Towards the equation of state with $N_f = 2$ twisted-mass lattice QCD

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General view, the phase diagram (see [1])

- Lattice QCD with $N_f = 2$ Wilson fermions modified with twisted mass term $i a\mu_0 \bar{\psi}\gamma_5\tau^3\psi$ and tree-level Symanzik improved gauge action is studied.

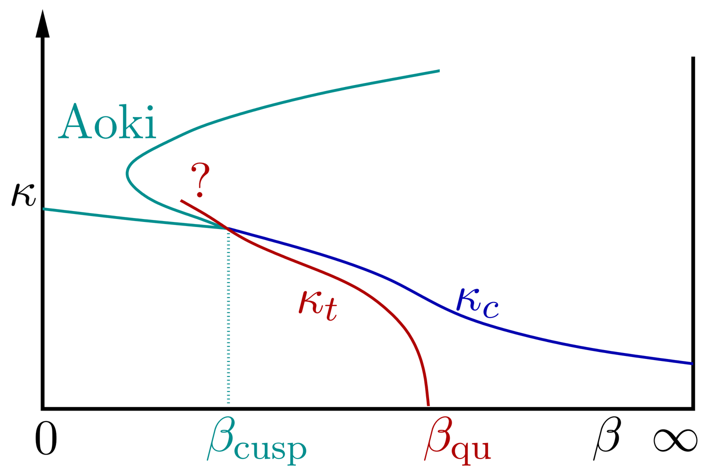
$$S_f[U,\psi,\overline{\psi}] = \sum_x \overline{\chi}(x)\left(1 - \kappa H[U] + 2i\kappa a\mu_0\gamma_5\tau^3\right)\chi(x)$$
$$\psi = \frac{1}{\sqrt{2}}(1 + i\gamma_5\tau^3)\chi \quad \text{and} \quad \overline{\psi} = \overline{\chi}\frac{1}{\sqrt{2}}(1 + i\gamma_5\tau^3) \qquad (1)$$

$$S_g^{\mathrm{tlSym}}[U] = \beta\Big(c_0 \sum_P [1 - \frac{1}{3}\mathrm{ReTr}\,(U_P)] + c_1 \sum_R [1 - \frac{1}{3}\mathrm{ReTr}\,(U_R)]\Big) \qquad (2)$$

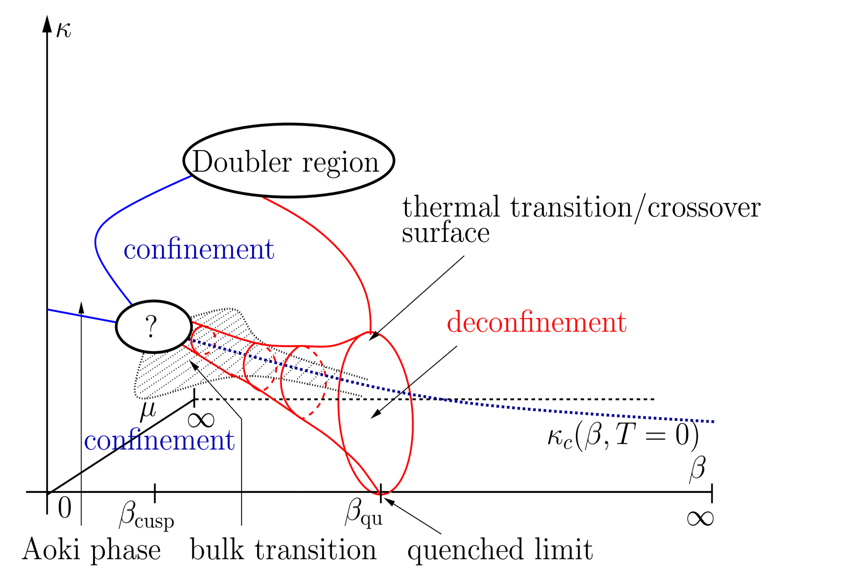
- Hopping parameter κ and twisted mass μ_0 are connected with the bare quark mass:

$$m_q = \sqrt{\frac{1}{4}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^2 + \mu_0^2} \qquad (3)$$

