Features of ghost-gluon, resp., ghost-quark bound states and non-perturbative BRST quartets

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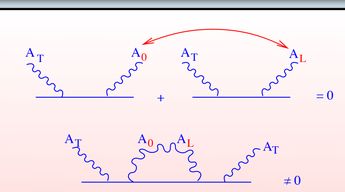
Motivation

The gluon propagator of Landau gauge QCD has been shown to be positivity violating, see *e.g.* [1] and references therein. This especially implies that the 1-gluon-state (treated as a physical state in perturbation theory) belongs to the states of negative norm in the indefinite-metric space of gauge theory. As such it can be identified as a parent state within a BRST quartet whose other members, however, have to be non-perturbative, *i.e.* bound, states. In the following we will identify possible members of this quartet and describe a strategy to provide evidence for their role in the formalism of covariantly gauge-fixed Yang-Mills theory. If successful this may provide a detailed picture of the kinematical aspects of gluon confinement in the Landau gauge. For the quark propagator the situation is less clear. Nevertheless, by following the same strategy we want to contribute to a clarification whether quarks are also positivity violating.

The perturbative BRST quartet mechanism

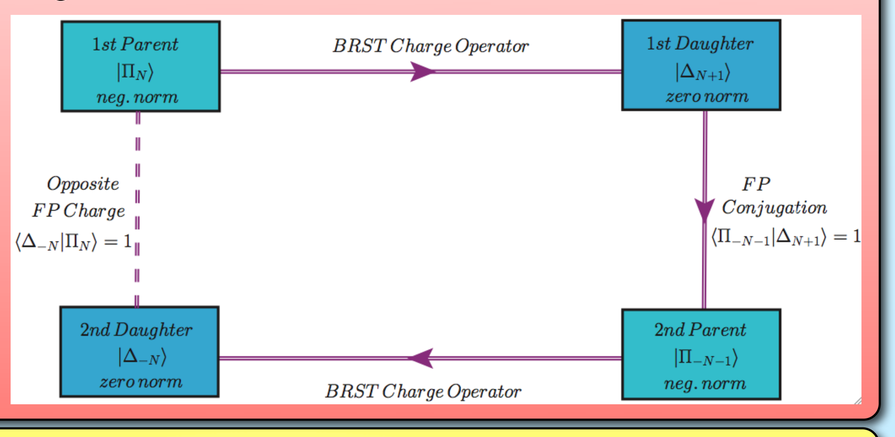


Landau gauge QED: In a physical state $|\Psi\rangle$ such that $\partial^\mu A^+_\mu |\Psi\rangle = 0$ the longitudinal and time-like photon cancel each other [2]

but they are needed for asymptotic completeness as exemplified in these diagrams:

The perturbative BRST quartet mechanism is the generalization of the Gupta-Bleuler mechanism to Yang-Mills theories and can be summarized as follows:

- ✓ The BRST symmetry of the renormalized covariantly gauge-fixed action is given by: $\delta_R A^a_\mu = \tilde{Z}_3 D^{ab}_\mu c^b \lambda_R, \quad \delta_R c^a = -\frac{1}{2}\tilde{Z}_1 g f^{abc} c^b c^c \lambda_R \quad \delta_R \bar{c}^a = B^a \lambda_R, \quad \delta_R B^a = 0.$
- ✓ Via Noether theorem a BRST charge operator Q_B is attributed such that $\delta\Phi = \{iQ_B, \Phi\}$ (ghost number graded algebra $Q_B{}^2 = 0$, $[iQ_C, Q_B] = Q_B$).
- ✓ BRST charge separates indefinite-metric state space in three subspaces: BRST singlets, BRST exact states (zero norm) and BRST non-singlets (negative norm).
- ✓ BRST non-invariant states form quartets: negative-norm state (parent 1) with Faddeev-Popov ghost charge N_{FP}, the zero-norm state being its BRST transformed state with ghost charge $N_{FP}+1$ (daughter 1), the Faddeev-Popov charge reflected state of daughter 1 with ghost charge $-N_{FP}-1$ (parent 2), and the BRST transformed state of the latter with ghost charge $-N_{FP}$ (daughter 2).
- ✓ There exists always one 'elementary' perturbative quartet: longitudinal gluon, ghost, antighost and a mixed state containing the timelike gluon. Under the assumption that the BRST singlets are the physical states [3] the 'elementary' quartet cancels in all these physical states and thus in the S-matrix.



Fields in the quartet of the transv. Gluon

★ As the gluon propagator displays positivity violation the transverse gluons are negative norm and thus parent states.

★ Their BRST transforms contain $\tilde{Z}_3 g f^{abc} A^c_\mu c^b$, and a bound state in this $N_{FP}=1$ channel is the natural candidate for the first daughter state.

★ The FP conjugated state of the latter, $\tilde{Z}_3 g f^{abc} A^c_\mu \bar{c}^b$, constitutes parent 2.

