J/ψ Suppression and Enhancement
The Statistical Hadronization Model

Jun Nian

Universität Heidelberg

Jan. 24, 2008
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4 Summary and reference
Quark-Gluon Plasma (QGP):

A plasma of strongly interacting, deconfined quarks and gluons.

A thermodynamic system (a system, which can be described by the equation of motion from statistical physics). For such a system there should be transition, eventually phase transition.

Hadronic Constituents  Transition  QGP
• 1965: Hagedron observed that the mass density of hadron states grows exponentially with energy, which led to the existence of a phase transition.

• 1973: asymptotic freedom (Nobelprize 2004)
  D.J.Gross, F.Wilczek, Phys. Rev. Lett. 30 (1973) 1343
  H.D.Politzer, Phys. Rev. Lett. 30 (1973) 1346

• 1975: deconfined quarks and gluons

• 1980: suggested by Shuryak, the new phase was denominated “Quark-Gluon Plasma” (QGP).

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Fig. 1. Schematic phase diagram of hadronic matter. $\rho_B$ is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

Fig. 6.14. Schematic phase diagram in temperature ($T$) and quark chemical potential ($\mu$) space. The line with the tricritical point, TCP, corresponds to $m_{ud} = 0$, and the line with the critical end point, CEP, corresponds to $m_{ud} > 0$. For the strange quark mass, $m_s > m_s^{tri}$ is assumed.
(Source: presentation given by M.Ries from the seminar “experimental particle and nuclear physics” in Uni. Heidelberg)
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Summary and reference

Phase diagrams - 4

(Source: W. Greiner, S. Schramm, E. Stein, (2002) Quantum Chromodynamics)
The only prerequisite condition to create QGP:

- Critical energy density: $\epsilon_c \approx 0.7\text{GeV/fm}^3$

(Source: presentation given by M.Ries from the seminar “experimental particle and nuclear physics” in Uni. Heidelberg)
3 possible places to find QGP:

- in the early universe ("Big Bang")
- at the center of compact stars
- in the initial stage of colliding heavy nuclei at high energies ("Little Bang")

(Source: presentation given by M.Ries from the seminar "experimental particle and nuclear physics" in Uni. Heidelberg)
Possible places to find QGP

3 possible places to find QGP:
- in the early universe (“Big Bang”)
- at the center of compact stars
- in the initial stage of colliding heavy nuclei at high energies (“Little Bang”)

(Source: presentation given by M.Ries from the seminar “experimental particle and nuclear physics” in Uni. Heidelberg)
The expected effects which cannot be produced in a normal nuclear surrounding:

- $J/\psi$ enhancement

- Very exotic objects, e.g. multibaryon states with a very large strangeness content.

(According to W.Greiner, S.Schramm, E Stein, (2002) *Quantum Chromodynamics*)
Brookhaven National Laboratory
Relativistic Heavy Ion Collider (RHIC)
Experiments:
   STAR, PHENIX,
   PHOBOS, BRAHMS
Au-Au, $\sqrt{s_{NN}} = 130, 200$ GeV
$\sim 10^3$ particles produced
1. Introduction to QGP

2. The statistical hadronization model
   - Introduction to the model
   - Grand canonical formalism
   - Canonical formalism

3. $J/\psi$ suppression and enhancement
   - Theoretical description
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4. Summary and reference
The statistical hadronization model (SHM):

- mainly used to calculate particle production;
- basic idea: all particles stem from a thermalized fireball at temperature $T$ and baryon chemical potential $\mu_b$;
- can be applied both to p+p collisions and to A+A collisions;
- a partition function (microcanonical, grand canonical, canonical) is used;
- with three parameters $V$, $T$ and $\mu_b$ particle yields can be explained.
Space-time evolution of heavy ion collisions in the Bjorken model:

1. \(0 < \tau < \tau_0\): Pre-equilibrium stage and thermalization
2. \(\tau = \tau_0\): Thermal equilibrium
3. \(\tau_0 < \tau < \tau_f\): Hydrodynamical evolution and freeze-out
4. \(\tau = \tau_f\): Chemical freeze-out
5. \(\tau > \tau_f\): Free-out and post-equilibrium
All assumptions of the model

Some Assumptions:

1. All heavy quarks (charm and bottom) are produced in primary hard collisions.
2. Their total number stays constant until hadronization.
3. Thermal (but not chemical) equilibration takes place in QGP, at least near the critical temperature $T_c$.
4. The quarkonia are all produced (nonperturbatively) when $T_c$ is reached and the system hadronizes.
5. The participating nucleons’ number $N_{part}$ is sufficiently large.
1. All heavy quarks (charm and bottom) are produced in primary hard collisions.
   - Hard QCD process $\rightarrow$ QGP $\rightarrow$ Hadron gas

2. Their total number stays constant until hadronization.

- The $c\bar{c}$ and $b\bar{b}$ pairs are created at the early stage of A+A reaction, and the number of $c\bar{c}$ and $b\bar{b}$ pairs remains approximately unchanged during the further evolution respectively.
3. Thermal (but not chemical) equilibration takes place in QGP, at least near the critical temperature $T_c$.

4. The quarkonia are all produced (nonperturbatively) when $T_c$ is reached and the system hadronizes.
Difference between grand canonical and canonical formalism

Grand Canonical Ensemble (GC):
- only valid in system with large number of produced particles;
- conservation of additive quantum numbers ($B$, $S$, $I_3$) can be implemented on average by use of chemical potential $\mu$;
- asymptotic realization of exact canonical approach.