- For maximal twist, i. e. $\kappa = \kappa_c(\beta; T = 0)$, automatic $\mathcal{O}(a)$ improvement is expected. See e.g. [3, 4].
- In order to explore the phase diagram HMC simulations were performed in a wide range $\beta \equiv 6/g_0^2 = 1.80, \ldots, 3.90$ on linear lattice sizes $N_s = 16$, 24 and $N_\tau = 8$.
- Phase diagram for $\mu_0 = 0$ and fixed N_τ divides into three regions:
  – the Aoki-Phase [5] for strong coupling, i.e. at small β,
  – a bulk 1st order transition region [6, 8] at intermediate β,
  – and the thermal transition and scaling region at larger β.
- Acc. to Eq. (3) for $\mu_0 \neq 0$ the thermal transition line is part of a conical surface $(\kappa_t(\beta), \mu_t(\beta))$.



Older view of the phase structure for standard Wilson fermions in the β-κ plane.

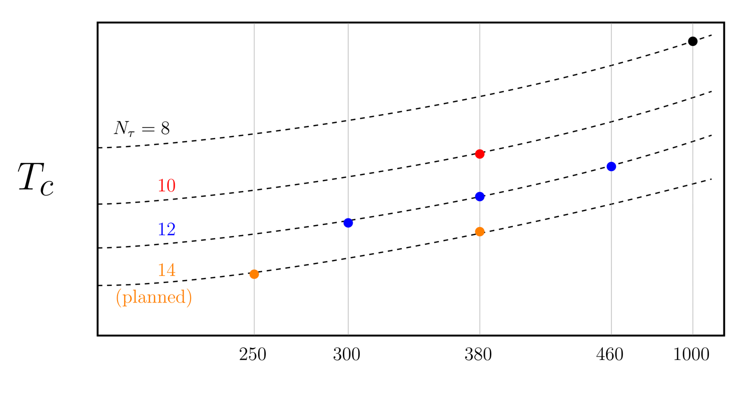


Present schematic view [1] of the phase diagram for tmQCD in β-κ-μ0 space, originally proposed by Creutz [7].

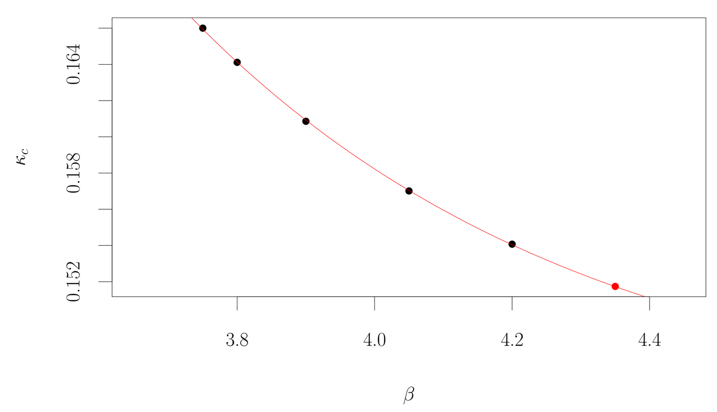
Setup for locating T_c (see [2])

- Scans in β at maximal twist.
- Keep pion mass m_π fixed while varying $T = 1/(N_\tau a(\beta))$.
- $a(\beta)$, $r_0/a(\beta)$, $\kappa_c(\beta)$ and $\mu_0(m_\pi, \beta)$ from **ETMC** data[9] (interpolations necessary).
- **Typical Statistics**:
  $N_\tau = 10$: O(4k) measurements per β-value
  $N_\tau = 12$: O(3k-10k) measurements per β-value
- Have scanned T-range $\approx [0.9 \ldots 1.05]\ T_c$
  For EoS: $\rightarrow$ **wider range needed**: $T \in [0.8 \ldots 1.7]\ T_c$
  (partly available at $m_\pi = 400$ MeV)
- Need also further $T = 0$ points (preliminary data available at $\beta = 4.35$, only one mass yet)

