Landau gauge gluon and ghost propagators at non-zero temperature on the lattice.

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Motivation and aims

- Compute Landau gauge gluon and ghost propagators in pure $SU(3)$ gauge theory within lattice discretization.
- Study of systematic effects: lattice artifacts, Gribov copies, finite-size effects.
- Main aims:
  - provide continuum limit results within some momentum range,
  - to see, in as far gluon and ghost propagators can be used as "order parameters" for the deconfinement transition or crossover.
- Results to be compared with outcome from Dyson-Schwinger (DS) and Functional Renormalization Group (FRG) equations and to be used for tuning truncations of the systems of equations.

Setup for the $SU(3)$ lattice simulation

- Use standard Wilson plaquette action:

$$S = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{3}\,\Re\mathrm{e}\,\mathrm{Tr}\Big(U_{x\mu}U_{x+\mu;\nu}U^\dagger_{x+\nu;\mu}U^\dagger_{x\nu}\Big)\right], \;\; \beta = 6/g_0^2.$$

- *Landau gauge fixing* by *maximizing* the gauge functional

$$F_U[g] = \frac{1}{3}\sum_x \sum_\mu \Re\mathrm{e}\,\mathrm{Tr}\Big(g_x U_{x\mu} g^\dagger_{x+\mu}\Big) \quad \text{w. r. to} \quad g_x \in SU(3)$$

  using *simulated annealing + overrelaxation*.
  Compare random ("first") Gribov copy (fc) with "best" copy (bc) taking global $\mathbf{Z(3)}$-flips betw. Polyakov loop sectors into account.
- *Choice of parameters:*
  Fix $\beta = 6.337$ corresponding to $T \equiv 1/aL_4 \simeq T_c$ for $L_4 = 12$ [1].
  Fix the 3-volume $V = (L_s\,a)^3 = (48\,a)^3 \simeq (2.7\text{ fm})^3$.
  Vary T with $L_4 = 4, 6, \ \ldots, \ 18$.
  Renormalize propagators at $\mu = 5$ GeV or 3 GeV.
  Scaling, Gribov copy and finite volume tests at $T = 0.86\,T_c$ and $T = 1.20\,T_c$.

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Lattice observables for $T > 0$:

- Lattice gauge potential

$$A_\mu(x+\hat\mu/2) = \frac{1}{2iag_0}(U_{x\mu} - U^\dagger_{x\mu})\,|_{traceless}\,.$$

- Transverse gluon propagator in momentum space

$$D_T = \frac{1}{(d-2)8}\,\langle \sum_{i=1}^{3} A_i^a(q)\ A_i^a(-q) - \frac{q_4^2}{\vec{q}^{\,2}}\,A_4^a(q)\ A_4^a(-q)\rangle.$$

- Longitudinal gluon propagator

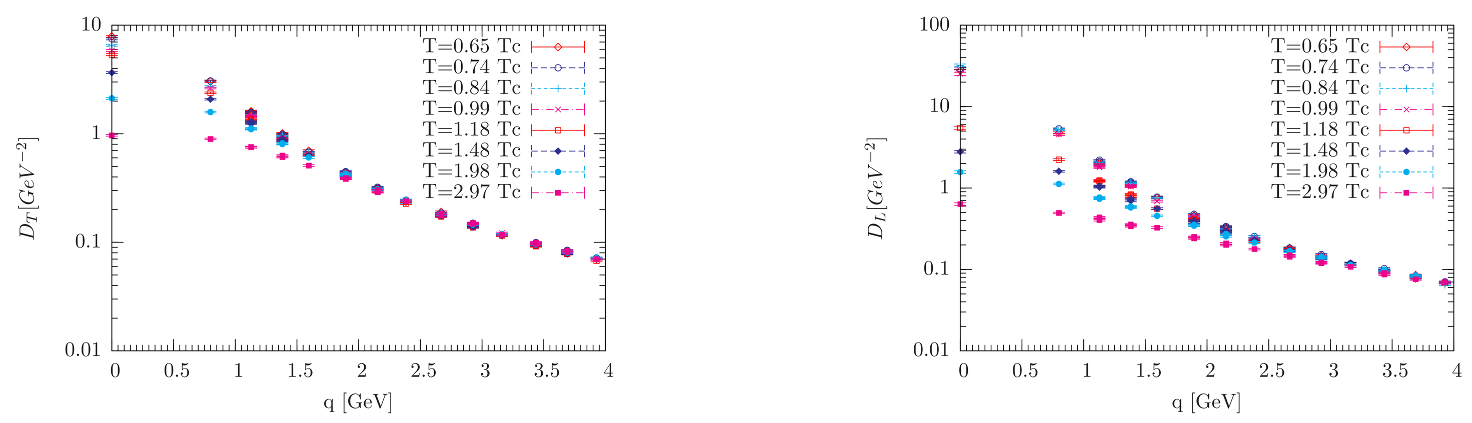
$$D_L = \frac{1}{8}(1+\frac{q_4^2}{\vec{q}^{\,2}})\,\langle\ A_4^a(q)\ A_4^a(-q)\ \rangle.$$