Canonical Ensemble (C):
- can be valid in system with small particle multiplicity;
- conservation law must be implemented locally on event-by-event basis, which leads to severe phase space reduction for particle production (“canonical suppression”);
- relevant in low energy A+A collisions, high energy p+p or $e^+ e^-$ collisions.
Grand canonical formalism: concrete derivation

Partition function: \( Z^{GC} = \text{Tr} \left[ e^{-\beta (\hat{H} - \mu \hat{N})} \right] \), where \( \hat{H} = \sum_p \varepsilon_p a_p^\dagger a_p \), \( \hat{N} = \sum_p a_p^\dagger a_p \).

Another form: \( Z^{GC} = \text{Tr} \left[ e^{-\beta (\hat{H} - \sum_j \mu_j Q_j)} \right] \),

where \( Q_j \) is some total quantum number (e.g. baryon number, electric charge, \( \cdots \)) of the system.

\[
\Rightarrow \quad Z^{GC} = \text{Tr} \prod_p e^{-\beta (\varepsilon_p - \mu)} a_p^\dagger a_p = \prod_p \sum_{n_p} e^{-\beta (\varepsilon_p - \mu) n_p} \equiv \prod_p Z_p,
\]

where

\[
Z_p \equiv \sum_{n_p} e^{-\beta (\varepsilon_p - \mu) n_p} = \begin{cases} \sum_{n_p=0}^\infty e^{-\beta (\varepsilon_p - \mu) n_p} = \frac{1}{1 - e^{-\beta (\varepsilon_p - \mu)}}, & \text{for boson} \\ \sum_{n_p=0}^1 e^{-\beta (\varepsilon_p - \mu) n_p} = 1 + e^{-\beta (\varepsilon_p - \mu)}, & \text{for fermion} \end{cases}
\]

\[
= (1 \mp e^{-\beta (\varepsilon_p - \mu)}) \mp 1 \equiv (1 \mp \lambda e^{-\beta \varepsilon_p}) \mp 1,
\]

where

\[
\lambda \equiv e^{\beta \mu} = \exp \left( \frac{\mu}{k_B T} \right) = \exp \left( \frac{B_i \mu_B + S_i \mu_S + Q_i \mu_Q}{T} \right). \quad (\text{set } k_B = 1)
\]

\( B_i, S_i, Q_i \): baryon number, strangeness and electric charge of one i-th particle.
For the i-th particle:
considering the degeneracy factor $g_i$, the partition function becomes
(the upper sign for boson, and the lower sign for fermion)

$$
\ln Z_i = g_i \sum_p \ln Z_p = g_i \frac{V}{(2\pi \hbar)^3} \int d^3 p \ln \left( 1 \mp \lambda_i e^{-\beta \varepsilon_i} \right)^{\mp 1}
$$

(set $\hbar = 1$) = \frac{g_i V}{2\pi^2} \cdot \int_0^\infty dp (\mp p^2) \cdot \ln \left( 1 \mp \lambda_i e^{-\beta \varepsilon_i} \right) \cdot (\text{Integral I})

\text{Integral I} = \int_0^\infty (\mp p^2) \cdot \sum_{k=1}^\infty \frac{(-1)^{k-1}}{k} \left( \mp \lambda_i e^{-\beta \varepsilon_i} \right)^k

= \sum_{k=1}^\infty \frac{(-1)^{k-1}}{k} \cdot (\mp 1)^{k+1} \cdot \lambda_i^k \cdot \int_0^\infty dp p^2 e^{-\beta \varepsilon_i k}

= \sum_{k=1}^\infty \frac{(\pm 1)^{k+1}}{k} \cdot \lambda_i^k \cdot \int_0^\infty dp p^2 e^{-\beta \varepsilon_i k} \cdot (\text{Integral II})
Grand canonical formalism: concrete derivation

\[ \text{Integral II} = \int_0^\infty dp \; p^2 \cdot e^{-\beta \varepsilon_i k} = \int_0^\infty dp \; p^2 \cdot e^{-\frac{k}{T} \sqrt{p^2 + m_i^2}} \]

\[ = \int_0^\infty dp \; p^2 \cdot e^{-\frac{km_j}{T} \sqrt{1 + \left( \frac{p}{m_j} \right)^2}} \]

\[ = \int_0^\infty dp \; p^2 \cdot e^{-z \cdot \cosh(t)} \quad (z \equiv \frac{km_j}{T}, \cosh(t) \equiv \sqrt{1 + \left( \frac{p}{m_j} \right)^2}) \]