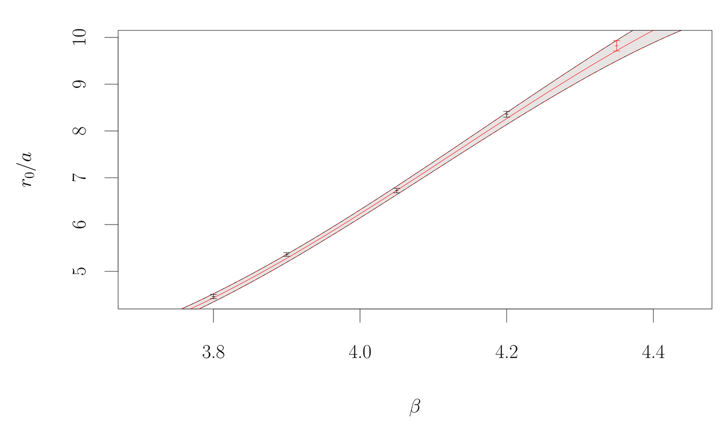
Simulated Pion Masses:



Interpolation of κ_c:



Interpolation of r_0/a:



Locating the thermal transition or crossover (cf. talk by L. Zeidlewicz)

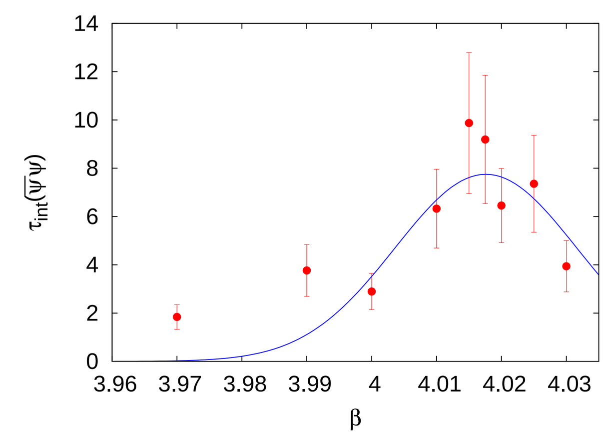
- **Observables**

Polyakov-Loop, Plaquette, $\langle\bar{\psi}\psi\rangle$, Pionnorm, their variances and integrated autocorrelation times τ_{int}. Mostly weak crossover signals as expected in intermediate mass range.

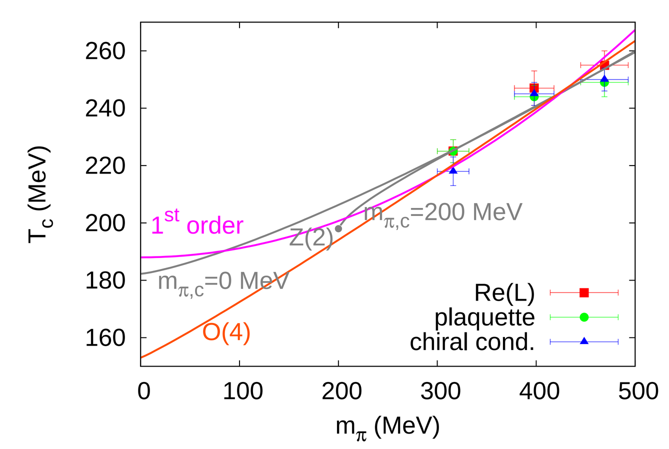
- **Result**

m_π[MeV]	≈ 320	≈ 400	≈ 470	≈ 400
N_τ	12	12	12	10
$\beta_c(\langle\bar{\psi}\psi\rangle)$	3.9400(96)	4.0150(11)	4.0298(11)	3.8804(04)
T_c[MeV]	218(5)	245(4)	250(4)	239(7)

Example: τ_{int} for $\langle\bar{\psi}\psi\rangle$:



T_c versus pion mass m_π



EoS: trace anomaly

- Use the integral method (see e.g. [10]) to get $p(T)$ and $\epsilon(T)$ from trace anomaly

$$\frac{\epsilon - 3p}{T^4} = -\frac{T}{V}\left\langle\frac{d\ln Z}{d\ln a}\right\rangle_{\mathrm{sub}}$$
$$= \left(a\frac{d\beta}{da}\right)\left(c_0\,\langle\mathrm{ReTr}U_P\rangle_{\mathrm{sub}} + c_1\,\langle\mathrm{ReTr}U_R\rangle_{\mathrm{sub}}\right.$$
$$\left. + \underbrace{\frac{\partial\kappa_c}{\partial\beta}\langle H\rangle_{\mathrm{sub}}}_{\approx 0 \text{ neglected so far}} + \left(2a\mu_0\frac{\partial\kappa_c}{\partial\beta} + \frac{\partial(a\mu_0)}{\partial\beta}\right)\langle\bar{\psi}\psi\rangle_{\mathrm{sub}}\right)$$

