

τ -Decay and Hadronic Spectral Functions

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Motivation

- not all low energy resonances can be explained within the constituent quark picture
- too many observed states and some of them even have exotic quantum numbers
- broad resonances and blurred thresholds, mixing of states with same quantum numbers
- for example: a_1 meson $J^{PC} = 1^{++}$ at around 1.2 GeV
 - i) mass and decay width?
 - ii) chiral partner of the ρ meson or just a $\rho\pi$ molecule-like state?
Wagner Leupold [arXiv:hep-ph/0801.0814]
- what about the other interactions that govern the phenomenology of our low energy color neutral objects?



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- what about the other interactions that govern the phenomenology of our low energy color neutral objects?
- in the constituent quark picture mesons are color neutral composite objects of quark and antiquark
- reduce complexity of QCD interaction by [effective hadron hadron interaction in models with hadronic dofs and symmetries known from the QCD Lagrangian](#).

$$\mathcal{L} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{baryon}} + \mathcal{L}_{\text{dilaton}} + \mathcal{L}_{\text{weak}}$$

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\ & + \text{chirally invariant vector and axialvector four-point interaction vertices} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{baryon}} = & \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi^\dagger \Psi_{2L}) \\ & - M (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} - \bar{\Psi}_{2R} \Psi_{1L}) \end{aligned}$$

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2} (\partial^\mu G)^2 - \frac{1}{4} \frac{m_G}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right)$$

$$\begin{aligned} \mathcal{L}_{\text{weak}} = & \delta_w \frac{g \cos \theta_C}{2} \text{Tr}[W_{\mu\nu} L^{\mu\nu}] + \delta_{\text{em}} \frac{e}{2} \text{Tr}[B_{\mu\nu} R^{\mu\nu}] + \frac{1}{4} \text{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] \\ & + \frac{g}{2\sqrt{2}} (W_\mu^- \bar{u}_{\nu\tau} \gamma_\mu (1 - \gamma_5) u_\tau + \text{h.c.}) \end{aligned}$$

$N_F = 2$ and $N_F = 3$ meson multiplets:

(Pseudo-)Scalars $\Phi_{ij} \simeq \langle q_L \bar{q}_R \rangle_{ij} \simeq \frac{1}{\sqrt{2}} (q_i \bar{q}_j - q_i \gamma_5 \bar{q}_j)$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0)}{\sqrt{2}} + \frac{i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0)}{\sqrt{2}} + \frac{i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}_0^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Lefthanded $L_{ij}^\mu \simeq \langle q_L \bar{q}_L \rangle_{ij} \simeq \frac{1}{\sqrt{2}} (q_i \gamma^\mu \bar{q}_j + q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$L^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 \\ K^{*-} + K_1^- & \bar{K}^{*0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu$$

Righthanded $R_{ij}^\mu \simeq \langle q_R \bar{q}_R \rangle_{ij} \simeq \frac{1}{\sqrt{2}} (q_i \gamma^\mu \bar{q}_j - q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 \\ K^{*-} - K_1^- & \bar{K}^{*0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu$$

Mesonic Lagrangian with Global Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors} \end{aligned}$$

$U(N_F)_L \times U(N_F)_R$ Transformation:

$$\Phi \rightarrow U_L \Phi U_R^\dagger, \quad L^\mu \rightarrow U_L L^\mu U_L^\dagger, \quad R^\mu \rightarrow U_R R^\mu U_R^\dagger$$

Covariant Derivative:

$$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu)$$

Explicit Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

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$U(1)_A$ -Anomaly

$$c_1 (\det \Phi - \det \Phi^\dagger)^2$$

non-vanishing quark masses, NO isospin breaking

$$\text{Tr}[H(\Phi + \Phi^\dagger)], \quad H = h_a t^a$$

remaining symmetry is $U(2)_V$

Spontaneous Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]
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Global Chiral Symmetry:

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Spontaneous breaking of global chiral symmetry by non-zero scalar condensate

$$\sigma \rightarrow \sigma + \phi, \quad \phi = Z f_\pi$$

- i) $m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3)$, $m_{a_1}^2 = m_1^2 + (g_1 \phi)^2 + \frac{\phi^2}{2}(h_1 + h_2 - h_3)$
- ii) 3 point interaction vertices and mixing terms in $(D^\mu \Phi)^\dagger D_\mu \Phi$ that are proportional to the VEV ϕ .