(* see below) \[ = \int_0^\infty dt \; m_i \cosh(t) \cdot m_i^2 \sinh^2(t) \cdot e^{-z \cdot \cosh(t)} \]

\[ = \int_0^\infty dt \; \frac{m_i^3}{4} \left[ \cosh(3t) - \cosh(t) \right] \cdot e^{-z \cdot \cosh(t)} \]

\[ = \int_0^\infty dt \; \frac{m_i^3}{4} \cosh(3t) \cdot e^{-z \cdot \cosh(t)} - \int_0^\infty dt \; \frac{m_i^3}{4} \cosh(t) \cdot e^{-z \cdot \cosh(t)} \]

\[ = \frac{m_i^3}{4} (K_3(z) - K_1(z)) = \frac{m_i^3}{4} \cdot \frac{4T}{km_j} \cdot K_2 \left( \frac{km_j}{T} \right) \]

\[ = \frac{m_i^2 T}{k} \cdot K_2 \left( \frac{km_j}{T} \right) \]

\[ \left[ K_n(z) = \int_0^\infty dt \; \cosh(nt) \cdot e^{-z \cdot \cosh(t)} , K_{n+1} - K_{n-1} = \frac{2n}{z} K_n(z) \right] \]

\[ \ast \quad \cosh(t) = \frac{e^t + e^{-t}}{2} = \sqrt{1 + \left( \frac{p}{m_i} \right)^2} \iff \frac{e^{2t} + e^{-2t} + 2}{4} = 1 + \left( \frac{p}{m_i} \right)^2 \]

\[ \iff \left( \frac{e^t - e^{-t}}{2} \right)^2 = \left( \frac{p}{m_i} \right)^2 \iff \sinh(t) = \frac{p}{m_i} \iff p = m_i \cdot \sinh(t) \]

\[ \iff p^2 = m_i^2 \cdot \sinh^2(t) \quad \& \quad dp = m_i \cdot \cosh(t) \; dt \]
\[
\ln Z_i = \frac{g_i V}{2\pi^2} \cdot (\text{Integral I})
\]
\[
= \frac{g_i V}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \cdot \lambda_i^k \cdot (\text{Integral II})
\]
\[
= \frac{g_i V}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \cdot \lambda_i^k \cdot \frac{m^2 T}{k} K_2 \left(\frac{km_i}{T}\right)
\]
\[
= \frac{VT g_i}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} \cdot \lambda_i^k \cdot m_i^2 K_2 \left(\frac{km_i}{T}\right)
\]
\[
\Rightarrow \langle N_i \rangle = \frac{1}{\beta} \frac{\partial \ln Z_i}{\partial \mu} = \frac{1}{\beta} \cdot \frac{VT g_i}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} \cdot \frac{\partial (\lambda_i^k)}{\partial \mu} \cdot m_i^2 K_2 \left(\frac{km_i}{T}\right)
\]
\[
[\lambda_i \equiv \exp \left(\frac{\mu}{T}\right)]
\]
\[
= \frac{1}{\beta} \cdot \frac{VT g_i}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} \cdot k \lambda_i^{k-1} \cdot \frac{\lambda_i}{T} \cdot m_i^2 K_2 \left(\frac{km_i}{T}\right)
\]
\[
= \frac{VT g_i}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \lambda_i^k m_i^2 K_2 \left(\frac{km_i}{T}\right)
\]
\[
\Rightarrow n_i (T, \mu) = \frac{\langle N_i \rangle}{V} = \frac{T g_i}{2\pi^2} \cdot \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \lambda_i^k m_i^2 K_2 \left(\frac{km_i}{T}\right)
\]

An important consequence:
there is a relation between the one-particle partition function and the one-particle density: \((k = 1)\)

\[
n_i = \frac{\ln Z_i}{V}
\]
Grand canonical formalism: summary

For a large number of particles:

Partition function: $Z^{GC}(T, V, \mu_Q) = Tr[e^{-\beta(H - \sum_i \mu_Q Q_i)}]

For particle $i$ of strangeness $S_i$, baryon number $B_i$, electric charge $Q_i$ and spin-isospin degeneracy factor $g_i$:

(1) for boson, and the lower sign for fermion)

\[
\ln Z_i(T, V, \bar{\mu}) = \frac{Vg_i}{2\pi^2} \int_0^{\infty} \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\beta \epsilon_i)],
\]

where

\[
\lambda_i(T, \bar{\mu}) = \exp \frac{B_i \mu_B + S_i \mu_S + Q_i \mu_Q}{T}.
\]

\[
\Rightarrow \ln Z_i(T, V, \bar{\mu}) = \frac{VTg_i}{2\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \lambda_i^k m_i^2 K_2 \left( \frac{km_i}{T} \right),
\]

where $K_2$ is the modified Bessel function of the second kind.

\[
\Rightarrow n_i(T, \bar{\mu}) = \frac{\langle N_i \rangle}{V} = \frac{Tg_i}{2\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \lambda_i^k m_i^2 K_2 \left( \frac{km_i}{T} \right).
\]
Grand canonical formalism: input parameters - 1

\[ Z_i(T, V, \mu_B, \mu_S, \mu_Q) \implies n_i(T, \vec{\mu}) \]

In order to compare with the experimental data, we must:

1. Fix some input parameters:

   - Strangeness conservation \( \langle S \rangle = 0 \implies \text{fix } \mu_S \)
   - Initial condition \( \frac{\langle Q \rangle}{\langle B \rangle} = \text{const} \implies \text{fix } \mu_Q \)

2. Consider relevant decays:

   \[
   n_i^{th} \equiv n_i^{tot}(T, \mu_B) = n_i(T, \mu_B) + \sum_j B(j \to i) \cdot n_j(T, \mu_B)
   \]