- Ghost propagator from Faddeev-Popov operator $M = -\partial D$

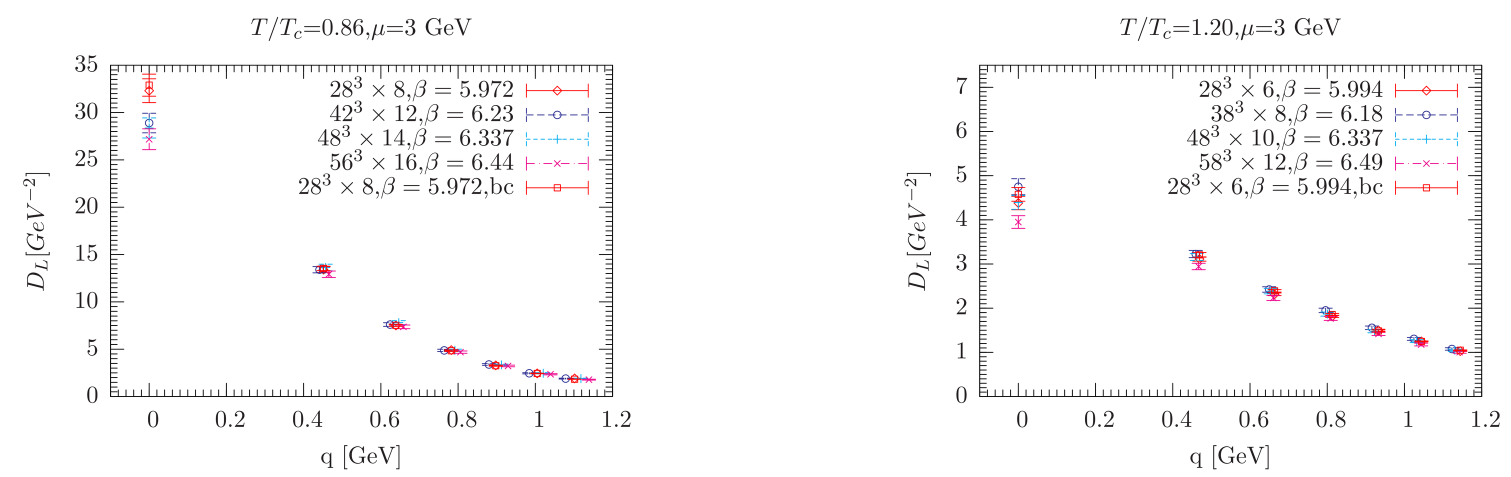
$$G^{ab}(q) = a^2 \sum_{x,y} < e^{-2\pi i k\cdot(x-y)/L}[M^{-1}]^{ab}_{xy} >= \delta^{ab} G(q),$$

- Momenta: $\quad q_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{L_\mu}\right), \quad k_\mu \in (-L_\mu/2, L_\mu/2]$. Here put $q_4 = 0$.