$U(2)_L \times U(2)_R$ Symmetry in the Baryonic Sector

S. Gallas, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 014004 arXiv:0907.5084 [hep-ph]

S. Gallas, F. Giacosa and G. Pagliara, Nucl. Phys. A 872 (2011) 13 arXiv:1105.5003 [hep-ph]

Baryons in the mirror assignment:

$$\begin{aligned} \mathcal{L}_{\text{baryon}} = & \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi^\dagger \Psi_{2L}) \\ & - M (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} - \bar{\Psi}_{2R} \Psi_{1L}) \end{aligned}$$

$U(2)_L \times U(2)_R$ Transformation

$$\Psi_{1R} \rightarrow U_R \Psi_{1R}, \quad \Psi_{1L} \rightarrow U_L \Psi_{1L}$$

$$\Psi_{2R} \rightarrow U_L \Psi_{2R}, \quad \Psi_{2L} \rightarrow U_R \Psi_{2L}$$

Covariant Derivative

$$D_{1R}^\mu = \partial^\mu - ic_1 R^\mu, \quad D_{1L}^\mu = \partial^\mu - ic_1 L^\mu$$

$$D_{2R}^\mu = \partial^\mu - ic_2 R^\mu, \quad D_{2L}^\mu = \partial^\mu - ic_2 L^\mu$$

- allows for **chirally invariant mass term** generated by the gluon and/or tetraquark condensate
- Nucleons N, N^* are real chiral partners $N(1650)$ is favoured as chiral partner of $N(939)$
- yields correct nuclear matter saturation

Scale Invariance and the Glueball

S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 84 (2011) 054007 arXiv:1103.3238 [hep-ph]

Scale invariance of the QCD Lagrangian is broken on the quantum level

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2} (\partial^\mu G)^2 - \frac{1}{4} \frac{m_G}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right)$$

\mathcal{L}_σ is in principle scale invariant, only mass terms and $U(1)_A$ -anomaly break scale invariance

$$x^\mu \rightarrow \lambda^{-1} x^\mu, \quad \varphi(x) \rightarrow \lambda \varphi(\lambda^{-1} x), \quad \Psi(x) \rightarrow \lambda^{\frac{3}{2}} \Psi(\lambda^{-1} x)$$

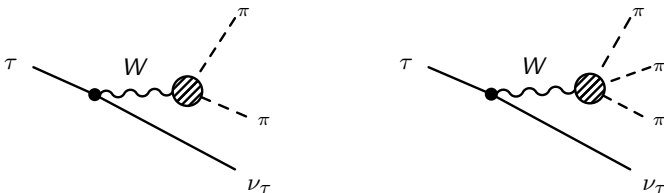
Scalar glueball is associated with fluctuations of the dilaton potential

Ground state of dilaton G_0 is related to the gluon condensate $G_0 = \Lambda = \frac{\sqrt{11}}{2m_G} C^2$

- favours $q\bar{q}$ interpretation of $f_0(1370)$ as chiral partner of the pion ($f_0(500)$ is disfavoured) and $f_0(1500)$ is 75% glueball
- scale invariance extended to $\mathcal{L}_{\text{meson}}, \mathcal{L}_{\text{baryon}}$ by parametrization of meson and baryon masses by G

τ -Decay

- semileptonic τ -decay involves strong and weak interactions



- describe effective electroweak interactions of hadrons in the vacuum.
- results can be further used to perform calculations at nonzero temperature and density (e.g. dilepton decay rate) and to understand more about the nature of resonances such as a_1 , e.g. $\bar{q}q$ or $\rho\pi$ -state?