3. Only consider the relative densities:

   \[
   \frac{n_i^{th}}{n_{ref}^{th}} = \frac{\langle N_i \rangle / V}{\langle N_{ref} \rangle / V} = \frac{\langle N_i \rangle}{\langle N_{ref} \rangle}
   \]
Grand canonical formalism: input parameters - 

\[ Z_i(T, V, \mu_B, \mu_S, \mu_Q) \implies n_i(T, \bar{\mu}) \]

To determine the input parameters \( T \) and \( \mu_B \):

- measure the multiplicities of three particles (e.g. \( \pi, \rho \) and \( K \)), then solve

\[
\frac{n_{\rho}^{th}}{n_{\pi}^{th}} = \frac{\langle \rho \rangle / V}{\langle \pi \rangle / V} = \frac{\langle \rho \rangle}{\langle \pi \rangle}, \quad \frac{n_{K}^{th}}{n_{\pi}^{th}} = \frac{\langle K \rangle / V}{\langle \pi \rangle / V} = \frac{\langle K \rangle}{\langle \pi \rangle}
\]
SPS, Pb-Pb collisions at $\sqrt{s_{NN}} = 40$ GeV, $T = 148$ MeV, $\mu_B = 400$ MeV:

\[
\chi^2 = \sum_i \frac{(R_i^{\text{exp.}} - R_i^{\text{model}})^2}{\sigma_i^2} = 1.1
\]

SPS, Pb-Pb collisions at $\sqrt{s_{NN}} = 158$ GeV, $T = 170$ MeV, $\mu_B = 255$ MeV:

Excluding $\phi$ and $d$: $\chi^2 = 2.0$;
Considering different efficiencies of $\Lambda$ and $\pi^-$ from weak decays: $\chi^2 = 1.5$

RHIC, Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV, $T = 174$ MeV, $\mu_B = 46$ MeV:

RHIC, Au-Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV, $T = 176$ MeV, $\mu_B = 41$ MeV for $\sqrt{s_{NN}} = 130$ GeV, $T = 177$ MeV, $\mu_B = 29$ MeV for $\sqrt{s_{NN}} = 200$ GeV:

\[
\chi^2 = 0.8 \quad \chi^2 = 1.1
\]

Why canonical formalism?

Grand Canonical Ensemble (GC):
- only valid in system with large number of produced particles;
- conservation of additive quantum numbers \((B, S, I_3)\) can be implemented on average by use of chemical potential \(\mu\).

Canonical Ensemble (C):
- can be valid in system with small particle multiplicity;
- conservation law must be implemented locally on event-by-event basis.

From experiments (SPS, RHIC): the heavy quarks’ multiplicities are very small in A+A collisions.
Grand Canonical Ensemble (GC):

- only valid in system with large number of produced particles;
- conservation of additive quantum numbers ($B$, $S$, $I^3$) can be implemented on average by use of chemical potential $\mu$.

Canonical Ensemble (C):

- can be valid in system with small particle multiplicity;
- conservation law must be implemented locally on event-by-event basis.

From experiments (SPS, RHIC): the heavy quarks’ multiplicities are very small in A+A collisions.
Aim: find an expression of particle density, in which some quantum number is exactly conserved.

Start from the grand canonical partition function:

\[ Z_{GC}(T, V, \mu_C) = \text{Tr}[e^{-\beta(\hat{H} - \mu_C \hat{C})}] \]

Eigenstates of \( \hat{H} \) and \( \hat{C} \):

\[ \hat{H} |C\rangle = E_C |C\rangle, \quad \hat{C} |C\rangle = C |C\rangle \]

\[ \Rightarrow Z_{GC}(T, V, \mu_C) = \sum_{C=-\infty}^{+\infty} e^{-\beta E_C} \cdot e^{\beta \mu_C C} = \sum_{C=-\infty}^{+\infty} Z_C \cdot (\lambda_C)^C, \]

where \( Z_C \equiv \text{Tr}_C[e^{-\beta \hat{H}}], \lambda_C \equiv e^{\beta \mu_C}. \)

1. If the value of \( C \) is fixed, \( Z_C \equiv \text{Tr}_C[e^{-\beta \hat{H}}] \) is a canonical partition function.

\[ Z_{C=C_0}(T, V) = \text{Tr}_{C_0}[e^{-\beta \hat{H}}] = \text{Tr}[e^{-\beta \hat{H}} \cdot P_{C_0}], \]

where \( P_{C=0} = P_{C=0}^2 = \delta_{CC_0} \) is the projection operator.

2. \[ Z_{GC}(T, V, \mu_C) \Rightarrow Z_{GC}(T, V, \lambda_C) \]

Define \( e^{i\phi} \equiv \lambda_C \):

\[ Z_{GC}(T, V, \mu_C) \Rightarrow Z_{GC}(T, V, \lambda_C) \Rightarrow \tilde{Z}(T, V, \phi) \]
Z_{C=C_0}(T, V) = Tr[e^{-\beta \hat{H}} \cdot \delta_{C_0}] = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-i(C-C_0)\phi} \cdot \tilde{Z}(T, V, \phi)

Redefine: \tilde{Z}_{\text{new}}(T, V, \phi) \equiv e^{i\phi} \cdot \tilde{Z}_{\text{old}}(T, V, \phi), \text{ then for fixed } C

Z_C(T, V) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot \tilde{Z}(T, V, \phi)

Consider a system consisting of \(m \ "C = 0",\) \(n - m \ "C = 1"\) and \(p - m \ "C = -1"\) particles, but without multicharm particles:

