# On the dynamically generated quark-gluon and scalar-gluon couplings at zero and non-vanishing temperatures

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*Quarks, Gluons, and Hadronic Matter under Extreme Conditions*

St. Goar, March 17, 2011

# Outline

1. Landau Gauge QCD Green's Functions @ $T = 0$
   - Infrared Structure of Landau gauge Yang-Mills theory
   - Quarks: Confinement vs. D$\chi$SB & $U_A(1)$ anomaly
   - Fundamentally charged scalar field
2. Landau Gauge QCD Green's Functions @ $T \neq 0$
   - Quark propagator
   - Scalar Propagator

## Why QCD Green's functions?

- ★ They embody confinement, D$\chi$SB, and the axial anomaly!
- ★ They provide input into hadron phenomenology:
  - bound-state equations for mesons and baryons
    masses, form factors, meson production, GPDs, decays, ...
- ★ They describe the phases of QCD!

Landau gauge:
Established knowledge on the **7** primitively divergent Green's functions at $T = 0$ & $\mu = 0$!

*cf.* the talks by L. Fister, J. Lücker, J.M. Pawlowski, ...

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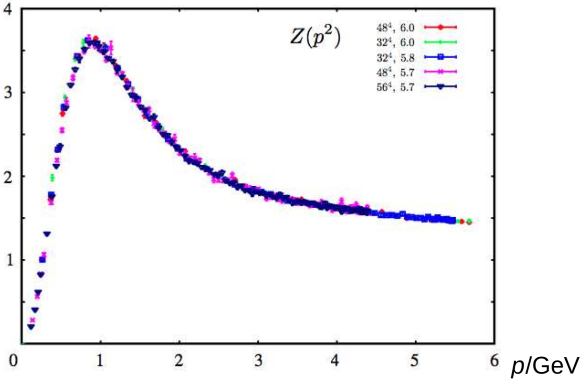
# Gluon Propagator

pure Yang-Mills, $T = 0$
Landau gauge Gluon Ren. Fct. $D^{tr}_{\text{Gluon}} = \mathbf{Z(p^2)}/p^2$

A. Sternbeck *et al.*, PoS LAT2006, 76



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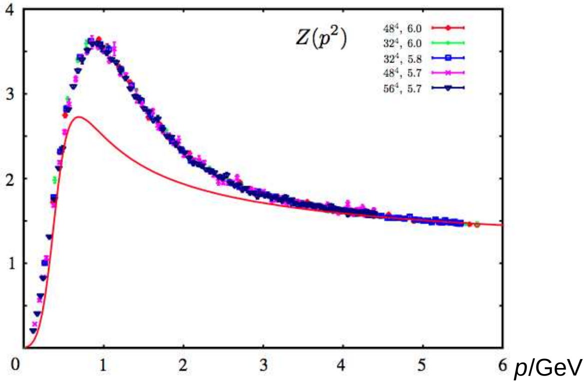


— L. von Smekal, A. Hauck, R.A., Phys. Rev. Lett. 79 (1997) 3591
Dyson-Schwinger eqs. (DSEs)

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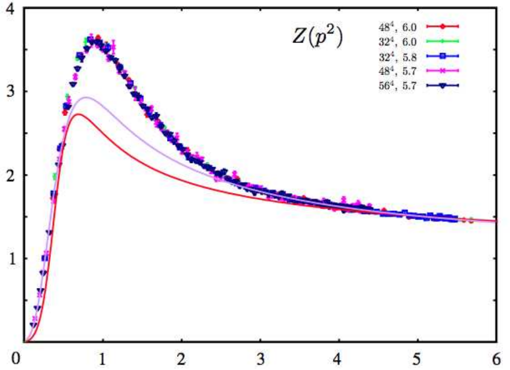
A. Sternbeck *et al.*, PoS LAT2006, 76




— L. von Smekal, A. Hauck, R.A., Phys. Rev. Lett. **79** (1997) 3591

— C. S. Fischer, R.A., Phys. Lett. **B536** (2002) 177;