Linear Sigma Model with Weak Interaction

$$\mathcal{L}_{\text{weak}} = \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + \delta_w \frac{g \cos \theta_C}{2} \text{Tr}[W_{\mu\nu} L^{\mu\nu}] + \delta_{\text{em}} \frac{e}{2} \text{Tr}[B_{\mu\nu} R^{\mu\nu}] \\ + \frac{1}{4} \text{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] + \frac{g}{2\sqrt{2}} (W_\mu^- \bar{u}_{\nu\tau} \gamma_\mu (1 - \gamma_5) u_\tau + \text{h.c.})$$

local $SU(2)_L \times U(1)_Y$ transformation:

$$\Phi \rightarrow U_L \Phi U_Y^\dagger, \quad L^\mu \rightarrow U_L L^\mu U_L^\dagger, \quad R^\mu \rightarrow U_Y R^\mu U_Y^\dagger$$

$$W^\mu \rightarrow U_L W^\mu U_L^\dagger + \frac{i}{g} U_L \partial^\mu U_L^\dagger, \quad B^\mu \rightarrow U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger$$

Cabibbo mixing ($N_f = 2$): $\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$

Weinberg mixing $\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_0^\mu \\ A^\mu \end{pmatrix}$

covariant derivative and field strength tensors:

$$D^\mu \Phi \equiv \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ie[A^\mu t_3, \Phi] - ig \cos \theta_C (W_1^\mu t_1 + W_2^\mu t_2) \Phi \\ - ig \cos \theta_W (Z^\mu \Phi + \tan^2 \theta_W \Phi Z^\mu)$$

$$L^{\mu\nu} \equiv \partial^\mu L^\nu - ie[A^\mu t_3, L^\nu] - ig[W_1^\mu t_1 + W_2^\mu t_2, L^\nu] \\ - \{\partial^\nu L^\mu - ie[A^\nu t_3, L^\mu] - ig[W_1^\nu t_1 + W_2^\nu t_2, L^\mu]\}$$

Axial-Vector Channel

$$\Gamma_{W^- \rightarrow \pi^- 2\pi^0}(s) \sim \frac{1}{s} \frac{2}{2 \cdot 3} \int \left| \text{Diagram} \right|^2 dm_{12}^2 dm_{23}^2$$

$$\begin{aligned} & \left| \text{Diagram} \right|^2 \\ &= \frac{1}{2} \left| 2 \text{Diagram}_1 + \text{Diagram}_2 + \text{Diagram}_3 \right. \\ & \left. + 2 \text{Diagram}_4 + \text{Diagram}_5 + \text{Diagram}_6 \right|^2 \end{aligned}$$

The diagrams are:

- Diagram 1:** A wavy line labeled W^- enters from the left and splits into two dashed lines labeled π^- and π^0 . The vertex is a shaded circle.
- Diagram 2:** A wavy line labeled W^- enters from the left and splits into a dashed line labeled π^- and a solid line labeled m_{12} . The solid line then splits into two dashed lines labeled $\pi^0(k_1)$ and $\pi^0(k_3)$. The vertex is a solid black dot.
- Diagram 3:** A wavy line labeled W^- enters from the left and splits into a dashed line labeled π^- and a solid line labeled m_{23} . The solid line then splits into two dashed lines labeled $\pi^0(k_3)$ and $\pi^0(k_1)$. The vertex is a solid black dot.
- Diagram 4:** A wavy line labeled W^- enters from the left and splits into a dashed line labeled π^- and a solid line labeled a_1^- . The solid line then splits into two dashed lines labeled π^0 and π^0 . The vertex is a solid black dot.
- Diagram 5:** A wavy line labeled W^- enters from the left and splits into a dashed line labeled π^- and a solid line labeled a_1^- . The solid line then splits into a dashed line labeled $\pi^0(k_3)$ and a solid line labeled m_{12} . The solid line then splits into two dashed lines labeled $\pi^0(k_1)$ and $\pi^-(k_2)$. The vertex is a solid black dot.
- Diagram 6:** A wavy line labeled W^- enters from the left and splits into a dashed line labeled π^- and a solid line labeled a_1^- . The solid line then splits into a dashed line labeled $\pi^0(k_1)$ and a solid line labeled m_{23} . The solid line then splits into two dashed lines labeled $\pi^0(k_3)$ and $\pi^-(k_2)$. The vertex is a solid black dot.