(All \(Z_k\)'s are one-particle partition functions.)

\[ \tilde{Z}(T, V, \phi) = \exp( \sum_{k=1}^{m} \ln Z_k^{C=0} + \sum_{k=m+1}^{n} \ln Z_k^{C=1} \cdot e^{i\phi} + \sum_{k=n+1}^{p} \ln Z_k^{C=-1} \cdot e^{-i\phi} ) \]

\[ \equiv \exp( \sum_k \ln Z_k^{C=0} ) \cdot \exp( e^{i\phi} \cdot C_1 ) \cdot \exp( e^{-i\phi} \cdot C_{-1} ) \]

\[ Z = \prod_i Z_i = e^{\ln Z_1} \cdot e^{\ln Z_2} \cdots e^{\ln Z_n} = e^{\ln Z_1 + \ln Z_2 + \cdots + \ln Z_n} \]
\[ Z_C = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot \tilde{Z} \]
\[ = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot e^{i\phi \cdot c_1 \cdot e^{i\phi \cdot c_{-1}}} \]
\[ \equiv Z_0 \cdot \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot e^{i\phi \cdot c_1 + e^{-i\phi \cdot c_{-1}}} \]
\[ = Z_0 \cdot \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot e^{\sqrt{C_1 C_{-1}} \left(e^{i\phi \sqrt{\frac{c_1}{c_{-1}}} + e^{-i\phi \sqrt{\frac{c_{-1}}{c_1}}}}\right)} \]
\[ (\text{see below }*) \]
\[ = Z_0 \cdot \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot \sum_{C' = -\infty}^{+\infty} \left(e^{i\phi \cdot \sqrt{\frac{c_1}{c_{-1}}}}\right)^{C'} \cdot I_{C'}(2\sqrt{C_1 C_{-1}}) \]
\[ = Z_0 \cdot \frac{1}{2\pi} \sum_{C' = -\infty}^{+\infty} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot e^{iC' \phi} \cdot \left(\frac{c_1}{c_{-1}}\right)^{C'/2} \cdot I_{C'}(2\sqrt{C_1 C_{-1}}) \]
\[ = Z_0 \cdot \left(\frac{c_1}{c_{-1}}\right)^{C/2} \cdot I_{C}(2\sqrt{C_1 C_{-1}}) \]

* For the modified Bessel function of the first kind:
\[ e^{\frac{x}{2}(t + \frac{1}{t})} = \sum_{C = -\infty}^{+\infty} t^C \cdot I_C(x). \]

Here \( x = 2\sqrt{C_1 C_{-1}} \), \( t = e^{i\phi \sqrt{\frac{c_1}{c_{-1}}}} \):
\[ e^{\frac{2\sqrt{C_1 C_{-1}}}{2}(e^{i\phi \sqrt{\frac{c_1}{c_{-1}}} + e^{-i\phi \sqrt{\frac{c_{-1}}{c_1}}})} = \sum_{C = -\infty}^{+\infty} \left(e^{i\phi \sqrt{\frac{c_1}{c_{-1}}}}\right)^{C} \cdot I_{C}(2\sqrt{C_1 C_{-1}}) \]
We implement a transformation by introducing fugacity parameters: (The same particle species has the same $\lambda_k$.)

$$\tilde{Z}(T, V, \phi) = \exp\left(\sum_{k=1}^{m} \ln Z_k^{C=0} + \sum_{k=m+1}^{n} \ln Z_k^{C=1} \cdot e^{i\phi} + \sum_{k=n+1}^{p} \ln Z_k^{C=-1} \cdot e^{-i\phi}\right)$$

$$\Rightarrow \tilde{Z}(T, V, \phi) = \exp\left(\sum_{k=1}^{m} \lambda_k \cdot \ln Z_k^{C=0} + \sum_{k=m+1}^{n} \lambda_k \cdot \ln Z_k^{C=1} \cdot e^{i\phi} + \sum_{k=n+1}^{p} \lambda_k \cdot \ln Z_k^{C=-1} \cdot e^{-i\phi}\right)$$

The density of the k-th particle: (assuming $N_k$ terms has the same $\lambda_k$)

$$n_k^{GC} = \frac{\lambda_k}{V} \cdot \frac{\partial}{\partial \lambda_k} \ln \tilde{Z}|_{\lambda_k=1} = N_k \cdot \frac{\ln Z_k}{V}$$

If we consider the conservation of the charm:

$$n_k^C = \frac{\lambda_k}{V} \cdot \frac{\partial}{\partial \lambda_k} \ln Z_C|_{\lambda_k=1}$$

with

$$Z_C = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iC\phi} \cdot \tilde{Z} = Z_0 \cdot \left(\frac{C_1}{C_{-1}}\right)^{C/2} \cdot I_C(2\sqrt{C_1C_{-1}})$$
First we calculate the density of a particle with $C = 1$:

$$n_k^C = \frac{\lambda_k}{V} \cdot \frac{\partial}{\partial \lambda_k} \ln Z_C|_{\lambda_k=1}, \quad Z_C = Z_0 \cdot \left( \frac{C_1}{C_{-1}} \right)^{C/2} \cdot I_C(2\sqrt{C_1C_{-1}}),$$