C. Lerche, L. von Smekal, Phys. Rev. **D65** (2002) 125006.

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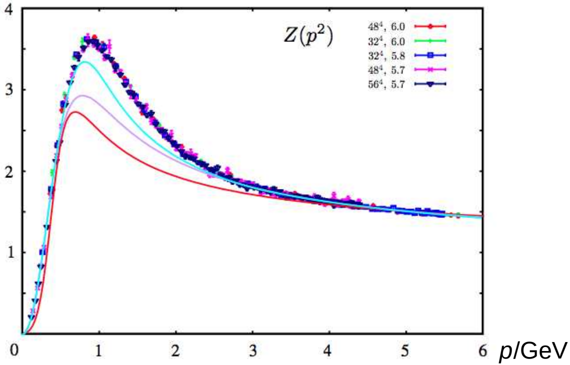


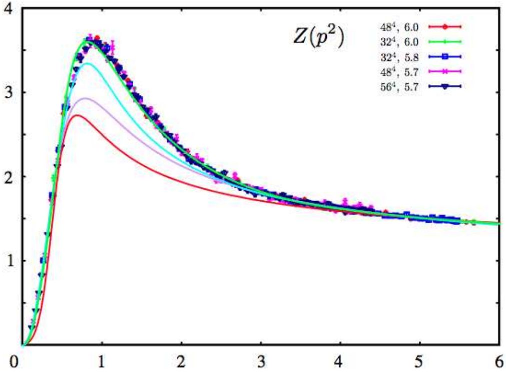

— L. von Smekal, A. Hauck, R.A., Phys. Rev. Lett. **79** (1997) 3591

— C. S. Fischer, R.A., Phys. Lett. **B536** (2002) 177

— J.M.Pawlowski, D.Litim, S.Nedelko, L.v.Smekal, PRL**93** (2004) 152002

Functional Renormalization Group (FRG) eqs.

A. Sternbeck *et al.*, PoS LAT2006, 76



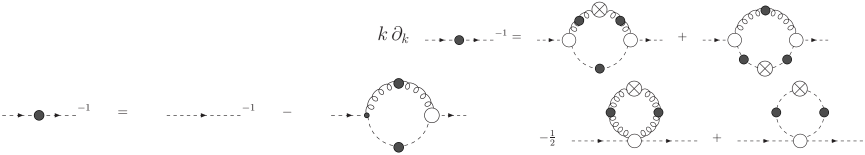

$p$/GeV

- — L. von Smekal, A. Hauck, R.A., Phys. Rev. Lett. **79** (1997) 3591
- — C. S. Fischer, R.A., Phys. Lett. **B536** (2002) 177
- — J.M.Pawlowski, D.Litim, S.Nedelko, L.v.Smekal, PRL**93** (2004) 152002
- — C.S. Fischer, A. Maas, J.M. Pawlowski, Ann. Phys. **324** (2009) 2408; J.M. Pawlowski, unpublished

# Infrared Exponents for Gluons and Ghosts

Use DSEs **and** FRG eqs:

→ Two different towers of equations for Green functions
E.g. ghost propagator

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MATHEMATICA-based derivation of functional equations:

R. A., M. Q. Huber, K. Schwenzer, Comp. Phys. Comm. **180** (2009) 965;
M. Q. Huber and J. Braun, *DoFun*, arXiv:1102.5307 [hep-th].

# Infrared Exponents for Gluons and Ghosts

IR-Analysis of two different towers of equations $\Rightarrow$

Unique **scaling** solution

(C.S. Fischer and J.M. Pawlowski, PRD **75** (2007) 025012; PRD **80** (2009) 025023)

as an endpoint of an one-parameter family of solutions with IR trivial Green functions (the **decoupling** solutions).

C.S. Fischer, A. Maas and J.M. Pawlowski, AoP **324** (2009) 2408.