Result: gluon propagators for various T



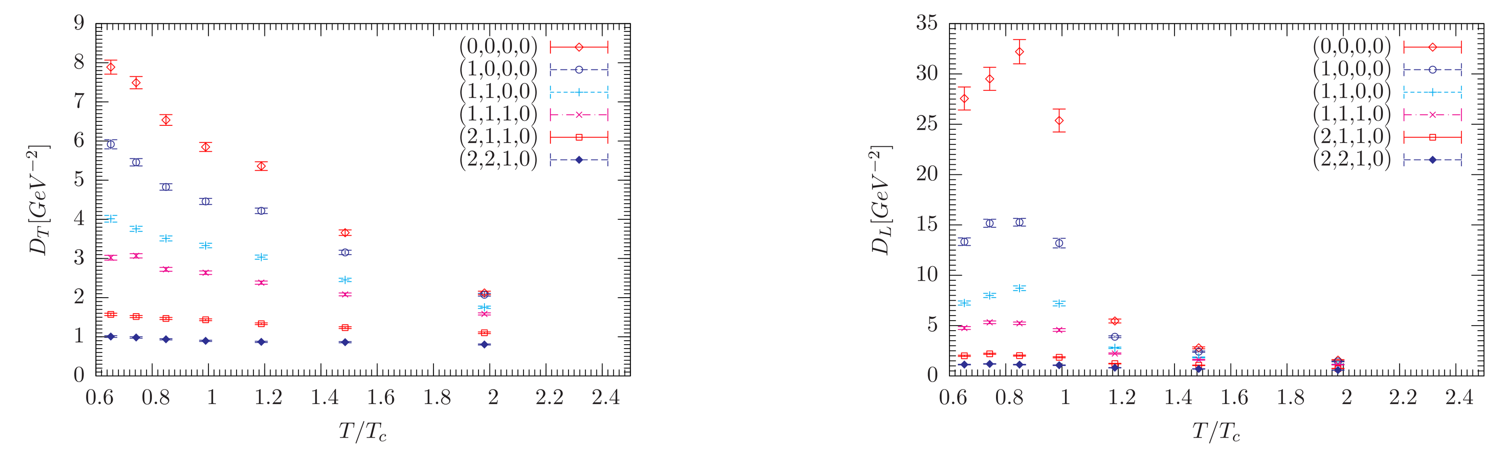
Scaling and Gribov copy test (here for D_L)



⟹ Scaling violation, Gribov copy effects visible at small q.

⟹ Effects for transversal gluon propagator even stronger.

Result: gluon propagators vs. T at lowest q

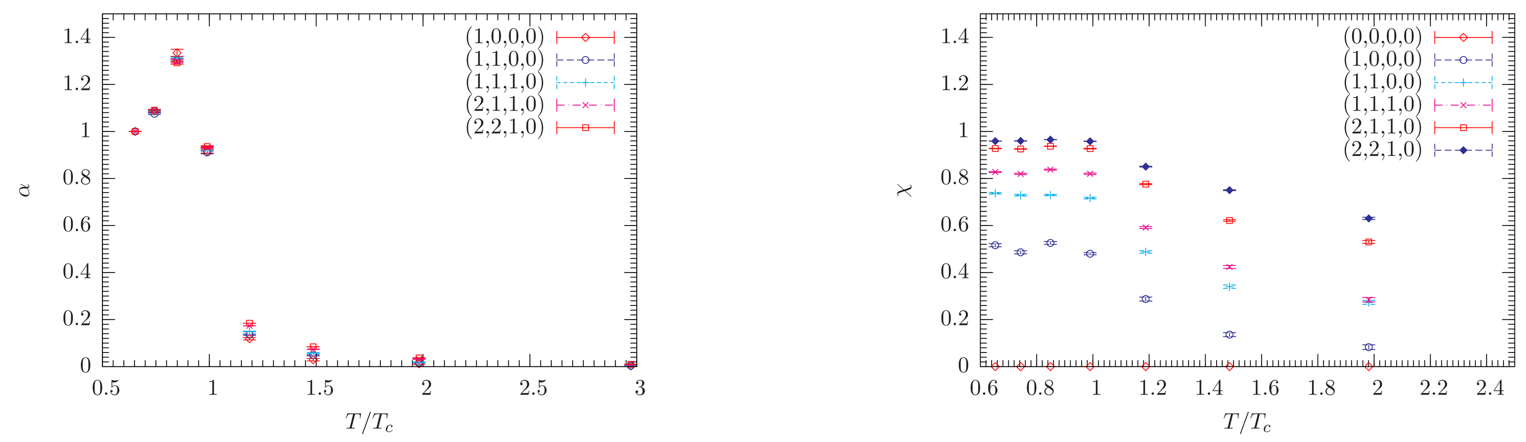


⟹ Transverse propagator depends smoothly on T.

⟹ Long. propagator at low momenta has a characteristic behaviour around T_c.
⟹ Ghost propagator almost independent of T (not shown here).

"Order parameters" from longitud. propagator

$$\alpha = \frac{D_L(0,T) - D_L(q,T)}{D_L(0,\frac{2}{3}T_c) - D_L(q,\frac{2}{3}T_c)} \qquad \chi = [D_L(0,T) - D_L(q,T)]/D_L(0,T)$$



Conclusions

- Longitudinal Landau gauge gluon propagator provides "order parameter(s)" for finite-temperature transition.
- Lattice artifacts, Gribov copy effects reasonably small at $q > 0.8$ GeV.
- Finite-volume effects have been studied up to $(64a)^3 \simeq (3.6\text{ fm})^3$. Negligible effect above $\simeq 0.5$ GeV.

References

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