$$C_1 \equiv \sum_{k=m+1}^{n} \lambda_k \cdot \ln Z_k^{C=1}, \quad C_{-1} \equiv \sum_{k=n+1}^{p} \lambda_k \cdot \ln Z_k^{C=-1}$$

$$n_k^C = \frac{\lambda_k}{V} \cdot \frac{\partial}{\partial \lambda_k} \ln Z_C|_{\lambda_k=1} = \frac{C}{2} \cdot \frac{1}{C_1} \cdot N_k \ln Z_k + \frac{1}{I_C(2\sqrt{C_1C_{-1}})} \cdot \frac{\partial I_C(2\sqrt{C_1C_{-1}})}{\partial \lambda_k}$$

$$= \frac{C}{2} \cdot \frac{1}{C_1} \cdot N_k \ln Z_k + \frac{1}{I_C(2\sqrt{C_1C_{-1}})} \cdot \left. \frac{\partial I_C(x)}{\partial x} \right|_{x=2\sqrt{C_1C_{-1}}} \cdot \sqrt{\frac{C_{-1}}{C_1}} \cdot N_k \ln Z_k$$

$$= N_k \ln Z_k \left[ \frac{C}{2C_1} + \frac{1}{I_C(x_0)} \cdot \frac{C_{-1}}{\sqrt{C_1C_{-1}}} \cdot \frac{I_{C-1}(x_0) + I_{C+1}(x_0)}{2} \right] (x_0 \equiv 2\sqrt{C_1C_{-1}})$$

$$= N_k \ln Z_k \left[ \frac{\sqrt{C_1C_{-1}}}{2C_1} \cdot \frac{I_{C-1}(x_0) - I_{C+1}(x_0)}{I_C(x_0)} + \frac{1}{I_C(x_0)} \cdot \frac{C_{-1}}{\sqrt{C_1C_{-1}}} \cdot \frac{I_{C-1}(x_0) + I_{C+1}(x_0)}{2} \right]$$

$$= N_k \ln Z_k \cdot \frac{C_{-1}}{\sqrt{C_1C_{-1}}} \cdot \frac{I_{C-1}(x_0)}{I_C(x_0)} \quad \text{(*) see next page)
The modified Bessel function of the first kind $I_n(x)$ has the properties:

$$I_n(x) = I_{-n}(x), \quad I_{n-1}(x) - I_{n+1}(x) = \frac{2n}{x} I_n(x), \quad I_{n-1}(x) + I_{n+1}(x) = 2I'_n(x).$$

In the last page we derived

$$\frac{\partial}{\partial \lambda_k} \ln Z_C = N_k \ln Z_k \cdot \frac{C_{-1}}{\sqrt{C_1 C_{-1}}} \cdot \frac{I_{C-1}(x_0)}{I_C(x_0)}.$$

This yields

$$n^C_k = \frac{N_k \ln Z_k}{V} \cdot \frac{C_{-1}}{\sqrt{C_1 C_{-1}}} \cdot \frac{I_{C-1}(x_0)}{I_C(x_0)}.$$

Similarly for a particle with $C = -1$:

$$n^C_k = \frac{N_k \ln Z_k}{V} \cdot \frac{C_1}{\sqrt{C_1 C_{-1}}} \cdot \frac{I_{C+1}(x_0)}{I_C(x_0)},$$

and for a particle with $C = 0$:

$$n^C_k = \frac{N_k \ln Z_k}{V}.$$
In QGP there are only open $c$ and $\bar{c}$ quarks, whose quantum number is $C = 1$ or $C = -1$, and $C_1 = C_{-1}$:

$$\Rightarrow n_c^C = n_{\bar{c}}^C = \frac{N_k \ln Z_c}{V} \cdot \frac{l_1(2n_c \cdot V)}{l_0(2n_c \cdot V)}$$

$$\Rightarrow n_c^C = n_{c}^{GC} \cdot \frac{l_1(2N_c^{GC})}{l_0(2N_c^{GC})} = n_{c}^{GC} \cdot \frac{l_1(N_{oc}^{GC})}{l_0(N_{oc}^{GC})}$$

I.e. using canonical formalism, the open charm densities (or yields) are suppressed by a factor $\frac{l_1}{l_0}$. 
Charm quarks’ evolution in heavy-ion collisions (in a not completely correct picture)

(Source: notes of the lecture “standard model” by Prof. Uwer from Uni. Heidelberg)

More precisely, what must be exactly conserved, is the number of open charms at the phase boundary.
More precisely, what must be exactly conserved, is the number of open charms at the phase boundary.
In grand canonical formalism:

\[ N_{c\bar{c}}^{dir} = \frac{1}{2} N_{oc}^{th} + N_{c\bar{c}}^{th} \]

1. Modification: canonical suppression factor \( l_1/l_0 \)

\[ N_{c\bar{c}}^{dir} = \frac{1}{2} N_{oc}^{th} \frac{l_1(N_{oc}^{th})}{l_0(N_{oc}^{th})} + N_{c\bar{c}}^{th}, \]

where \( l_0 \) and \( l_1 \) are modified Bessel functions of the first kind.

2. Modification: fugacity parameter \( g_c(N_{part}, T, \mu_B) \), to account for deviations from chemical equilibrium.

\[ N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{l_1(g_c N_{oc}^{th})}{l_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \]
The yields of different charm hadrons produced in heavy ion collisions:

- For open charm mesons and hyperons:
  \[ N_i = g_c N_i^{th} \frac{l_1(g_c N_{oc}^{th})}{l_0(g_c N_{oc}^{th})} \]

- For charmonia:
  \[ N_j = g_c^2 N_j^{th} \]
5. The participating nucleons’ number $N_{part}$ is sufficiently large.