A.C. Aguilar, D. Binosi , J. Papavassiliou, PRD **78** (2008) 025010 and refs. therein.

**Difference of fundamental interest**

**but phenomenologically irrelevant!**

Use scaling solution for qualitative arguments . . .

**Scaling solution:**
$n$ external ghost & antighost legs and $m$ external gluon legs
(one external scale $p^2$; **solves functional eqs. and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Ghost propagator IR divergent
- Gluon propagator IR suppressed
- Ghost-Gluon vertex IR finite
- 3- & 4- Gluon vertex IR divergent

★ IR fixed point for the coupling from each vertex

# General Infrared Exponents for Gluons and Ghosts

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# Picturing Gluon Confinement

DSE scaling solution of Yang-Mills theory:

- Gluon propagator vanishes on the light cone, and
- *n*-point gluon vertex functions diverge on the light cone!

$\Rightarrow$ Attempts to kick a gluon free (*i.e.* to produce a real gluon) immediately results in production of infinitely many virtual soft gluons!

$\Rightarrow$ perfect color charge screening
+ positivity violation
(which implies BRST quartet cancelation, *cf.* poster by N. Alkofer):

**Gluon confinement!**

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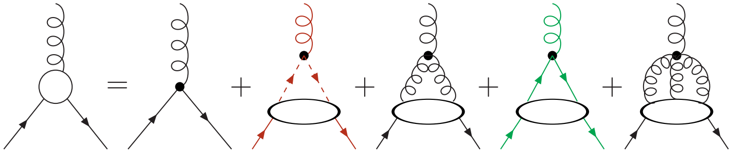
# Dynamically induced scalar quark confinement

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals Phys. **324** (2009) 106.

Quark-gluon vertex:



**Quark diagram**: Hadronic contributions ('unquenching')

**Ghost diagram: Infrared leading!**

# Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitely broken:
quark propagator infrared finite

$$S(p) = \frac{\not p + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not p}{M^2} + \frac{Z_f}{M}$$

**AND**

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G^i_\mu \,, \quad G^1_\mu = \gamma_\mu \,, \; G^2_\mu = \hat{p}_\mu \,, \; G^3_\mu = \ldots$$

**WITH** $\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa}$

INFRARED DIVERGENT

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**I N F R A R E D   D I V E R G E N T**

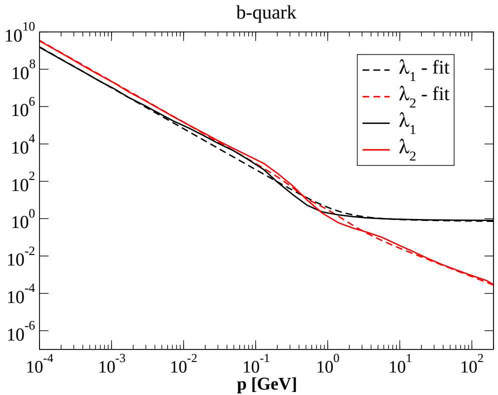
# Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitely broken:

$$\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa} \text{ i.e.}$$

**Quark-Gluon vertex IR divergent!**

Scalar component $\lambda_2$ in IR even **larger** than vector component $\lambda_1$!

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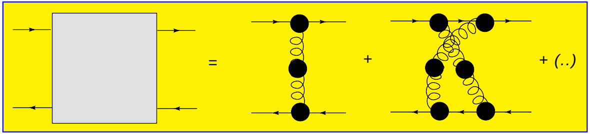
# Dynamically induced scalar quark confinement

"Quenched´´ quark-antiquark potential




infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{pr}} \quad \sim \quad |\mathbf{r}|$$