Axial-Vector Channel

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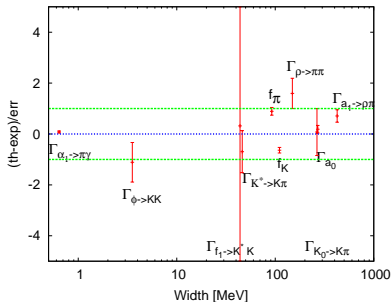
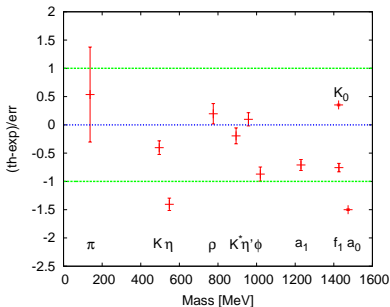
The diagrams represent various particle exchange channels in the axial-vector channel. The first diagram is a contact term where a W^- boson decays into π^- and π^0 . The second and third diagrams show W^- exchange between a pion and a ρ meson, with ρ mesons decaying into two pions. The fourth and fifth diagrams show a_1^- exchange between a pion and a ρ meson, with ρ mesons decaying into two pions.

- $\Gamma(W^- \rightarrow \pi^- 2\pi^0) \simeq 1\%$ and $\Gamma(a_1^- \rightarrow \pi^- 2\pi^0) \simeq 1\%$
- in principle also contributions of σ resonance $\Gamma_{W^- \rightarrow \sigma 2\pi^0 \pi^-} \simeq 0$

Quest for the Parameters

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke

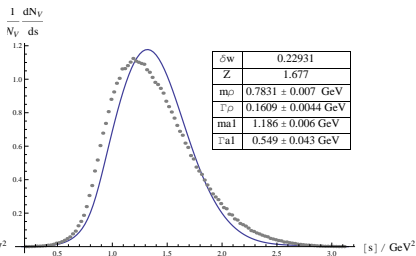
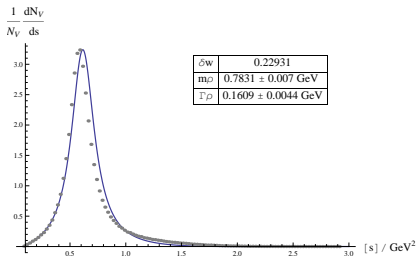
Scalar mesons in a linear sigma model with (axial-)vector mesons



[arXiv:hep-ph/1208.0585]

- global fit of 13 parameters; test model
- 21 decay widths and masses

Results with $m_\rho, \Gamma_\rho, m_{a_1}, \Gamma_{a_1}$ and pion renormalisation constant Z as obtained from $N_f = 3$



- Only one free parameter δ_w which describes the mixing between the charged weak bosons and the (axial-)vector mesons!
- $\Gamma_{\rho^- \rightarrow \pi^- \pi^0}, \Gamma_{a_1^- \rightarrow \rho^- \pi^-}$
- We wanted to know if ρ and a_1 can be described as chiral partners. Yes!

$$W\rho \text{ mixing} \sim \delta_w s \quad Wa_1 \text{ mixing} \sim (\delta_w s + g_1 \phi^2)$$

