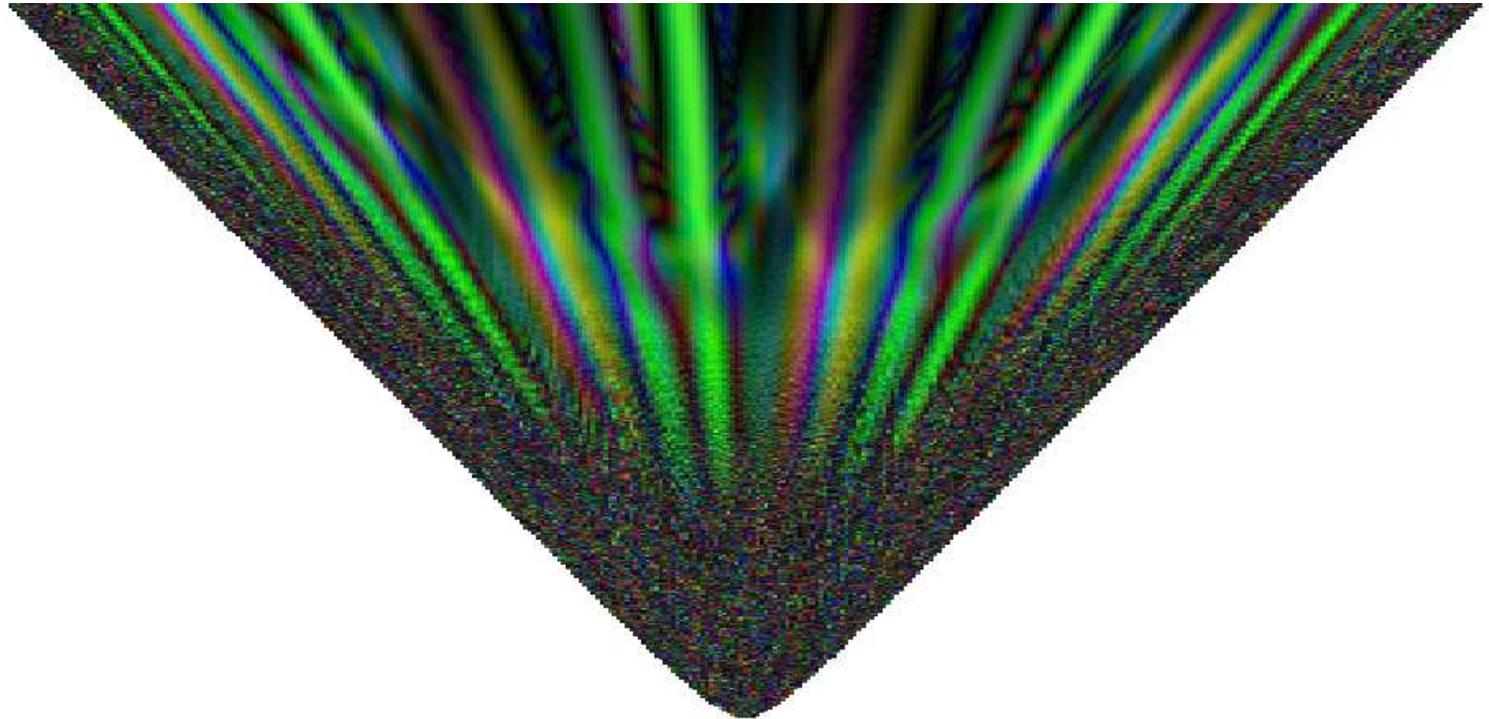
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[Strickland 2008] Visualization of the space-time development of color correlations in a non-Abelian plasma instabilities in Bjorken expansion.

# Backup - Equation of motions

## Conjugate Momenta

$$\partial_\tau E_i = +\tau j^i + \frac{1}{\tau} D_\eta^2 A^i + \tau g^2 i [A^{j \neq i}, i [A^{j \neq i}, A^i]] \quad (6)$$

$$\partial_\tau E^\eta = -\tau j^\eta + \frac{ig}{\tau} [A^i, D_\eta A^i] \quad (7)$$

## Gauss law

$$j^\tau = +\frac{1}{\tau} D_\eta E^\eta - \frac{ig}{\tau} [A_i, E^i] \quad (8)$$

with

$$E^i \equiv \tau \partial_\tau A_i, \quad E^\eta \equiv \frac{1}{\tau} \partial_\tau A_\eta \quad (9)$$

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# Backup - Unstable transverse modes