**i.e. linear, dominantly scalar, quark confinement!**

# $U_A(1)$: $\eta'$ mass from IR divergent Green functions

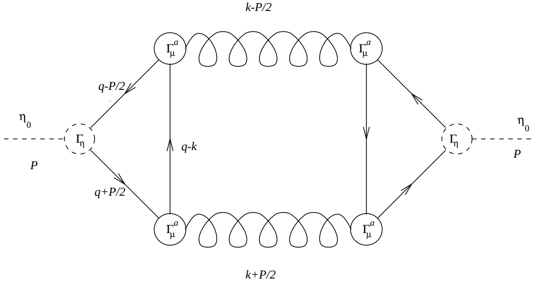
R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

$U_A(1)$ symmetry anomalous $\Rightarrow \eta'$ mass $\gg \pi$ mass

**Where is this encoded in the Green functions?**

(J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.)

E.g. in:




$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

# $\eta'$ mass from IR divergent Green functions

Infinitely many diagrams ($n$-gluon exchange) contribute!

**Diamond diagram only** with DSE results for the gluon and quark propagators and quark-gluon vertex:

$\chi^2 \approx (160\mathrm{MeV})^4$ vs. phenomenological value $(180\mathrm{MeV})^4$

results in: $m_\eta = 479$MeV, $m_{\eta'} = 906$MeV, $\theta = -23^0$.

Conclusion:
(Fluct.) topologically non-trivial fields $\Leftrightarrow$ IR singularities of GF!

Quark confinement $\Rightarrow$ $U_A(1)$anomaly!

# Picturing quark confinement

**Implication of YM-DSE scaling solution for quark sector:**

- quark propagator IR trivial (D$\chi$SB),
- quark-gluon vertex functions including a self-consistently generated scalar quark-gluon coupling (D$\chi$SB!) diverge on the quark “mass´´ shell!

$\Rightarrow$ Attempts to kick a quark free (*i.e.* to produce a real quark) immediately results in production of infinitely many virtual soft gluons!

$\Rightarrow$ linearly rising potential
*i.e.*, infrared slavery:

**Quark confinement!**

String formation? Relation to Chiral Symmetry [Breaking]? ...? ...?

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# Fundamentally charged scalar field

⚠

Coupled system of quark propagator and quark-gluon vertex fragile!

- $T = 0$: Twelve tensor components of quark-gluon vertex!
- $T \neq 0$: **32** (mostly lin. ind.) tensors!!!

+ $T = 0$: Only two independent components of scalar-gluon vertex.
+ $T \neq 0$: Only four components!

$\Rightarrow$ **Employ fundamentally charged scalar field as a "laboratory" for quark dynamics!**

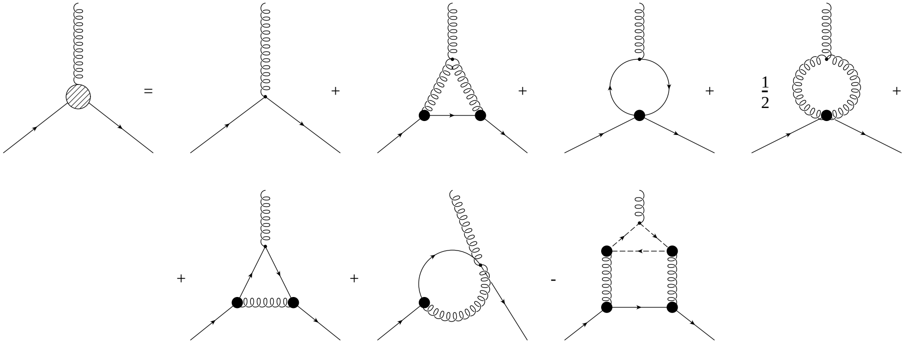
See also related lattice calculations
A. Maas, Eur. Phys. J. **C71** (2011) 1548 [arXiv:1007.0729 [hep-lat]];
PoS **FacesQCD** (2011) 033 [arXiv:1102.0901 [hep-lat]].