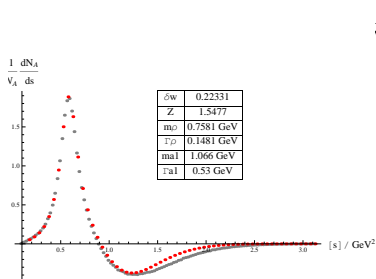
- the parameters have errors within range $\sim 5\%$ therefore we can still improve our results

Inclusive Spectral Functions VMA and VPA

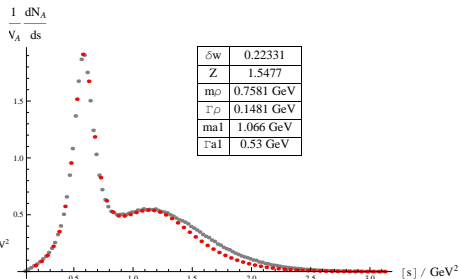
- Starting values have a strong effect on results in each channel.
- inclusive spectral functions

Inclusive Spectral Functions VMA and VPA

- Starting values have a strong effect on results in each channel.
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V - A

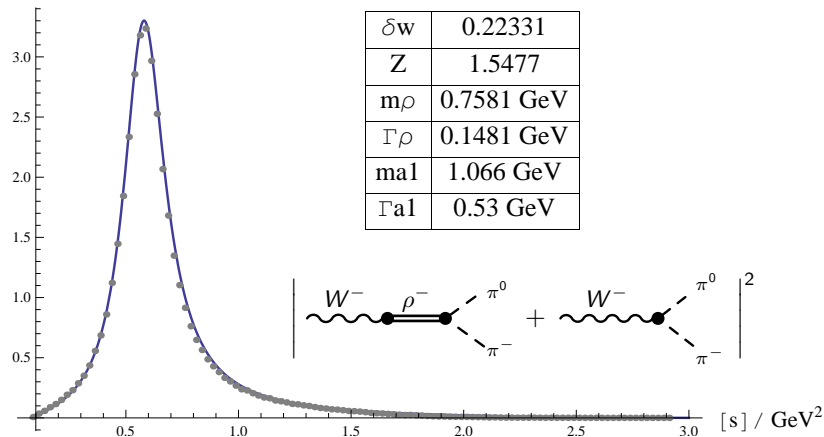


V + A

Vector Channel Spectral Function $\tau \rightarrow 2\pi\nu_\tau$

Coherent sum $|W \xrightarrow{\text{direct}} 2\pi + W \xrightarrow{\rho} 2\pi|^2$

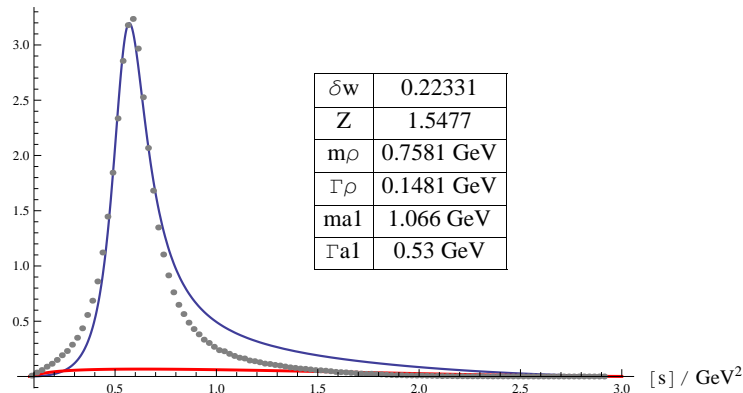
$$\frac{1}{\sqrt{V}} \frac{dN_V}{ds}$$



Vector Channel Spectral Function $\tau \rightarrow \pi^- \pi^0 \nu_\tau$

Isolated contributions $W^- \rightarrow \pi^- \pi^0$ and $W^- \rightarrow \rho^- \rightarrow \pi^- \pi^0$