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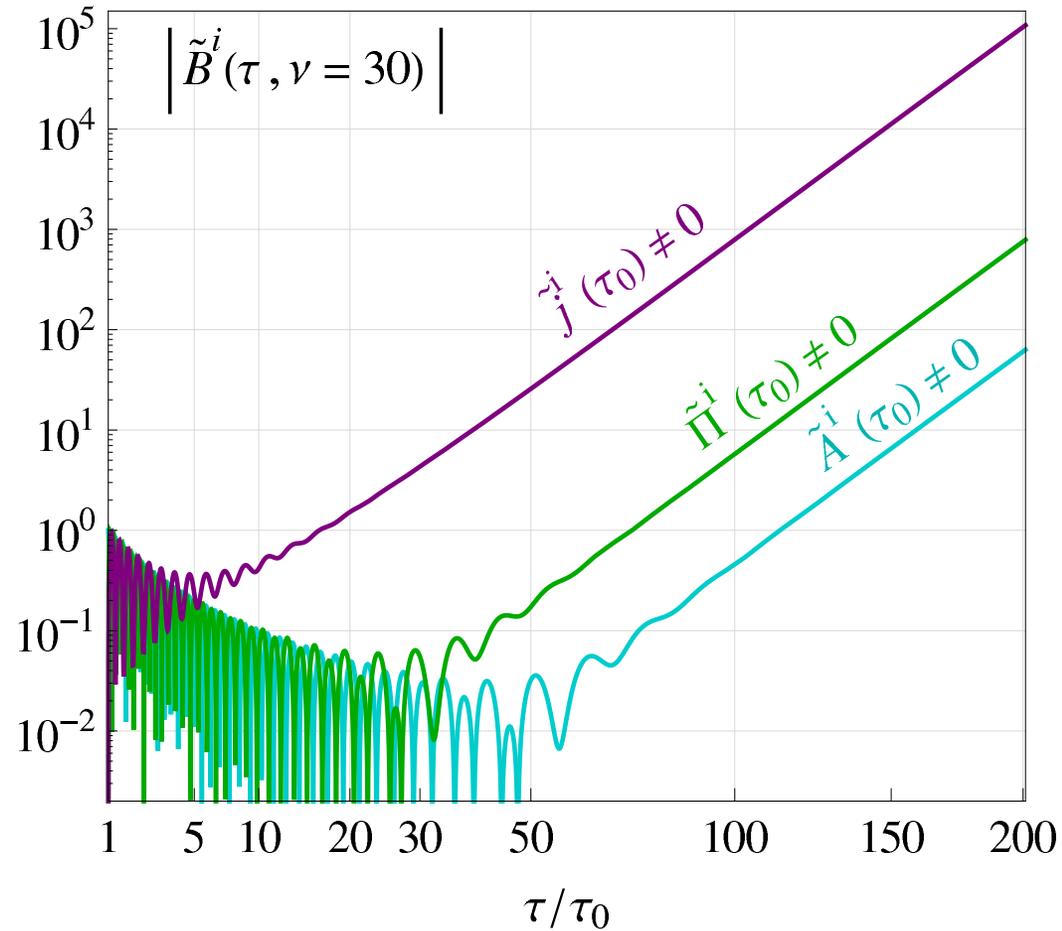
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[Rebhan, Steineder 2009] Influence of different initial conditions for a specific mode with  $\nu = 30$ .

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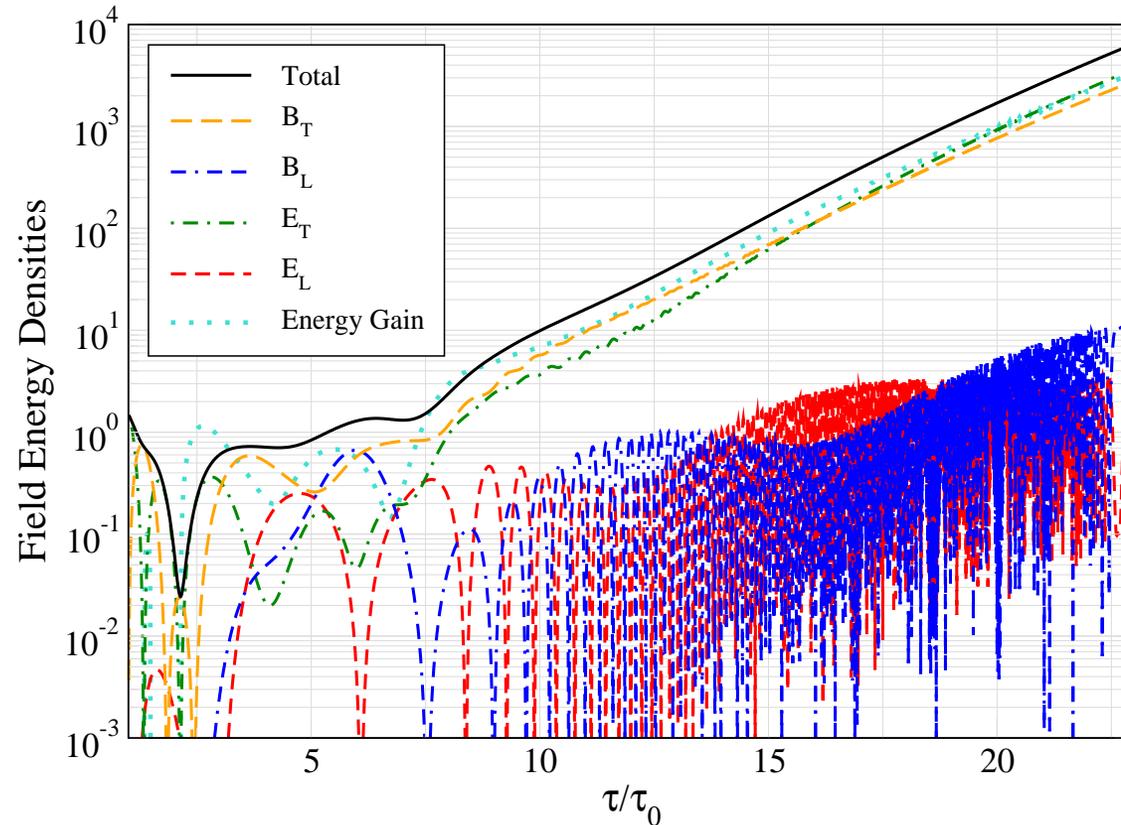
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[Rebhan, Strickland, Attems 2008] The proper-time dependence of the chromo-field energy densities from a run with a single non-Abelian mode seeded with random noise.