# Fundamentally charged scalar field

L. Fister, R.A., K. Schwenzer, Phys. Lett. **B688** (2010) 237;

L. Fister, Diploma Thesis Graz 2009, arXiv:1002.1649[hep-th].

DSE for scalar-gluon vertex:



IR analyis of DSEs:

- Uniform IR scaling:
  - Scaling and decoupling solutions found!
  - Massive case /scaling solution:
    Same IR power law for scalar-gluon as for quark-gluon vertex!

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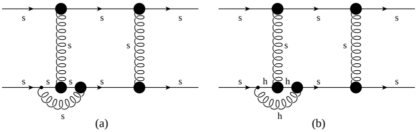
  - Massive case /scaling solution:
    Same IR power law for scalar-gluon as for quark-gluon vertex!
- Kinematic singularities in quenched approximation:
  - Soft-gluon same as uniform divergence!
  - Leading diagrams:

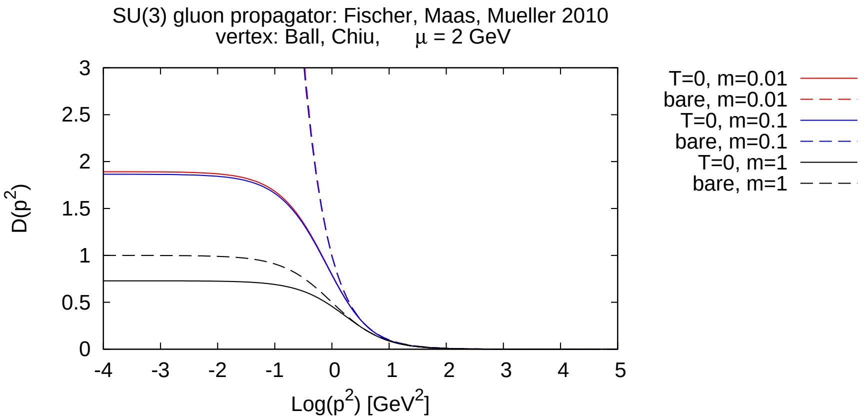



Linear confinement as $V(r) \propto r$!

**Actually holds for matter fields in any representation of the Lorentz group: Confinement of fundamental $SU(N)$ charges!**

# Fundamentally charged scalar field

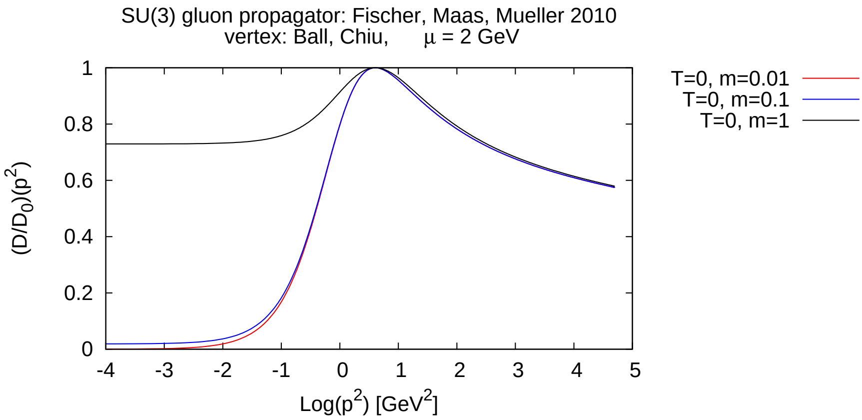
## Scalar propagator with Ball-Chiu scalar-gluon vertex:




**If available agreement with lattice data!**

# Fundamentally charged scalar field

Scalar propagator **dressing function** with BC scalar-gluon vertex:

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# $T \neq 0$

# Landau gauge propagators:

$T \neq 0$:

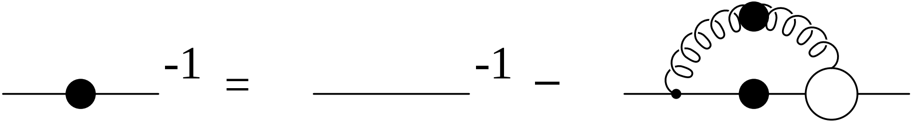
**Landau gauge propagators:**

- $D^{\mathrm{Gluon}}_{\mu\nu} = \frac{\mathbf{Z_T(p^2)}}{p^2} P^T_{\mu\nu}(p) + \frac{\mathbf{Z_L(p^2)}}{p^2} P^L_{\mu\nu}(p)$
- $S(p) = \frac{1}{-i\vec{\gamma}\vec{p}\mathbf{A(p)} - i\gamma_4\omega_p \mathbf{C(p)} + \mathbf{B(p)}}$
- $D_S(p) = \frac{\mathbf{Z_S(p)}}{p^2 + m^2_{bare}}$

$T = 0$: $Z_T = Z_L = Z$ and $C = A$, functions of $p^2$ only

$T \neq 0$: Propagator functions depending on $\vec{p}^2$ and $\omega_p$ ($p = (\omega_p, \vec{p})$)
transv. to medium $\rightarrow$ chromomag. / long. to medium $\rightarrow$ chromoel.

$T \rightarrow \infty$: 3-dimensional Yang-Mills theory + “Higgs´´ ($A_4$)




State of the art: See talk of Jan Lücker from Tuesday!

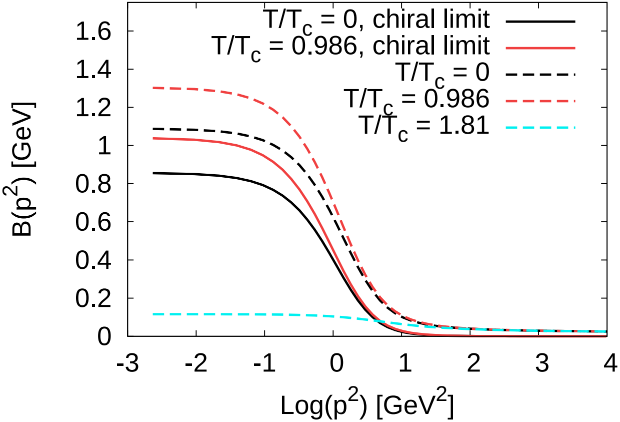
Input:

+ Dressed, fully $T$-dependent, unquenched gluon propagator.
- *Quark-gluon vertex not dynamical* but STI-motivated ansatz

$$\Gamma_\mu^a = t^a \gamma_\mu \Gamma(k^2) \left( \delta_{\mu,4} \frac{1}{2}(C(p) + C(q)) + \delta_{\mu,i} \frac{1}{2}(A(p) + A(q)) \right)$$

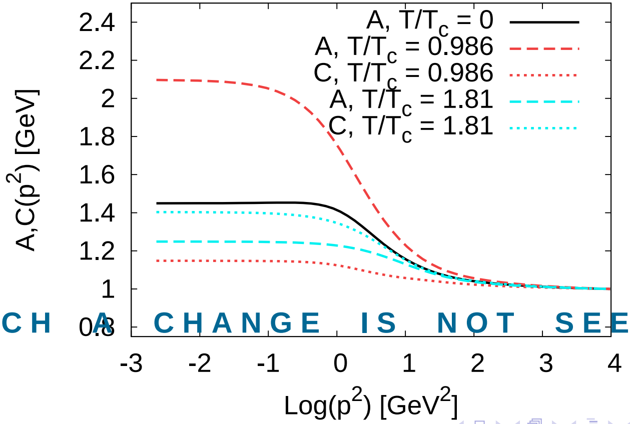
# Quark Propagator $T \neq 0$

Phase / crossover transition:
dynamically generated $\chi$SB function $B$ vanishes!



# Quark Propagator $T \neq 0$

Deconfinement transition:
Change in quark renormalization functions $Z_4 = 1/C$, resp., $Z_i = 1/A$?