$$\frac{\Gamma(W^- \rightarrow \pi^- \pi^0)}{\Gamma(W^- \rightarrow \rho^- \rightarrow \pi^- \pi^0)} \simeq 0.02$$

$$\frac{1}{\sqrt{V}} \frac{dN_V}{ds}$$


Vector Decay Constant

- Vector Decay Constant f_ρ : $\langle 0 | j^\mu | \rho^\mu \rangle = m_\rho f_\rho \varepsilon^\mu$
- describes mixing of the ρ meson with the vector current,
- weak coupling with vector meson $\frac{g \cos \theta_C}{2} \delta_w S$

$$\rightarrow f_\rho m_\rho = \sqrt{2} \delta_w m_\rho$$

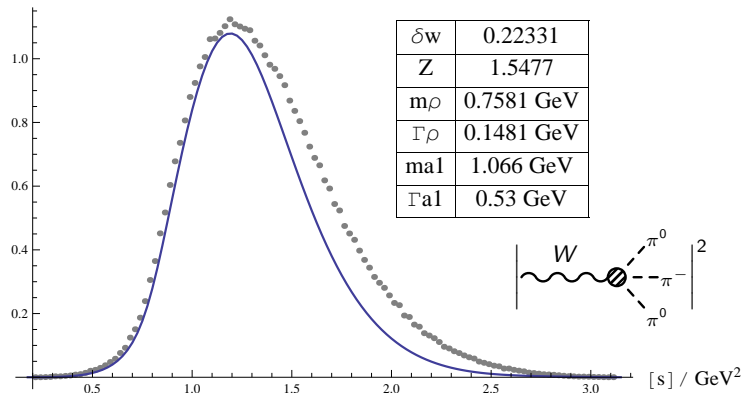
- $f_\rho^{\text{exp.}} \sim 214 \text{ MeV}$, $f_\rho^{\text{L}\sigma\text{M}} = 239 \text{ MeV}$
- Also this result can be improved by including direct contributions in the axial-vector channel.

Axial Vector Channel Spectral Function

$$\tau^- \rightarrow \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$$

Coherent Sum $|W^- \rightarrow \rho\pi \rightarrow 3\pi + W \xrightarrow{a_1} \rho\pi \rightarrow 3\pi|^2$

$$\frac{1}{V_A} \frac{dN_A}{ds}$$



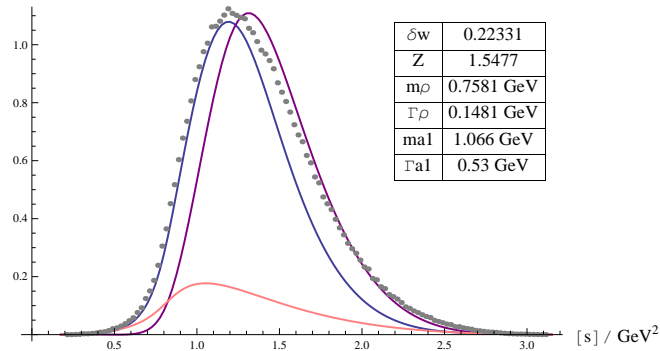
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Isolated contributions $W^- \rightarrow \pi^- \pi^0$ and $W^- \rightarrow \rho^- \rightarrow \pi^- \pi^0$

$$\frac{\Gamma(W^- \xrightarrow{\text{direct}} \pi^- 2\pi^0)}{\Gamma(W^- \xrightarrow{\text{coh.}} \rho^- \pi^0 \rightarrow \pi^- 2\pi^0)} \simeq 0.02$$

$$\frac{1}{N_A} \frac{dN_A}{ds}$$



Conclusion

- We described the decay of the τ lepton in an effective hadronic model
- Can we use effective chiral models to describe the phenomenology of the low energy resonances? **Yes!**
When the model is comprehensive enough.
- Is a_1 a $\bar{q}q$ state? **Yes!**
- Are ρ and a_1 chiral partners? **Yes!**
Very nice example of Vector Meson Dominance.