★ The BRST transform of the 2nd parent, $\tilde{Z}_3 g f^{abc}\left(-\tilde{Z}_3 \bar{c}^b D^{cd}_\mu c^d + B^b A^c_\mu\right)$, should also contain a bound state.

TASK: Prove the existence of these bound states!
Look for daughter 1 (i.e. gluon-ghost) bound state first !

Fields in the quartet of the quark

★ In anology to the gluon we assume that quarks are negative norm and thus parent states.

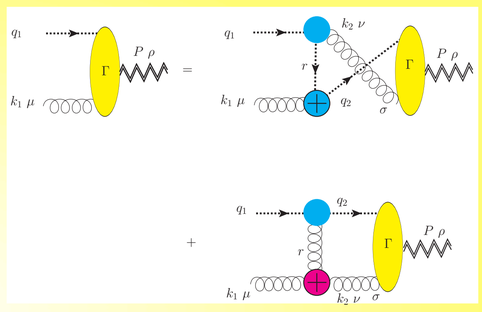
★ Their BRST transforms contain $-\tilde{Z}_1 i g t^a c^a q$, and a bound state in this $N_{FP}=1$ channel is the natural candidate for the first daughter state.

★ The FP conjugated state of the latter, $-\tilde{Z}_1 i g t^a \bar{c}^a q$, constitutes parent 2.

★ The BRST transform of the 2nd parent, $-\tilde{Z}_1 i g t^a\left(B^a q - \tilde{Z}_1 i g \bar{c}^a t^b c^b q\right)$, should also contain a bound state.

TASK: Prove the existence of these bound states!
Look for daughter 1 (i.e. quark-ghost) bound state first !

Bethe-Salpeter Equation for ghost-gluon [4]



★ Landau gauge is ghost-antighost symmetric: If 1st daughter contains a massless bound state also 2nd parent does.

★ IR analysis for scaling solution ghost-gluon scattering kernel defines truncation:
(i) Neglect n>4-point functions, and
(ii) keep only IR leading terms, *i.e.*, ghost and gluon exchange.

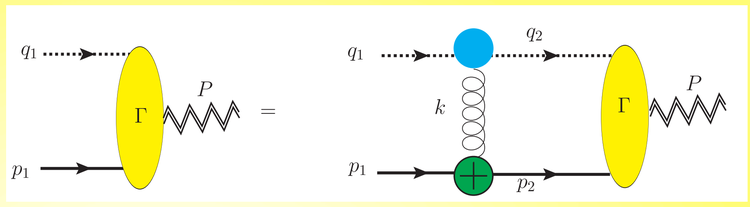
Choose correct interaction channels and reduce to homogeneous Bethe-Salpeter (BS) equation by expansion around pole produced by bound state.

★ Important result: BS equation is infrared consistent!

★ As ghost-gluon vertex is almost momentum-independent and ghost exchange is dominant, ladder approximation for ghost-gluon bound state is justified.

★ Interaction kernel is proportional to the strong coupling from the ghost-gluon vertex, $\alpha(p^2)$ with $\alpha(0) = 8.92/N_c$, and therefore likely supercritically strong.

Bethe-Salpeter Equation for ghost-quark [4]



★ IR analysis for scaling solution ghost-quark scattering kernel and same truncation as for gluons: Gluon exchange.

★ BS equation is IR consistent due to the fully dressed IR divergent quark-gluon vertex in the scaling scenario.

★ Interaction kernel again proportional to strong coupling.

Numerical solution (work in progress):

★ Gluon propagator from scaling solution of DSEs [5].

★ Quark-gluon vertex and quark propagator from self-consistent DSEs [6].

★ Bound states not expected to be exactly degenerate due to truncation error.

★ Solve homogeneous and then inhomogeneous equations, reconstruct propagator and test for positivity violation.

Summary & Outlook

- ☑ Field content of non-perturbative BRST quartets generated by transverse gluons or quarks in Landau gauge identified.
- ☑ Homogeneous Bethe-Salpeter equation for ghost-gluon and ghost-quark bound states in respective colour representations derived and IR consistency for scaling solution incl. IR divergent quark-gluon vertex shown [4].
- ☐ Solve numerically the IR-consistent truncated homogeneous / inhomogeneous Bethe-Salpeter equations for ghost-gluon and ghost-quark bound states and test for positivity violation.
- ☐ 2nd daughter in Landau gauge: Both terms decouple due to vanishing gauge parameter! Only bound state of longitudinal gluon ($\propto B^a$) and transverse gluon, resp., quark, relevant?
- ☐ Can the quartet partners of transverse gluons be identified with Goldstone fermions/bosons of some broken symmetry, *cf.* [7]? Is BRST dynamically broken? Are the corresponding Goldstone modes the long-searched unphysical zero-gap excitations needed for the failure of cluster decomposition?
- ☐ Are quarks positivity violating?

References

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