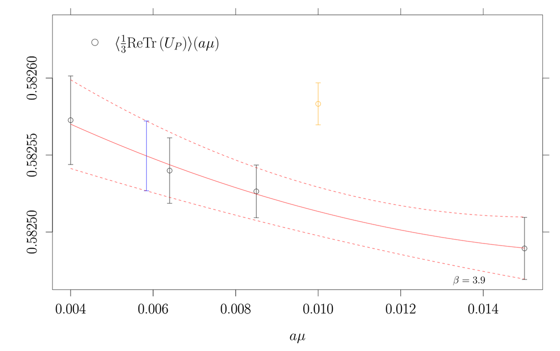
where:

$$\langle\ldots\rangle_{\mathrm{sub}} \equiv \langle\ldots\rangle_{T>0} - \langle\ldots\rangle_{T=0}$$

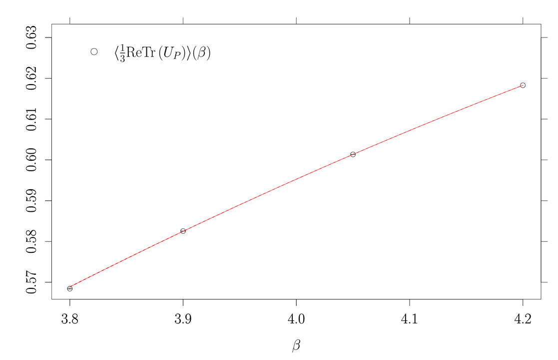
- Have to use interpolation for $\left(\frac{r_0}{a}\right)(\beta)$ to get:

$$\left(a\frac{d\beta}{da}\right) = -\left(\frac{r_0}{a}\right)\left(\frac{d\left(\frac{r_0}{a}\right)}{d\beta}\right)^{-1}$$

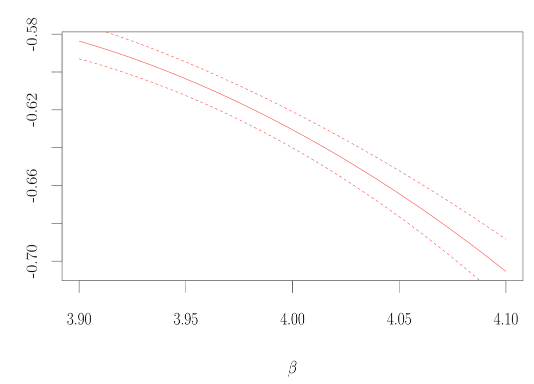
Plaquette interpolation in $a\mu_0$:



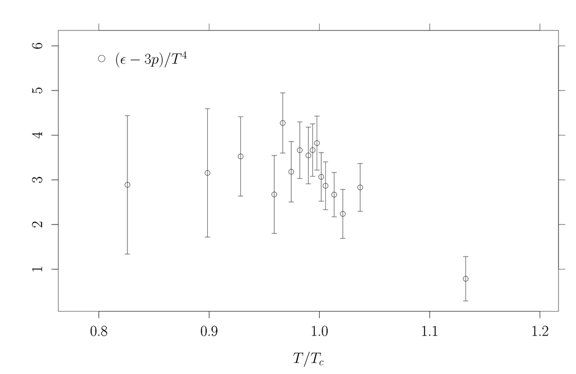
Plaquette interpolation in β:



Interpolation of β-function:



Trace anomaly (preliminary):



Conclusions

- The finite temperature transition has been located down to $m_\pi = O(300\text{MeV})$. Our results at $N_\tau = 10$, 12 support a weak crossover behaviour.
- The chiral limit is consistent with $O(4)$ universality, but does not exclude other scenarios.
- Do not see a splitting of chiral and deconfinement transitions.
- First (and preliminary) results for the trace anomaly presented.

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