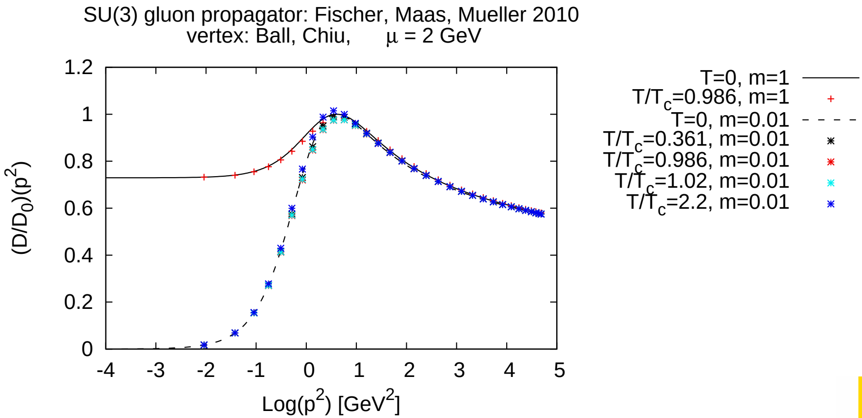



**SUCH A CHANGE IS NOT SEEN!**

# Scalar Propagator $T \neq 0$

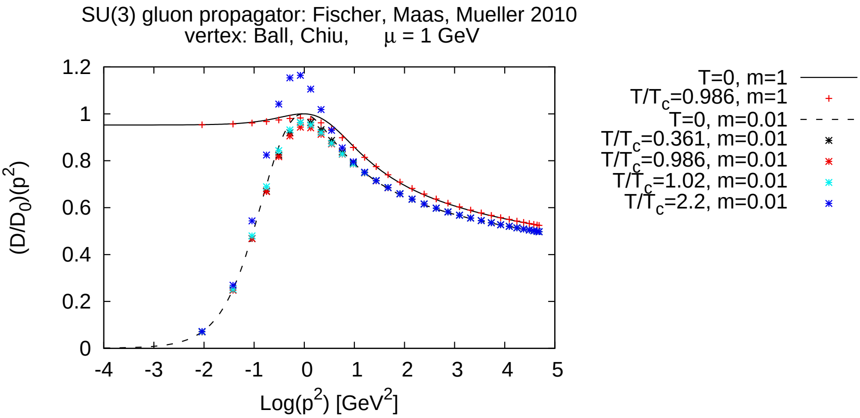
Input scalar propagator DSE:

+ Dressed fully $T$-dependent gluon propagator.
- Scalar-gluon vertex: $T$-independent STI-motivated ansatz.



# Scalar Propagator $T \neq 0$

Lowering renormalization scale:



# Green's Function $T \neq 0$

*Result for scalar propagator only very weakly $T$-dependent?!?*

## Possible interpretation:

Confinement / deconfinement transition not seen
because matter-gluon vertex is not dynamical?

## Current investigation:

- Dressed Polyakov loop (*cf.* C. Gattringer *et al.*) from scalar propagator.
- Solution of (truncated) **coupled** scalar propagator and scalar-gluon vertex DSEs.

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**Landau gauge QCD Green functions**

- Landau gauge YM theory: Gluons confined (positivity violated).
- Chiral symmetry dynamically broken! In 2- and **3**-point function!
- Quark confinement: In IR dominantly of scalar type!
- $U_A(1)$ anomaly: topology $\leftrightarrow$ infrared domain

- Drastic changes of long. gluon / quark propagators and (dual) condensates at $T_c$
- Scalar propagator with $T$-ind. scalar-gluon vertex: No qualitative change at $T_c$!?!

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- self-consistently calculated scalar-gluon-vertex &
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  $\Rightarrow$ nature of (phase) transition?
- self-consistently calculated quark-gluon-vertex
  ($\chi$SB-breaking part!?!)
  & dual quark condensate (resp., dressed Polyakov loop)
  in different phases

. . . still a lot of work . . .

Thermodynamic observables at all $T$ and $\mu$!

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