We’ve just got for charmonia:  \[ N_j = g_c^2 N_{j}^{th} \]

\[ \Rightarrow N_{\psi'} = g_c(N_{part})^2 N_{\psi'}^{th}, \quad N_{J/\psi} = g_c(N_{part})^2 N_{J/\psi}^{th} \]

\[ \Rightarrow \frac{N_{\psi'}}{N_{J/\psi}} = \frac{N_{\psi}^{th}}{N_{J/\psi}^{th}}, \]

which doesn’t depend on $N_{part}$.

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which doesn’t depend on $N_{\text{part}}$.

"Corona Effect"

1. Introduction to QGP

2. The statistical hadronization model
   - Introduction to the model
   - Grand canonical formalism
   - Canonical formalism

3. $J/\psi$ suppression and enhancement
   - Theoretical description
   - Data analysis
   - Some new results

4. Summary and reference
Some concepts in relativistic kinematics - 1

- Rapidity:
  \[ y = \frac{1}{2} \ln \left( \frac{E + p_l}{E - p_l} \right) \]

- Pseudo-rapidity:
  \[ \eta = \frac{1}{2} \ln \left( \frac{|p| + p_l}{|p| - p_l} \right) = -\ln(\tan \frac{\theta}{2}) \]

\[ \begin{align*}
\text{1. When } &\beta \equiv \frac{v}{c} \to 1, y \approx \eta. \\
\text{2. } &\eta = 0 \iff p_l = 0 \iff \vec{p}_3 \perp \vec{p}_1
\end{align*} \]

An example:
\[ |\eta| \leq \frac{1}{2} \iff -\frac{1}{2} \leq \ln(\tan \frac{\theta}{2}) \leq \frac{1}{2} \iff e^{-\frac{1}{2}} \leq \tan \frac{\theta}{2} \leq e^{\frac{1}{2}} \]
\[ \iff \arctan(e^{-\frac{1}{2}}) \leq \frac{\theta}{2} \leq \arctan(e^{\frac{1}{2}}) \iff 62.4762^\circ \leq \theta \leq 117.5238^\circ \]
Some concepts in relativistic kinematics - 2

(Source: Notes of the lecture “compressed nuclear matter” by C.Höhne in GSI Summer Student Program 2007)

- Centrality:
  - describes quantitatively, how central (or peripheral) a collision is.
  - is related to the concept “impact parameter”.

- Centrality class: (non)overlapping percentage of colliding ions, e.g. 0 \sim 20\%, 20 \sim 40\%, \ldots

- Centrality dependence: a physical quantity as a function of $N_{part}$ or $E_T$. 
What is $J/\psi$ suppression?


- **X-Axis:** $E_T$, $N_{\text{part}}$, · · ·
- **Y-Axis:** $\frac{dN_{J/\psi}/d\eta}{N_{\text{part}}}$, $R_{AA}^{J/\psi} \equiv \frac{dN_{\text{AA}}^{J/\psi}/d\eta}{N_{\text{coll}} \cdot dN_{J/\psi}^{pp}/d\eta}$, $\frac{B_{\mu\mu} \cdot \sigma(J/\psi)}{\sigma(DY)}$, · · ·
Qualitative description of $J/\psi$ suppression and enhancement

(Source: notes of the lecture “standard model” by Prof. Uwer from Uni. Heidelberg)
Quantitative description of $J/\psi$ suppression using SHM: Calculation procedure

Under the assumptions mentioned before:

$$N_{cc}^{dir} = \frac{1}{2} g_c n_{oc}^{th} \frac{l_1(g_c n_{oc}^{th})}{l_0(g_c n_{oc}^{th})} + g_c^2 n_{cc}^{th}, \quad N_{J/\psi} = g_c^2 N_{J/\psi}$$

- From pQCD: $\sigma(pp \rightarrow c\bar{c})$; nuclear overlap function $T_{AA}(N_{part})$
  $$N_{cc}^{dir} = \sigma(pp \rightarrow c\bar{c}) \cdot T_{AA}(N_{part})$$

- From grand canonical formalism: $n_{oc}^{th}$, $n_{cc}^{th}$, $n_{ch}^{th}$; measure $dN_{ch}/dy|_{y=0}$;
  $$dN_{ch}/dy|_{y=0} = n_{ch}^{th} \cdot V_{\Delta y=1} \quad \Rightarrow \quad V_{\Delta y=1}$$
  $$n_{oc}^{th} = n_{oc}^{th} \cdot V_{\Delta y=1}, \quad n_{cc}^{th} = n_{cc}^{th} \cdot V_{\Delta y=1}$$

- $N_{cc}^{dir}$, $N_{oc}^{th}$, $N_{cc}^{th}$ $\Rightarrow$ $g_c$

- $J/\psi$ yields: $N_{J/\psi} = g_c^2 \cdot N_{J/\psi}^th$
When suppression? When enhancement?

\[ N_{cc}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{l_1(g_c N_{oc}^{th})}{l_0(g_c N_{oc}^{th})} + g_c^2 N_{cc}^{th} \]

(Source: A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, nucl-th/0611023v2)
Centrality dependence of $N_{J/\psi} / N_{\text{part}}$ at SPS

Centrality dependence of $R_{AA}^{J/\psi}$ at RHIC and LHC

**Influence of the input parameters at RHIC**

Rapidity dependence of $R_{AA}^{J/\psi}$ at RHIC


- In other models (e.g. melting model, co-moving model, · · ·): constant or minimum at midrapidity
- Only in the statistical hadronization model: maximum at midrapidity

The first direct evidence for generation of $J/\psi$ mesons at the phase boundary!
Rapidity dependence of $R_{AA}^{J/\psi}$ at RHIC


- In other models (e.g. melting model, co-moving model, · · ·): constant or minimum at midrapidity
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The first direct evidence for generation of $J/\psi$ mesons at the phase boundary!
Recent revision of the SHM - 1: “Corona Effect”

\[ N_{J/\psi} = N_{J/\psi}^{\text{core}} + N_{J/\psi}^{\text{corona}}, \]

where

\[ N_{J/\psi} = g_c^2 \cdot n_{J/\psi}^{\text{th}} \cdot \nu_{\text{core}}, \]

\[ N_{J/\psi}^{\text{corona}} = N_{\text{coll}}^{\text{corona}} \cdot \sigma_{J/\psi}^{pp} / \sigma_{\text{inel}}^{pp}. \]

(Source: A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel, nucl-th/0611023v2)
Recent revision of the SHM - 2: $c\bar{c}$ annihilation in QGP

- QGP not in chemical equilibrium.
  $\Rightarrow c + \bar{c} \rightarrow g + g$ and $g + g \rightarrow c + \bar{c}$ are asymmetric.
- Production of $c\bar{c}$ pairs in QGP is negligible when $T < 1\text{GeV}$.
  (see hep-ph/0001008)
- We can estimate:

$$N_{c\bar{c}}^{anni} = \int_{\tau_0}^{\tau_c} \frac{dN_{c\bar{c}}}{d\tau} V(\Delta y = 1, \tau) d\tau,$$

where

$$\frac{dN_{c\bar{c}}}{d\tau} = n_c n_{\bar{c}} \langle \sigma_{c\bar{c} \rightarrow gg} \cdot v_r \rangle, \quad V(\Delta y = 1, \tau) = A_{\perp} \tau,$$

$$n_c = \frac{dN_c/dy(\tau)}{V(\Delta y = 1, \tau)} \leq \frac{dN_c/dy(\tau_0)}{V(\Delta y = 1, \tau)}.$$

$\Rightarrow$ 

$$N_{c\bar{c}}^{anni} \leq \left( \frac{dN_c}{dy}(\tau_0) \right)^2 \frac{1}{A_{\perp}} \int_{\tau_0}^{\tau_c} \frac{d\tau}{\tau} \langle \sigma_{c\bar{c} \rightarrow gg} \cdot v_r \rangle.$$
Recent revision of the SHM - 2: $c\bar{c}$ annihilation in QGP

(Source: A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel, nucl-th/0611023v2)

Consequences:

- Charm quark annihilation in QGP can be safely neglected.
- Charmonia are not likely to be formed in QGP, and not before QGP formation, either.
- The statistical hadronization model (SHM) and the recombination model (RM) are different, because

$$\sigma(c\bar{c} \rightarrow J/\psi)_{RM} > \sigma(c\bar{c} \rightarrow gg)_{SHM}.$$
New analysis of centrality dependence at SPS

(Source: A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel, nucl-th/0611023v2)
New analysis of centrality dependence at RHIC

(Source: A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, nucl-th/0611023v2)
Revised predictions for centrality and rapidity dependence at LHC

(Source: A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, nucl-th/0611023v2)
1. Introduction to QGP

2. The statistical hadronization model
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4. Summary and reference
• Statistical hadronization model (SHM): assumptions, grand canonical formalism, canonical formalism.

• To $J/\psi$ suppression: SHM is quite successful (centrality dependence at SPS and RHIC, rapidity dependence at RHIC).

• To $J/\psi$ enhancement: SHM gives predictions (centrality and rapidity dependence at LHC).

• Recent revision of SHM: Corona Effect, $c\bar{c}$ annihilation in QGP.

• Next generation of SHM: maybe to think about continuous transition from core region to $pp$ region.


- **Books:**
  R.C.Hwa(Editor), X.N.Wang(Editor) (2004) *Quark-Gluon Plasma 3*
  W.Greiner, S.Schramm, E.Stein, (2002) *Quantum Chromodynamics*
  O.Nachtmann, (1991) *Phänomene und Konzepte der Elementarteilchenphysik*

- **Papers:**
  P.Braun-Munzinger, K.Redlich, J.Stachel, nucl-th/0304013v1
  A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel, nucl-th/0611023v2

- **Scripts:**
  Notes of the lecture “quark-gluon plasma physics” by Prof. Stachel and Prof. Braun-Munzinger in CERN Summer Student Program 2005:
  [link]
  Notes of the lecture “compressed nuclear matter” by C.Höhne in GSI Summer Student Program 2007:
  [link]
  Notes of the lecture “standard model” by Prof. Uwer from Uni. Heidelberg:
  [link]
  Presentation given by M.Ries from the seminar “experimental particle and nuclear physics” in Uni. Heidelberg:
  [link]